3.6 Derivatives of Logarithmic and Inverse Trigonometric Functions

Derivatives of Logarithmic Functions

$f(x) = \ln x$ or $f(x) = \log_b x$

$f'(x) = \frac{1}{x}$ $\quad$ $f'(x) = \frac{1}{x \ln b}$

1

$$\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}$$

2

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

3

$$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$$

or

$$\frac{d}{dx}[\ln g(x)] = \frac{g'(x)}{g(x)}$$

EXAMPLE 1 Differentiate $y = \ln(x^3 + 1)$.

$$y' = \frac{1}{x^3+1} * (3x^2 + 0) = \frac{3x^2}{x^3+1}$$

2. $g(t) = \ln(3 + t^2)$

$$g'(t) = \frac{1}{3+t^2} * (0 + 2t) = \frac{2t}{3+t^2}$$

4. $f(x) = x \ln x - x$

① ② opp

productrule

① ②' + ② ①'

$$f'(x) = x * \frac{1}{x} + \ln x * 1 - 1$$

$$f'(x) = 1 + \ln x - 1 = \ln x$$

EXAMPLE 2 Find $\frac{d}{dx}\ln(\sin x)$. $= \frac{1}{\sin x} * \cos x = \frac{\cos x}{\sin x} = \cot x$

5. $f(x) = \sin(\ln x) = \underbrace{\cos(\ln x)}_{\text{تفاضل } \sin(\ln x)} * \underbrace{\frac{1}{x}}_{\text{تفاضل } \ln x} = \frac{\cos(\ln x)}{x}$

16. $P(v) = \frac{\ln v}{1 - v} \longrightarrow$ quotient rule

$$P'(v) = \frac{\overset{\text{مقام}}{(1-v)} * \overset{\text{تفاضل بسط}}{\frac{1}{v}} - \ln v * [0 - 1]}{(1-v)^2}$$

$$= \frac{\frac{1-v}{v} + \ln v}{(1-v)^2} \longleftarrow \text{درجة كاملة}$$

$(①②' + ②①')$

22. $g(x) = e^{\overbrace{x^2}^{①}\overbrace{\ln x}^{②}}$

$$g'(x) = e^{x^2 \ln x} * \left[x^{\cancel{2}} * \frac{1}{\cancel{x}} + \ln x * 2x \right]$$

$$= e^{x^2 \ln x} \left[x + 2x \ln x \right]$$

EXAMPLE 3 Differentiate $f(x) = \sqrt{\ln x}$. $= (\ln x)^{1/2} \longrightarrow$ power rule

قوس

انقص ① $\longrightarrow$ $\frac{1}{2} - 1$

$$f'(x) = \frac{1}{2} (\ln x)^{\frac{1}{2} - 1} * \frac{1}{x}$$

انزل الاس

القوس كما هو

نفاصل ما بداخل القوس

$$= \frac{(\ln x)^{-1/2}}{2x} = \frac{1}{2x(\ln x)^{1/2}} = \frac{1}{2x\sqrt{\ln x}}$$

EXAMPLE 4 Differentiate $f(x) = \log_{10}(2 + \sin x)$.

$$f'(x) = \frac{1}{(2+\sin x)\ln 10} * [0 + \cos x]$$

$$= \frac{\cos x}{(2+\sin x)\ln 10}$$

$$\log_b u \quad \rightarrow \quad = \frac{1}{u \ln b} * u'$$

EXAMPLE 5 Find $\dfrac{d}{dx} \ln \dfrac{x+1}{\sqrt{x-2}}$.

$$= \ln(x+1) - \ln\sqrt{x-2}$$

$$= \ln(x+1) - \ln(x-2)^{1/2}$$

$$f(x) = \ln(x+1) - \frac{1}{2}\ln(x-2)$$

$$f'(x) = \frac{1}{x+1} * [1+0] - \frac{1}{2} * \frac{1}{(x-2)} * [1-0]$$

$$= \frac{1}{x+1} - \frac{1}{2(x-2)}$$

$$\ln\frac{P}{M} = \ln P - \ln M$$

$$\ln P \cdot M = \ln P + \ln M$$

$$\ln M^{P} = P \ln M$$

EXAMPLE 6 Find $f'(x)$ if $f(x) = \ln|x|$.

$f(x) = \begin{cases} \ln(x) & x > 0 \rightarrow f'(x) = \frac{1}{x} \\ \ln(-x) & x < 0 \rightarrow f'(x) = \frac{1}{-x} \cdot -1 = \frac{1}{x} \end{cases}$

$f'(x) = \frac{1}{x}$ $\quad x \neq 0$

29–32 Find y' and y''.

31. $y = \ln|\sec x|$

$y = \begin{cases} \ln(\sec x) \rightarrow y' = \frac{1}{\sec x} \cdot \sec x \tan x = \tan x \rightarrow y'' = \sec x \tan x \\ \ln(-\sec x) \rightarrow y' = \frac{1}{-\sec x} \cdot -\sec x \tan x = \tan x \rightarrow y'' = \sec x \tan x \end{cases}$

$y' = \tan x$ & $y'' = \sec x \tan x$.

✗ $(\tan x)' = \sec^2 x$
$\Rightarrow y'' = \sec^2 x$

✗ $y'' = \sec^2 x$ (not sec x tan x)

39–40 Find an equation of the tangent line to the curve at the given point.

$y - y_1 = m(x - x_1)$

40. $y = x^2 \ln x, \quad (1, 0)$ ① ② x_1 y_1

$m_{tan} = y'$

$y' = x^2 * \frac{1}{x} + \ln x * 2x$

$y' = x + 2x \ln x$

$m_{tan} = y'|_{x=1} = 1 + 2*1*\ln 1 = 1$

$\boxed{m_{tan} = 1}$

$y - y_1 = m(x - x_1)$

$y - 0 = 1(x - 1)$

$\boxed{y = x - 1}$