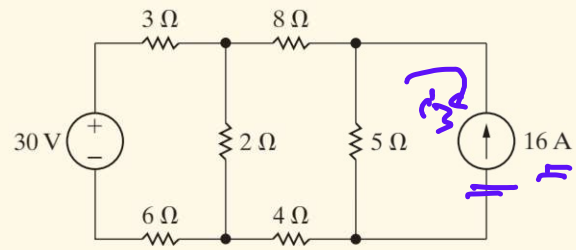
✓ ASSESSMENT PROBLEMS

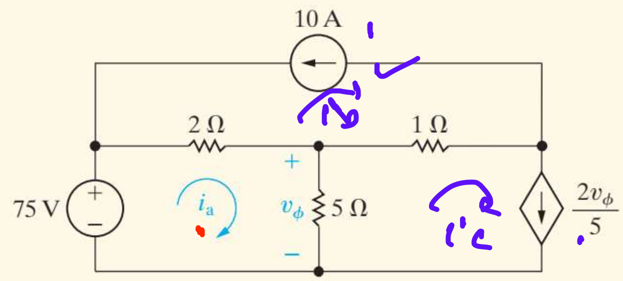
Objective 2—Understand and be able to use the mesh-current method

4.10 Use the mesh-current method to find the power dissipated in the 2 Ω resistor in the circuit shown.



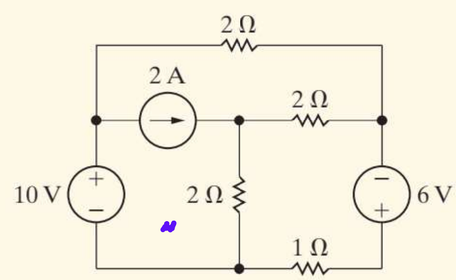
Answer: 72 W.

4.11 Use the mesh-current method to find the mesh current i_a in the circuit shown.



Answer: 15 A.

4.12 Use the mesh-current method to find the power dissipated in the 1 Ω resistor in the circuit shown.



Answer: 36 W.

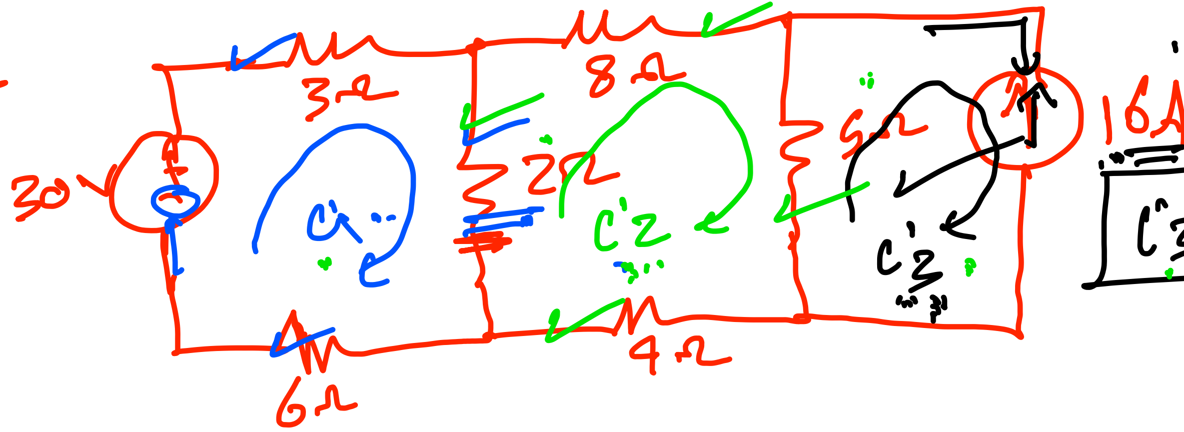
NOTE: Also try Chapter Problems 4.43, 4.47, 4.49, and 4.52.

4.8 The Node-Voltage Method Versus the Mesh-Current Method

The greatest advantage of both the node-voltage and mesh-current methods is that they reduce the number of simultaneous equations that must be manipulated. They also require the analyst to be quite systematic in terms of organizing and writing these equations. It is natural to ask, then, "When is the node-voltage method preferred to the mesh-current method and vice versa?" As you might suspect, there is no clear-cut answer. Asking a number of questions, however, may help you identify the more efficient method before plunging into the solution process:

- Does one of the methods result in fewer simultaneous equations to solve?
- Does the circuit contain supernodes? If so, using the node-voltage method will permit you to reduce the number of equations to be solved.

4.10



$\boxed{i'_3 = -16A}$

i'_1: $\boxed{11i'_1 - 2i'_1 = 30}$

i_2: $-2i'_1 + 19i'_2 - 5i'_3 = 0$ → $= 5i'_3 = 5\times -16 = -80$

$\boxed{-2i'_1 + 19i'_2 = -80}$

Mode → 5 → 1

$i'_1 = 2A$

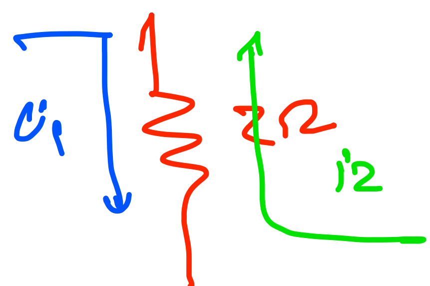
$i'_2 = -4A$

$P = \Delta i'^2 R$

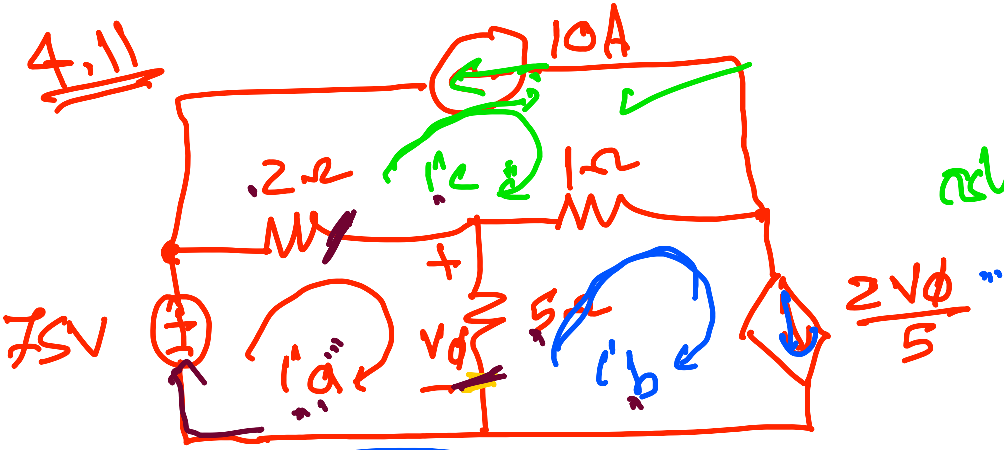
$P = (i'_1 - i'_2)^2 R$

$P = (2 - (-4))^2 \times 2 \Rightarrow (2+4)^2 \times 2$

$P = 6^2 \times 2 = \underline{\underline{72W}}$



4.11



التيار تعتبر مع عكس عقرب الساعة

$i_c = -10A$

$i_b = + \frac{2V_\phi}{5}$

$V_\phi = \Delta i R$

$V_\phi = 5(i_a - i_b)$

$i_b = \frac{2}{\cancel{5}} \cdot \cancel{5}(i_a - i_b)$

$\cancel{i_b} = 2i_a - 2i_b - i_b$

$\boxed{2i_a - 3i_b = 0} \rightarrow (1)$

i_a

$7i_a - 5i_b - 2i_c - 75 = 0$

$2i_c + 75 = 2(-10) + 75$ -20

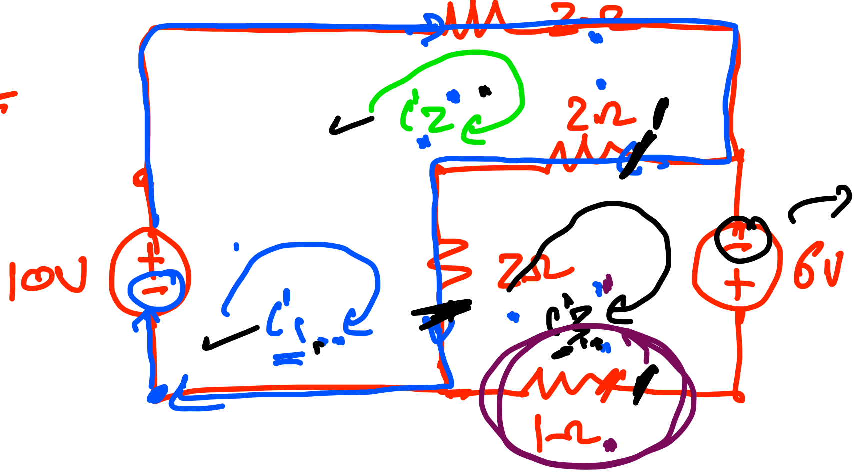
$\boxed{7i_a - 5i_b = 55} \rightarrow (2)$

$i_a = 15A$

$i_b = 10A$

Mode → 5 → 1

4.12



super mesh

↳ تيار : $\boxed{i'_1 - i'_2 + 0 i'_3 = 2} \rightarrow (1)$

↳ جهد : $-10 + 2i'_2 + 2(i'_2 - i'_3) + 2(i'_1 - i'_3) = 0$

$\boxed{2i'_1 + 4i'_2 - 4i'_3 = 10} \rightarrow (2)$

i'_3 $\boxed{-2i'_1 - 2i'_2 + 5i'_3 = 6} \rightarrow (3)$

Mode → 5 → 2

$i'_1 = 7A$ $\quad i'_2 = 5A$ $\quad i'_3 = 6A$

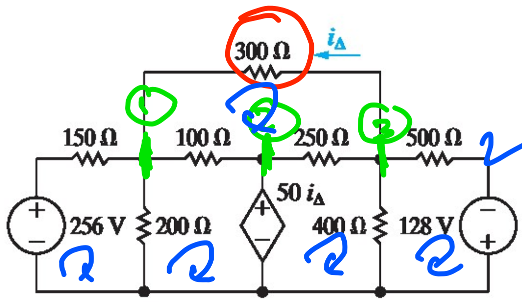
$P_{1\Omega} = i_3^2 R = 6^2 \times 1 = 36\,W$

- Does the circuit contain supermeshes? If so, using the mesh-current method will permit you to reduce the number of equations to be solved.
- Will solving some portion of the circuit give the requested solution? If so, which method is most efficient for solving just the pertinent portion of the circuit?

Perhaps the most important observation is that, for any situation, some time spent thinking about the problem in relation to the various analytical approaches available is time well spent. Examples 4.6 and 4.7 illustrate the process of deciding between the node-voltage and mesh-current methods.

Example 4.6 Understanding the Node-Voltage Method Versus Mesh-Current Method

Find the power dissipated in the 300 Ω resistor in the circuit shown in Fig. 4.29.



Mesh: 5eq
Node: 3eq

Figure 4.29 ▲ The circuit for Example 4.6.

Solution

To find the power dissipated in the 300 Ω resistor, we need to find either the current in the resistor or the voltage across it. The mesh-current method yields the current in the resistor; this approach requires solving five simultaneous mesh equations, as depicted in Fig. 4.30. In writing the five equations, we must include the constraint $i_\Delta = -i_b$.

Before going further, let's also look at the circuit in terms of the node-voltage method. Note that, once we know the node voltages, we can calculate either the current in the 300 Ω resistor or the voltage across it. The circuit has four essential nodes, and therefore only three node-voltage equations are required to describe the circuit. Because of the dependent voltage source between two essential nodes, we have to sum the currents at only two nodes. Hence the problem is reduced to writing two node-voltage equations and a constraint equation. Because the node-voltage method requires only three simultaneous equations, it is the more attractive approach.

Once the decision to use the node-voltage method has been made, the next step is to select a reference node. Two essential nodes in the circuit in Fig. 4.29 merit consideration. The first is the reference node in Fig. 4.31. If this node is selected, one of the unknown node voltages is the voltage across the 300 Ω resistor, namely, v_2 in Fig. 4.31. Once we know this voltage, we calculate the power in the 300 Ω resistor by using the expression

$$p_{300\Omega} = v_2^2/300.$$

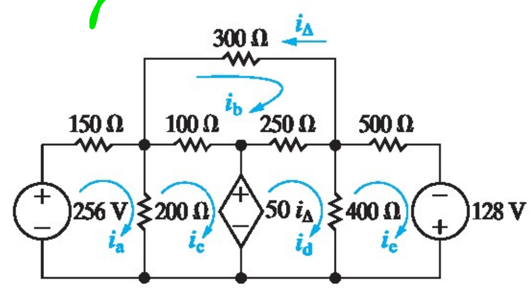


Figure 4.30 ▲ The circuit shown in Fig. 4.29, with the five mesh currents.

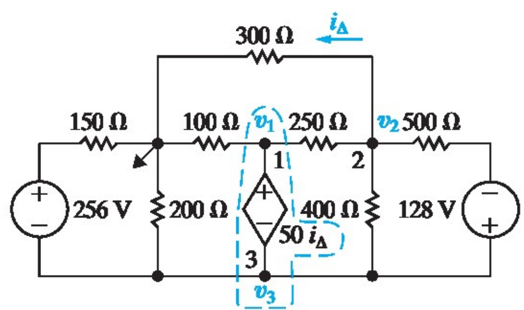


Figure 4.31 ▲ The circuit shown in Fig. 4.29, with a reference node.

Note that, in addition to selecting the reference node, we defined the three node voltages v_1, v_2, and v_3 and indicated that nodes 1 and 3 form a super-node, because they are connected by a dependent voltage source. It is understood that a node voltage is a rise from the reference node; therefore, in Fig. 4.31, we have not placed the node voltage polarity references on the circuit diagram.

The second node that merits consideration as the reference node is the lower node in the circuit, as shown in Fig. 4.32. It is attractive because it has the most branches connected to it, and the node-voltage equations are thus easier to write. However, to find either the current in the 300 Ω resistor or the voltage across it requires an additional calculation once we know the node voltages v_a and v_c. For example, the current in the 300 Ω resistor is $(v_c - v_a)/300$, whereas the voltage across the resistor is $v_c - v_a$.

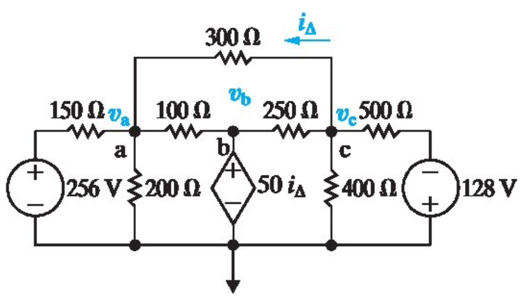


Figure 4.32 ▲ The circuit shown in Fig. 4.29 with an alternative reference node.

We compare these two possible reference nodes by means of the following sets of equations. The first set pertains to the circuit shown in Fig. 4.31, and the second set is based on the circuit shown in Fig. 4.32.

- Set 1 (Fig 4.31)

  At the supernode,

$$\frac{v_1}{100}+\frac{v_1-v_2}{250}+\frac{v_3}{200}+\frac{v_3-v_2}{400}+\frac{v_3-(v_2+128)}{500}+\frac{v_3+256}{150}=0.$$

At v_2,

$$\frac{v_2}{300}+\frac{v_2-v_1}{250}+\frac{v_2-v_3}{400}+\frac{v_2+128-v_3}{500}=0.$$

From the supernode, the constraint equation is

$$v_3 = v_1 - 50i_\Delta = v_1 - \frac{v_2}{6}.$$

- Set 2 (Fig 4.32)

  At v_a,

$$\frac{v_a}{200}+\frac{v_a-256}{150}+\frac{v_a-v_b}{100}+\frac{v_a-v_c}{300}=0.$$

At v_c,

$$\frac{v_c}{400}+\frac{v_c+128}{500}+\frac{v_c-v_b}{250}+\frac{v_c-v_a}{300}=0.$$

From the supernode, the constraint equation is

$$v_b = 50i_\Delta = \frac{50(v_c-v_a)}{300} = \frac{v_c-v_a}{6}.$$

You should verify that the solution of either set leads to a power calculation of 16.57 W dissipated in the 300 Ω resistor.

Example 4.7 Comparing the Node-Voltage and Mesh-Current Methods

Find the voltage v_o in the circuit shown in Fig. 4.33.

Solution

At first glance, the node-voltage method looks appealing, because we may define the unknown voltage as a node voltage by choosing the lower terminal of the dependent current source as the reference node. The circuit has four essential nodes and two voltage-controlled dependent sources, so the node-voltage method requires manipulation of three node-voltage equations and two constraint equations.

Let's now turn to the mesh-current method for finding v_o. The circuit contains three meshes, and we can use the leftmost one to calculate v_o. If we let i_a denote the leftmost mesh current, then $v_o = 193 - 10i_a$. The presence of the two current sources reduces the problem to manipulating a single supermesh equation and two constraint equations. Hence the mesh-current method is the more attractive technique here.

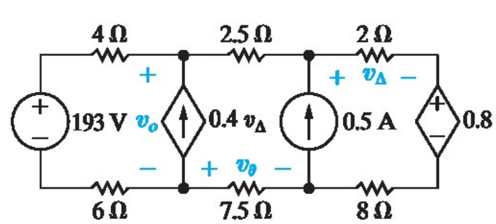


Figure 4.33 ▲ The circuit for Example 4.7.

Node:
Mesh:

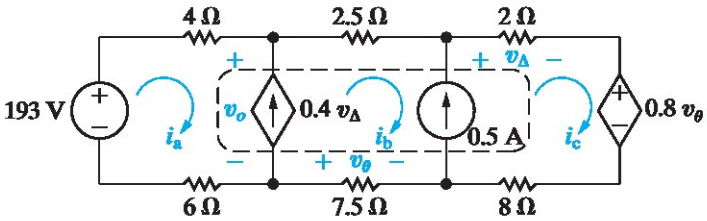


Figure 4.34 ▲ The circuit shown in Fig. 4.33 with the three mesh currents.

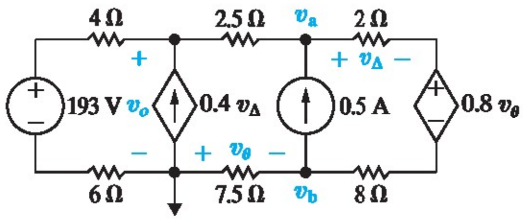


Figure 4.35 ▲ The circuit shown in Fig. 4.33 with node voltages.

To help you compare the two approaches, we summarize both methods. The mesh-current equations are based on the circuit shown in Fig. 4.34, and the node-voltage equations are based on the circuit shown in Fig. 4.35. The supermesh equation is

$$193 = 10i_a + 10i_b + 10i_c + 0.8v_\theta,$$

and the constraint equations are

$$i_b - i_a = 0.4v_\Delta = 0.8i_c;$$

$$v_\theta = -7.5i_b; \text{ and}$$

$$i_c - i_b = 0.5.$$

We use the constraint equations to write the supermesh equation in terms of i_a:

$$160 = 80i_a, \quad \text{or} \quad i_a = 2\text{ A},$$

$$v_o = 193 - 20 = 173\text{ V}.$$

The node-voltage equations are

$$\frac{v_o - 193}{10} - 0.4v_\Delta + \frac{v_o - v_a}{2.5} = 0,$$

$$\frac{v_a - v_o}{2.5} - 0.5 + \frac{v_a - (v_b + 0.8v_\theta)}{10} = 0,$$

$$\frac{v_b}{7.5} + 0.5 + \frac{v_b + 0.8v_\theta - v_a}{10} = 0.$$

The constraint equations are

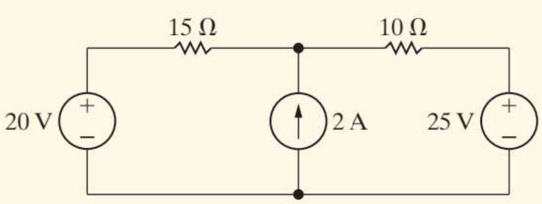
$$v_\theta = -v_b, \quad v_\Delta = \left[\frac{v_a - (v_b + 0.8v_\theta)}{10}\right]2.$$

We use the constraint equations to reduce the node-voltage equations to three simultaneous equations involving v_o, v_a, and v_b. You should verify that the node-voltage approach also gives $v_o = 173$ V.

✓ASSESSMENT PROBLEMS

Objective 3—Deciding between the node-voltage and mesh-current methods

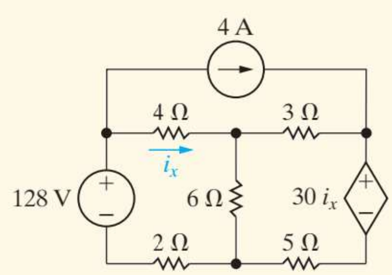
4.13 Find the power delivered by the 2 A current source in the circuit shown.



Answer: 70 W.

NOTE: Also try Chapter Problems 4.54 and 4.56.

4.14 Find the power delivered by the 4 A current source in the circuit shown.

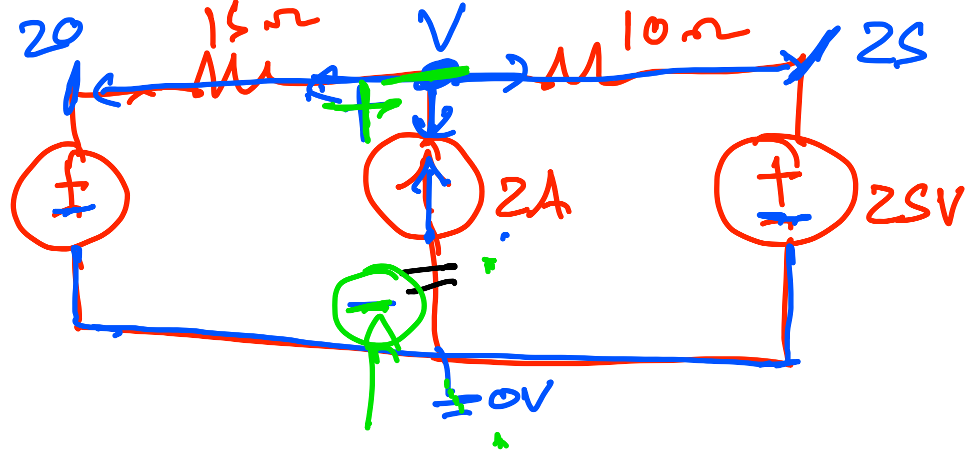


Answer: 40 W.

4.9 Source Transformations

Even though the node-voltage and mesh-current methods are powerful techniques for solving circuits, we are still interested in methods that can be used to simplify circuits. Series-parallel reductions and Δ-to-Y transformations are

4.13



Node : 1 Eq

Mesh : 2 Eq supermesh

Node

$$\frac{V-20}{15} + -2 + \frac{V-25}{10} = 0$$

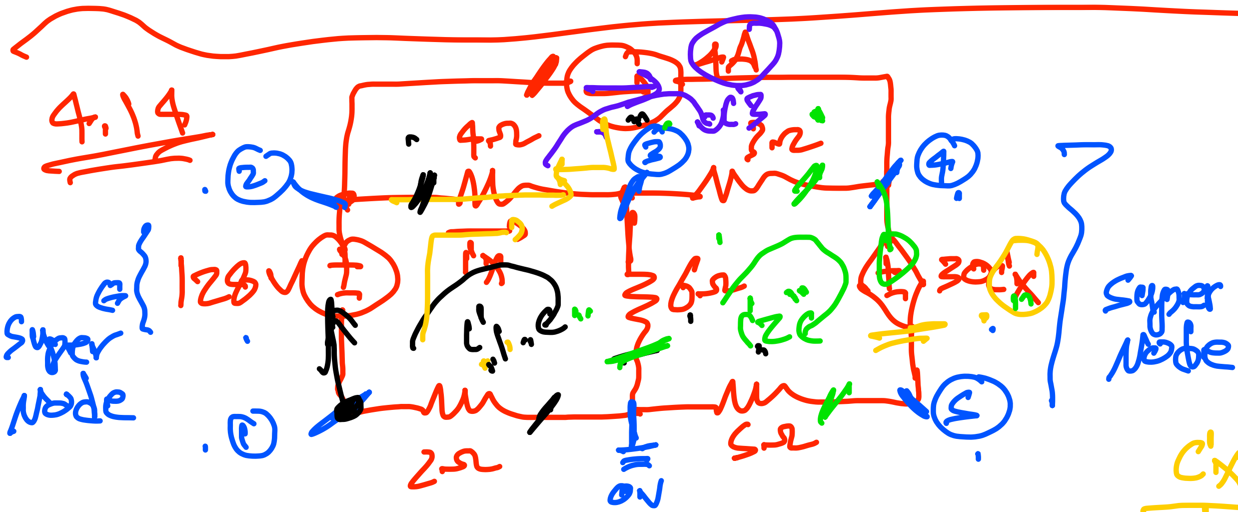
$$\frac{1}{6}V - \frac{20}{15} - 2 - \frac{25}{10} = 0$$

$$\cancel{6}\left[\frac{1}{\cancel{6}}V = \frac{35}{\cancel{6}}\right]\cancel{6}$$

$$\boxed{V = 35\,V}$$

$$P = -iV = -2 \times 35 = -70\,W$$ "del"

4.14



Node : 5eq / 2s

Mesh : ~~3eq~~ 2eq

$$\boxed{i_3 = 4A}$$

$$i_x = i_1 - i_3$$

$$\boxed{i_x = (i_1 - 4)}\ 30$$

$$i_1 : \quad 12i_1 - 6i_2 - 4i_3 = 128 + 4(4)$$

$$\boxed{12i_1 - 6i_2 = 144} \rightarrow (1)$$

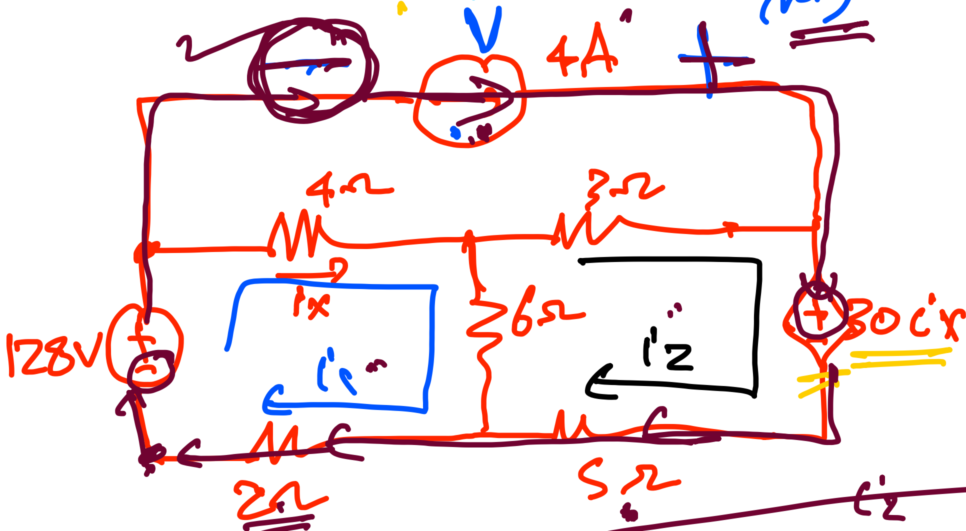
$$i_2 : \quad -6i_1 + 14i_2 - 3i_3 + 30i_1 - 30(4) = 0 \qquad 3(4) + 30(4)$$

$$\boxed{24i_1 + 14i_2 = 132} \rightarrow (2)$$

Mode $\longrightarrow$ S $\longrightarrow$ P

$i'_1 = 9A$ $\qquad$ $i'_2 = -6A$

موجب



$i'_x = i'_1 - 4 = 9 - 4$

$i'_x = 5A$

$30 i'_x = 150V$

$P = -i'V$

KVL

$$-128 + (-V) + 150 + 5(-6) + 2(9) = 0$$

$$V = -128 + 150 - 30 + 18 \quad \therefore \boxed{V = +10V}$$

عكس موجب

$$P = -4 \times 10 = -40W \quad \text{"del"}$$