Substituting Eqs. 4.16 and 4.17 into Eq. 4.15 yields

$$v_b\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{(1+\beta)R_E}\right]=\frac{V_{CC}}{R_1}+\frac{V_0}{(1+\beta)R_E}. \tag{4.18}$$

Solving Eq. 4.18 for v_b yields

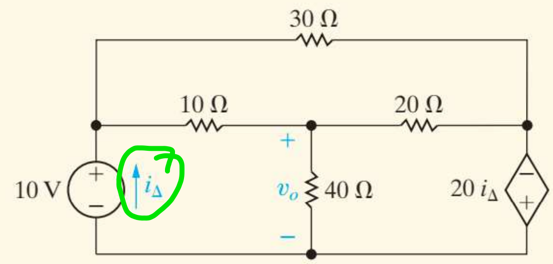
$$v_b=\frac{V_{CC}R_2(1+\beta)R_E+V_0R_1R_2}{R_1R_2+(1+\beta)R_E(R_1+R_2)}. \tag{4.19}$$

Using the node-voltage method to analyze this circuit reduces the problem from manipulating six simultaneous equations (see Problem 2.27) to manipulating three simultaneous equations. You should verify that, when Eq. 4.19 is combined with Eqs. 4.16 and 4.17, the solution for i_B is identical to Eq. 2.25. (See Problem 4.30.)

✓ASSESSMENT PROBLEMS

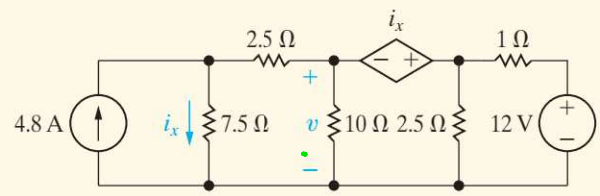
Objective 1—Understand and be able to use the node-voltage method

4.4 Use the node-voltage method to find v_o in the circuit shown.



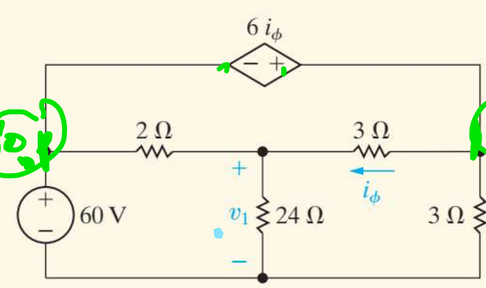
Answer: $\boxed{24 \text{ V.}}$

4.5 Use the node-voltage method to find v in the circuit shown.



Answer: $\boxed{8 \text{ V.}}$

4.6 Use the node-voltage method to find v_1 in the circuit shown.



Answer: 48 V.

NOTE: Also try Chapter Problems 4.22, 4.23, and 4.26.

4.5 Introduction to the Mesh-Current Method

As stated in Section 4.1, the mesh-current method of circuit analysis enables us to describe a circuit in terms of $b_e-(n_e-1)$ equations. Recall that a mesh is a loop with no other loops inside it. The circuit in Fig. 4.1(b) is shown again in Fig. 4.18, with current arrows inside each loop to distinguish it. The

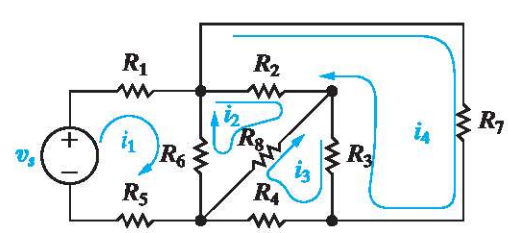
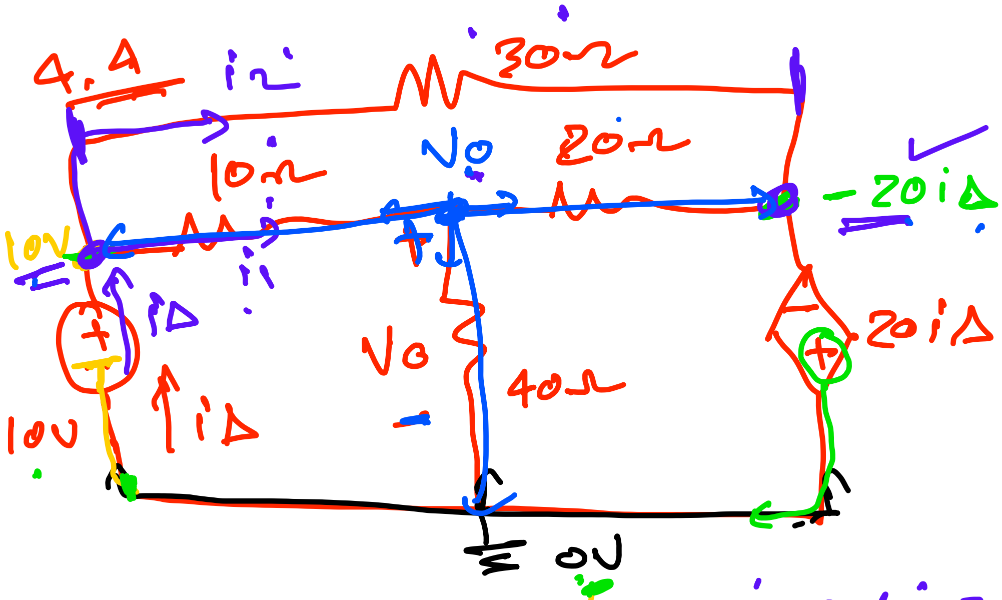


Figure 4.18 ▲ The circuit shown in Fig. 4.1(b), with the mesh currents defined.



Find Vo

$i_\Delta = \frac{\Delta V}{R} = \frac{0-10}{?}$

* KCL $\quad \sum i_{in} = \sum i_{out}$

$i_\Delta = i_1 + i_2$

$i_1 = \frac{10 - V_o}{10}$, $i_2 = \frac{10 - (-20 i_\Delta)}{30}$

$1 i_\Delta = 1 - \frac{V_o}{10} + \frac{1}{3} + \frac{2}{3} i_\Delta$

$-\frac{2}{3} i_\Delta$

$3\left[\frac{1}{3} i_\Delta = \frac{4}{3} - \frac{V_o}{10}\right]$

$\therefore \boxed{i_\Delta = 4 - \frac{3V_o}{10}}$ *

V_o

$\frac{V_o - 10}{10} + \frac{V_o - 0}{40} + \frac{V_o - (-20 i_\Delta)}{20} = 0$

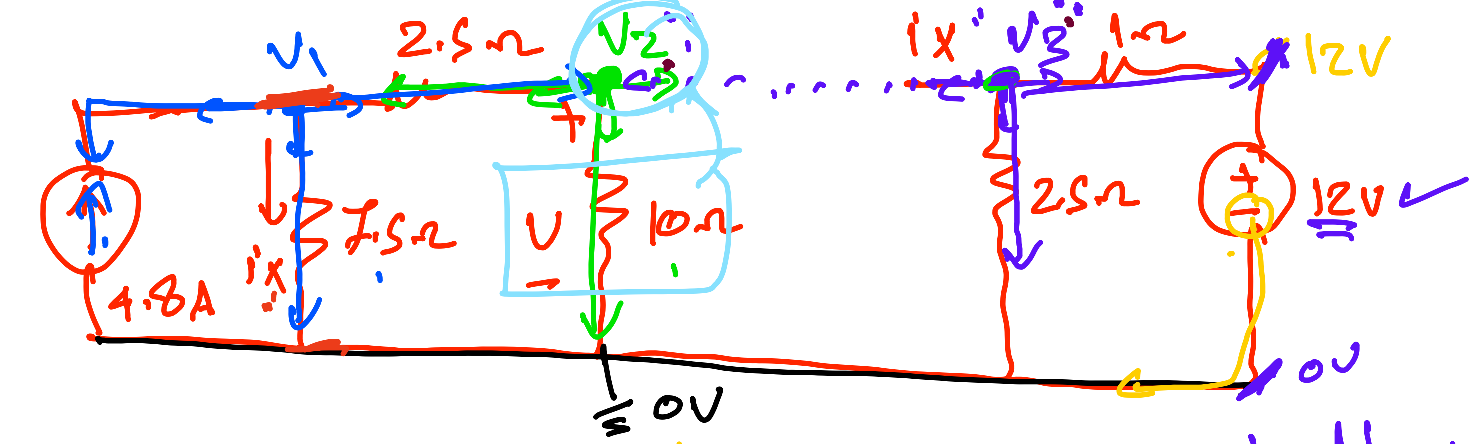
$\boxed{\frac{3V_o}{10} + i_\Delta = 4}$

$\boxed{\frac{1}{40} V_o + i_\Delta = 1}$

Mode → 5 → 1

$x = V_o = 24V$

$y = i_\Delta = -3.2A$



Super Node : V_2, V_3 have a battery

ohm's

$$\boxed{V_3 - V_2} = i_x \qquad i_x = \frac{V_1 - 0}{7.5}$$

2nd

$$\underline{V_2 + V_3} : \frac{V_2 - V_1}{2.5} + \frac{V_2 - 0}{10} + \frac{V_3 - 0}{2.5} + \frac{V_3 - 12}{1} = 0$$

$\underline{V_1}$

$$-4.8 + \frac{V_1 - 0}{7.5} + \frac{V_1 - V_2}{2.5} = 0$$

$$\boxed{\frac{1}{7.5}V_1 + 1V_2 - 1V_3 = 0} \rightarrow (1)$$

$$\boxed{\frac{-1}{2.5}V_1 + \frac{1}{2}V_2 + \frac{7}{5}V_3 = 12} \rightarrow (2)$$

$$\boxed{\frac{8}{15}V_1 - \frac{1}{2.5}V_2 + 0V_3 = 4.8} \rightarrow (3)$$

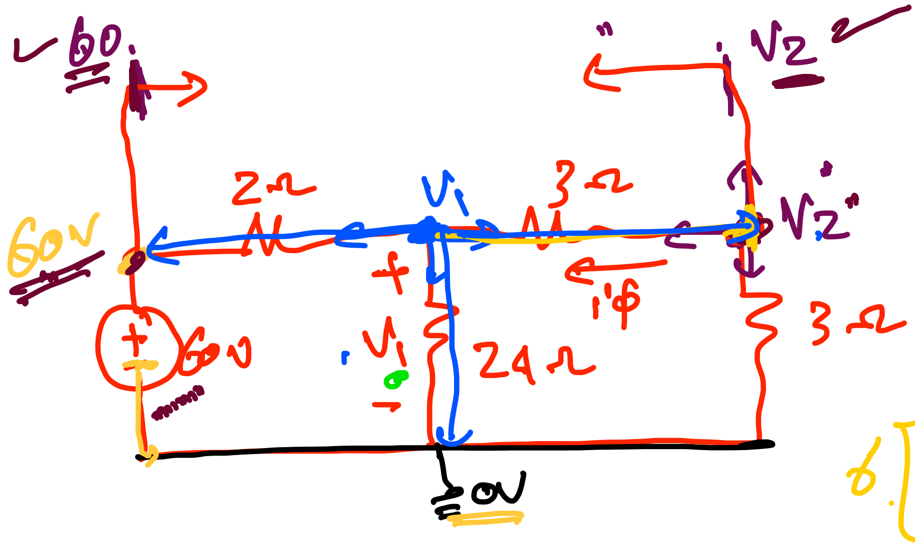
Mode
5
2

$V_1 = 15V$

$\boxed{V_2 = 8V = V}$

$V_3 = 10V$

$V = 8V$



$6\left[i_\phi = \frac{V_2 - V_1}{3} \right] \times 6$

$6 i_\phi = 2V_2 - 2V_1$

Super Node:

$V_2 - 60 = 6 i_\phi$

No equation "(0)"

$1V_2 - 60 = 2V_2 - 2V_1$

$-2V_2$

$+2V_1$

$2V_1 - 1V_2 = 60 \rightarrow (1)$

$\frac{7}{8}V_1 - \frac{1}{3}V_2 = 30 \rightarrow (2)$

Mode → 5 → 1

$\underline{V_1}$

$\frac{V_1 - 60}{12} + \frac{V_1 - 0}{24} + \frac{V_1 - V_2}{3} = 0$

$$\boxed{\begin{aligned} V_1 &= 48V \\ V_2 &= 36V \end{aligned}}$$

circuit in Fig. 4.18 contains seven essential branches where the current is unknown and four essential nodes. Therefore, to solve it via the mesh-current method, we must write four $[7 - (4 - 1)]$ mesh-current equations.

A **mesh current** is the current that exists only in the perimeter of a mesh. On a circuit diagram it appears as either a closed solid line or an almost-closed solid line that follows the perimeter of the appropriate mesh. An arrowhead on the solid line indicates the reference direction for the mesh current. Figure 4.18 shows the four mesh currents that describe the circuit in Fig. 4.1(b). Note that by definition, mesh currents automatically satisfy Kirchhoff's current law. That is, at any node in the circuit, a given mesh current both enters and leaves the node.

Figure 4.18 also shows that identifying a mesh current in terms of a branch current is not always possible. For example, the mesh current i_2 is not equal to any branch current, whereas mesh currents i_1, i_3, and i_4 can be identified with branch currents. Thus measuring a mesh current is not always possible; note that there is no place where an ammeter can be inserted to measure the mesh current i_2. The fact that a mesh current can be a fictitious quantity doesn't mean that it is a useless concept. On the contrary, the mesh-current method of circuit analysis evolves quite naturally from the branch-current equations.

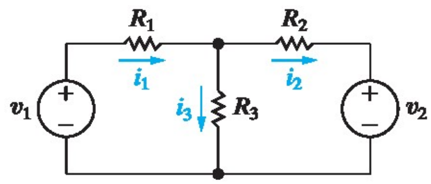


Figure 4.19 ▲ A circuit used to illustrate development of the mesh-current method of circuit analysis.

We can use the circuit in Fig. 4.19 to show the evolution of the mesh-current technique. We begin by using the branch currents (i_1, i_2, and i_3) to formulate the set of independent equations. For this circuit, $b_e = 3$ and $n_e = 2$. We can write only one independent current equation, so we need two independent voltage equations. Applying Kirchhoff's current law to the upper node and Kirchhoff's voltage law around the two meshes generates the following set of equations:

$$i_1 = i_2 + i_3, \tag{4.20}$$

$$v_1 = i_1R_1 + i_3R_3, \tag{4.21}$$

$$-v_2 = i_2R_2 - i_3R_3. \tag{4.22}$$

We reduce this set of three equations to a set of two equations by solving Eq. 4.20 for i_3 and then substituting this expression into Eqs. 4.21 and 4.22:

$$v_1 = i_1(R_1 + R_3) - i_2R_3, \tag{4.23}$$

$$-v_2 = -i_1R_3 + i_2(R_2 + R_3). \tag{4.24}$$

We can solve Eqs. 4.23 and 4.24 for i_1 and i_2 to replace the solution of three simultaneous equations with the solution of two simultaneous equations. We derived Eqs. 4.23 and 4.24 by substituting the $n_e - 1$ current equations into the $b_e - (n_e - 1)$ voltage equations. The value of the mesh-current method is that, by defining mesh currents, we automatically eliminate the $n_e - 1$ current equations. Thus the mesh-current method is equivalent to a systematic substitution of the $n_e - 1$ current equations into the $b_e - (n_e - 1)$ voltage equations. The mesh currents in Fig. 4.19 that are equivalent to eliminating the branch current i_3 from Eqs. 4.21 and 4.22 are shown in Fig. 4.20. We now apply Kirchhoff's voltage law around the two meshes, expressing all voltages across resistors in terms of the mesh currents, to get the equations

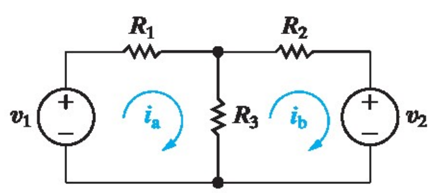


Figure 4.20 ▲ Mesh currents i_a and i_b.

$$v_1 = i_aR_1 + (i_a - i_b)R_3, \tag{4.25}$$

$$-v_2 = (i_b - i_a)R_3 + i_bR_2. \tag{4.26}$$

Collecting the coefficients of i_a and i_b in Eqs. 4.25 and 4.26 gives

$$v_1 = i_a(R_1 + R_3) - i_bR_3, \tag{4.27}$$

$$-v_2 = -i_aR_3 + i_b(R_2 + R_3). \tag{4.28}$$

Note that Eqs. 4.27 and 4.28 and Eqs. 4.23 and 4.24 are identical in form, with the mesh currents i_a and i_b replacing the branch currents i_1 and i_2. Note also that the branch currents shown in Fig. 4.19 can be expressed in terms of the mesh currents shown in Fig. 4.20, or

$$i_1 = i_a, \tag{4.29}$$

$$i_2 = i_b, \tag{4.30}$$

$$i_3 = i_a - i_b. \tag{4.31}$$

The ability to write Eqs. 4.29–4.31 by inspection is crucial to the mesh-current method of circuit analysis. Once you know the mesh currents, you also know the branch currents. And once you know the branch currents, you can compute any voltages or powers of interest.

Example 4.4 illustrates how the mesh-current method is used to find source powers and a branch voltage.

Example 4.4 Using the Mesh-Current Method

a) Use the mesh-current method to determine the power associated with each voltage source in the circuit shown in Fig. 4.21.

b) Calculate the voltage v_o across the 8 Ω resistor.

Solution

a) To calculate the power associated with each source, we need to know the current in each source. The circuit indicates that these source currents will be identical to mesh currents. Also, note that the circuit has seven branches where

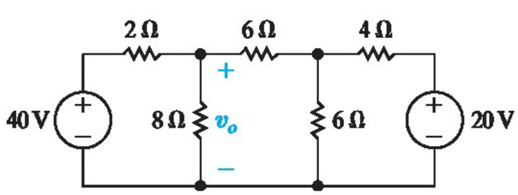


Figure 4.21 ▲ The circuit for Example 4.4.

the current is unknown and five nodes. Therefore we need three $[b - (n - 1) = 7 - (5 - 1)]$ mesh-current equations to describe the circuit. Figure 4.22 shows the three mesh currents used to describe the circuit in Fig. 4.21. If we assume that the voltage drops are positive, the three mesh equations are

$$-40 + 2i_a + 8(i_a - i_b) = 0,$$

$$8(i_b - i_a) + 6i_b + 6(i_b - i_c) = 0,$$

$$6(i_c - i_b) + 4i_c + 20 = 0. \tag{4.32}$$

Your calculator can probably solve these equations, or you can use a computer tool. Cramer's method is a useful tool when solving three or more simultaneous equations by hand. You can review this important tool in Appendix A. Reorganizing Eqs. 4.32 in anticipation of using your calculator, a computer program, or Cramer's method gives

$$10i_a - 8i_b + 0i_c = 40;$$

$$-8i_a + 20i_b - 6i_c = 0;$$

$$0i_a - 6i_b + 10i_c = -20. \tag{4.33}$$

The three mesh currents are

$$i_a = 5.6 \text{ A},$$

$$i_b = 2.0 \text{ A},$$

$$i_c = -0.80 \text{ A}.$$

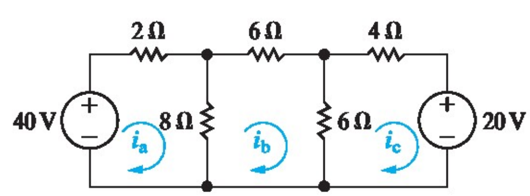


Figure 4.22 ▲ The three mesh currents used to analyze the circuit shown in Fig. 4.21.

The mesh current i_a is identical with the branch current in the 40 V source, so the power associated with this source is

$$p_{40V} = -40i_a = -224 \text{ W}.$$

The minus sign means that this source is delivering power to the network. The current in the 20 V source is identical to the mesh current i_c; therefore

$$p_{20V} = 20i_c = -16 \text{ W}.$$

The 20 V source also is delivering power to the network.

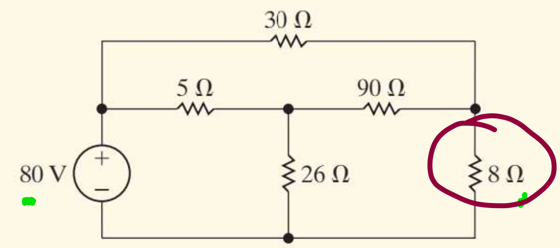
b) The branch current in the 8 Ω resistor in the direction of the voltage drop v_o is $i_a - i_b$. Therefore

$$v_o = 8(i_a - i_b) = 8(3.6) = 28.8 \text{ V}.$$

✓ASSESSMENT PROBLEM

Objective 2—Understand and be able to use the mesh-current method

4.7 Use the mesh-current method to find (a) the power delivered by the 80 V source to the circuit shown and (b) the power dissipated in the 8 Ω resistor.



Answer: (a) 400 W;
(b) 50 W.

NOTE: Also try Chapter Problems 4.32 and 4.36.

4.6 The Mesh-Current Method and Dependent Sources

If the circuit contains dependent sources, the mesh-current equations must be supplemented by the appropriate constraint equations. Example 4.5 illustrates the application of the mesh-current method when the circuit includes a dependent source.

Example 4.5 Using the Mesh-Current Method with Dependent Sources

Use the mesh-current method of circuit analysis to determine the power dissipated in the 4 Ω resistor in the circuit shown in Fig. 4.23.

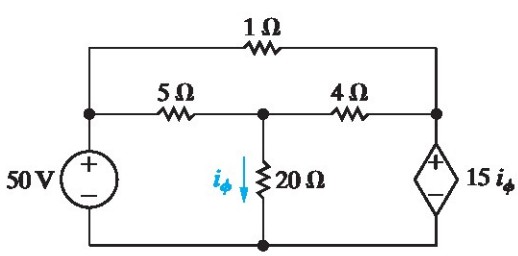


Figure 4.23 ▲ The circuit for Example 4.5.

Solution

This circuit has six branches where the current is unknown and four nodes. Therefore we need three mesh currents to describe the circuit. They are defined on the circuit shown in Fig. 4.24. The three mesh-current equations are

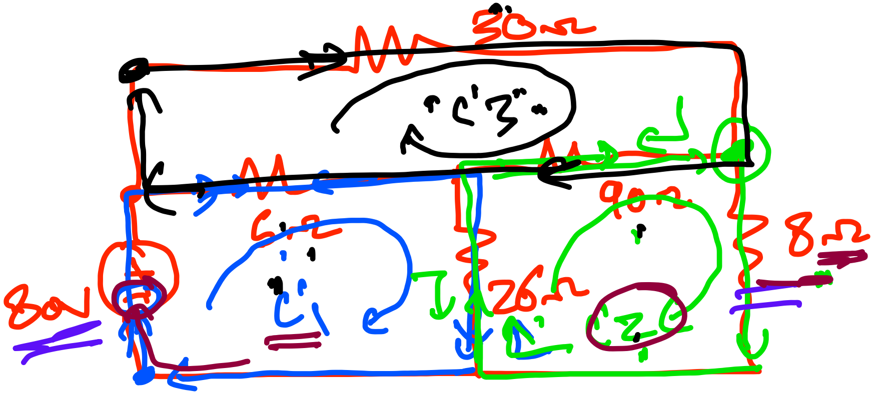
$$50 = 5(i_1 - i_2) + 20(i_1 - i_3),$$

$$0 = 5(i_2 - i_1) + 1i_2 + 4(i_2 - i_3),$$

$$0 = 20(i_3 - i_1) + 4(i_3 - i_2) + 15i_\phi. \quad (4.34)$$

We now express the branch current controlling the dependent voltage source in terms of the mesh currents as

$$i_\phi = i_1 - i_3, \quad (4.35)$$



Mesh Analysis

current تيار

كم مربع عندي ؟!

3 ⟵ 3 تيار ⟵ هو كتابة المعادلة

$V = iR$ حدود

i_1

$$-80 + 5(i_1 - i_3) + 26(i_1 - i_2) = 0$$

$$\boxed{31 i_1 - 26 i_2 - 5 i_3 = 80} \rightarrow (1)$$

i_2

$$8 i_2 + 26(i_2 - i_1) + 90(i_2 - i_3) = 0$$

$$\boxed{-26 i_1 + 124 i_2 - 90 i_3 = 0} \rightarrow (2)$$

i_3

$$30 i_3 + 90(i_3 - i_2) + 5(i_3 - i_1) = 0$$

$$\boxed{-5 i_1 - 90 i_2 + 125 i_3 = 0} \rightarrow (3)$$

Mode
5
2

$i_1 = 5A$

$i_2 = 2.5A$

$i_3 = 2A$

$$P_{80V} = -i_1 V = -5 \times 80 = -400\,W$$ "del"

$$P_{8\Omega} = i_2^2 R = 2.5^2 \times 8 = +50\,W$$ "diss"

which is the supplemental equation imposed by the presence of the dependent source. Substituting Eq. 4.35 into Eqs. 4.34 and collecting the coefficients of i_1, i_2, and i_3 in each equation generates

$$50 = 25i_1 - 5i_2 - 20i_3,$$

$$0 = -5i_1 + 10i_2 - 4i_3,$$

$$0 = -5i_1 - 4i_2 + 9i_3.$$

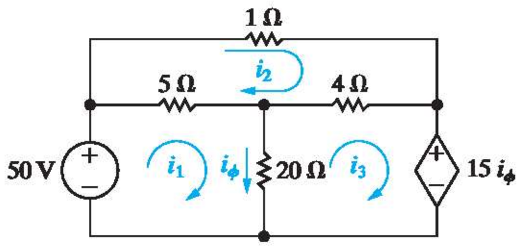


Figure 4.24 ▲ The circuit shown in Fig. 4.23 with the three mesh currents.

Because we are calculating the power dissipated in the 4 Ω resistor, we compute the mesh currents i_2 and i_3:

$$i_2 = 26 \text{ A},$$

$$i_3 = 28 \text{ A}.$$

The current in the 4 Ω resistor oriented from left to right is $i_3 - i_2$, or 2 A. Therefore the power dissipated is

$$p_{4\Omega} = (i_3 - i_2)^2(4) = (2)^2(4) = 16 \text{ W}.$$

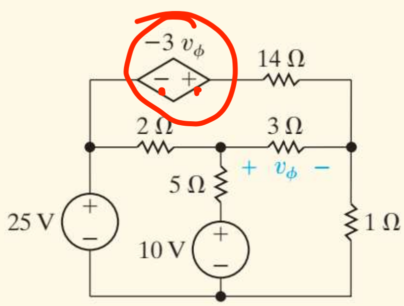
What if you had not been told to use the mesh-current method? Would you have chosen the node-voltage method? It reduces the problem to finding one unknown node voltage because of the presence of two voltage sources between essential nodes. We present more about making such choices later.

✓ ASSESSMENT PROBLEMS

Objective 2—Understand and be able to use the mesh-current method

4.8 a) Determine the number of mesh-current equations needed to solve the circuit shown.

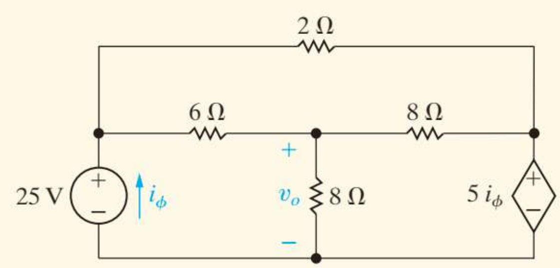
b) Use the mesh-current method to find how much power is being delivered to the dependent voltage source.



Answer: (a) 3;

(b) −36 W.

4.9 Use the mesh-current method to find v_o in the circuit shown.



Answer: 16 V.

NOTE: Also try Chapter Problems 4.39 and 4.40.

4.7 The Mesh-Current Method: Some Special Cases

When a branch includes a current source, the mesh-current method requires some additional manipulations. The circuit shown in Fig. 4.25 depicts the nature of the problem.

We have defined the mesh currents i_a, i_b, and i_c, as well as the voltage across the 5 A current source, to aid the discussion. Note that the circuit contains five essential branches where the current is unknown and four essential nodes. Hence we need to write two [5 − (4 − 1)] mesh-current

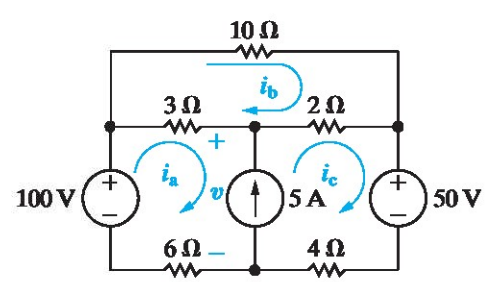
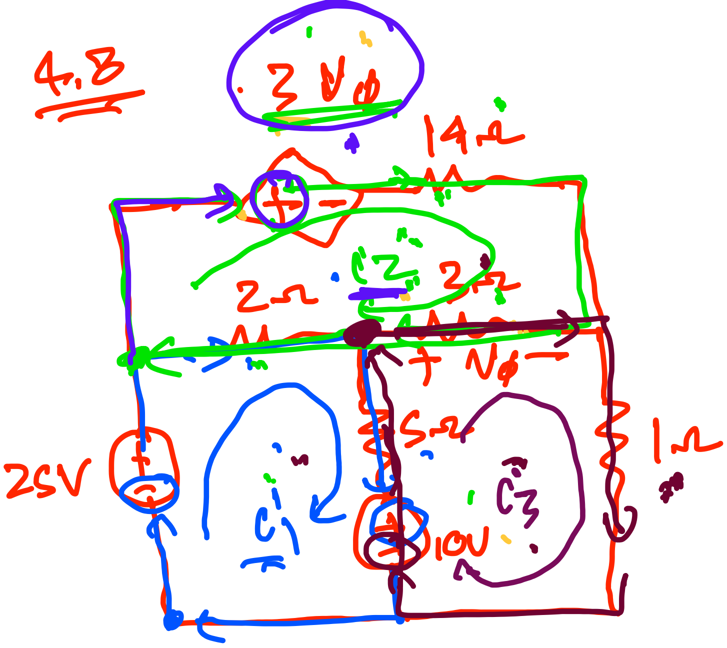


Figure 4.25 ▲ A circuit illustrating mesh analysis when a branch contains an independent current source.

4.8



Note $V_\phi = \Delta i' R$

$$\boxed{V_\phi = 3(i_3 - i_2)}$$

$$3(V_\phi = 3i_3 - 3i_2)$$

$$\boxed{3V_\phi = 9i_3 - 9i_2}$$

$9 \times 3 - 9 \times -1$

$3V_\phi = 36V$

i_1

$$-25 + 2(i_1 - i_2) + 5(i_1 - i_3) + 10 = 0 \qquad 25 - 10$$

$$\boxed{7i_1 - 2i_2 - 5i_3 = 15} \to (1)$$

i_2

$$9i_3 - 9i_2 + 14i_2 + 3(i_2 - i_3) + 2(i_2 - i_1) = 0$$

$$\boxed{-2i_1 + 10i_2 + 6i_3 = 0} \to (2)$$

i_3

$$3(i_3 - i_2) + 1i_3 - 10 + 5(i_3 - i_1) = 0$$

$$\boxed{-5i_1 - 3i_2 + 9i_3 = 10} \to (3)$$

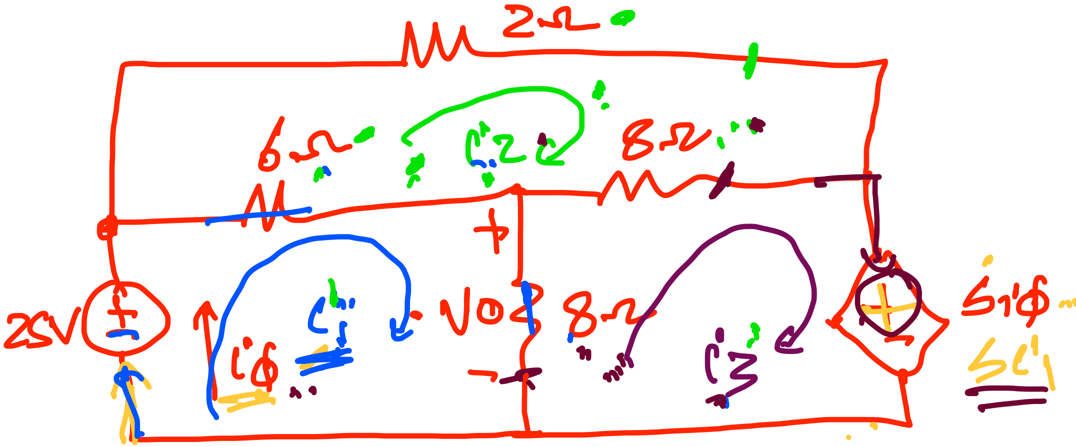
$i_1 = 4A$

$i_2 = -1A$

$i_3 = 3A$

Mode → 5 → 2

$$P_\Delta = +i_2 V = -1 \times 36 = -36\ W$$



Note
$i_\phi \to i'_1$

i'_1

$$\boxed{14i'_1 - 6i'_2 - 8i'_3 = 25} \to (1)$$

i'_2

$$\boxed{-6i'_1 + 16i'_2 - 8i'_3 = 0} \to (2)$$

i'_3

$$-8i'_1 - 8i'_2 + 16i'_3 + 5i'_1 = 0$$

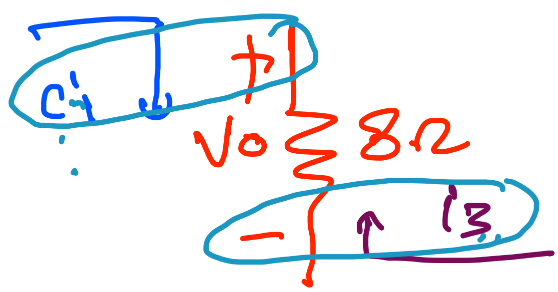
$$\boxed{-3i'_1 - 8i'_2 + 16i'_3 = 0} \to (3)$$

Mode → 5 → 2

$i'_1 = 4A$

$i'_2 = 2.5A$

$i'_3 = 2A$



$$V_o = \Delta i' R = (i'_1 - i'_3)R$$

$$V_o = (4-2)\times 8 = 2\times 8 = \underline{160}$$