CHAPTER

3

Simple Resistive Circuits

CHAPTER CONTENTS

✓CHAPTER OBJECTIVES

1. Be able to recognize resistors connected in series and in parallel and use the rules for combining series-connected resistors and parallel-connected resistors to yield equivalent resistance.
2. Know how to design simple voltage-divider and current-divider circuits.
3. Be able to use voltage division and current division appropriately to solve simple circuits.
4. Be able to determine the reading of an ammeter when added to a circuit to measure current; be able to determine the reading of a voltmeter when added to a circuit to measure voltage.
5. Understand how a Wheatstone bridge is used to measure resistance.
6. Know when and how to use delta-to-wye equivalent circuits to solve simple circuits.

Our analytical toolbox now contains Ohm's law and Kirchhoff's laws. In Chapter 2 we used these tools in solving simple circuits. In this chapter we continue applying these tools, but on more-complex circuits. The greater complexity lies in a greater number of elements with more complicated interconnections. This chapter focuses on reducing such circuits into simpler, equivalent circuits. We continue to focus on relatively simple circuits for two reasons: (1) It gives us a chance to acquaint ourselves thoroughly with the laws underlying more sophisticated methods, and (2) it allows us to be introduced to some circuits that have important engineering applications.

The sources in the circuits discussed in this chapter are limited to voltage and current sources that generate either constant voltages or currents; that is, voltages and currents that are invariant with time. Constant sources are often called **dc sources.** The *dc* stands for *direct current*, a description that has a historical basis but can seem misleading now. Historically, a direct current was defined as a current produced by a constant voltage. Therefore, a constant voltage became known as a direct current, or dc, voltage. The use of *dc* for *constant* stuck, and the terms *dc current* and *dc voltage* are now universally accepted in science and engineering to mean constant current and constant voltage.

Practical Perspective

Resistive Touch Screens

Some mobile phones and tablet computers use resistive touch screens, created by applying a transparent resistive material to the glass or acrylic screens. Two screens are typically used, separated by a transparent insulating layer. The resulting touch screen can be modeled by a grid of resistors in the x-direction and a grid of resistors in the y-direction, as shown in the figure on the right.

A separate electronic circuit applies a voltage drop across the grid in the x-direction, between the points a and b in the circuit, then removes that voltage and applies a voltage drop across the grid in the y-direction (between points c and d), and continues to repeat this process. When the screen is touched, the two resistive layers are pressed together, creating a voltage that is sensed in the x-grid and another voltage that is sensed in the y-grid. These two voltages precisely locate the point where the screen was touched.

How is the voltage created by touching the screen related to the position where the screen was touched? How are the properties of the grids used to calculate the touch position? We will answer these questions in the Practical Perspective at the end of this chapter. The circuit analysis required to answer these questions uses some circuit analysis tools developed next.

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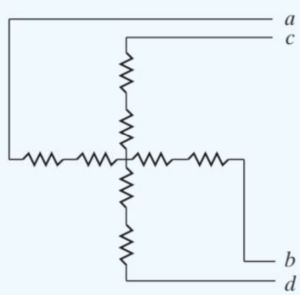


3.1 Resistors in Series

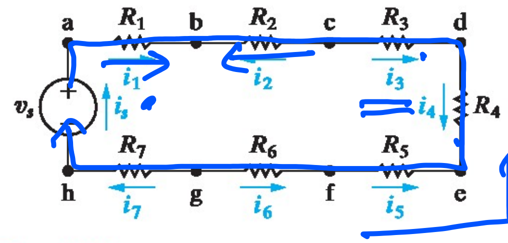


Figure 3.1 ▲ Resistors connected in series.

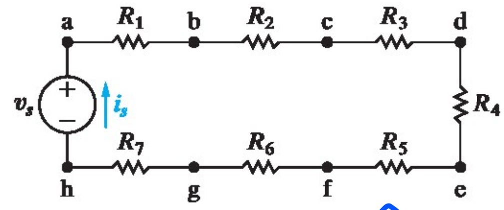


Figure 3.2 ▲ Series resistors with a single unknown current i_s.

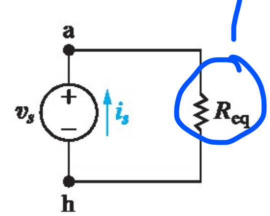


Figure 3.3 ▲ A simplified version of the circuit shown in Fig. 3.2.

In Chapter 2, we said that when just two elements connect at a single node, they are said to be in series. **Series-connected circuit elements** carry the same current. The resistors in the circuit shown in Fig. 3.1 are connected in series. We can show that these resistors carry the same current by applying Kirchhoff's current law to each node in the circuit. The series interconnection in Fig. 3.1 requires that

$$i_s = i_1 = -i_2 = i_3 = i_4 = -i_5 = -i_6 = i_7, \tag{3.1}$$

which states that if we know any one of the seven currents, we know them all. Thus we can redraw Fig. 3.1 as shown in Fig. 3.2, retaining the identity of the single current i_s.

To find i_s, we apply Kirchhoff's voltage law around the single closed loop. Defining the voltage across each resistor as a drop in the direction of i_s gives

$$-v_s + i_sR_1 + i_sR_2 + i_sR_3 + i_sR_4 + i_sR_5 + i_sR_6 + i_sR_7 = 0, \tag{3.2}$$

or

$$v_s = i_s(R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7). \tag{3.3}$$

The significance of Eq. 3.3 for calculating i_s is that the seven resistors can be replaced by a single resistor whose numerical value is the sum of the individual resistors, that is,

$$R_{eq} = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7 \tag{3.4}$$

and

$$v_s = i_sR_{eq}. \tag{3.5}$$

Thus we can redraw Fig. 3.2 as shown in Fig. 3.3.

In general, if k resistors are connected in series, the equivalent single resistor has a resistance equal to the sum of the k resistances, or

Combining resistors in series ▶

$$R_{eq} = \sum_{i=1}^{k} R_i = R_1 + R_2 + \cdots + R_k. \tag{3.6}$$

$R_{eq} > R$

Note that the resistance of the equivalent resistor is always larger than that of the largest resistor in the series connection.

Another way to think about this concept of an equivalent resistance is to visualize the string of resistors as being inside a black box. (An electrical engineer uses the term **black box** to imply an opaque container; that is, the contents are hidden from view. The engineer is then challenged to model the contents of the box by studying the relationship between the voltage and current at its terminals.) Determining whether the box contains k resistors or a single equivalent resistor is impossible. Figure 3.4 illustrates this method of studying the circuit shown in Fig. 3.2.

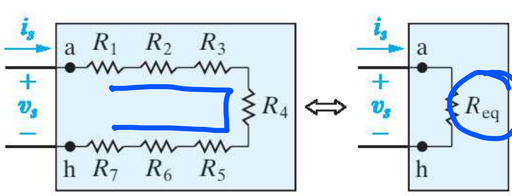


Figure 3.4 ▲ The black box equivalent of the circuit shown in Fig. 3.2.

$V_s = V_1 = V_2 = V_3 \ldots$

3.2 Resistors in Parallel

When two elements connect at a single node pair, they are said to be in parallel. **Parallel-connected circuit elements** have the same voltage across their terminals. The circuit shown in Fig. 3.5 illustrates resistors connected in parallel. Don't make the mistake of assuming that two elements are parallel connected merely because they are lined up in parallel in a circuit diagram. The defining characteristic of parallel-connected elements is that they have the same voltage across their terminals. In Fig. 3.6, you can see that R_1 and R_3 are not parallel connected because, between their respective terminals, another resistor dissipates some of the voltage.

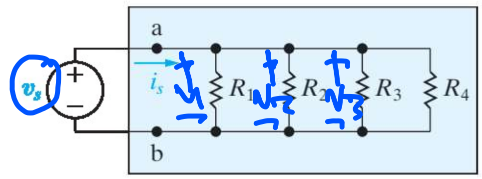


Figure 3.5 ▲ Resistors in parallel.

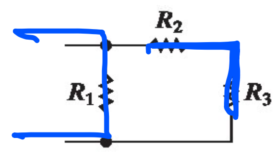


Figure 3.6 ▲ Nonparallel resistors.

$R_1 \;//\; (R_2 + R_3)$

Resistors in parallel can be reduced to a single equivalent resistor using Kirchhoff's current law and Ohm's law, as we now demonstrate. In the circuit shown in Fig. 3.5, we let the currents i_1, i_2, i_3, and i_4 be the currents in the resistors R_1 through R_4, respectively. We also let the positive reference direction for each resistor current be down through the resistor, that is, from node a to node b. From Kirchhoff's current law,

$$i_s = i_1 + i_2 + i_3 + i_4. \tag{3.7}$$

The parallel connection of the resistors means that the voltage across each resistor must be the same. Hence, from Ohm's law,

$$\underbrace{i_1R_1}_{V_1} = \underbrace{i_2R_2}_{V_2} = \underbrace{i_3R_3}_{V_3} = \underbrace{i_4R_4}_{V_4} = v_s. \tag{3.8}$$

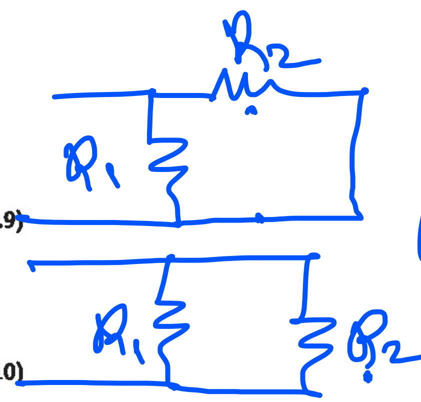
Therefore,

$$i_1 = \frac{v_s}{R_1},$$

$$i_2 = \frac{v_s}{R_2},$$

$$i_3 = \frac{v_s}{R_3}, \quad \text{and}$$

$$i_4 = \frac{v_s}{R_4}. \tag{3.9}$$



$R_1 \;//\; R_2$

Substituting Eq. 3.9 into Eq. 3.7 yields

$$i_s = v_s\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right), \tag{3.10}$$

from which

$$\frac{i_s}{v_s} = \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}. \tag{3.11}$$

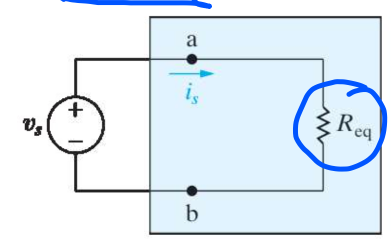


Figure 3.7 ▲ Replacing the four parallel resistors shown in Fig. 3.5 with a single equivalent resistor.

Equation 3.11 is what we set out to show: that the four resistors in the circuit shown in Fig. 3.5 can be replaced by a single equivalent resistor. The circuit shown in Fig. 3.7 illustrates the substitution. For k resistors connected in parallel, Eq. 3.11 becomes

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$$\frac{1}{R_{eq}} = \sum_{i=1}^{k} \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_k}. \tag{3.12}$$

◀ **Combining resistors in parallel**

Note that the resistance of the equivalent resistor is always smaller than the resistance of the smallest resistor in the parallel connection. Sometimes,

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} \cdots}$$

$R_{eq} < R_{\text{أصغر}}$

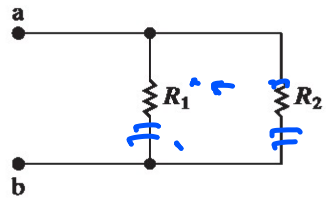


Figure 3.8 ▲ Two resistors connected in parallel.

$R_{eq} = \frac{\text{[illegible]}}{\text{[illegible]}}$

using conductance when dealing with resistors connected in parallel is more convenient. In that case, Eq. 3.12 becomes

$$G_{eq} = \sum_{i=1}^{k} G_i = G_1 + G_2 + \cdots + G_k. \qquad (3.13)$$

Many times only two resistors are connected in parallel. Figure 3.8 illustrates this special case. We calculate the equivalent resistance from Eq. 3.12:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2}, \qquad (3.14)$$

or

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}. \qquad (3.15)$$

Thus for just two resistors in parallel the equivalent resistance equals the product of the resistances divided by the sum of the resistances. Remember that you can only use this result in the special case of just two resistors in parallel. Example 3.1 illustrates the usefulness of these results.

Example 3.1 Applying Series-Parallel Simplification

Find i_s, i_1, and i_2 in the circuit shown in Fig. 3.9.

Solution

We begin by noting that the 3 Ω resistor is in series with the 6 Ω resistor. We therefore replace this series combination with a 9 Ω resistor, reducing the circuit to the one shown in Fig. 3.10(a). We now can replace the parallel combination of the 9 Ω and 18 Ω resistors with a single resistance of $(18 \times 9)/(18 + 9)$, or 6 Ω. Figure 3.10(b) shows this further reduction of the circuit. The nodes x and y marked on all diagrams facilitate tracing through the reduction of the circuit.

From Fig. 3.10(b) you can verify that i_s equals 120/10, or 12 A. Figure 3.11 shows the result at this point in the analysis. We added the voltage v_1 to help clarify the subsequent discussion. Using Ohm's law we compute the value of v_1:

$$v_1 = (12)(6) = 72 \text{ V}. \qquad (3.16)$$

But v_1 is the voltage drop from node x to node y, so we can return to the circuit shown in Fig. 3.10(a) and again use Ohm's law to calculate i_1 and i_2. Thus,

$$i_1 = \frac{v_1}{18} = \frac{72}{18} = 4 \text{ A}, \qquad (3.17)$$

$$i_2 = \frac{v_1}{9} = \frac{72}{9} = 8 \text{ A}. \qquad (3.18)$$

We have found the three specified currents by using series-parallel reductions in combination with Ohm's law.

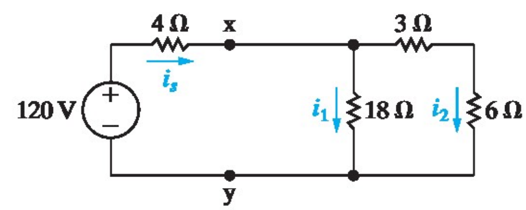


Figure 3.9 ▲ The circuit for Example 3.1.

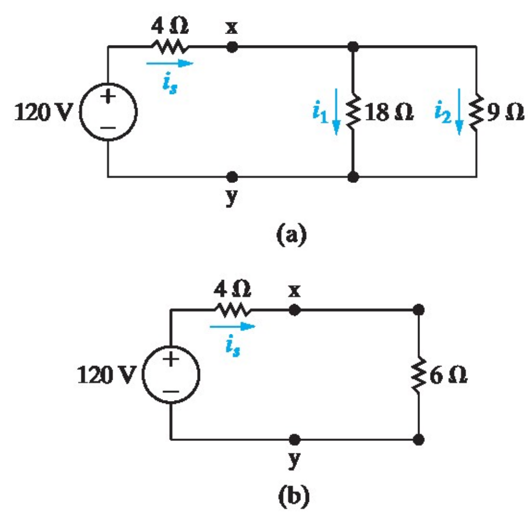


Figure 3.10 ▲ A simplification of the circuit shown in Fig. 3.9.

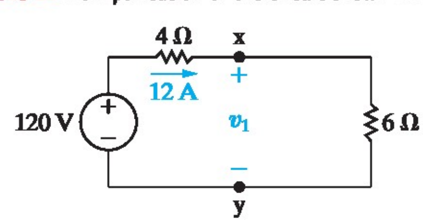


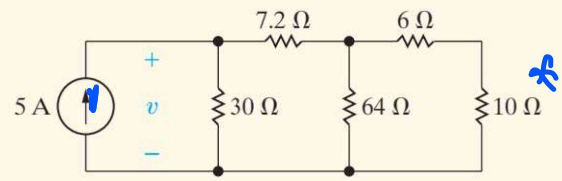
Figure 3.11 ▲ The circuit of Fig. 3.10(b) showing the numerical value of i_s.

Before leaving Example 3.1, we suggest that you take the time to show that the solution satisfies Kirchhoff's current law at every node and Kirchhoff's voltage law around every closed path. (Note that there are three closed paths that can be tested.) Showing that the power delivered by the voltage source equals the total power dissipated in the resistors also is informative. (See Problems 3.1 and 3.2.)

✓ASSESSMENT PROBLEM

Objective 1—Be able to recognize resistors connected in series and in parallel

3.1 For the circuit shown, find (a) the voltage v, (b) the power delivered to the circuit by the current source, and (c) the power dissipated in the 10 Ω resistor.



Answer: (a) 60 V;
(b) 300 W;
(c) 57.6 W.

NOTE: Also try Chapter Problems 3.3–3.6.

3.3 The Voltage-Divider and Current-Divider Circuits

⟹ series

At times—especially in electronic circuits—developing more than one voltage level from a single voltage supply is necessary. One way of doing this is by using a **voltage-divider circuit**, such as the one in Fig. 3.12.

We analyze this circuit by directly applying Ohm's law and Kirchhoff's laws. To aid the analysis, we introduce the current i as shown in Fig. 3.12(b). From Kirchhoff's current law, R_1 and R_2 carry the same current. Applying Kirchhoff's voltage law around the closed loop yields

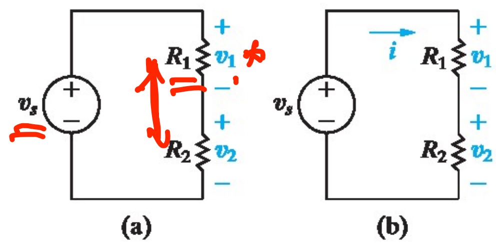


Figure 3.12 ▲ (a) A voltage-divider circuit and (b) the voltage-divider circuit with current i indicated.

$$v_s = iR_1 + iR_2, \tag{3.19}$$

or

$$i = \frac{v_s}{R_1 + R_2}. \tag{3.20}$$

$$V_1 = \frac{R_1}{R_1 + R_2} * V_s$$

Now we can use Ohm's law to calculate v_1 and v_2:

$$v_1 = iR_1 = v_s\frac{R_1}{R_1 + R_2}, \tag{3.21}$$

$$v_2 = iR_2 = v_s\frac{R_2}{R_1 + R_2}. \tag{3.22}$$

Equations 3.21 and 3.22 show that v_1 and v_2 are fractions of v_s. Each fraction is the ratio of the resistance across which the divided voltage is defined to the sum of the two resistances. Because this ratio is always less than 1.0, the divided voltages v_1 and v_2 are always less than the source voltage v_s.

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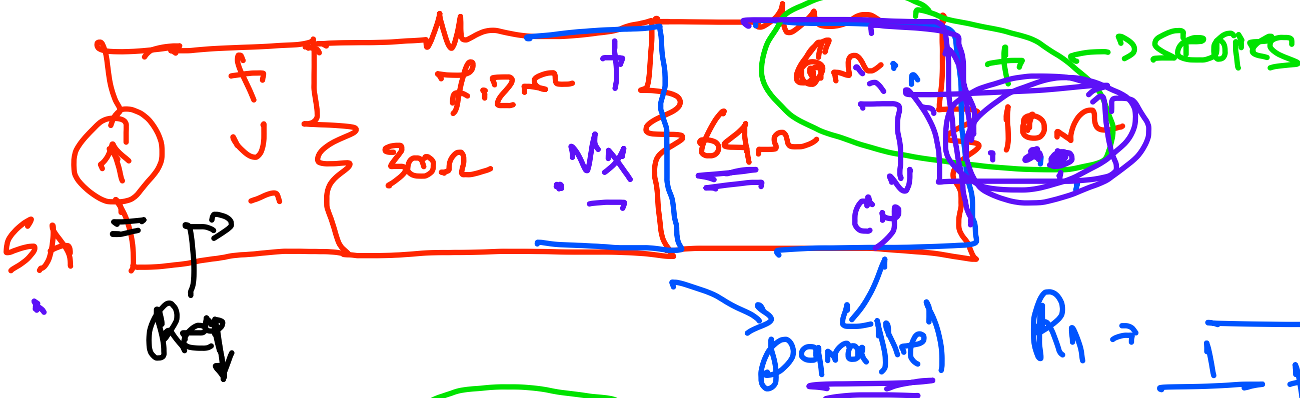
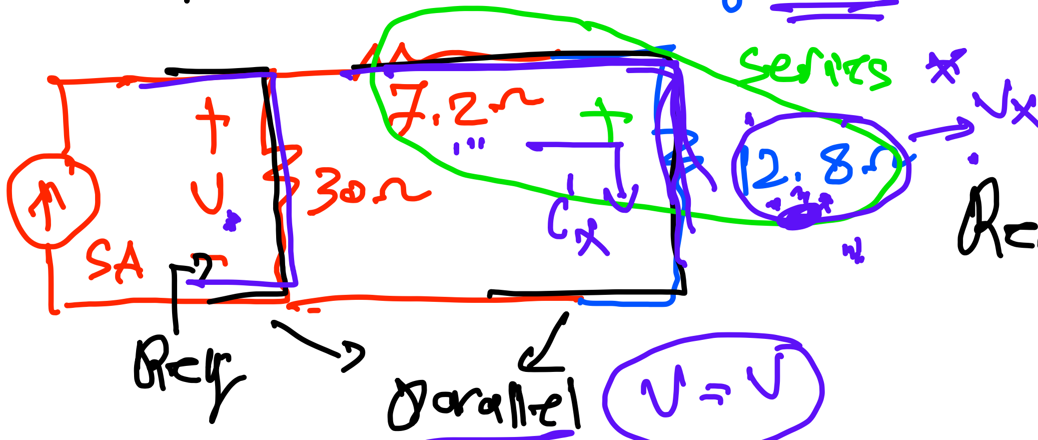


$$R_1 = \frac{1}{\frac{1}{64} + \frac{1}{6+10=16}} = 12.8\Omega$$



V = V

$$R_{eq} = \frac{1}{\frac{1}{30} + \frac{1}{7.2+12.8=20}} = 12\Omega$$

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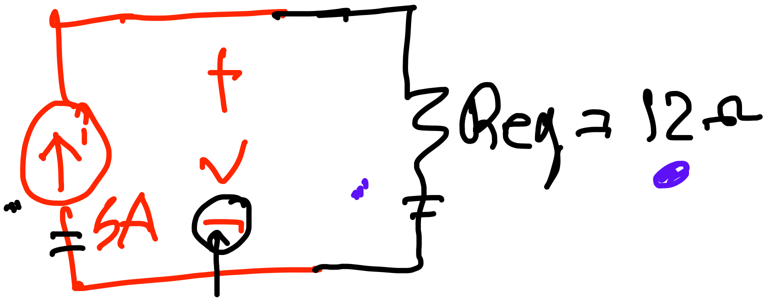


a) $V = I R_{eq} = 5 \times 12 = 60\,V$

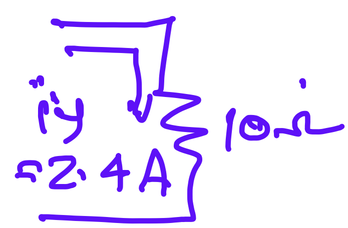
b) $P_I = -IV = -5 \times 60 = -300\,W$ P: delivered

$P = 300\,W$ " delivered "

c) $i'_x = \frac{V}{R} = \frac{60}{20} = 3\,A$

$V_x = i'_x R = 3 \times 12.8 = 38.4\,V$

$i_y = \frac{V_x}{R} = \frac{38.4}{16} = 2.4\,A$



$P_{10\Omega} = i^2 R = 2.4^2 \times 10 = 57.6\,W$

Parallel ⟹ V | series ⟹ نفس

If you desire a particular value of v_2, and v_s is specified, an infinite number of combinations of R_1 and R_2 yield the proper ratio. For example, suppose that v_s equals 15 V and v_2 is to be 5 V. Then $v_2/v_s = \frac{1}{3}$ and, from Eq. 3.22, we find that this ratio is satisfied whenever $R_2 = \frac{1}{2}R_1$. Other factors that may enter into the selection of R_1, and hence R_2, include the power losses that occur in dividing the source voltage and the effects of connecting the voltage-divider circuit to other circuit components.

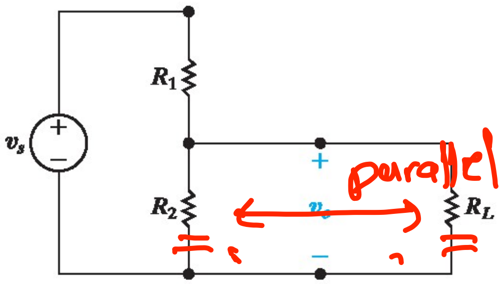
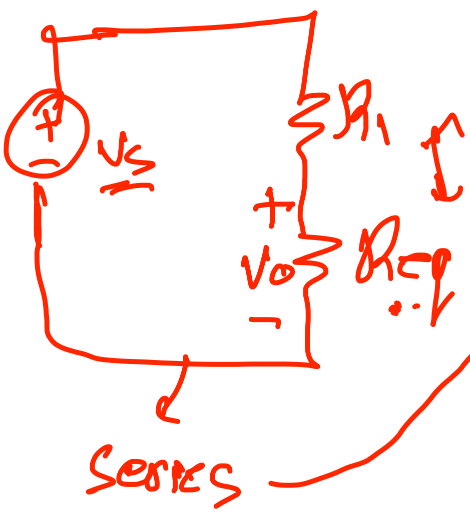


Figure 3.13 ▲ A voltage divider connected to a load R_L.

Consider connecting a resistor R_L in parallel with R_2, as shown in Fig. 3.13. The resistor R_L acts as a load on the voltage-divider circuit. A **load** on any circuit consists of one or more circuit elements that draw power from the circuit. With the load R_L connected, the expression for the output voltage becomes

$$v_o = \frac{R_{eq}}{R_1 + R_{eq}} v_s, \tag{3.23}$$



where

$$R_{eq} = \frac{R_2 R_L}{R_2 + R_L}. \tag{3.24}$$

Substituting Eq. 3.24 into Eq. 3.23 yields

$$v_o = \frac{R_2}{R_1[1 + (R_2/R_L)] + R_2} v_s. \tag{3.25}$$

Note that Eq. 3.25 reduces to Eq. 3.22 as $R_L \to \infty$, as it should. Equation 3.25 shows that, as long as $R_L \gg R_2$, the voltage ratio v_o/v_s is essentially undisturbed by the addition of the load on the divider.

Another characteristic of the voltage-divider circuit of interest is the sensitivity of the divider to the tolerances of the resistors. By *tolerance* we mean a range of possible values. The resistances of commercially available resistors always vary within some percentage of their stated value. Example 3.2 illustrates the effect of resistor tolerances in a voltage-divider circuit.

Example 3.2 Analyzing the Voltage-Divider Circuit

The resistors used in the voltage-divider circuit shown in Fig. 3.14 have a tolerance of ±10%. Find the maximum and minimum value of v_o.

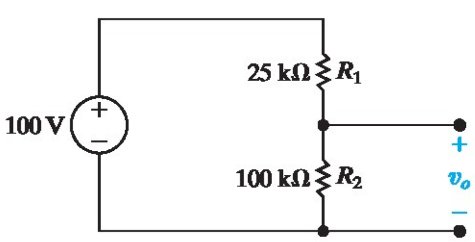


Figure 3.14 ▲ The circuit for Example 3.2.

Solution

From Eq. 3.22, the maximum value of v_o occurs when R_2 is 10% high and R_1 is 10% low, and the minimum value of v_o occurs when R_2 is 10% low and R_1 is 10% high. Therefore

$$v_o(\max) = \frac{(100)(110)}{110 + 22.5} = 83.02 \text{ V},$$

$$v_o(\min) = \frac{(100)(90)}{90 + 27.5} = 76.60 \text{ V}.$$

Thus, in making the decision to use 10% resistors in this voltage divider, we recognize that the no-load output voltage will lie between 76.60 and 83.02 V.

The Current-Divider Circuit

⇒ parallel

The **current-divider circuit** shown in Fig. 3.15 consists of two resistors connected in parallel across a current source. The current divider is designed to divide the current i_s between R_1 and R_2. We find the relationship between the current i_s and the current in each resistor (that is, i_1 and i_2) by directly applying Ohm's law and Kirchhoff's current law. The voltage across the parallel resistors is

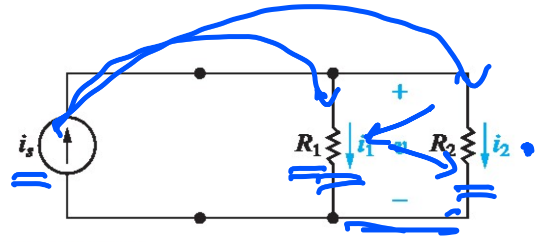


Figure 3.15 ▲ The current-divider circuit.

$$v = i_1R_1 = i_2R_2 = \frac{R_1R_2}{R_1 + R_2}i_s. \quad (3.26)$$

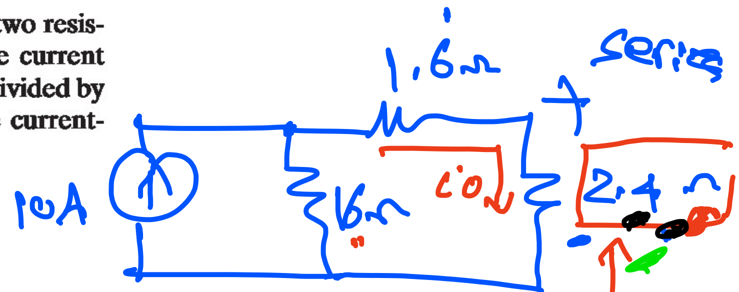
$$i_1 = \frac{R_2}{R_1+R_2} \times i_s$$

From Eq. 3.26,

$$\boxed{\begin{aligned} i_1 &= \frac{R_2}{R_1 + R_2}i_s, \quad (3.27) \\ i_2 &= \frac{R_1}{R_1 + R_2}i_s. \quad (3.28) \end{aligned}}$$

$$i_2 = \frac{R_1}{R_1+R_2} \times i_s$$

Equations 3.27 and 3.28 show that the current divides between two resistors in parallel such that the current in one resistor equals the current entering the parallel pair multiplied by the other resistance and divided by the sum of the resistors. Example 3.3 illustrates the use of the current-divider equation.



Example 3.3 Analyzing a Current-Divider Circuit

Find the power dissipated in the 6 Ω resistor shown in Fig. 3.16.

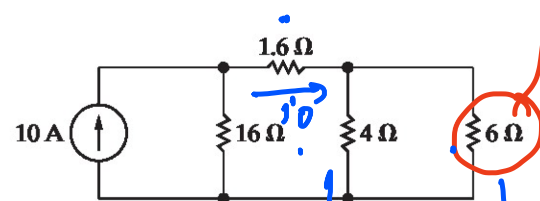


Figure 3.16 ▲ The circuit for Example 3.3.

parallel

$$\frac{1}{\frac{1}{6}+\frac{1}{4}} = 2.4$$

Solution

First, we must find the current in the resistor by simplifying the circuit with series-parallel reductions. Thus, the circuit shown in Fig. 3.16 reduces to the one shown in Fig. 3.17. We find the current i_o by using the formula for current division:

$$i_o = \frac{16}{16 + 4}(10) = 8 \text{ A}.$$

Note that i_o is the current in the 1.6 Ω resistor in Fig. 3.16. We now can further divide i_o between the 6 Ω and 4 Ω resistors. The current in the 6 Ω resistor is

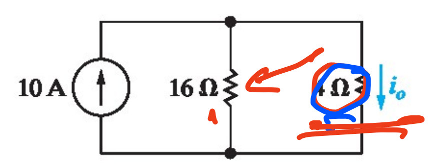
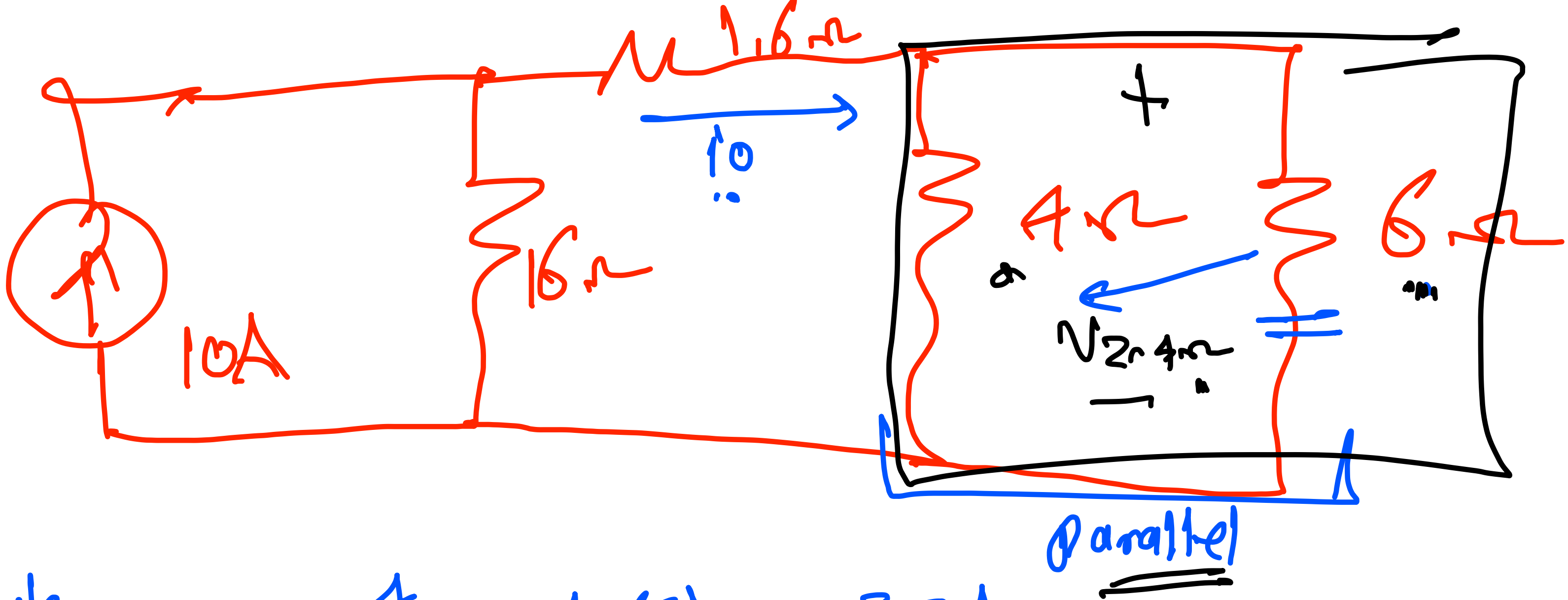


Figure 3.17 ▲ A simplification of the circuit shown in Fig. 3.16.

$$i_6 = \frac{4}{6 + 4}(8) = 3.2 \text{ A},$$

and the power dissipated in the 6 Ω resistor is $p = (3.2)^2(6) = 61.44$ W.



$$i_{6Ω} = \frac{4}{4+8} \times i_0 (8) = 3.2A$$

$$P = i^2 R = 3.2^2 \times 6 = 61.44W$$

$$V_{2.4} = i_0 \times R = 8 \times 2.4 = 19.2V$$

$$P = \frac{V^2}{R} = \frac{19.2^2}{6} = 61.44w$$

✓ASSESSMENT PROBLEMS

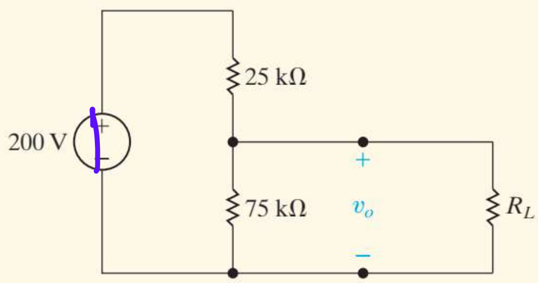
Objective 2—Know how to design simple voltage-divider and current-divider circuits

3.2 a) Find the no-load value of v_o in the circuit shown.

b) Find v_o when R_L is 150 kΩ.

c) How much power is dissipated in the 25 kΩ resistor if the load terminals are accidentally short-circuited?

d) What is the maximum power dissipated in the 75 kΩ resistor?



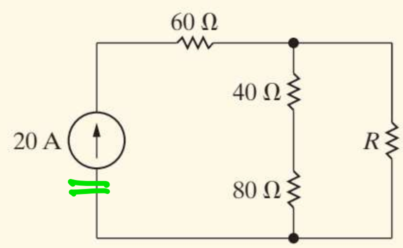
Answer: (a) 150 V;
(b) 133.33 V;
(c) 1.6 W;
(d) 0.3 W.

NOTE: Also try Chapter Problems 3.12, 3.14, and 3.16.

3.3 a) Find the value of R that will cause 4 A of current to flow through the 80 Ω resistor in the circuit shown.

b) How much power will the resistor R from part (a) need to dissipate?

c) How much power will the current source generate for the value of R from part (a)?



Answer: (a) 30 Ω;
(b) 7680 W;
(c) 33,600 W.

$\frac{R_1}{\Sigma R}$ $\frac{R_2}{\Sigma R}$

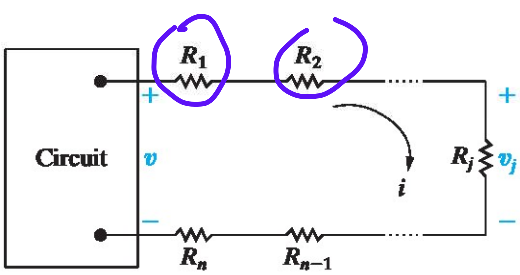


Figure 3.18 ▲ Circuit used to illustrate voltage division.

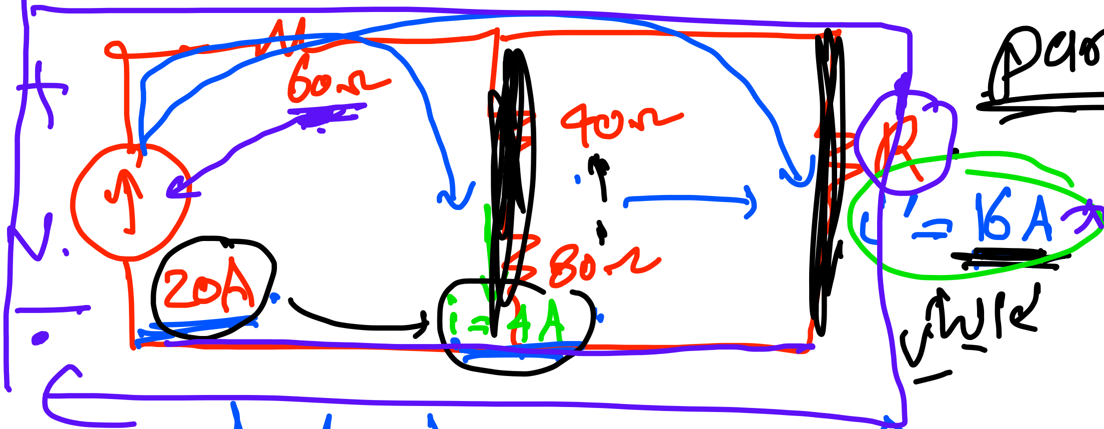
3.4 Voltage Division and Current Division

We can now generalize the results from analyzing the voltage divider circuit in Fig. 3.12 and the current-divider circuit in Fig. 3.15. The generalizations will yield two additional and very useful circuit analysis techniques known as **voltage division** and **current division**. Consider the circuit shown in Fig. 3.18.

The box on the left can contain a single voltage source or any other combination of basic circuit elements that results in the voltage v shown in the figure. To the right of the box are n resistors connected in series. We are interested in finding the voltage drop v_j across an arbitrary resistor R_j in terms of the voltage v. We start by using Ohm's law to calculate i, the current through all of the resistors in series, in terms of the current v and the n resistors:

$$i = \frac{v}{R_1 + R_2 + \cdots + R_n} = \frac{v}{R_{eq}}. \tag{3.29}$$

The equivalent resistance, R_{eq}, is the sum of the n resistor values because the resistors are in series, as shown in Eq. 3.6. We apply Ohm's



current divider

$$i = \frac{R}{R+40+80} * \frac{20}{1} = 4$$

$$\frac{20R}{R+120} = \frac{4}{1}$$

$$20R = 4R + 480$$
$$-4R$$

$$\frac{16R}{16} = \frac{480}{16}$$

$$\boxed{R = 30\Omega}$$

parallel ⇒

$$V = i'R$$

$$V = V$$

$$4*(\overset{120}{40+80}) = 16R$$

$$\frac{480}{16} = \frac{16R}{16}$$

$$\boxed{R = 30\Omega}$$

$$P = i'^2 R = 16^2 * 30$$

$$P = 7680\ W$$

Using KVL $\sum V = 0$

$$-V + V_{60\Omega} + V_R = 0 \Rightarrow V = V_{60\Omega} + V_R$$

$$V = 60*20 + 30*16 \Rightarrow V = 1200 + 480 = 1680V$$

$$P_{20A} = -iV = -20*1680 = (-)33600\ W$$

generate/deliv

law a second time to calculate the voltage drop v_j across the resistor R_j, using the current i calculated in Eq. 3.29:

Series

$$\boxed{v_j = iR_j = \frac{R_j}{R_{eq}}v.} \qquad (3.30)$$

◀ **Voltage-division equation**

Note that we used Eq. 3.29 to obtain the right-hand side of Eq. 3.30. Equation 3.30 is the voltage division equation. It says that the voltage drop v_j across a single resistor R_j from a collection of series-connected resistors is proportional to the total voltage drop v across the set of series-connected resistors. The constant of proportionality is the ratio of the single resistance to the equivalent resistance of the series connected set of resistors, or R_j/R_{eq}.

Now consider the circuit shown in Fig. 3.19. The box on the left can contain a single current source or any other combination of basic circuit elements that results in the current i shown in the figure. To the right of the box are n resistors connected in parallel. We are interested in finding the current i_j through an arbitrary resistor R_j in terms of the current i. We start by using Ohm's law to calculate v, the voltage drop across each of the resistors in parallel, in terms of the current i and the n resistors:

$$v = i(R_1 \| R_2 \| \ldots \| R_n) = iR_{eq}. \qquad (3.31)$$

The equivalent resistance of n resistors in parallel, R_{eq}, can be calculated using Eq. 3.12. We apply Ohm's law a second time to calculate the current i_j through the resistor R_j, using the voltage v calculated in Eq. 3.31:

Parllel

$$\boxed{i_j = \frac{v}{R_j} = \frac{R_{eq}}{R_j}i.} \qquad (3.32)$$

◀ **Current-division equation**

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots}$$

Note that we used Eq. 3.31 to obtain the right-hand side of Eq. 3.32. Equation 3.32 is the current division equation. It says that the current i through a single resistor R_j from a collection of parallel-connected resistors is proportional to the total current i supplied to the set of parallel-connected resistors. The constant of proportionality is the ratio of the equivalent resistance of the parallel-connected set of resistors to the single resistance, or R_{eq}/R_j. Note that the constant of proportionality in the current division equation is the inverse of the constant of proportionality in the voltage division equation!

Example 3.4 uses voltage division and current division to solve for voltages and currents in a circuit.

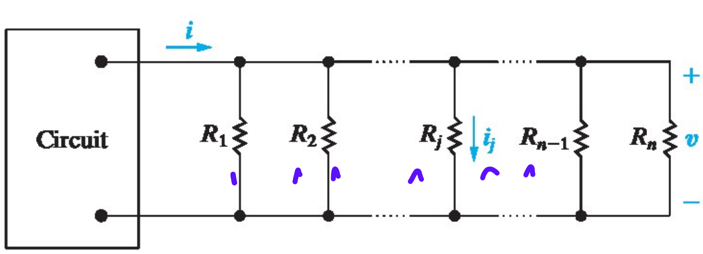
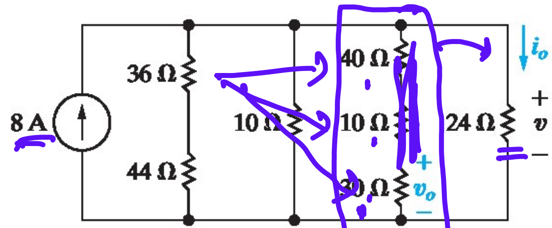


Figure 3.19 ▲ Circuit used to illustrate current division.

Example 3.4 **Using Voltage Division and Current Division to Solve a Circuit**

Parallel = 48V



series

Figure 3.20 ▲ The circuit for Example 3.4.

Use current division to find the current i_o and use voltage division to find the voltage v_o for the circuit in Fig. 3.20.

Solution

We can use Eq. 3.32 if we can find the equivalent resistance of the four parallel branches containing resistors. Symbolically,

$$R_{eq} = (36 + 44)\|10\|(40 + 10 + 30)\|24$$

$$= 80\|10\|80\|24 = \frac{1}{\frac{1}{80} + \frac{1}{10} + \frac{1}{80} + \frac{1}{24}} = 6\ \Omega.$$

Applying Eq. 3.32,

$$i_o = \frac{6}{24}(8\text{ A}) = 2\text{ A}.$$

We can use Ohm's law to find the voltage drop across the 24 Ω resistor:

$$v = (24)(2) = \boxed{48\text{ V.}}$$

This is also the voltage drop across the branch containing the 40 Ω, the 10 Ω, and the 30 Ω resistors in series. We can then use voltage division to determine the voltage drop v_o across the 30 Ω resistor given that we know the voltage drop across the series-connected resistors, using Eq. 3.30. To do this, we recognize that the equivalent resistance of the series-connected resistors is 40 + 10 + 30 = 80 Ω:

$$v_o = \frac{30}{80}(48\text{ V}) = 18\text{ V}.$$

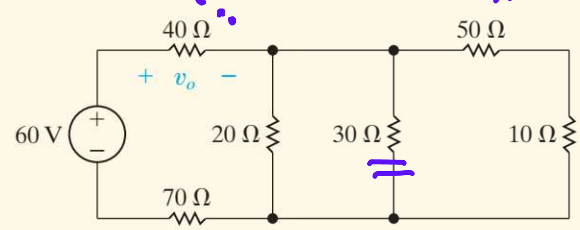
✓ASSESSMENT PROBLEM

Objective 3—Be able to use voltage and current division to solve simple circuits

3.4 a) Use voltage division to determine the voltage v_o across the 40 Ω resistor in the circuit shown.

b) Use v_o from part (a) to determine the current through the 40 Ω resistor, and use this current and current division to calculate the current in the 30 Ω resistor.

c) How much power is absorbed by the 50 Ω resistor?



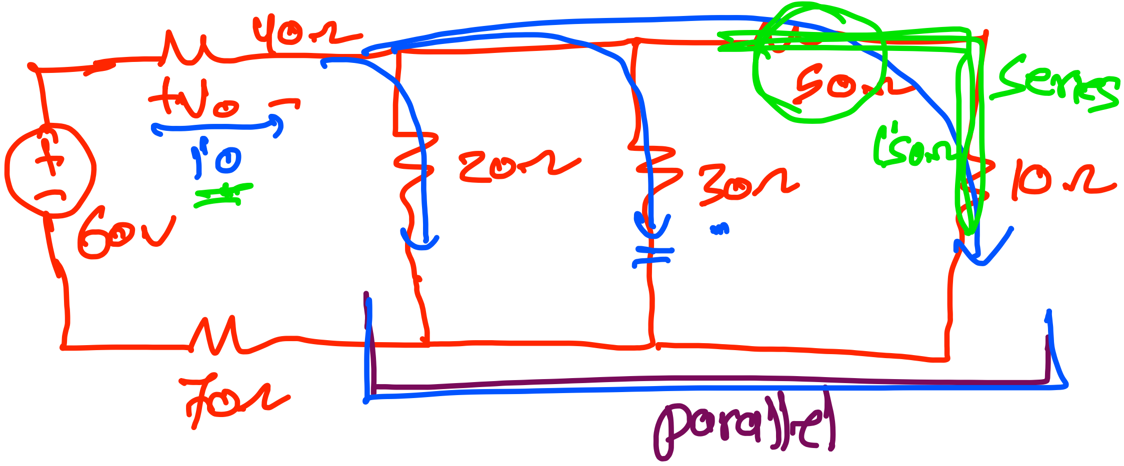
Answer: (a) 20 V;
(b) 166.67 mA;
(c) 347.22 mW.

NOTE: Also try Chapter Problems 3.25 and 3.26.

3.5 Measuring Voltage and Current

When working with actual circuits, you will often need to measure voltages and currents. We will spend some time discussing several measuring devices here and in the next section, because they are relatively simple to analyze and offer practical examples of the current- and voltage-divider configurations we have just studied.

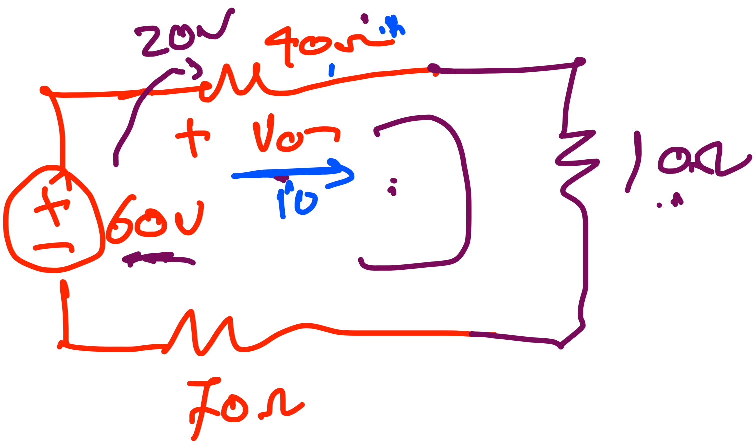
An **ammeter** is an instrument designed to measure current; it is placed in series with the circuit element whose current is being measured. A **voltmeter** is an instrument designed to measure voltage; it is placed in parallel with the element whose voltage is being measured. An ideal ammeter or voltmeter has no effect on the circuit variable it is designed to measure.



$P = i^2R$

$P = V^2/R$

$$R_p = \frac{1}{\frac{1}{20} + \frac{1}{30} + \frac{1}{50+10}} = 10\,\Omega$$



~ series ~ ⟹ voltage div

$$V_0 = \frac{40}{40+10+70} \times 60$$

$$V_0 = 20\,V$$

$$i = \frac{R_{eq}}{R_i} \times i_s$$

$$i_0 = \frac{V}{R} = \frac{20}{40} = 0.5\,A$$

$$i_{30\Omega} = \frac{10}{30} \times 0.5 = 166.67\,mA$$

$$i_{50\Omega} = \frac{10}{50+10} \times 0.5 = 83.33\,mA$$

$$P = i^2R = (83.33 \times 10^{-3})^2 \times 50 = 347.22\,mW$$

That is, an ideal ammeter has an equivalent resistance of 0 Ω and functions as a short circuit in series with the element whose current is being measured. An ideal voltmeter has an infinite equivalent resistance and thus functions as an open circuit in parallel with the element whose voltage is being measured. The configurations for an ammeter used to measure the current in R_1 and for a voltmeter used to measure the voltage in R_2 are depicted in Fig. 3.21. The ideal models for these meters in the same circuit are shown in Fig. 3.22.

There are two broad categories of meters used to measure continuous voltages and currents: digital meters and analog meters. **Digital meters** measure the continuous voltage or current signal at discrete points in time, called the sampling times. The signal is thus converted from an analog signal, which is continuous in time, to a digital signal, which exists only at discrete instants in time. A more detailed explanation of the workings of digital meters is beyond the scope of this text and course. However, you are likely to see and use digital meters in lab settings because they offer several advantages over analog meters. They introduce less resistance into the circuit to which they are connected, they are easier to connect, and the precision of the measurement is greater due to the nature of the readout mechanism.

Analog meters are based on the d'Arsonval meter movement which implements the readout mechanism. A d'Arsonval meter movement consists of a movable coil placed in the field of a permanent magnet. When current flows in the coil, it creates a torque on the coil, causing it to rotate and move a pointer across a calibrated scale. By design, the deflection of the pointer is directly proportional to the current in the movable coil. The coil is characterized by both a voltage rating and a current rating. For example, one commercially available meter movement is rated at 50 mV and 1 mA. This means that when the coil is carrying 1 mA, the voltage drop across the coil is 50 mV and the pointer is deflected to its full-scale position. A schematic illustration of a d'Arsonval meter movement is shown in Fig. 3.23.

An analog ammeter consists of a d'Arsonval movement in parallel with a resistor, as shown in Fig. 3.24. The purpose of the parallel resistor is to limit the amount of current in the movement's coil by shunting some of it through R_A. An analog voltmeter consists of a d'Arsonval movement in series with a resistor, as shown in Fig. 3.25. Here, the resistor is used to limit the voltage drop across the meter's coil. In both meters, the added resistor determines the full-scale reading of the meter movement.

From these descriptions we see that an actual meter is nonideal; both the added resistor and the meter movement introduce resistance in the circuit to which the meter is attached. In fact, any instrument used to make physical measurements extracts energy from the system while making measurements. The more energy extracted by the instruments, the more severely the measurement is disturbed. A real ammeter has an equivalent resistance that is not zero, and it thus effectively adds resistance to the circuit in series with the element whose current the ammeter is reading. A real voltmeter has an equivalent resistance that is not infinite, so it effectively adds resistance to the circuit in parallel with the element whose voltage is being read.

How much these meters disturb the circuit being measured depends on the effective resistance of the meters compared with the resistance in the circuit. For example, using the rule of 1/10th, the effective resistance of an ammeter should be no more than 1/10th of the value of the smallest resistance in the circuit to be sure that the current being measured is nearly the same with or without the ammeter. But in an analog meter, the value of resistance is determined by the desired full-scale reading we wish to make, and it cannot be arbitrarily selected. The following examples illustrate the calculations involved in determining the resistance needed in an analog ammeter or voltmeter. The examples also consider the resulting effective resistance of the meter when it is inserted in a circuit.

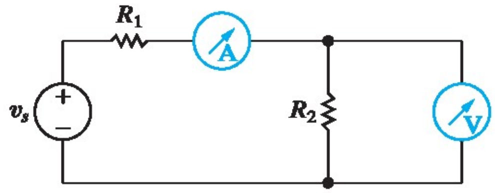


Figure 3.21 ▲ An ammeter connected to measure the current in R_1, and a voltmeter connected to measure the voltage across R_2.

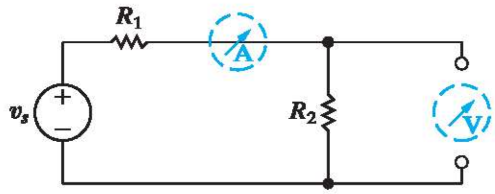


Figure 3.22 ▲ A short-circuit model for the ideal ammeter, and an open-circuit model for the ideal voltmeter.

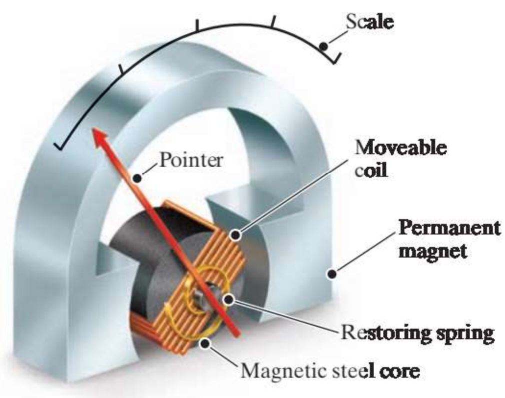


Figure 3.23 ▲ A schematic diagram of a d'Arsonval meter movement.

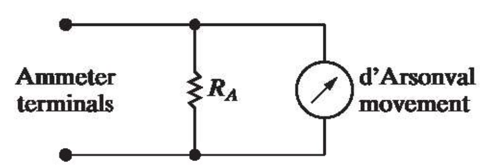


Figure 3.24 ▲ A dc ammeter circuit.

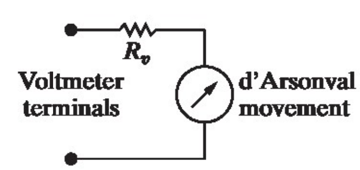


Figure 3.25 ▲ A dc voltmeter circuit.

Example 3.5 Using a d'Arsonval Ammeter

a) A 50 mV, 1 mA d'Arsonval movement is to be used in an ammeter with a full-scale reading of 10 mA. Determine R_A.

b) Repeat (a) for a full-scale reading of 1 A.

c) How much resistance is added to the circuit when the 10 mA ammeter is inserted to measure current?

d) Repeat (c) for the 1 A ammeter.

Solution

a) From the statement of the problem, we know that when the current at the terminals of the ammeter is 10 mA, 1 mA is flowing through the meter coil, which means that 9 mA must be diverted through R_A. We also know that when the movement carries 1 mA, the drop across its terminals is 50 mV. Ohm's law requires that

$$9 \times 10^{-3} R_A = 50 \times 10^{-3},$$

or

$$R_A = 50/9 = 5.555\ \Omega.$$

b) When the full-scale deflection of the ammeter is 1 A, R_A must carry 999 mA when the movement carries 1 mA. In this case, then,

$$999 \times 10^{-3} R_A = 50 \times 10^{-3},$$

or

$$R_A = 50/999 \approx 50.05\ \text{m}\Omega.$$

c) Let R_m represent the equivalent resistance of the ammeter. For the 10 mA ammeter,

$$R_m = \frac{50\ \text{mV}}{10\ \text{mA}} = 5\ \Omega,$$

or, alternatively,

$$R_m = \frac{(50)(50/9)}{50 + (50/9)} = 5\ \Omega.$$

d) For the 1 A ammeter

$$R_m = \frac{50\ \text{mV}}{1\ \text{A}} = 0.050\ \Omega,$$

or, alternatively,

$$R_m = \frac{(50)(50/999)}{50 + (50/999)} = 0.050\ \Omega.$$

Example 3.6 Using a d'Arsonval Voltmeter

a) A 50 mV, 1 mA d'Arsonval movement is to be used in a voltmeter in which the full-scale reading is 150 V. Determine R_v.

b) Repeat (a) for a full-scale reading of 5 V.

c) How much resistance does the 150 V meter insert into the circuit?

d) Repeat (c) for the 5 V meter.

Solution

a) Full-scale deflection requires 50 mV across the meter movement, and the movement has a resistance of 50 Ω. Therefore we apply Eq. 3.22 with $R_1 = R_v$, $R_2 = 50$, $v_s = 150$, and $v_2 = 50$ mV:

$$50 \times 10^{-3} = \frac{50}{R_v + 50}(150).$$

Solving for R_v gives

$$R_v = 149{,}950\ \Omega.$$

b) For a full-scale reading of 5 V,

$$50 \times 10^{-3} = \frac{50}{R_v + 50}(5),$$

or

$$R_v = 4950\ \Omega.$$

c) If we let R_m represent the equivalent resistance of the meter,

$$R_m = \frac{150\ \text{V}}{10^{-3}\ \text{A}} = 150{,}000\ \Omega,$$

or, alternatively,

$$R_m = 149{,}950 + 50 = 150{,}000\ \Omega.$$

d) Then,

$$R_m = \frac{5\ \text{V}}{10^{-3}\ \text{A}} = 5000\ \Omega,$$

or, alternatively,

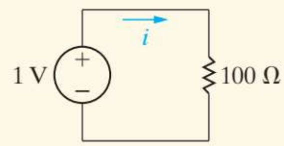
$$R_m = 4950 + 50 = 5000\ \Omega.$$

✓ASSESSMENT PROBLEMS

Objective 4—Be able to determine the reading of ammeters and voltmeters

3.5 a) Find the current in the circuit shown.

b) If the ammeter in Example 3.5(a) is used to measure the current, what will it read?

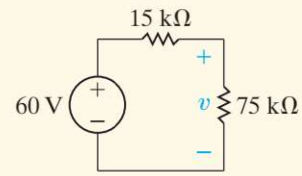


Answer: (a) 10 mA;

(b) 9.524 mA.

3.6 a) Find the voltage v across the 75 kΩ resistor in the circuit shown.

b) If the 150 V voltmeter of Example 3.6(a) is used to measure the voltage, what will be the reading?



Answer: (a) 50 V;

(b) 46.15 V.

NOTE: Also try Chapter Problems 3.34 and 3.37.

3.6 Measuring Resistance—The Wheatstone Bridge

Many different circuit configurations are used to measure resistance. Here we will focus on just one, the Wheatstone bridge. The Wheatstone bridge circuit is used to precisely measure resistances of medium values, that is, in the range of 1 Ω to 1 MΩ. In commercial models of the Wheatstone bridge, accuracies on the order of ±0.1% are possible. The bridge circuit consists of four resistors, a dc voltage source, and a detector. The resistance of one of the four resistors can be varied, which is indicated in Fig. 3.26 by the arrow through R_3. The dc voltage source is usually a battery, which is indicated by the battery symbol for the voltage source v in Fig. 3.26. The detector is generally a d'Arsonval movement in the microamp range and is called a galvanometer. Figure 3.26 shows the circuit arrangement of the resistances, battery, and detector where R_1, R_2, and R_3 are known resistors and R_x is the unknown resistor.

To find the value of R_x, we adjust the variable resistor R_3 until there is no current in the galvanometer. We then calculate the unknown resistor from the simple expression

$$R_x = \frac{R_2}{R_1}R_3. \tag{3.33}$$

The derivation of Eq. 3.33 follows directly from the application of Kirchhoff's laws to the bridge circuit. We redraw the bridge circuit as Fig. 3.27 to show the currents appropriate to the derivation of Eq. 3.33. When i_g is zero, that is, when the bridge is balanced, Kirchhoff's current law requires that

$$i_1 = i_3, \tag{3.34}$$

$$i_2 = i_x. \tag{3.35}$$

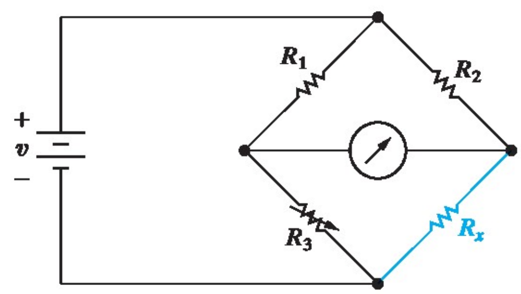


Figure 3.26 ▲ The Wheatstone bridge circuit.

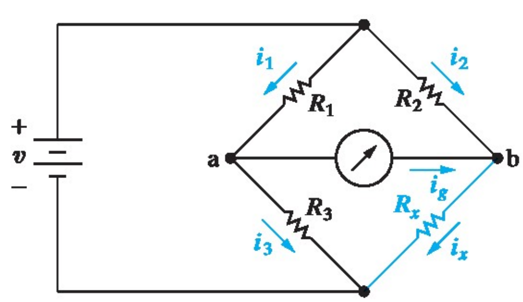


Figure 3.27 ▲ A balanced Wheatstone bridge ($i_g = 0$).

Now, because i_g is zero, there is no voltage drop across the detector, and therefore points a and b are at the same potential. Thus when the bridge is balanced, Kirchhoff's voltage law requires that

$$i_3R_3 = i_xR_x, \tag{3.36}$$
$$i_1R_1 = i_2R_2. \tag{3.37}$$

Combining Eqs. 3.34 and 3.35 with Eq. 3.36 gives

$$i_1R_3 = i_2R_x. \tag{3.38}$$

We obtain Eq. 3.33 by first dividing Eq. 3.38 by Eq. 3.37 and then solving the resulting expression for R_x:

$$\frac{R_3}{R_1} = \frac{R_x}{R_2}, \tag{3.39}$$

from which

$$R_x = \frac{R_2}{R_1}R_3. \tag{3.40}$$

Now that we have verified the validity of Eq. 3.33, several comments about the result are in order. First, note that if the ratio R_2/R_1 is unity, the unknown resistor R_x equals R_3. In this case, the bridge resistor R_3 must vary over a range that includes the value R_x. For example, if the unknown resistance were 1000 Ω and R_3 could be varied from 0 to 100 Ω, the bridge could never be balanced. Thus to cover a wide range of unknown resistors, we must be able to vary the ratio R_2/R_1. In a commercial Wheatstone bridge, R_1 and R_2 consist of decimal values of resistances that can be switched into the bridge circuit. Normally, the decimal values are 1, 10, 100, and 1000 Ω so that the ratio R_2/R_1 can be varied from 0.001 to 1000 in decimal steps. The variable resistor R_3 is usually adjustable in integral values of resistance from 1 to 11,000 Ω.

Although Eq. 3.33 implies that R_x can vary from zero to infinity, the practical range of R_x is approximately 1 Ω to 1 MΩ. Lower resistances are difficult to measure on a standard Wheatstone bridge because of thermoelectric voltages generated at the junctions of dissimilar metals and because of thermal heating effects—that is, i^2R effects. Higher resistances are difficult to measure accurately because of leakage currents. In other words, if R_x is large, the current leakage in the electrical insulation may be comparable to the current in the branches of the bridge circuit.

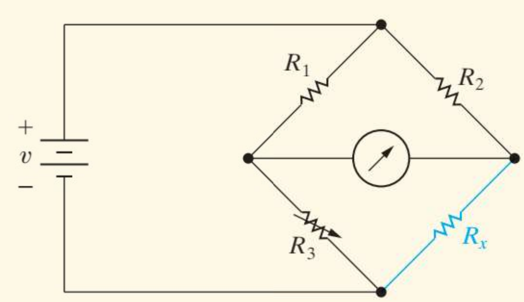
✓ ASSESSMENT PROBLEM

Objective 5—Understand how a Wheatstone bridge is used to measure resistance

3.7 The bridge circuit shown is balanced when $R_1 = 100\ \Omega$, $R_2 = 1000\ \Omega$, and $R_3 = 150\ \Omega$. The bridge is energized from a 5 V dc source.

a) What is the value of R_x?

b) Suppose each bridge resistor is capable of dissipating 250 mW. Can the bridge be left in the balanced state without exceeding the power-dissipating capacity of the resistors, thereby damaging the bridge?



Answer: (a) 1500 Ω;
(b) yes.

NOTE: Also try Chapter Problem 3.51.

3.7 Delta-to-Wye (Pi-to-Tee) Equivalent Circuits

The bridge configuration in Fig. 3.26 introduces an interconnection of resistances that warrants further discussion. If we replace the galvanometer with its equivalent resistance R_m, we can draw the circuit shown in Fig. 3.28. We cannot reduce the interconnected resistors of this circuit to a single equivalent resistance across the terminals of the battery if restricted to the simple series or parallel equivalent circuits introduced earlier in this chapter. The interconnected resistors can be reduced to a single equivalent resistor by means of a delta-to-wye (Δ-to-Y) or pi-to-tee (π-to-T) equivalent circuit.[1]

The resistors R_1, R_2, and R_m (or R_3, R_m and R_x) in the circuit shown in Fig. 3.28 are referred to as a **delta (Δ) interconnection** because the interconnection looks like the Greek letter Δ. It also is referred to as a **pi interconnection** because the Δ can be shaped into a π without disturbing the electrical equivalence of the two configurations. The electrical equivalence between the Δ and π interconnections is apparent in Fig. 3.29.

The resistors R_1, R_m, and R_3 (or R_2, R_m and R_x) in the circuit shown in Fig. 3.28 are referred to as a **wye (Y) interconnection** because the interconnection can be shaped to look like the letter Y. It is easier to see the Y shape when the interconnection is drawn as in Fig. 3.30. The Y configuration also is referred to as a **tee (T) interconnection** because the Y structure can be shaped into a T structure without disturbing the electrical equivalence of the two structures. The electrical equivalence of the Y and the T configurations is apparent from Fig. 3.30.

Figure 3.31 illustrates the Δ-to-Y (or π-to-T) equivalent circuit transformation. Note that we cannot transform the Δ interconnection into the Y interconnection simply by changing the shape of the interconnections. Saying the Δ-connected circuit is equivalent to the Y-connected circuit means that the Δ configuration can be replaced with a Y configuration to make the terminal behavior of the two configurations identical. Thus if each circuit is placed in a black box, we can't tell by external measurements whether the box contains a set of Δ-connected resistors or a set of Y-connected resistors. This condition is true only if the resistance between corresponding terminal pairs is the same for each box. For example, the resistance between terminals a and b must be the same whether we use the Δ-connected set or the Y-connected set. For each pair of terminals in the Δ-connected circuit, the equivalent resistance can be computed using series and parallel simplifications to yield

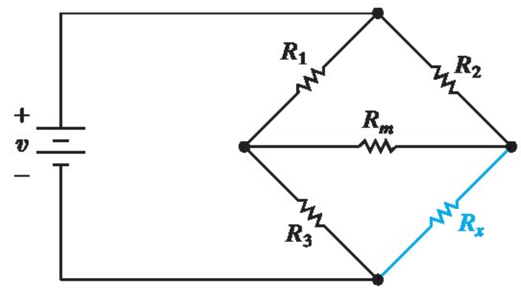


Figure 3.28 ▲ A resistive network generated by a Wheatstone bridge circuit.

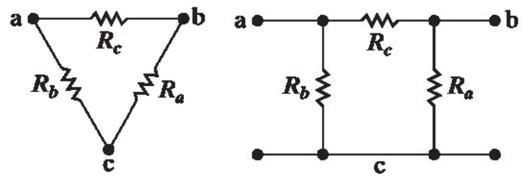


Figure 3.29 ▲ A Δ configuration viewed as a π configuration.

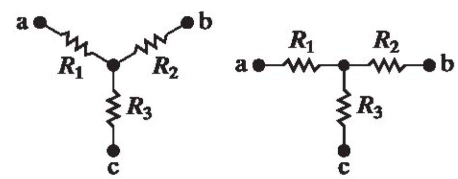


Figure 3.30 ▲ A Y structure viewed as a T structure.

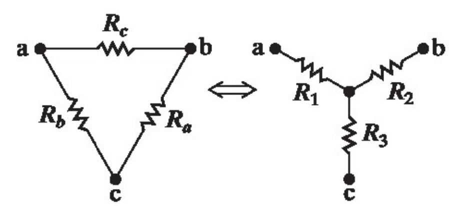
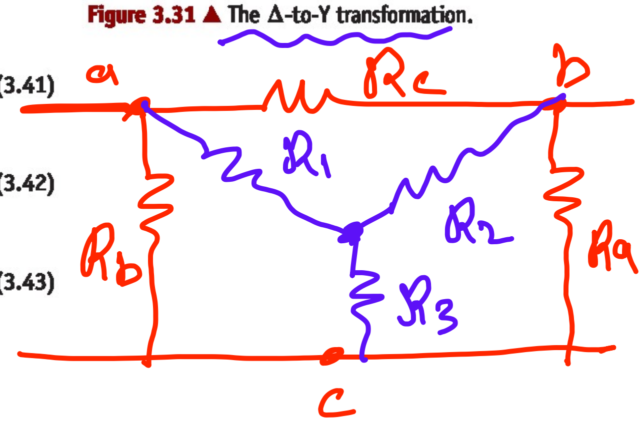


Figure 3.31 ▲ The Δ-to-Y transformation.

$$R_{ab} = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} = R_1 + R_2, \qquad (3.41)$$

$$R_{bc} = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} = R_2 + R_3, \qquad (3.42)$$

$$R_{ca} = \frac{R_b(R_c + R_a)}{R_a + R_b + R_c} = R_1 + R_3. \qquad (3.43)$$



[1] Δ and Y structures are present in a variety of useful circuits, not just resistive networks. Hence the Δ-to-Y transformation is a helpful tool in circuit analysis.

$\Delta \rightarrow Y$

$R =$ $\dfrac{\text{ضرب "المجاورين"}}{\text{مجموع}}$

Straightforward algebraic manipulation of Eqs. 3.41–3.43 gives values for the Y-connected resistors in terms of the Δ-connected resistors required for the Δ-to-Y equivalent circuit:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}, \tag{3.44}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}, \tag{3.45}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}. \tag{3.46}$$

Reversing the Δ-to-Y transformation also is possible. That is, we can start with the Y structure and replace it with an equivalent Δ structure. The expressions for the three Δ-connected resistors as functions of the three Y-connected resistors are

$Y \rightarrow \Delta$

$R =$ $\dfrac{\text{ضرب ثنائيات}}{\text{المقاومة البعيدة}}$

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}, \tag{3.47}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}, \tag{3.48}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}. \tag{3.49}$$

Example 3.7 illustrates the use of a Δ-to-Y transformation to simplify the analysis of a circuit.

Example 3.7 Applying a Delta-to-Wye Transform

Find the current and power supplied by the 40 V source in the circuit shown in Fig. 3.32.

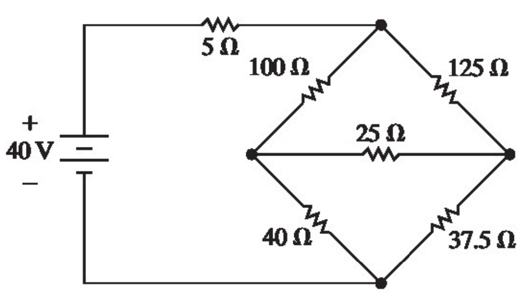


Figure 3.32 ▲ The circuit for Example 3.7.

Solution

We are interested only in the current and power drain on the 40 V source, so the problem has been solved once we obtain the equivalent resistance across the terminals of the source. We can find this equivalent resistance easily after replacing either the upper Δ (100, 125, 25 Ω) or the lower Δ (40, 25, 37.5 Ω) with its equivalent Y. We choose to replace the upper Δ. We then compute the three Y resistances, defined in Fig. 3.33, from Eqs. 3.44 to 3.46. Thus,

$$R_1 = \frac{100 \times 125}{250} = 50\ \Omega,$$

$$R_2 = \frac{125 \times 25}{250} = 12.5\ \Omega,$$

$$R_3 = \frac{100 \times 25}{250} = 10\ \Omega.$$

Substituting the Y-resistors into the circuit shown in Fig. 3.32 produces the circuit shown in Fig. 3.34. From Fig. 3.34, we can easily calculate the resistance across the terminals of the 40 V source by series-parallel simplifications:

$$R_{eq} = 55 + \frac{(50)(50)}{100} = 80\ \Omega.$$

The final step is to note that the circuit reduces to an 80 Ω resistor across a 40 V source, as shown in Fig. 3.35, from which it is apparent that the 40 V source delivers 0.5 A and 20 W to the circuit.

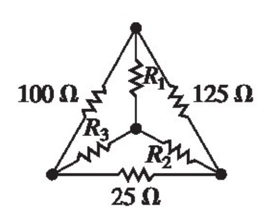


Figure 3.33 ▲ The equivalent Y resistors.

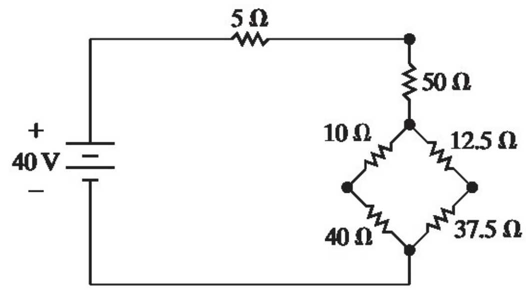


Figure 3.34 ▲ A transformed version of the circuit shown in Fig. 3.32.

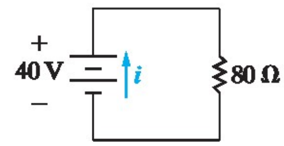
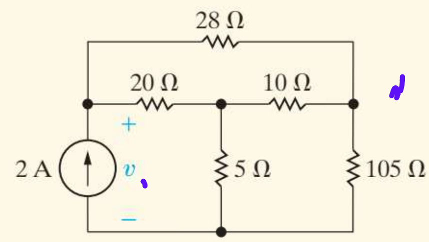


Figure 3.35 ▲ The final step in the simplification of the circuit shown in Fig. 3.32.

✓ASSESSMENT PROBLEM

Objective 6—Know when and how to use delta-to-wye equivalent circuits

3.8 Use a Y-to-Δ transformation to find the voltage v in the circuit shown.



Answer: 35 V.

NOTE: Also try Chapter Problems 3.60, 3.62, and 3.63.

Practical Perspective

Resistive Touch Screens

Begin by analyzing the resistive grid in the x-direction. We model the resistance of the grid in the x-direction with the resistance R_x, as shown in Fig. 3.34. The x-location where the screen is touched is indicated by the arrow. The resulting voltage drop across the resistance αR_x is V_x. Touching the screen effectively divides the total resistance, R_x, into two separate resistances αR_x and $(1 - \alpha)R_x$.

From the figure you can see that when the touch is on the far right side of the screen, $\alpha = 0$, and $V_x = 0$. Similarly, when the touch is on the far left side of the screen, $\alpha = 1$, and $V_x = V_s$. If the touch is in between the two edges of the screen, the value of α is between 0 and 1 and the two parts of the resistance R_x form a voltage divider. We can calculate the voltage V_x using the equation for voltage division:

$$V_x = \frac{\alpha R_x}{\alpha R_x + (1 - \alpha)R_x} V_s = \frac{\alpha R_x}{R_x} V_s = \alpha V_s.$$

We can find the value of α, which represents the location of the touch point with respect the far right side of the screen, by dividing the voltage