

EE 210 ELECTRIC CIRCUITS II

Chapter 10

Sinusoidal Steady-State Power Calculations

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Recap: Sinusoidal Sources



- A sinusoidal voltage (current) source produces a voltage (current) that varies sinusoidally in time.

$v(t) = V_m \cos(\omega t + \phi)$

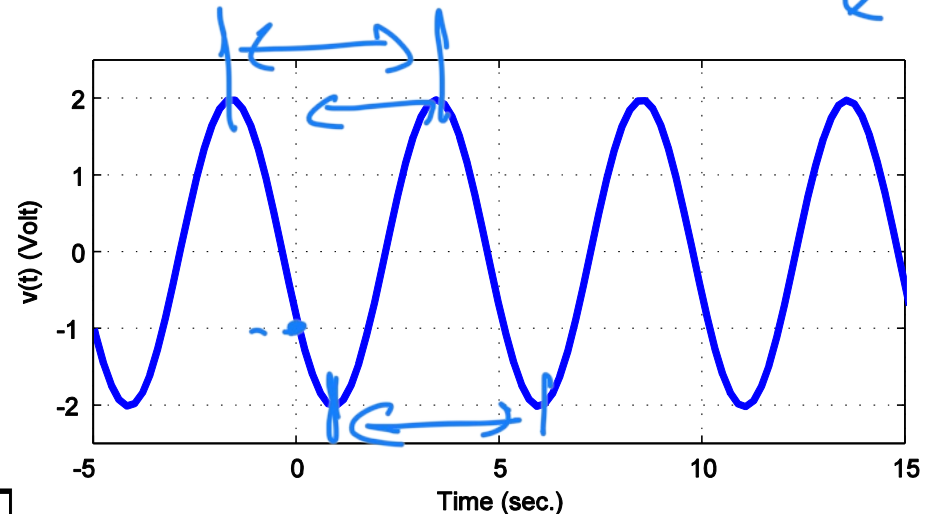
Amplitude

Frequency (radians/sec)

Phase

$\omega = 2\pi f$

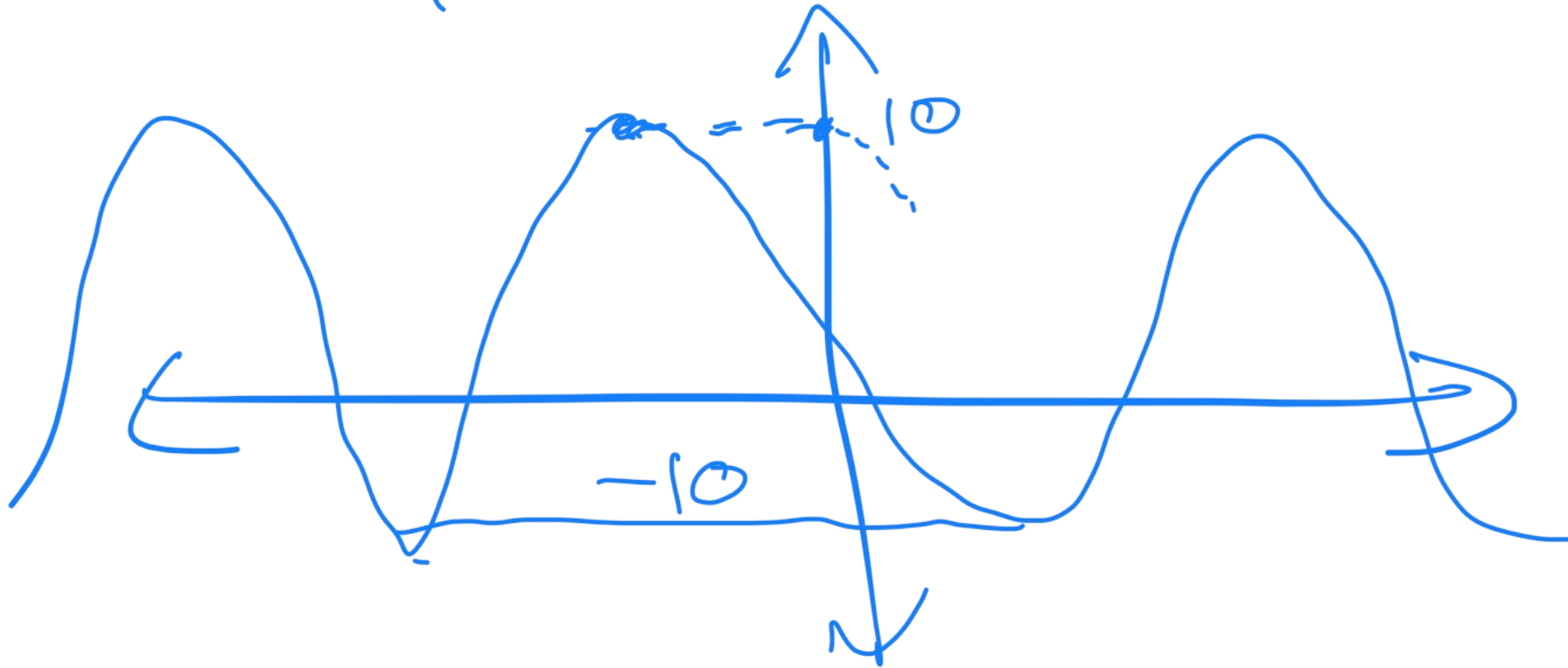
A diagram illustrating the components of the sinusoidal voltage equation $v(t) = V_m \cos(\omega t + \phi)$. The equation is shown in a light blue box. Three arrows point from the terms V_m , ω , and ϕ to three separate boxes below: 'Amplitude', 'Frequency (radians/sec)', and 'Phase'. A handwritten blue note ' $\omega = 2\pi f$ ' is written below the frequency box. There are also some handwritten blue scribbles to the left of the diagram.



f : freq $= \frac{1}{T}$

T : period

$$v(t) = \underbrace{V_m}_{10} \cos(\omega t + \underbrace{\theta}_{=})$$



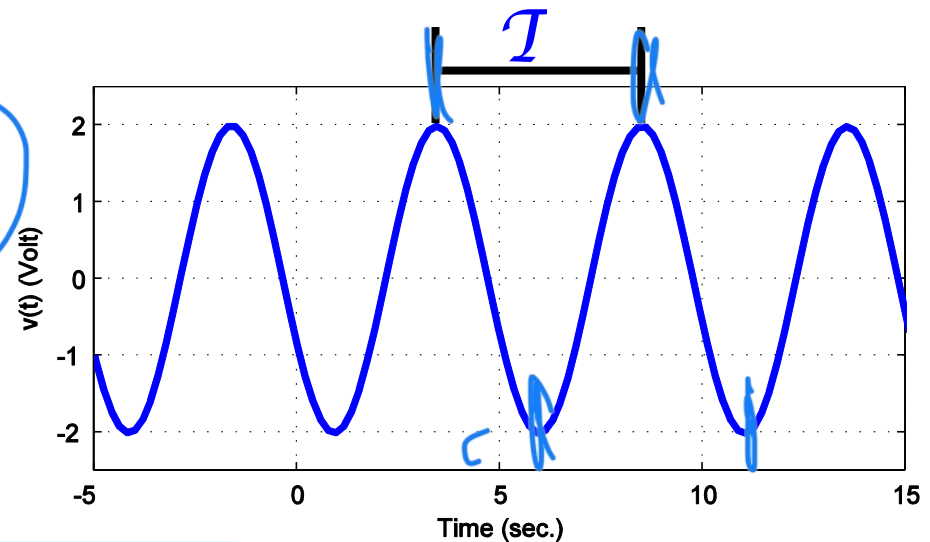
Recap: Frequency and Period

- A function that repeats itself is called **periodic**. The amount of time required to complete one full cycle is called the **period**.
- The number of cycles per second is called the **frequency**.

Period (sec.) → $T = \frac{1}{f}$

Frequency (Hertz) → f

Frequency (rad./sec.) → $\omega = 2\pi f$



Recap: Root-Mean-Square (RMS) Value

- The rms value of a periodic function is defined as the square-root of the mean value of the squared function.

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt}$$

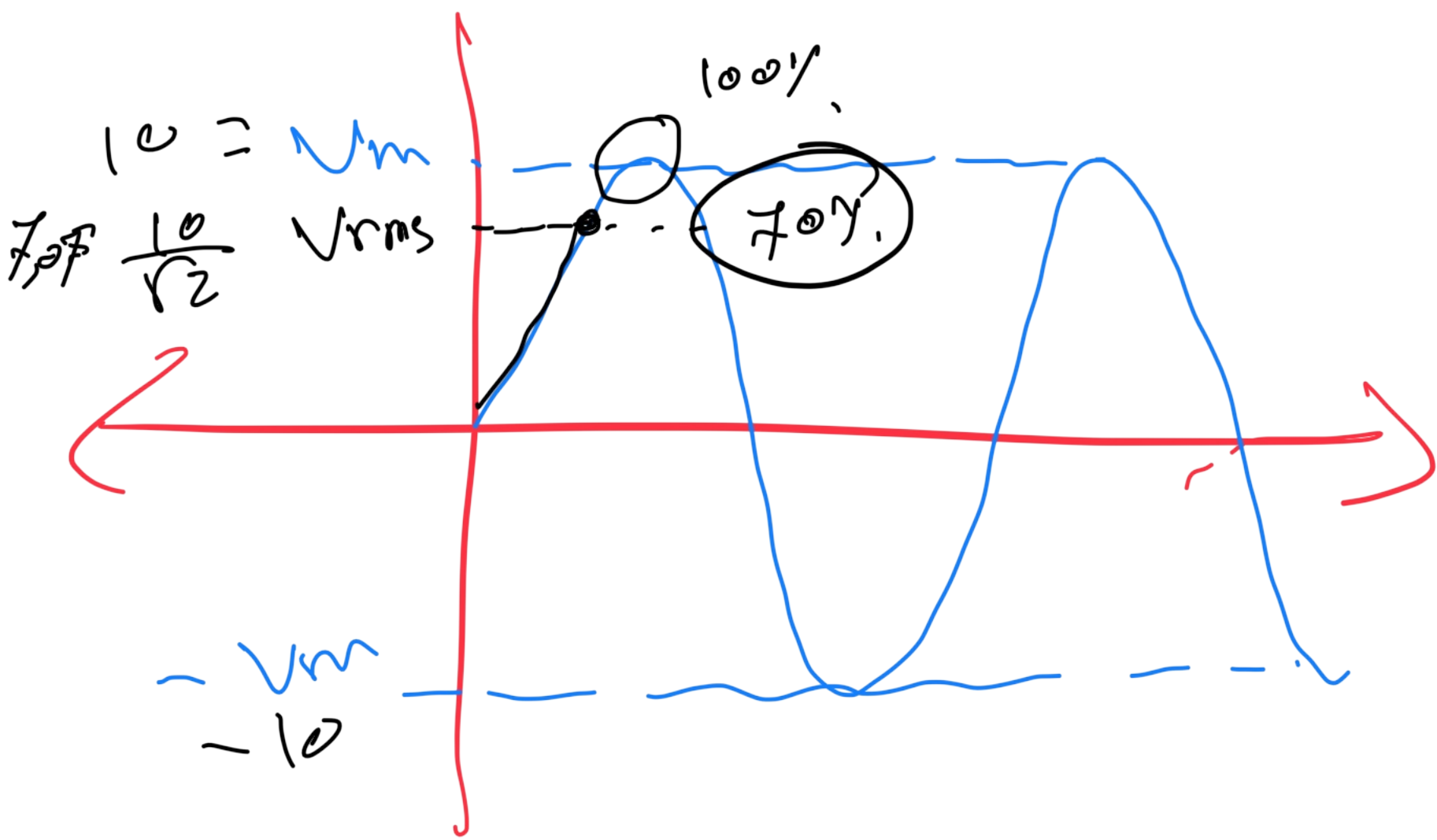


- For sinusoidal signal sources, the RMS value can be shown to be

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

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$$V_{rms} = \frac{V_m}{\sqrt{2}} \quad \leftarrow$$

$$V_m = \sqrt{2} V_{rms}$$

$$V_{rms} < \underline{\underline{V_m}}$$

..

Recap: Phasor



complex power

- Phasor is a complex number that carries the amplitude and phase information of a sinusoidal function.

time domain

$$\mathcal{V} = \mathcal{P} \left\{ \mathcal{V}_m \cos(\omega t + \phi) \right\} = \mathcal{V}_m e^{j\phi}$$

- The sinusoidal function is obtained from the inverse phasor transform as

$$v(t) = \mathcal{P}^{-1} \{ \mathcal{V} \} = \Re \{ \mathcal{V} e^{j\omega t} \}$$

Recap: Polar and Rectangular Forms

- Euler Identity shows us how to go from polar form to the rectangular form:

$$e^{j\phi} = \cos(\phi) + j \sin(\phi)$$

$$\hat{r}, \phi' = \sqrt{-1}$$

- One can then go from rectangular to polar form as

$$V = x + jy = V_m e^{j\phi}$$

with

$$V_m = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

amplitude

Recap: Addition and Multiplication

- Multiplication is more convenient in polar form:

$$\underbrace{V_{m1} e^{j\phi_1}}_{V_1} \cdot \underbrace{V_{m2} e^{j\phi_2}}_{V_2} = \underbrace{V_{m1} V_{m2}}_V e^{j(\phi_1 + \phi_2)}_V$$

- Taking a power is simple in polar form:

$$\left(V_m e^{j\phi} \right)^k = V_m^k e^{jk\phi}$$

- Addition is more convenient in rectangular form:

$$\underbrace{V_{m1} e^{j\phi_1}}_{V_1} + \underbrace{V_{m2} e^{j\phi_2}}_{V_2} = V_{m1} \cos(\phi_1) + V_{m2} \cos(\phi_2) + j(V_{m1} \sin(\phi_1) + V_{m2} \sin(\phi_2))$$

Chapter 10 Sinusoidal Steady-State Power Calculations

In Chapter 9, we calculated the steady state voltages and currents in electric circuits driven by sinusoidal sources.

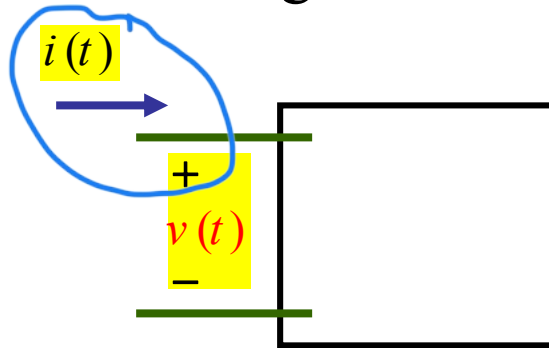
We used phasor method to find the steady state voltages and currents.

In this Chapter, we consider power in such circuits.

The techniques we develop are useful for analyzing many of the electric devices we encounter daily, because sinusoidal sources are predominant means of providing electric power in our homes, school, and businesses.

10.1 Instantaneous Power

Consider the following circuit represented by a black box.



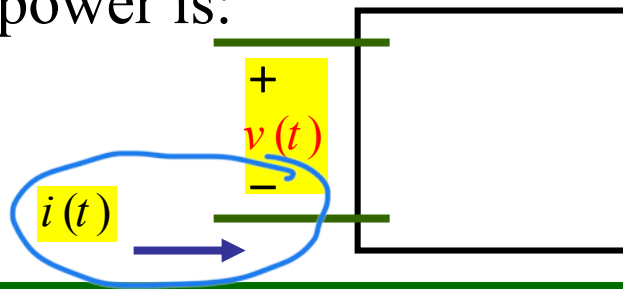
$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$v(t) = V_m \cos(\omega t + \theta_v)$$

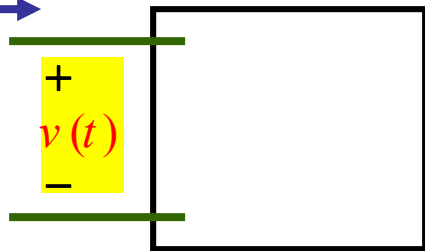
The instantaneous power assuming **passive sign convention**
(Current in the direction of **voltage drop** + □ -)

$$p(t) = v(t)i(t) \quad (\text{Watts}) \quad (+)$$

If the current is in the direction of **voltage rise** (- □ +) the instantaneous power is:



$$p(t) = -v(t)i(t)$$

$i(t)$ 

$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$v(t) = V_m \cos(\omega t + \theta_v)$$



$$i(t) = I_m \cos(\omega t)$$

$$v(t) = V_m \cos(\omega t + \theta_v - \theta_i)$$

$$p(t) = v(t)i(t) = \{V_m \cos(\omega t + \theta_v - \theta_i)\} \{I_m \cos(\omega t)\}$$

$$= V_m I_m \cos(\omega t + \theta_v - \theta_i) \cos(\omega t)$$

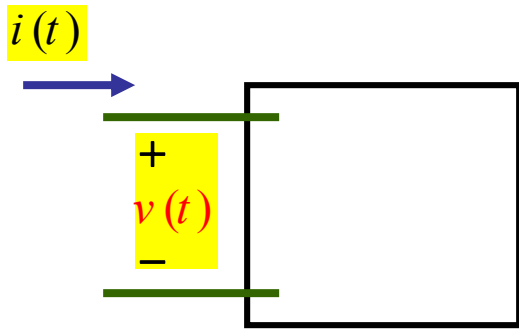
Since

$$\cos \alpha \cos \beta = \frac{1}{2} \cos(\alpha - \beta) + \frac{1}{2} \cos(\alpha + \beta)$$

Therefore

$$p(t) = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(2\omega t + \theta_v - \theta_i)$$

$$p = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(2\omega t + \theta_v - \theta_i)$$



$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$v(t) = V_m \cos(\omega t + \theta_v)$$



$$i(t) = I_m \cos(\omega t)$$

$$v(t) = V_m \cos(\omega t + \theta_v - \theta_i)$$

Therefore
$$p(t) = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(2\omega t + \theta_v - \theta_i)$$

Since
$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

Therefore
$$\cos(2\omega t + \theta_v - \theta_i) = \cos(\theta_v - \theta_i) \cos(2\omega t) - \sin(\theta_v - \theta_i) \sin(2\omega t)$$



$$p(t) = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) \cos(2\omega t) - \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) \sin(2\omega t)$$

Average

power

$i(t)$

+

$v(t)$

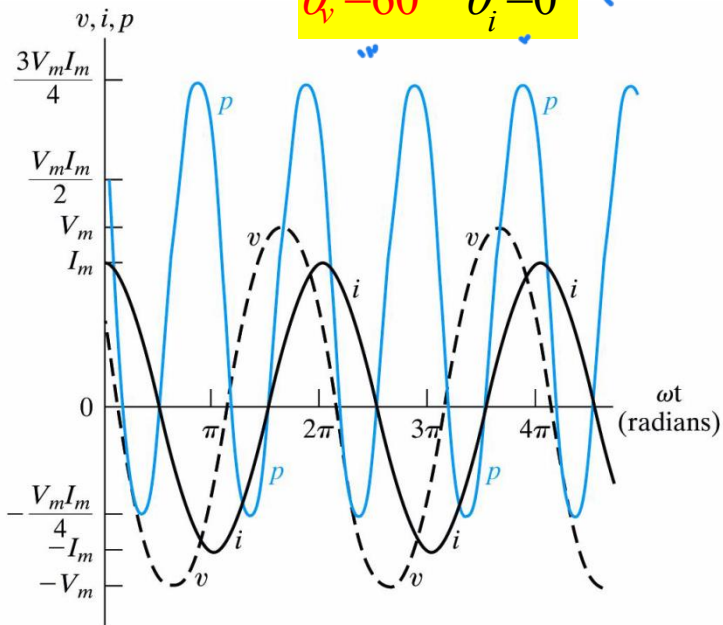
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$$i(t) = I_m \cos(\omega t)$$

$$v(t) = V_m \cos(\omega t + \theta_v - \theta_i)$$

$$p(t) = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) \cos(2\omega t) - \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) \sin(2\omega t)$$

$$\theta_v = 60^\circ \quad \theta_i = 0^\circ$$



You can see that the frequency of the Instantaneous power is twice the frequency of the voltage or current

10.2 Average and Reactive Power

Recall the Instantaneous power $p(t)$

$$p(t) = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) \cos(2\omega t) - \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) \sin(2\omega t)$$

$$p(t) = \underline{P} + \underline{P} \cos(2\omega t) - Q \sin(2\omega t)$$

where

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

$$Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i)$$

Average Power (**Real Power**)

Reactive Power

10.2 Average and Reactive Power

Average Power P is sometimes called **Real power** because it describes the power in a circuit that is transformed from electric to non electric (**Example Heat**).

It is easy to see why P is called Average Power because

$$\frac{1}{T} \int_{t_0}^{t_0+T} p(t) dt = \frac{1}{T} \int_{t_0}^{t_0+T} \{ \underline{P} + \underline{P \cos(2\omega t)} - \underline{Q \sin(2\omega t)} \} dt = P$$

Power for purely resistive Circuits

$$p(t) = P + P \cos(2\omega t) - Q \sin(2\omega t)$$

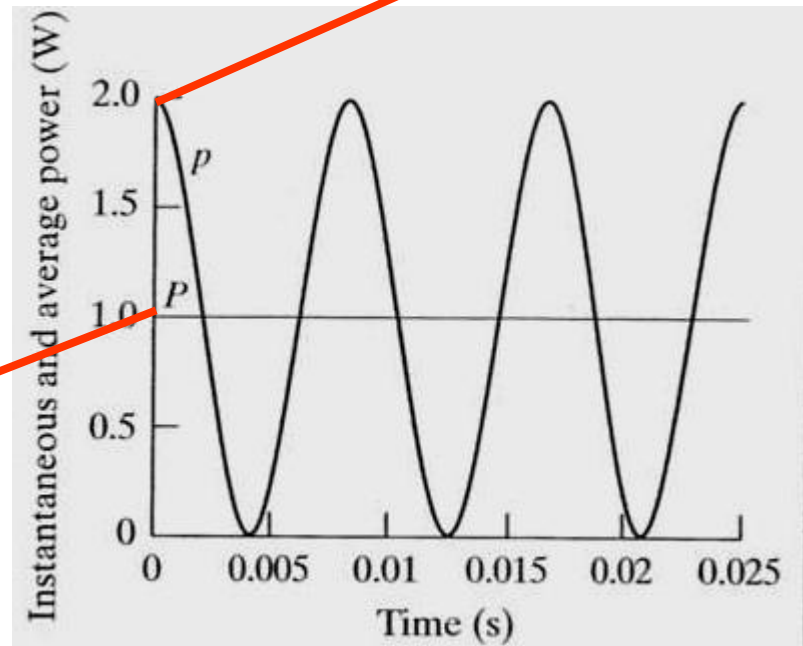
$$\theta_v = \theta_i \Rightarrow P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) = \frac{V_m I_m}{2} \cos(0) = \frac{V_m I_m}{2}$$

$$Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) = \frac{V_m I_m}{2} \sin(0) = 0$$

$$\Rightarrow p(t) = \frac{V_m I_m}{2} + \frac{V_m I_m}{2} \cos(2\omega t)$$

The instantaneous power can never be negative.

$$\frac{V_m I_m}{2}$$



Power can not be extracted from a purely resistive network.

Power for purely Inductive Circuits

$$p(t) = P + P \cos(2\omega t) - Q \sin(2\omega t)$$

$$\theta_v = \theta_i + 90^\circ$$

$$\Rightarrow \theta_v - \theta_i = 90^\circ \Rightarrow P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

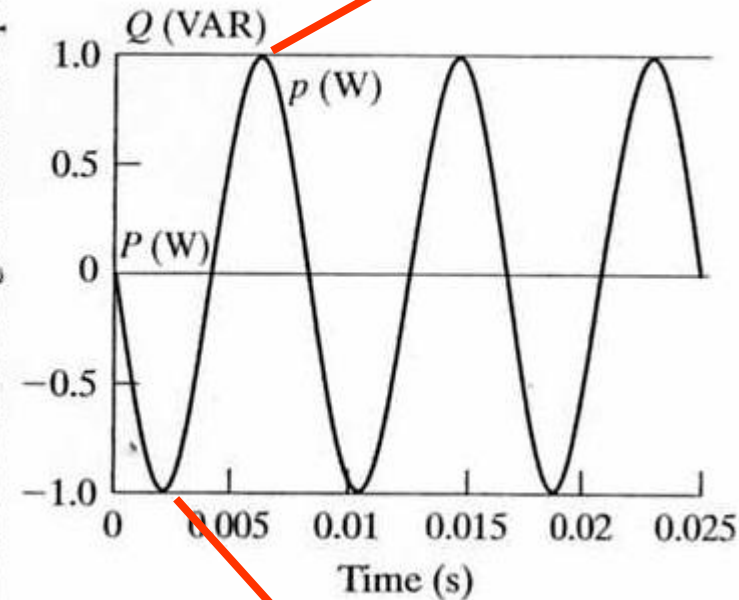
$$= \frac{V_m I_m}{2} \cos(90^\circ) = 0$$

$$Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) = \frac{V_m I_m}{2} \sin(90^\circ) = \frac{V_m I_m}{2}$$

$$\Rightarrow p(t) = - \frac{V_m I_m}{2} \sin(2\omega t)$$

The instantaneous power $p(t)$ is continuously **exchanged** between the circuit and the source driving the circuit. **The average power is zero.**

Instantaneous, average and reactive power



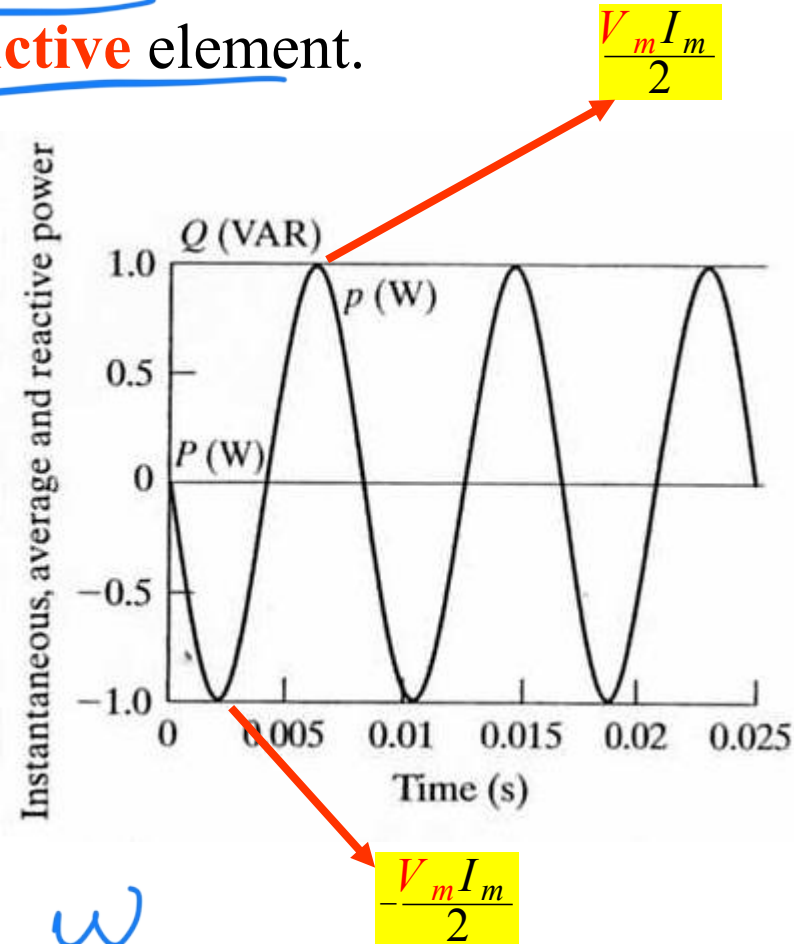
Power for purely Inductive Circuits

When $p(t)$ is **positive**, energy is being **stored** in the **magnetic field** associated with the **inductive** element.

When $p(t)$ is **negative**, energy is being **extracted** from the **magnetic field**.

The power associated with **purely inductive** circuits is the reactive power Q .

The dimension of **reactive power** Q is the same as the average power P . To distinguish them we use the unit **VAR** (**V**olt **A**mpere **R**eactive) for **reactive power**.



$$Q = \text{VAR}$$

Power for purely Capacitive Circuits

$$p(t) = P + P \cos(2\omega t) - Q \sin(2\omega t)$$

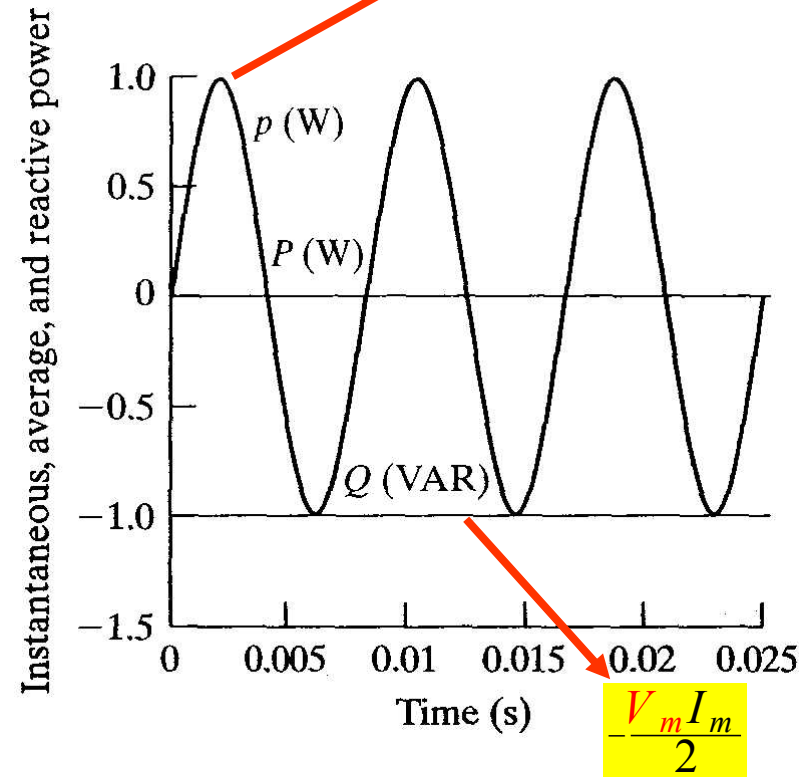
$$\theta_v = \theta_i - 90^\circ$$

$$\Rightarrow \theta_v - \theta_i = -90^\circ \Rightarrow P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) = \frac{V_m I_m}{2} \cos(-90^\circ) = 0$$

$$Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) = \frac{V_m I_m}{2} \sin(-90^\circ) = -\frac{V_m I_m}{2}$$

$$\Rightarrow p(t) = \frac{V_m I_m}{2} \sin(2\omega t)$$

The instantaneous power $p(t)$ is continuously **exchanged** between the circuit and the source driving the circuit. **The average power is zero.**

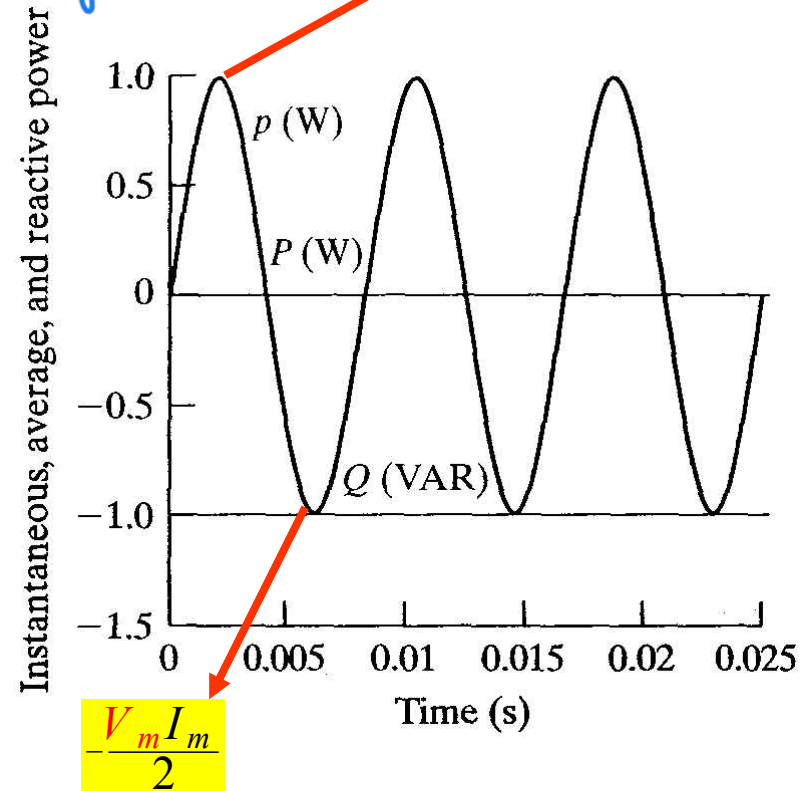


Power for purely Capacitive Circuits

When $p(t)$ is **positive**, energy is being **stored** in the **electric field** associated with the **capacitive** element.

When $p(t)$ is **negative**, energy is being **extracted** from the **electric field**.

The power associated with **purely capacitive** circuits is the reactive power Q (VAR).



The power factor

Recall the Instantaneous power $p(t)$

$$p(t) = \underbrace{\frac{V_m I_m}{2} \cos(\theta_v - \theta_i)}_{P \text{ average power}} + \underbrace{\frac{V_m I_m}{2} \cos(\theta_v - \theta_i)}_{P \text{ average power}} \cos(2\omega t) - \underbrace{\frac{V_m I_m}{2} \sin(\theta_v - \theta_i)}_{Q \text{ reactive power}} \sin(2\omega t)$$
$$= P + P \cos(2\omega t) - Q \sin(2\omega t)$$

The angle $\theta_v - \theta_i$ plays a role in the computation of both **average** and **reactive** power

The angle $\theta_v - \theta_i$ is referred to as the power factor angle

We now define the following : The **power factor** $\text{pf} = \cos(\theta_v - \theta_i)$

The **reactive factor** $\text{rf} = \sin(\theta_v - \theta_i)$

The power factor

$$\text{pf} = \cos(\theta_v - \theta_i)$$

Knowing the power factor **pf** does not tell you the power factor angle, because

$$\cos(\theta_v - \theta_i) = \cos(\theta_i - \theta_v)$$

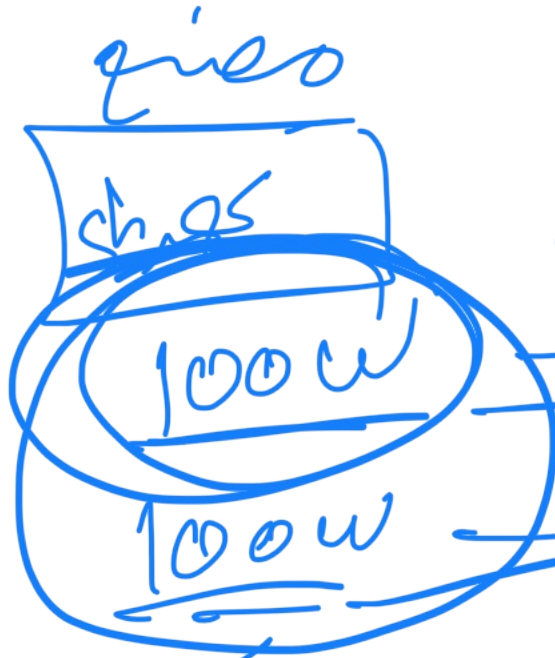
To completely describe this angle, we use the descriptive phrases **lagging power factor** and **leading power factor**

Lagging power factor implies that **current** lags **voltage** hence an inductive load

Leading power factor implies that **current** leads **voltage** hence a capacitive load

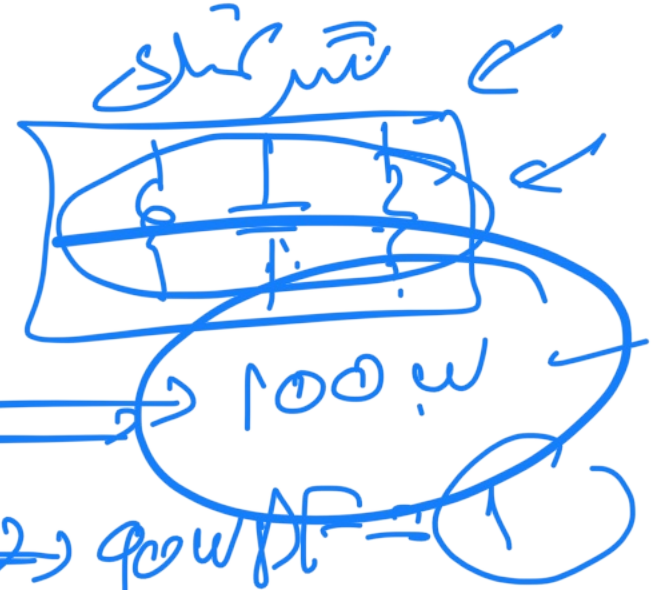
PF

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PF=100%

PF=0.9 = 90%

70

70

Average Power

$$\hat{i} \hat{v} \neq i v$$

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

$$0 \leq PF \leq 1$$

$$PF = \cos(\theta_v - \theta_i)$$

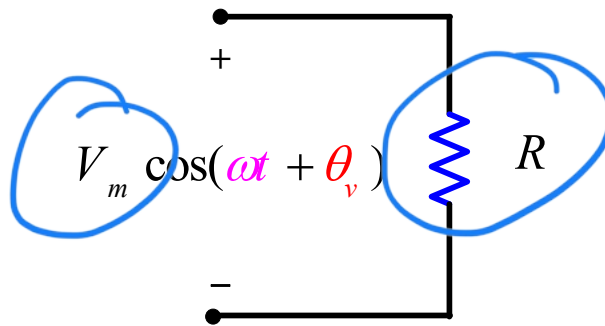
$\theta_v = \theta_i \rightarrow$ purely resistive $R \rightarrow PF = 1$

$\theta_v > \theta_i \rightarrow$ inductor $\Rightarrow PF$ lagging

$\theta_v < \theta_i \rightarrow$ capacitive $\Rightarrow PF$ leading

10.3 The rms Value and Power Calculations

Assume that a sinusoidal voltage is applied to the terminals of a resistor as shown



Handwritten derivation of the average power formula:

$$P \rightarrow V_{rms} \rightarrow \frac{N^2}{D^2} \rightarrow I_{rms}^2 R$$

Suppose we want to determine the average power delivered to the resistor

10.3 The rms Value and Power Calculations

Suppose we want to determine the average power delivered to the resistor

$$P = \frac{1}{T} \int_{t_0}^{t_0+T} p(t) dt = \frac{1}{T} \int_{t_0}^{t_0+T} \frac{\left\{ V_m \cos(\omega t + \theta_v) \right\}^2}{R} dt = \frac{1}{R} \left[\frac{1}{T} \int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \theta_v) dt \right]$$

However since $V_{rms} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \theta_v) dt}$



$$P = \frac{V_{rms}^2}{R}$$

If the resistor carry sinusoidal current $P = R I_{rms}^2$

Recall the Average and Reactive power

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

$$Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i)$$

Which can be written as

$$P = \frac{V_m I_m}{\sqrt{2}\sqrt{2}} \cos(\theta_v - \theta_i)$$

$$Q = \frac{V_m I_m}{\sqrt{2}\sqrt{2}} \sin(\theta_v - \theta_i)$$

Therefore the Average and Reactive power can be written in terms of the **rms** value as

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

$$Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

The **rms** value is also referred to as the **effective value** **eff**

Therefore, the average and reactive power can be written in terms of the **eff** value as:

$$P = V_{eff} I_{eff} \cos(\theta_v - \theta_i)$$

$$Q = V_{eff} I_{eff} \sin(\theta_v - \theta_i)$$

eff
rms

$$V_{rms} = \frac{V_m}{\sqrt{2}} \rightarrow \underline{\underline{V_m}} = \sqrt{2} V_{rms}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} \rightarrow \underline{\underline{I_m}} = \sqrt{2} I_{rms}$$

Average power $P = \frac{V_m I_m \cos(\theta_v - \theta_i)}{2}$

$$P = \frac{V_m I_m \cos(\theta_v - \theta_i)}{\underline{\underline{2}}}$$

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

Example 10.3

- a) A sinusoidal voltage having a maximum amplitude of 625 V is applied to the terminals of 50 Ω resistor. Find the average power delivered to the resistor.
- b) Repeat (a) by first finding the current in the resistor.

a) $\underline{V_m} = 625 \text{ V} \checkmark$ $R = 50 \Omega$

$P = \frac{V_{rms}^2}{R} \leftarrow$ $\rightarrow V_{rms} = \frac{625}{\sqrt{2}} = \underline{441.9 \text{ V}}$

$P = \frac{441.9^2}{50} = \underline{\underline{3906.25 \text{ W}}}$

$$b) \quad V = IR \quad \rightarrow \quad I_m = \frac{V_m}{R}$$

$$I_m = \frac{625}{50} = 12,5 \text{ A}$$

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

$$P = \frac{625 \times 12,5}{2} = \boxed{3906,25 \text{ W}}$$

Example 10.3

Answer:

a) $P=3906.25$ W. b) $I=12.5$ A, $P=3906.25$ W

10.4 Complex Power

Previously, we found it convenient to introduce sinusoidal voltage and current in terms of the complex number, the **phasor**.

Definition

Let the **complex power** be the complex sum of real power and reactive power

$$S = \underline{P} + j\underline{Q}$$

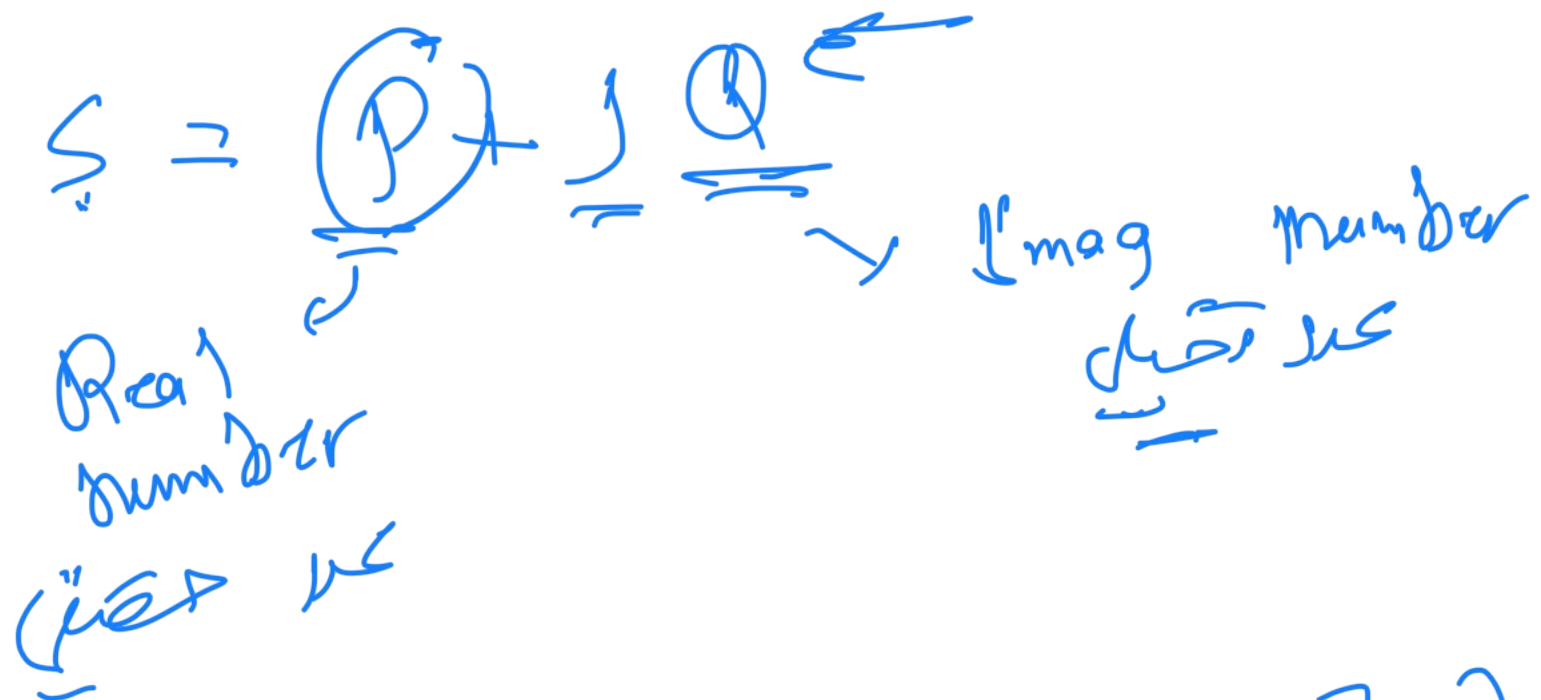
were

S is the complex power

P is the average power

Q is the reactive power

$$j = 1^{\circ} = \sqrt{-1}$$



$$P = \operatorname{Re}\{s\}, \quad Q = \operatorname{Im}\{s\}$$

$$s = 500 + j 1400$$

$$P = 500 \text{ W}$$

$$Q = 1400 \text{ VAR}$$

Advantages of using complex power

$$S = P + jQ$$

– We can compute the average and reactive power from the complex power S

$$P = \Re\{S\} \quad Q = \Im\{S\}$$

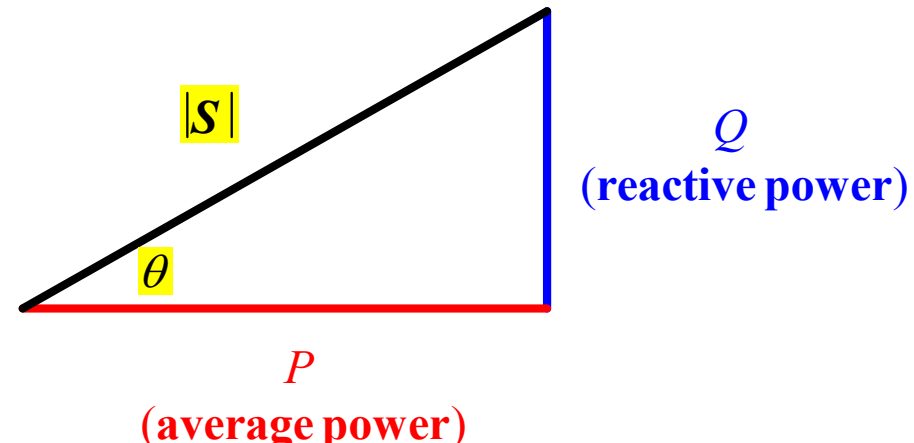
– complex power S provide a geometric interpretation

$$S = P + jQ = |S| e^{j\theta}$$

where

$|S| = \sqrt{P^2 + Q^2}$ is called the apparent power

apparent power

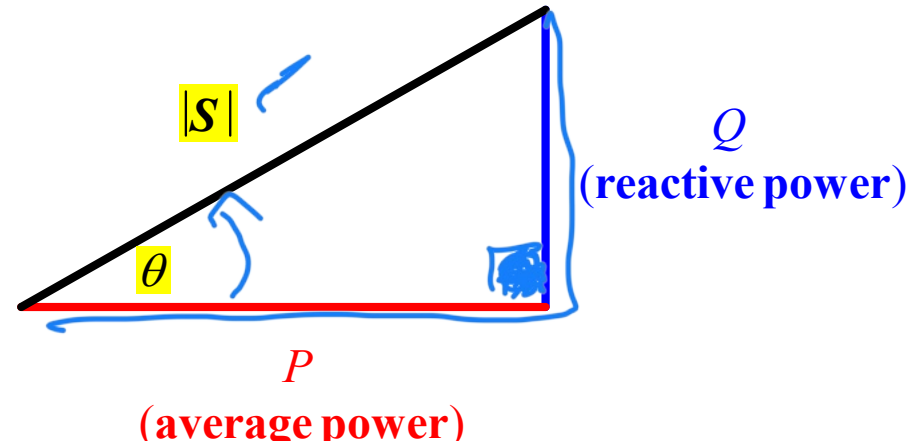


Advantages of using complex power

$$S = P + jQ$$

$$S = P + jQ$$

$$= |S| \angle \theta$$



$|S| = \sqrt{P^2 + Q^2}$ is called the **apparent power**

$$\theta = \tan^{-1}\left(\frac{Q}{P}\right) = \tan^{-1}\left(\frac{V_m I_m \sin(\theta_v - \theta_i)}{V_m I_m \cos(\theta_v - \theta_i)}\right) = \tan^{-1}\left(\frac{\sin(\theta_v - \theta_i)}{\cos(\theta_v - \theta_i)}\right) = \tan^{-1}(\tan(\theta_v - \theta_i)) = \underbrace{\theta_v - \theta_i}_{\text{power factor angle}}$$

The geometric relations for a right triangle mean the four power triangle dimensions

($|S|$, P , Q , θ) can be determined if **any two** of the four are known.

Example 10.4

An electrical load operates at 240 V rms. The load absorbs an average power of 8 kW at a lagging power factor of 0.8.

a) Calculate the complex power of the load.

b) Calculate the impedance of the load.

a) $P=8\text{kW}, \rightarrow S = P/\cos 36.87=10 \text{ kVA}$

$$Q=8 \tan(36.87)=6 \text{ kVAR} \quad S=P+jQ=8+j6 \text{ kVA}$$

b)

$$P = V_{eff} I_{eff} \cos(\theta_v - \theta_i) = (240) I_{eff} (0.8) = 8000 \text{ W}$$

$$\Rightarrow I_{eff} = 41.67 \text{ A}$$

$$Z = \frac{V}{I_{eff}} = \frac{240}{41.67} = 5.76 \Omega$$

$$V_{rms} = 240 \text{ V} \checkmark$$

$$P = \underline{\underline{8 \text{ kW}}}$$

$$PF = 0,8 \quad \text{lagging} \checkmark$$

$$a) \quad S = \underline{\underline{P}} + j \underline{\underline{Q}} \quad \text{PF} = \cos \theta$$

$$P = V_{rms} \underline{\underline{I_{rms}}} \text{ PF} \Rightarrow \underline{\underline{I_{rms}}} = \frac{P}{V_{rms} \text{ PF}}$$

$$I_{rms} = \frac{8000}{240 \times 0,8} = \underline{\underline{\frac{125}{3} \text{ A}}} = 41,67 \text{ A}$$

$$Q = \check{V}_{rms} \check{I}_{rms} \sin(\theta)$$

$$PF = \cos \theta \rightarrow \theta = \cos^{-1}(PF)$$

$$Q = 240 \times \frac{125}{3} \sin(\cos^{-1}(0.8))$$

$$Q = 6000 \text{ VAR}$$

$$S = 8 + j6 \text{ KVA}$$

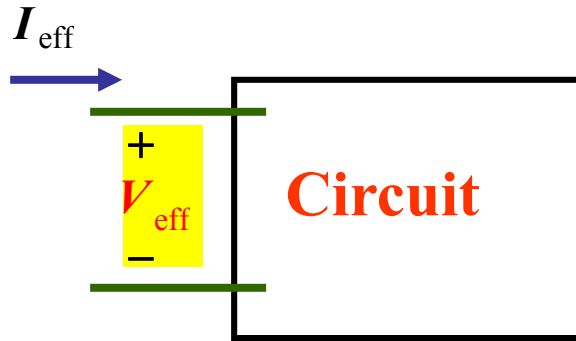
$$b) \quad Z = \frac{V_{rms}}{I_{rms}} \Rightarrow \frac{240}{1.75/3}$$

$$Z = 5.76 \, \Omega$$

10.5 Power Calculations

$$\begin{aligned} S &= P + jQ = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + j \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) \\ &= \frac{V_m I_m}{2} [\cos(\theta_v - \theta_i) + j \sin(\theta_v - \theta_i)] = \frac{V_m I_m}{2} e^{j(\theta_v - \theta_i)} = V_{\text{eff}} I_{\text{eff}} e^{j(\theta_v - \theta_i)} \\ &= V_{\text{eff}} e^{j\theta_v} I_{\text{eff}} e^{-j\theta_i} = V_{\text{eff}} I_{\text{eff}}^* \end{aligned}$$

where I_{eff}^* Is the conjugate of the current phasor I_{eff}



Also $S = \frac{1}{2} VI^*$

$$S \Rightarrow P + jQ \rightarrow \textcircled{1}$$

$$P = V_{rms} I_{rms} \underline{\cos(\theta_v - \theta_i)} \text{ PF}$$

$$Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

$$S \Rightarrow \frac{1}{2} V_m I_m^* = V_{rms} I_{rms}^*$$

Complex number

$$Z = \underline{Z} + \underline{Z}'$$

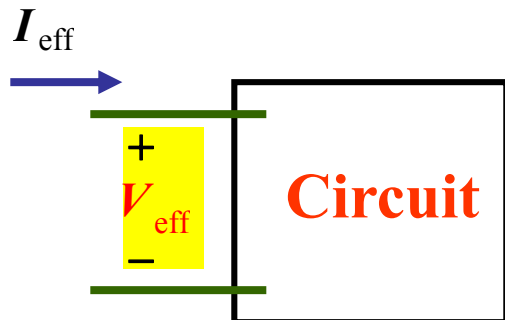
$$\rightarrow Z^* = Z - Z_c'$$

$$Z \rightarrow \underline{2\sqrt{2} \text{ } \underline{1.43}}$$

$$\rightarrow z^* = z\sqrt{z} \quad \text{L-43}$$

* → conjugate → $\bar{z}$ is the conjugate of z .

Alternate Forms for Complex Power



The complex power was **defined** as

$$S = P + jQ$$

Then complex power was **calculated** to be

However there are several useful variations as follows:

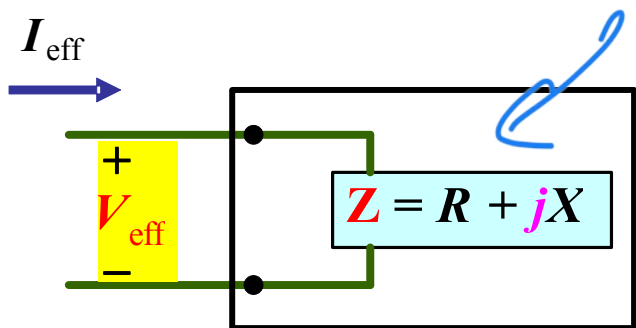
$$S = V_{\text{eff}} I_{\text{eff}}^*$$

OR

$$S = \frac{1}{2} VI^*$$

eff-rms

First variation



$$S = V_{\text{eff}} I_{\text{eff}}^* = (Z I_{\text{eff}}) I_{\text{eff}}^* = Z I_{\text{eff}} I_{\text{eff}}^* = Z |I_{\text{eff}}|^2$$

$$= (R + jX) |I_{\text{eff}}|^2 = \underbrace{R |I_{\text{eff}}|^2}_P + \underbrace{jX |I_{\text{eff}}|^2}_Q$$

$$\Rightarrow P = R |I_{\text{eff}}|^2 = R I_{\text{eff}}^2 = \frac{1}{2} R I_m^2$$

$$Q = X |I_{\text{eff}}|^2 = X I_{\text{eff}}^2 = \frac{1}{2} X I_m^2$$

$$S = P + jQ$$

$$Z = R + jX$$



$+jX \rightarrow$ inductor
 $-jX \rightarrow$ capacitor

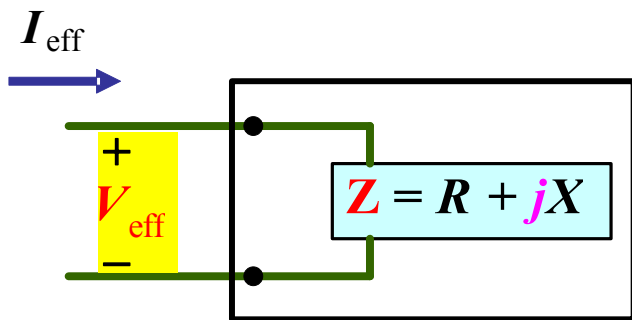
$$P = |I_{eff}|^2 R$$

$$= \frac{|V_{eff}|^2}{R}$$

$$Q = |I_{eff}|^2 X$$

$$= \frac{|V_{eff}|^2}{X}$$

Second variation



$$\begin{aligned}
 S &= V_{\text{eff}} I_{\text{eff}}^* = V_{\text{eff}} \left(\frac{V_{\text{eff}}}{Z} \right)^* = \frac{V_{\text{eff}} V_{\text{eff}}^*}{Z^*} = \frac{|V_{\text{eff}}|^2}{Z^*} \\
 &= \frac{|V_{\text{eff}}|^2}{R - jX} = \frac{|V_{\text{eff}}|^2}{R - jX} \frac{R + jX}{R + jX} = \frac{R + jX}{R^2 + X^2} |V_{\text{eff}}|^2 \\
 &= \underbrace{\frac{R}{R^2 + X^2} |V_{\text{eff}}|^2}_P + j \underbrace{\frac{X}{R^2 + X^2} |V_{\text{eff}}|^2}_Q
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow P &= \frac{R}{R^2 + X^2} |V_{\text{eff}}|^2 = \frac{R}{R^2 + X^2} V_{\text{eff}}^2 = \frac{1}{2} \frac{R}{R^2 + X^2} V_m^2 \\
 Q &= \frac{X}{R^2 + X^2} |V_{\text{eff}}|^2 = \frac{X}{R^2 + X^2} V_{\text{eff}}^2 = \frac{1}{2} \frac{X}{R^2 + X^2} V_m^2
 \end{aligned}$$

If $Z = R$ (pure resistive) $X = 0 \Rightarrow P = \frac{R}{R^2 + X^2} |V_{\text{eff}}|^2 = \frac{|V_{\text{eff}}|^2}{R} \quad Q = 0$

If $Z = X$ (pure reactive) $R = 0 \Rightarrow P = 0 \quad Q = \frac{X}{R^2 + X^2} |V_{\text{eff}}|^2 = \frac{|V_{\text{eff}}|^2}{X}$

Example 10.5

In the circuit shown a load having an impedance of $39 + j26 \Omega$ is fed from a voltage source through a line having an impedance of $1 + j4 \Omega$. The effective, or rms, value of the source voltage is 250 V.

a) Calculate the load current I_L and voltage V_L .

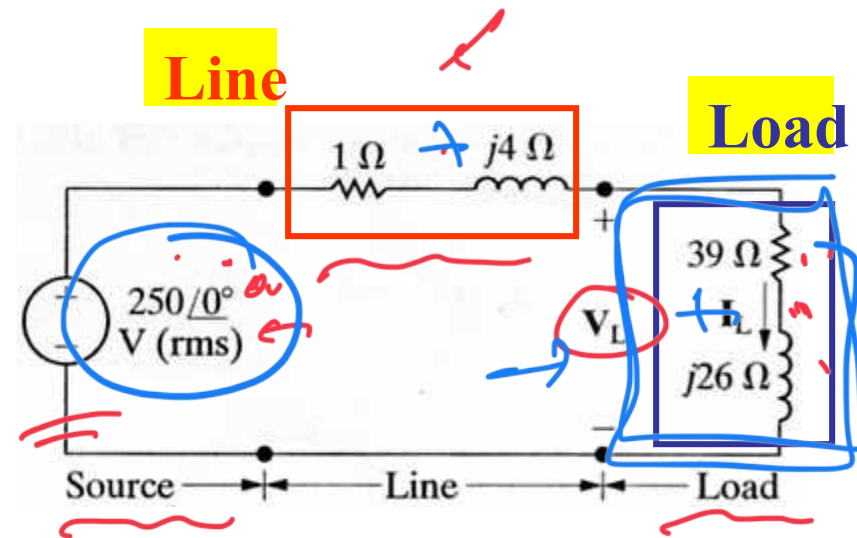
SOLUTION

a) The line and load impedances are in series across the voltage source, so the load current equals the voltage divided by the total impedance, or

$$\begin{aligned} I_L &= \frac{250 \angle 0^\circ}{40 + j30} = 4 - j3 \\ &= 5 \angle -36.87^\circ \text{ A (rms)} \end{aligned}$$

rms because the voltage is given in terms of rms.

$$\Rightarrow V_L = (39 + j26)I_L = 234 - j13 = 234.36 \angle -3.18^\circ \text{ V (rms)}$$



Series $\rightarrow Z = Z_L + Z_{load}$

$$Z = 1 + j4 + 39 + j26$$

$$Z_T = 40 + j30 \Omega \leftarrow$$

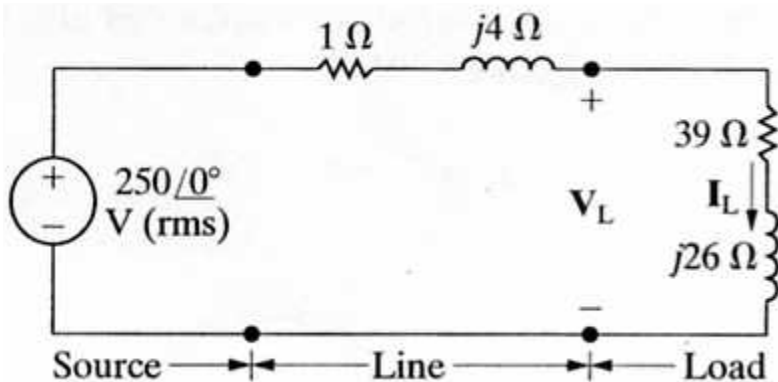


$$I_L = \frac{V_{rms}}{Z} = \frac{250 \angle 0}{(40 + j30)}$$

$$I_L = \underline{4 - j3 \text{ A}} = \underline{5 \angle -36.87^\circ \text{ A}_{rms}}$$

$$V_2 = I_Z = (39 + j26)(4 - j3)$$

$$V_2 = \underline{234 - j13 \text{ V}_{rms}} = \underline{234.36 \angle -3.178^\circ \text{ V}_{rms}}$$



$$\mathbf{I_L} = 4 - j3 = 5 \angle -36.87^\circ \text{ A (rms)}$$

$$\mathbf{V_L} = 234 - j13 = 234.36 \angle -3.18^\circ \text{ V (rms)}$$

b) Calculate the average and reactive power delivered to the load.

$$S = \mathbf{V_L} \mathbf{I_L}^* = (234 - j13)(4 + j3) = 975 + j650 \text{ VA}$$

→ $P = 975 \text{ W}$ $Q = 650 \text{ var}$

Another solution The load average power is the power absorbed by the load resistor 39Ω

Recall the average Power for purely resistive Circuits $P = \frac{V^R_m I^R_m}{2} = V_{eff}^R I_{eff}^R$

where V_{eff}^R and I_{eff}^R Are the **rms** voltage across the resistor and the **rms** current through the resistor

$$P = V_{eff}^R I_{eff}^R = R I_{eff}^2$$

b) $S = V_L \underline{I_L^*}$ Complex power

$$S = (234 - j13)(4 - j3)$$

$$= (234 - j13)(4 + j3)$$

$$S = \underline{975} + j\underline{650}$$

$P = 975 \text{ W}$
Average power

$Q = 650 \text{ VAR}$
Reactive power

$$Z_L = \underline{\underline{39}} + j \underline{\underline{26}} \Omega$$

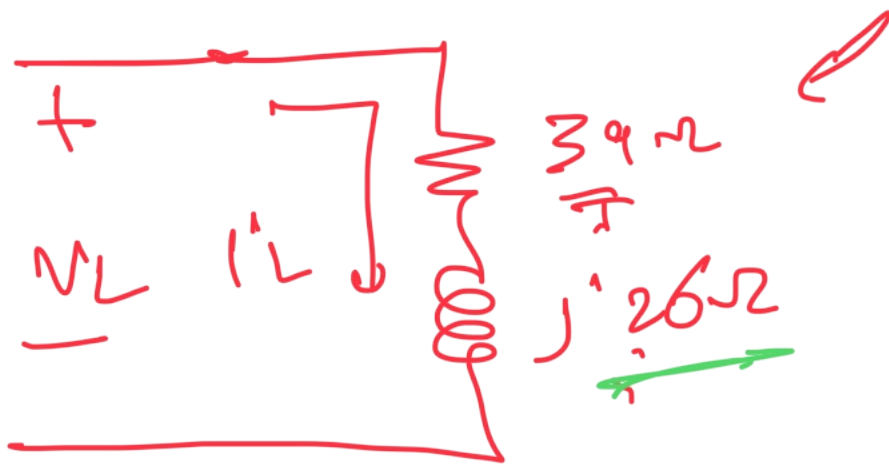


$$R = \underline{\underline{39}} \Omega$$

$$X_L = 26 \Omega$$

$$I_L = \textcircled{5} \angle -36.87^\circ \text{ Arms} \quad + \quad |I_L| = \textcircled{5}$$

$$\underline{\underline{V_L}} = \underline{\underline{234.36}} \angle -31.8^\circ \text{ rms} \quad |V_L| = \underline{\underline{234.36}}$$



series $\Rightarrow$ voltage divider

$$\underline{V_R} = \frac{39}{39 + j26} \times (\underline{234 - j13})^{NL}$$

$$\underline{V_R} = 156 - j117$$

$$\underline{V_R} = \underline{195} \angle -36.87^\circ$$

V_{rms}

$$\rightarrow |V_R| = 195$$

$$P = \frac{|V_{eff r}|^2}{R} = \frac{195^2}{39} = \underline{975 \text{ W}}$$

$$P = |I_{eff}|^2 R = 5^2 \times 39 = \underline{975 \text{ W}}$$

$$P \Rightarrow \text{Netfa}(\text{Ieff}) = 195 \times S$$

$$\Rightarrow 9750$$

$$Q = |V_{eff}|^2$$

$$V_L = \frac{j26}{j26+39} \rightarrow (234-j13)$$

$$K_L = \textcircled{5}$$

$$V_L = 78 + j104$$

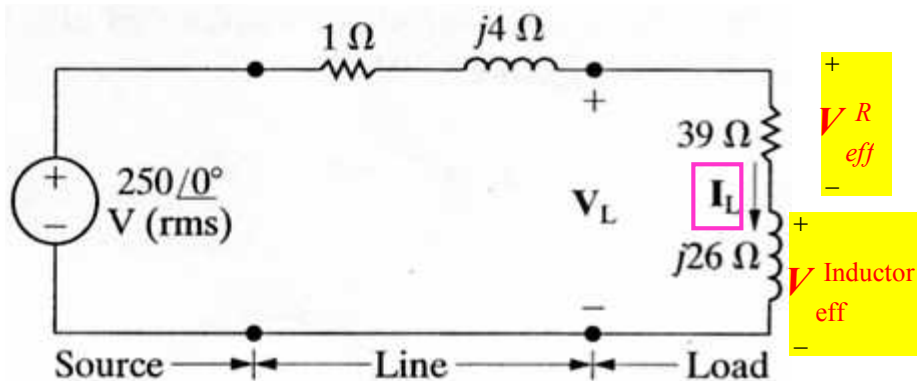
$$V_{L_{eff}} = \underline{130} \angle 53.13 \rightarrow |V_L| = \underline{130}$$

$$Q = \frac{|V_{eff}|^2}{X} = |I_{eff}|^2 \rightarrow$$

$$Q = \text{Var}(\text{LRF}) = 130 \times 5 = 650 \text{ VAR}$$

$$Q = \frac{|\text{Var}|^2}{f} = \frac{130^2}{26} = 650 \text{ VAR}$$

$$Q = |\text{LRF}|^2 \times 5 = 5^2 \times 26 = 650 \text{ VAR}$$



$$\mathbf{I}_L = 4 - j3 = 5 \angle -36.87^\circ \text{ A (rms)}$$

$$\mathbf{V}_L = 234 - j13 = 234.36 \angle -3.18^\circ \text{ V (rms)}$$

$$\mathbf{S} = \mathbf{V}_L \mathbf{I}_L^* = (234 - j13)(4 + j3) = 975 + j650 \text{ VA}$$

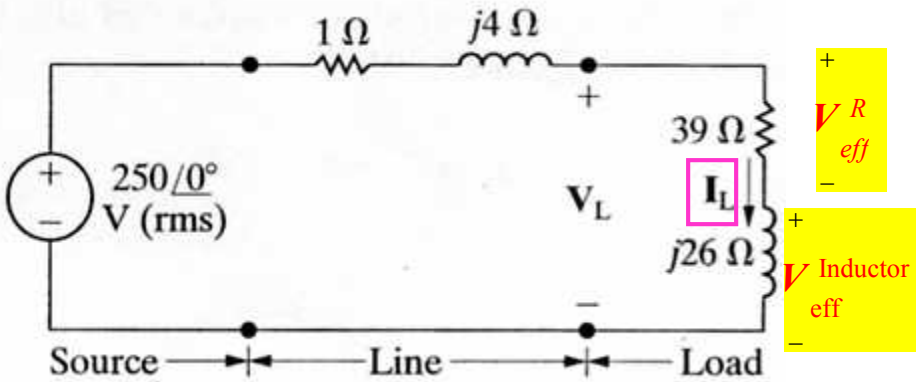
$$\begin{aligned} \Rightarrow P &= 975 \text{ W} \\ Q &= 650 \text{ var} \end{aligned}$$

From Power for purely resistive Circuits

$$P = \frac{1}{2} V_m I_m = V_{\text{eff}} I_{\text{eff}} \Rightarrow P = V_{\text{eff}}^R \parallel I_{\text{eff}}^R = V_{\text{eff}}^R I_{\text{eff}}^R$$

$$V_{\text{eff}}^R = \frac{39}{39 + j26} V_L = \frac{39}{39 + j26} 234.36 e^{-j3.18^\circ} = 195 e^{j36.87^\circ} \Rightarrow V_{\text{eff}}^R = 195$$

$$\Rightarrow P = V_{\text{eff}}^R I_{\text{eff}}^R = (195)(5) = 975 \text{ W}$$



$$I_L = 4 - j3 = 5 \angle -36.87^\circ \text{ A (rms)}$$

$$V_L = 234 - j13 = 234.36 \angle -3.18^\circ \text{ V (rms)}$$

$$S = V_L I_L^* = (234 - j13)(4 + j3) = 975 + j650 \text{ VA}$$

From Power for purely resistive Circuits

$$\Rightarrow P = 975 \text{ W}$$

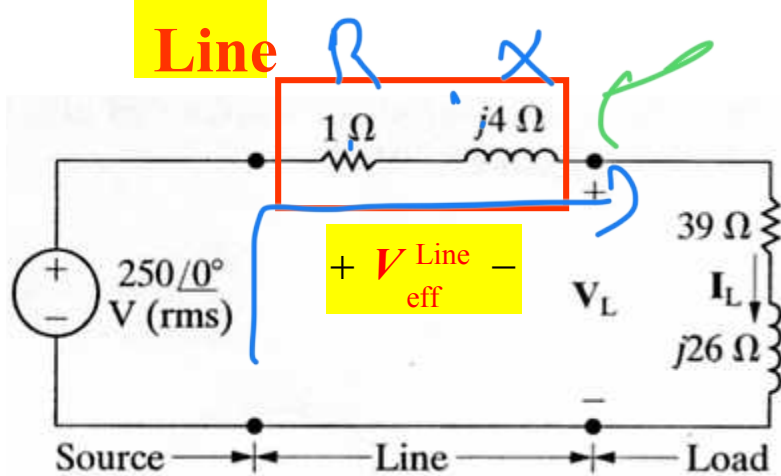
$$Q = 650 \text{ var}$$

OR $P = V_{eff}^R I_{eff}^R = (R I_{eff}^R) I_{eff}^R = R (I_{eff}^R)^2 = (39)(5^2) = (39)(25) = 975 \text{ W}$

$$Q = V_{eff}^L I_{eff}^L \Rightarrow Q = V_{eff}^L I_{eff}^L \quad V_{eff}^L = \frac{j26}{39 + j26} V_L = \frac{j26}{39 + j26} 234.36 e^{-j3.18^\circ} = 78 + j104 = 130 e^{j53.13^\circ}$$

$$\Rightarrow V_{eff}^L = 130 \Rightarrow Q = (130)(5) = 650 \text{ VAR}$$

OR $Q = X I_{eff}^2 = 650 \text{ var}$



$$I_L = 4 - j3 = 5 \angle -36.87^\circ \text{ A (rms)}$$

$$V_L = 234 - j13 = 234.36 \angle -3.18^\circ \text{ V (rms)}$$

$$S = P + jQ$$

c) Calculate the average and reactive power delivered to the line.

$$P = I_{\text{eff}}^2 R \rightarrow P = (5)^2 (1) = 25 \text{ W}$$

$$Q = I_{\text{eff}}^2 X \rightarrow Q = (5)^2 (4) = 100 \text{ VAR}$$

$$P = |I_L|^2 R \rightarrow$$

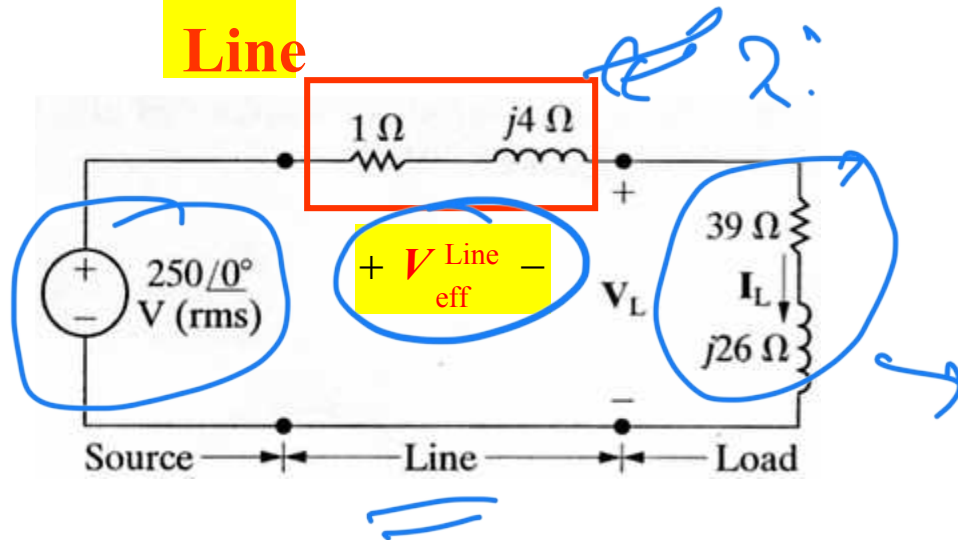
$$5^2 \times 1 = 25 \text{ W}$$

$$Q = |I_L|^2 X \rightarrow$$

$$5^2 \times 4 = 100 \text{ VAR}$$

$$S = 25 + j100 \text{ VA}$$

Line



$$I_L = 4 - j3 = 5 \angle -36.87^\circ \text{ A (rms)}$$

$$V_L = 234 - j13 = 234.36 \angle -3.18^\circ \text{ V (rms)}$$

$$234.36 \angle -3.18$$

OR using complex power

$$S_{\text{Line}} = V_{\text{eff}}^{\text{Line}} I_{\text{eff}}^* \quad V_{\text{eff}}^{\text{Line}} = \frac{1+j4}{(1+j4)+(39+j26)} (250)$$

OR

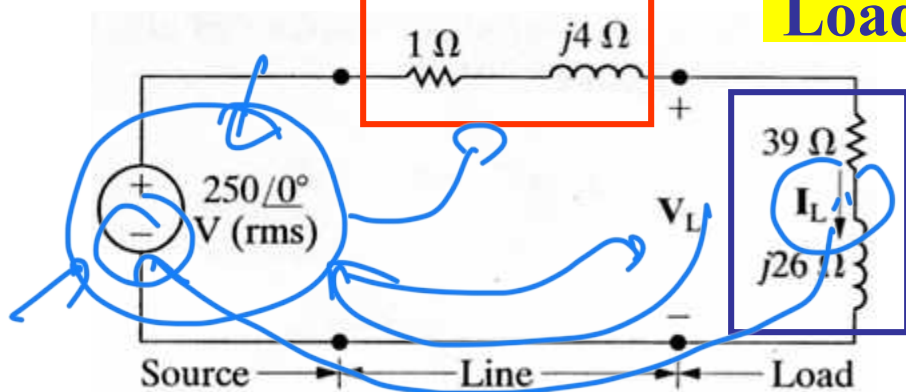
$$V_{\text{eff}}^{\text{Line}} = 250 - V_L$$

$$V_{\text{eff}}^{\text{Line}} = 20.6 \angle 39.1^\circ \text{ V rms}$$

$$\Rightarrow S_{\text{Line}} = V_{\text{eff}}^{\text{Line}} I_{\text{eff}}^* = 20.6 \angle 39.1^\circ \cdot 5 \angle 36.87^\circ = 103 \angle 75.97^\circ = 25 + j100 \text{ VA}$$

Line

Load



$$I_L = 4 - j3 = 5 \angle -36.87^\circ \text{ A (rms)}$$

$$V_L = 234 - j13 = 234.36 \angle -3.18^\circ \text{ V (rms)}$$

d) Calculate the average and reactive power supplied by the source.

$$S_{\text{Absorb}} = S_{\text{Line}} + S_{\text{Load}} = \underbrace{(25 + j100)}_{\text{From part (c)}} + \underbrace{(975 + j650)}_{\text{From part (b)}} = (25 + 975) + j(100 + 650) = 1000 + j750 \text{ VA}$$

$$\Rightarrow S_{\text{Supply}} = -S_{\text{Absorb}} = -(1000 + j750) \text{ VA}$$

$$\begin{aligned} S_{\text{Supply}} &= -250 \angle 0^\circ (I_L^*) = -250 \angle 0^\circ 5 \angle 36.87^\circ = -1250 \angle 36.87^\circ \text{ VA} \\ &= -1000 - j750 \text{ VA} \end{aligned}$$

OR

$$S_{\text{total}} \Rightarrow S_{\text{line}} + S_{\text{load}}$$

$$= \underline{\underline{2S + j100}} + \underline{\underline{97S + j650}}$$

$$S = -(1000 + j750)$$

absorbed + stored

supplied $\Rightarrow$ $\tilde{z}$

$$S = -1000 - j750 \text{ VA}$$

$$S = -V_s I_s^*$$

$$S = - [250 \angle 0] [4 - j3]^*$$

$$S = - \underline{250} \times (4 + j3)$$

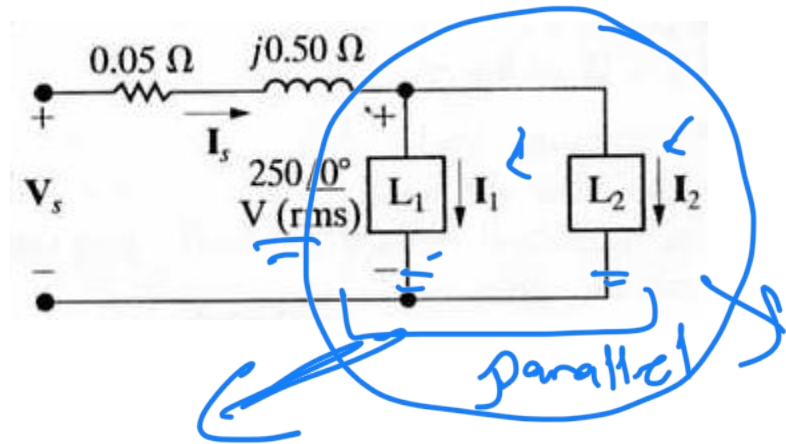
$$S = -1000 - j750 \text{ VA}$$

↑
=

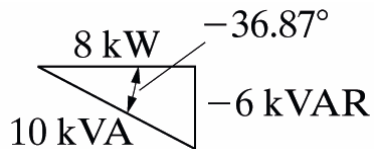
Example 10.6 Calculating Power in Parallel Loads

The two loads in the circuit shown can be described as follows: Load 1 absorbs an average power of 8 kW at a leading power factor of 0.8. Load 2 absorbs 20 kVA at a lagging power factor of 0.6.

a) Determine the power factor of the two loads in parallel.

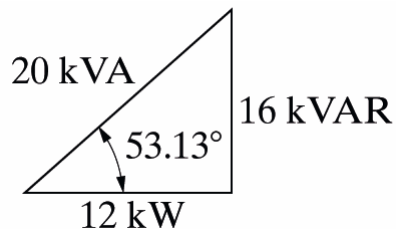


$$I_s = I_1 + I_2 \quad S = (250)I_s^* = (250)(I_1 + I_2)^* = (250)I_1^* + (250)I_2^* = S_1 + S_2$$



$$S_1 = 8000 - j \frac{8000(.6)}{(.8)}$$

$$= 8000 - j6000 \text{ VA}$$



$$S_2 = 20,000(.6) + j20,000(.8)$$

$$= 12,000 + j16,000 \text{ VA}$$

load @

$$P = \underline{8 \text{ kW}}$$

$$PF = \underline{0.8} \text{ leading}$$

$$S = P + jQ$$

+ Q $\rightarrow$ lagging

- Q $\rightarrow$ leading

$$PF = \cos \theta = 0.8 \rightarrow \theta = \cos^{-1} 0.8$$

$$\frac{Q}{P} = \frac{V_{rms} I_{rms} \sin \theta}{V_{rms} I_{rms} \cos \theta}$$

$$\frac{Q}{P} \Rightarrow \frac{\sin \theta}{\cos \theta}$$

$$\Rightarrow Q = P \tan \theta$$

$$\underline{\underline{Q = 6 \text{ kVAR}}}$$

$$Q = 8000 \text{ from } (\cos^{-1} 0.8)$$

$$Q = \underline{\underline{6000 \text{ VAR}}}$$

$$S = 8 - j6 \quad 12 \text{ VA}$$

load @

$$I_s = 20 \text{ kVA}$$

PF = 0.8 lagging

$\theta = \cos^{-1} 0.8$

Apparent power



Apparent power $\underline{I_s} \rightarrow \underline{V_{rms} I_{rms}}$

Complex power $S = V_{rms} I_{rms}^*$

$$P = \boxed{V_{rms} I_{rms} \cos \theta = \text{PF}}$$

$$Q = \boxed{V_{rms} I_{rms} \sin \theta}$$

$$P = 1 \angle \cos \theta$$

$$Q = 1 \angle \sin \theta$$

$$P = 20000 \times 0.6 = 12 \text{ kW}$$

$$Q = 20000 \sin(\cos^{-1} 0.6) = 16 \text{ kVAR}$$

$$S_z = 12 + j16 \text{ kVA}$$

$$S = S_1 + S_2$$

$$= 8 - j6 + 12 + j16$$

$$S_{\uparrow} = 20 + j10 \text{ KVA}$$

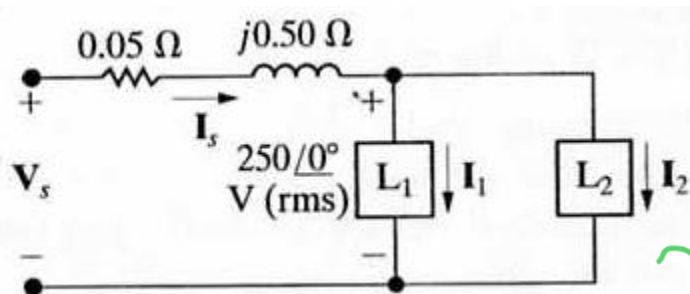
$$S_T = \underline{22360.7} \text{ VA} \quad \underline{26.565} \text{ VA}$$

$$PF = \cos(26.565)$$

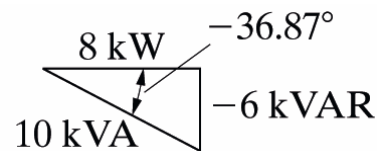
$$= \underline{\underline{0.8944}}$$

↳ lagging ✓

↳ leading ✗

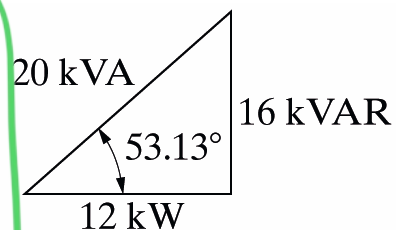


$$\mathbf{I}_s = \mathbf{I}_1 + \mathbf{I}_2$$



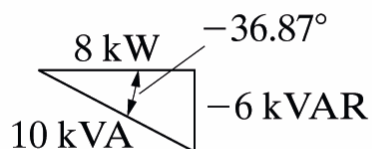
$$S_1 = 8000 - j \frac{8000(.6)}{(.8)}$$

$$= 8000 - j6000 \text{ VA}$$

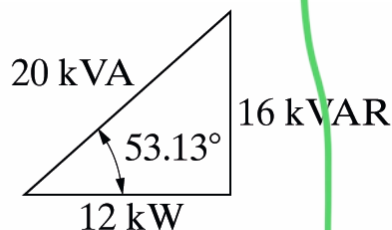


$$S_2 = 20,000(.6) + j20,000(.8)$$

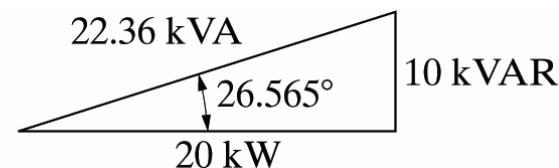
$$= 12,000 + j16,000 \text{ VA}$$



+



=

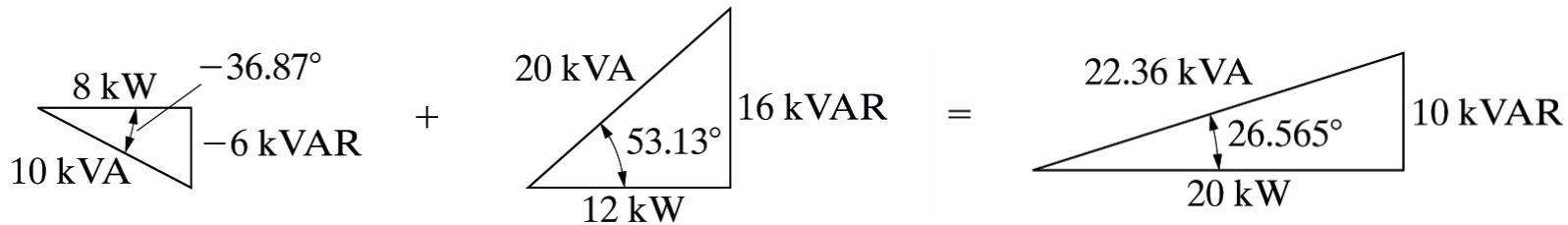


$$S = 20,000 + j10,000 \text{ VA}$$



$$\mathbf{I}_s^* = \frac{20,000 + j10,000}{250} = 80 + j40 \text{ A}$$

$$\mathbf{I}_s = 80 - j40 = 89.44 \angle -26.57^\circ \text{ A}$$



$$S = 20,000 + j10,000 \text{ VA}$$

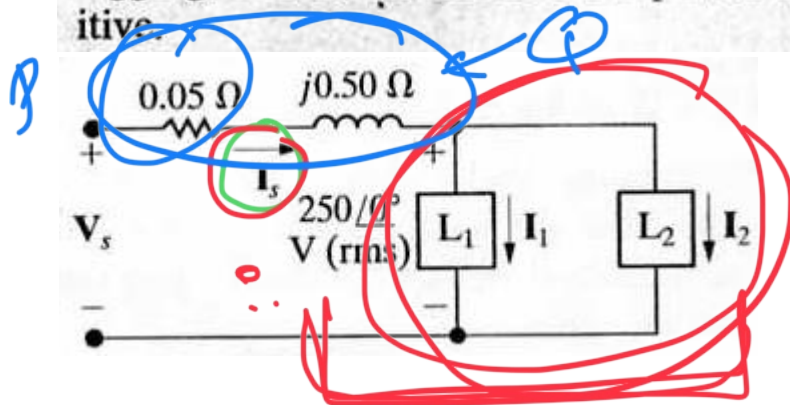
➔

$$\mathbf{I}_s^* = \frac{20,000 + j10,000}{250} = 80 + j40 \text{ A} \quad \mathbf{I}_s = 80 - j40 = 89.44 \angle -26.57^\circ \text{ A}$$

$$\text{pf} = \cos(\theta_v - \theta_i)$$

$$\text{pf} = \cos(0 + 26.57^\circ) = 0.8944 \text{ lagging}$$

The power factor of the two loads in parallel is lagging because the net reactive power is positive.



b) Determine the apparent power required to supply the loads, the magnitude of the current, $\mathbf{I}_s$, and the average power loss in the transmission line.

$$\mathbf{I}_s = 80 - j40 = 89.44 \angle -26.57^\circ \text{ A}$$

$$S = 20 + j10 \text{ kVA} \quad \checkmark$$

$$V = 250 \text{ V rms} \quad \checkmark$$

$$S = V_{\text{rms}} I_{\text{rms}}^*$$

$$I_{\text{rms}}^* = \frac{S}{V_{\text{rms}}} = \frac{20000 + j10000}{250}$$

$$I_{\text{rms}}^* = 80 + j40 \text{ A}$$

$$\Rightarrow \boxed{I_{rms} = 80 - j40 \text{ A}}$$

$$= \underline{89,44} \angle -26,56^\circ \text{ A}$$

$$|I_{rms}| = 89,44 \text{ A}$$

$$P = |I|^2 R = 89,44^2 \times 0,05 = 4000$$

$$Q = |I|^2 X = 89,44^2 \times 0,5 = 4000 \text{ VAR}$$

$$\boxed{S_L = 4000 + j4000 \text{ VA}}$$

$$S_1 = 8000 - j6000 \text{ VA} \quad S_2 = 12000 + j16000 \text{ VA} \quad S = 20000 + j10000 \text{ VA}$$

The apparent power which must be supplied to these loads is

$$|S| = |20000 + j10000| \text{ VA} = 22.36 \text{ kVA}$$

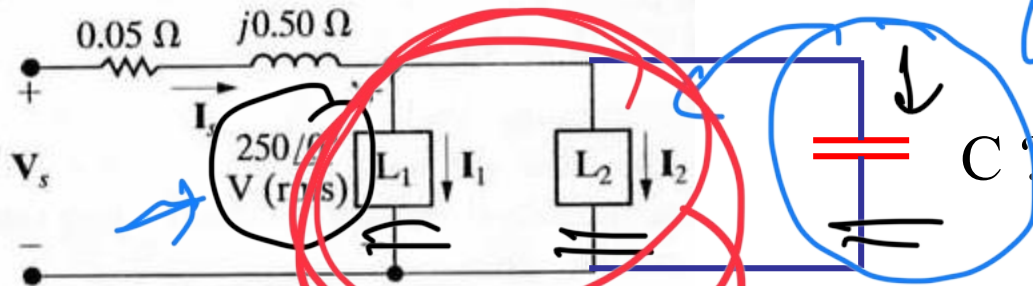
The magnitude of the current that supplies this apparent power is

$$|I_s| = |80 - j40| = 89.44 \text{ A}$$

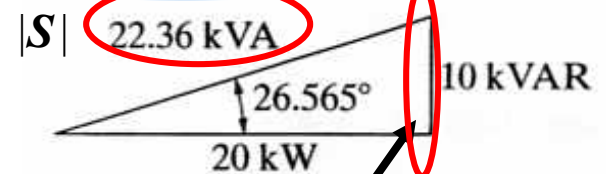
The average power lost in the line results from the current flowing through the line resistance

$$P_{\text{line}} = |I_s|^2 R = (89.44)^2 (0.05) = 400 \text{ W}$$

Note that the power supplied totals $20,000 + 400 = 20,400 \text{ W}$, even though the loads require a total of only $20,000 \text{ W}$.



DF = 0.9999 lag



c) Given that the frequency of the source is 60 Hz, compute the value of the capacitor that would correct the power factor to 1 if placed in parallel with the two loads. Recompute the values in (b) for the load with the corrected power factor.

424.4 μF
PF = 1

As we can see from the power triangle We can correct the power factor to 1 if we place a capacitor in parallel with the existing load

Will cancel this

the capacitive reactance

$$X = \frac{|V_{eff}|^2}{Q} = \frac{(250)^2}{-10,000} = -6.25 \Omega$$

Recall that $X = -\frac{1}{\omega C}$ $\omega = 2\pi(60) = 376.99 \text{ rad/s}$ $C = \frac{-1}{\omega X} = \frac{-1}{(376.99)(-6.25)} = 424.4 \mu\text{F}$

$$\underline{S_T} = 200000 + \frac{j100000}{\omega} \quad \text{VA}$$

0

$$S_T = \underbrace{S_1 + S_2}_{S_T} + S_c$$

$$\underbrace{200000}_{S_T}$$

$$+ \cancel{j100000}$$

$$\rightarrow \cancel{j100000}$$

$$S = 200000 \rightarrow \text{new VA}$$

$$\boxed{PF = 1}$$

$$\begin{array}{ccc}
 PF & \begin{array}{c} 100 \\ 1000 \end{array} & \begin{array}{c} \longrightarrow 100 \\ \longrightarrow 90 \end{array} & \begin{array}{c} PF \rightarrow 1 \\ PF = 0,9 \end{array}
 \end{array}$$

$$Q \leftarrow \frac{\sqrt{VAF^2}}{X} = \sqrt{R^2 X}$$

$$X = \frac{VAF R}{\textcircled{Q}} \quad \rightarrow \quad \frac{250^2}{10000}$$

$X = 6,25 \Omega$

$$X_L = \omega L$$

$$\textcircled{X_C} = \frac{1}{\omega C}$$

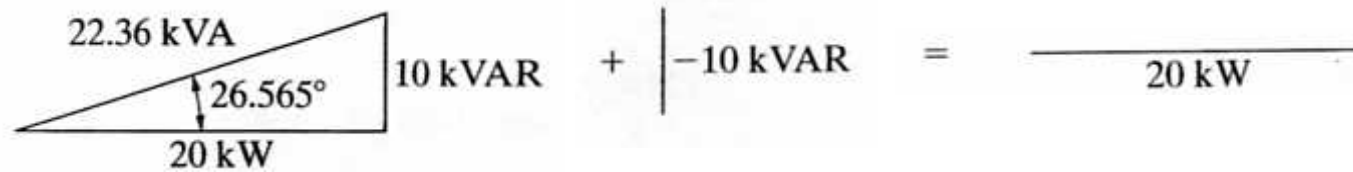
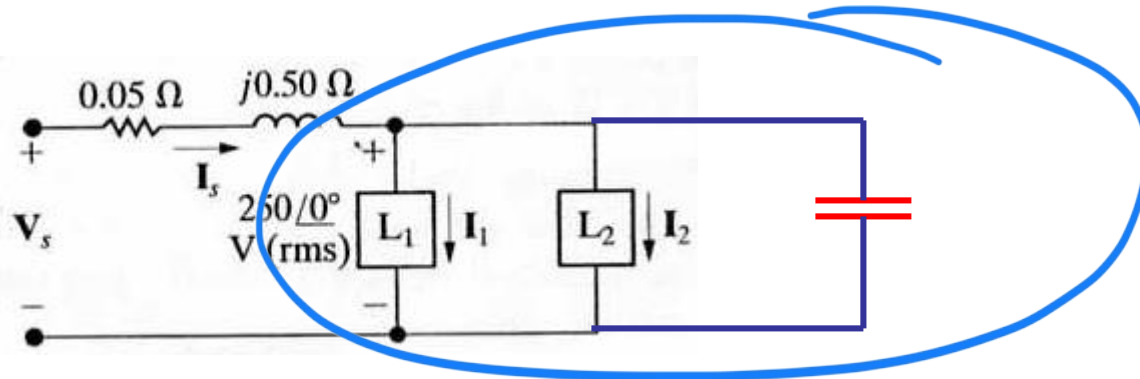
$$C = \frac{1}{\omega X_C}$$

$$\omega = 2\pi f$$

↓
Hz

$$C = \frac{1}{2\pi \times 60 \times 625}$$

$$C = 424,4 \text{ nF}$$



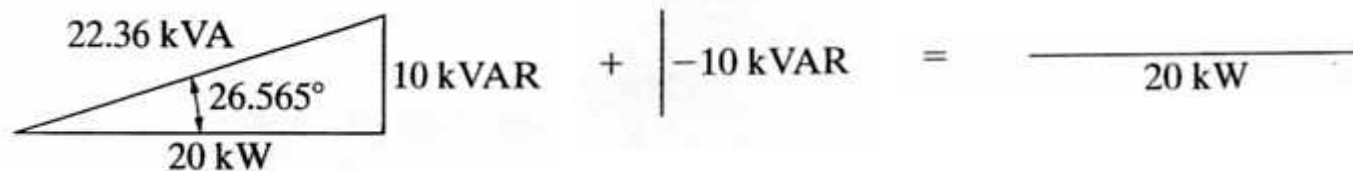
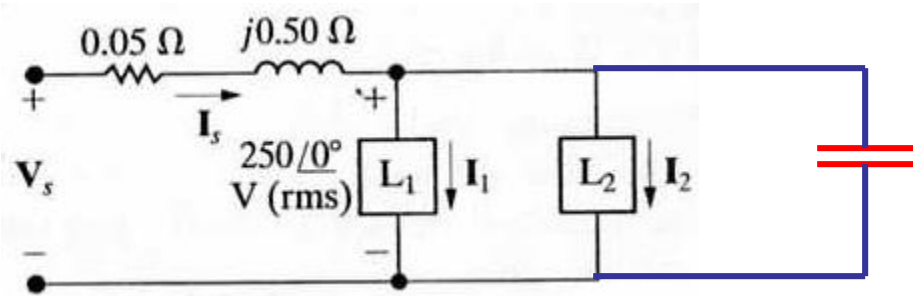
When the power factor is 1, the apparent power and the average power are the same

$$|S| = P = 20 \text{ kVA}$$

← PF = 1

The magnitude of the current that supplies this apparent power is

$$|I_s| = \frac{20,000}{250} = 80 \text{ A}$$



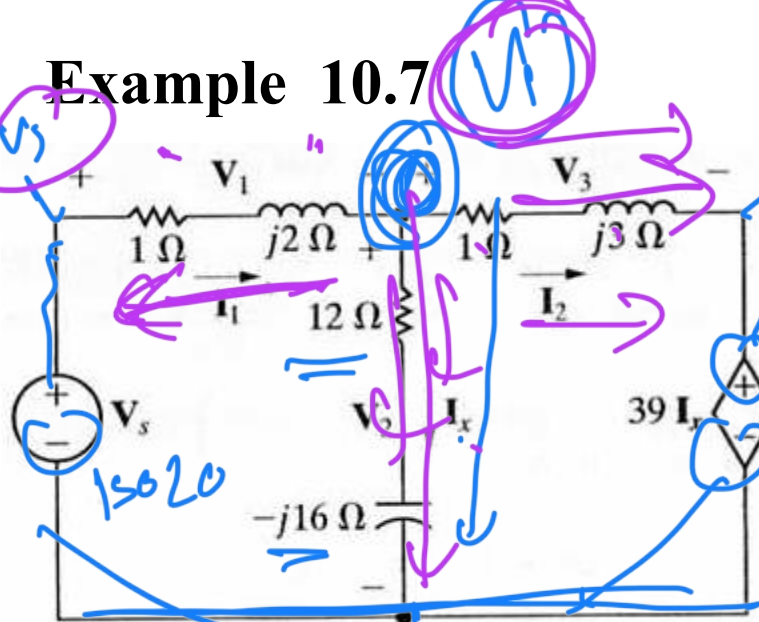
The average power lost in the line is thus reduced to

$$P_{\text{line}} = |I_s|^2 R = (80)^2 (0.05) = 320 \text{ W}$$

Now, the power supplied totals $20,000 + 320 = 20,320 \text{ W}$

The addition of the capacitor has reduced the line loss from **400 W** to **320 W**

Example 10.7



$$V_s = 150 \angle 0^\circ \text{ V}$$

$$V_1 = (78 - j104) \text{ V}$$

$$V_2 = (72 + j104) \text{ V}$$

$$V_3 = (150 - j130) \text{ V}$$

$$I_1 = (-26 - j52) \text{ A}$$

$$I_x = (-2 + j6) \text{ A}$$

$$I_2 = (-24 - j58) \text{ A}$$

a) Calculate the total average and reactive power delivered to each impedance in the circuit

$$S_1 = \frac{1}{2} V_1 I_1^* = \frac{1}{2} (78 - j104) (-26 + j52) = 1690 + j3380 \text{ VA} = P_1 + jQ_1$$

→ $P_1 = 1690 \text{ W}$ and $Q_1 = 3380 \text{ VAR}$

Global Analysis

$$I_x = \frac{V_1 - 0}{12 - j16}$$

$$\frac{V_1 - 150}{1 + j2} + \frac{V_1}{12 - j10} + \frac{V_1 - 30V_1}{\cancel{12 - j10} \cdot \underbrace{(1 + j3)}} = 0$$

$$\cancel{\frac{V_1}{1 + j2}} - \frac{150}{1 + j2} + \cancel{\frac{V_1}{12 - j10}} + \frac{V_1}{1 + j3}$$

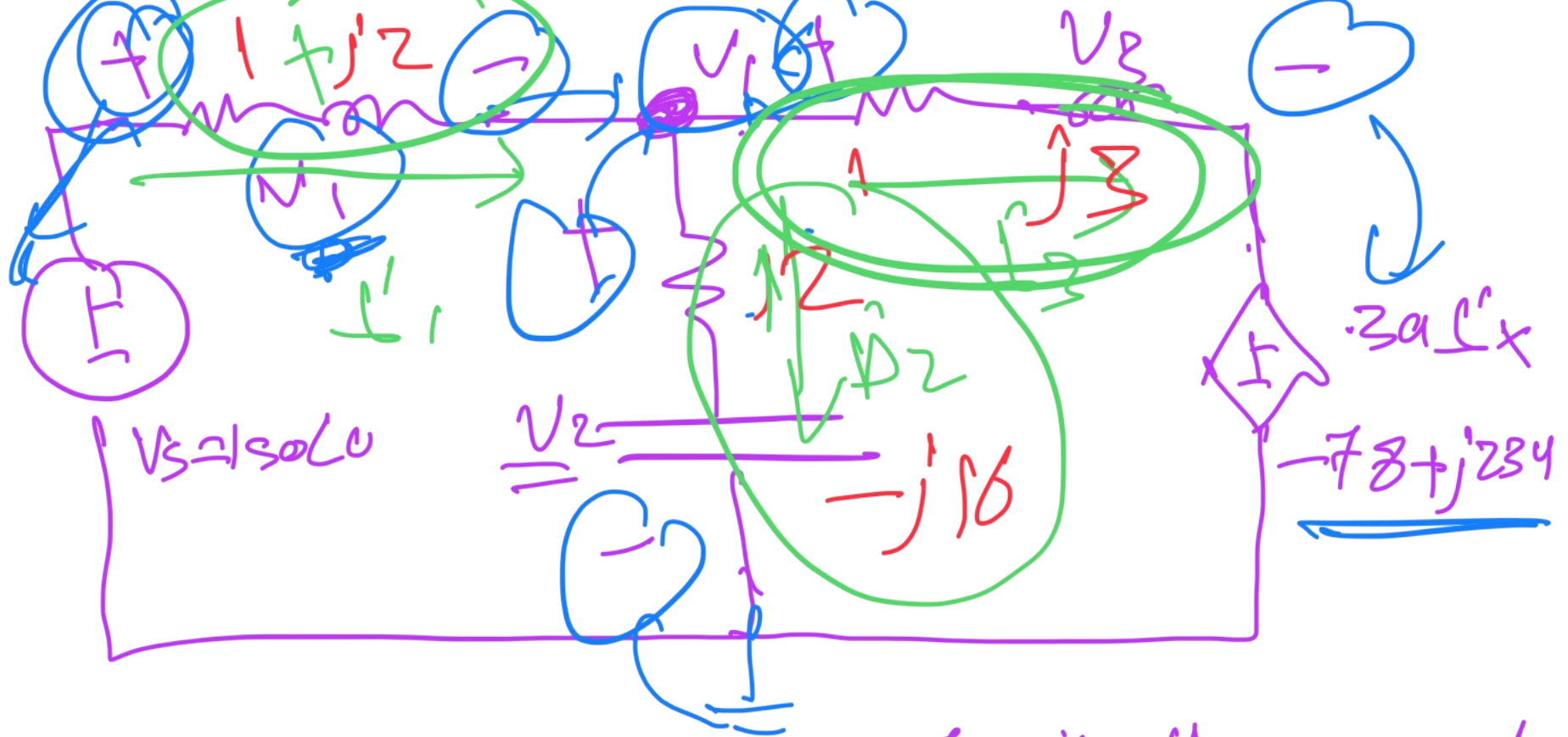
$$\underbrace{- \frac{30V_1}{(12 - j10)(1 + j3)}}_{= 0}$$

$$\frac{150}{1 + j2}$$

$$V_1 = \left[\frac{-51 - j93}{200} \right] = 30 - j60$$

$$V_1 = [30 - j60] \frac{200}{-51 - j93}$$

$$V_1 = 72 + j104 \text{ V}$$



$$I_x = \frac{V_1}{12 - j16} = \frac{72 + j104}{12 - j16} = -2 + j6$$

$$V_{ab} = V_a - V_b$$

$$V_1 = V_s - V_i$$

$$V_1 = 150 - (72 + j104)$$

$$V_1 = \underline{78 - j104} \text{ V}$$

$$V_2 = \underline{72 + j104} \text{ V}$$

$$V_3 = (72 + j104) - (-78 + j234)$$

$$V_3 = \underline{150 - j130 \text{ V}}$$

$$I_1 = \frac{V_1}{Z} = \frac{78 - j104}{1 + j2} =$$

$$I_2 = \frac{V_2}{Z} = \frac{72 + j104}{12 - j16} =$$

$$I_3 = \frac{V_3}{Z} = \frac{150 - j130}{1 + j3} =$$

$$I_1 = -26 - j52 \text{ A} \leftarrow$$

$$I_2 = -2 + j6 \text{ A}$$

$$I_3 = -24 - j58 \text{ A}$$

$$S = \frac{1}{2} V_m I_m^*$$

$$= V_{rms} I_{rms}^*$$

$$S_1 = \frac{1}{2} (78 - j104) (-26 + j52)$$

$$S_1 = 1690 + j3380 \text{ VA}$$

$$S_2 = \frac{1}{2} (72 + j104) (-2 - j6)$$

$$S_2 = 240 - j320 \text{ VA}$$

$$S_3 = \frac{1}{2} (150 - j130) (-24 + j58)$$

$$S_3 = 1970 + j5910 \text{ VA}$$

Another solution

$$P = \frac{1}{2} R |\mathbf{I}_R|^2 = \frac{1}{2} \frac{|\mathbf{V}_R|^2}{R} \rightarrow P_1 = \frac{1}{2} (1) |\mathbf{I}_1|^2 = \frac{1}{2} (1) \left(\sqrt{(-26)^2 + (-52)^2} \right)^2 = \underline{\underline{1690 \text{ W}}}$$

$$\text{OR } \mathbf{V}_R = \frac{1}{1+j2} (\mathbf{V}_1) \rightarrow P_1 = \frac{1}{2} \frac{|\mathbf{V}_R|^2}{1} = \underline{\underline{1690 \text{ W}}}$$

Similarly

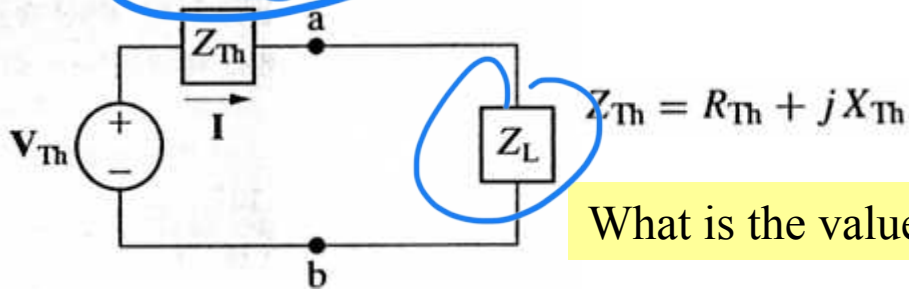
$$Q = \frac{1}{2} X |\mathbf{I}_X|^2 = \frac{1}{2} \frac{|\mathbf{V}_X|^2}{X} \rightarrow Q_1 = \frac{1}{2} (2) |\mathbf{I}_1|^2 = \frac{1}{2} (2) \left(\sqrt{(-26)^2 + (-52)^2} \right)^2 = \underline{\underline{3380 \text{ VAR}}}$$

$$\text{OR } \mathbf{V}_X = \frac{j2}{1+j2} (\mathbf{V}_1) \rightarrow Q_1 = \frac{1}{2} \frac{|\mathbf{V}_X|^2}{2} = \underline{\underline{3380 \text{ VAR}}}$$

$$> Q_L = Q_H$$

10.5 Maximum Power Transfer

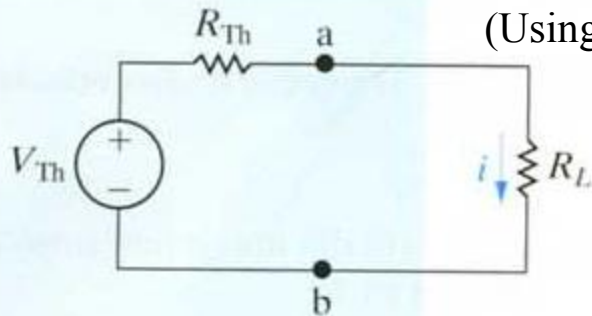
$\Rightarrow$ Thévenin



What is the value of Z_L that will absorb maximum power

For maximum power transfer :

Recall



(Using rms values)

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

$R_L = R_{Th}$

$X_L = X_{Th}^*$

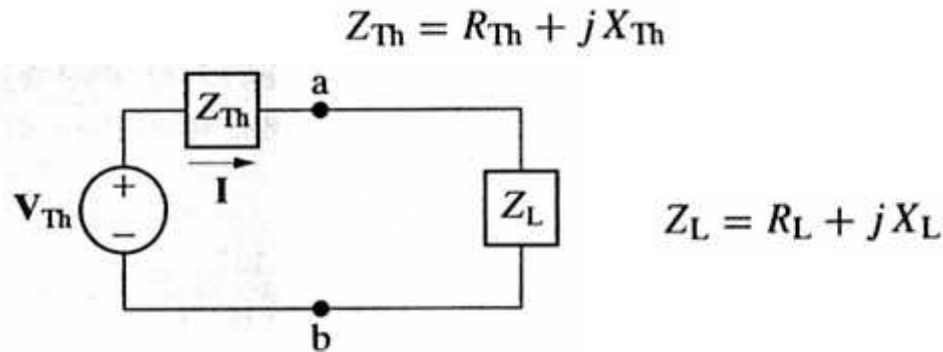
$$\frac{dp}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 - R_L \cdot 2(R_{Th} + R_L)}{(R_{Th} + R_L)^4} \right] = 0$$



$R_L = R_{Th}$

$Z_L = Z_{Th}^*$

Similarly



load current I is
$$I = \frac{V_{Th}}{(R_{Th} + R_L) + j(X_{Th} + X_L)}$$

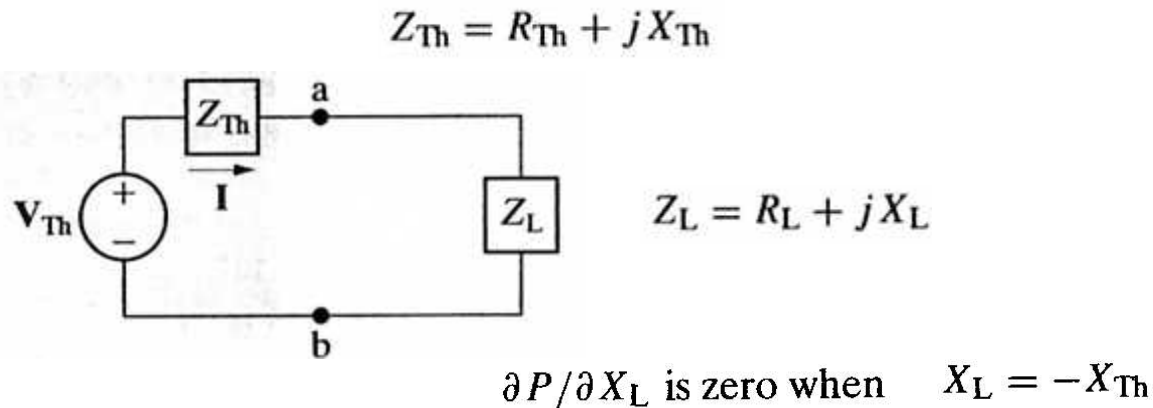
The average power delivered to the load is
$$P = |I|^2 R_L \qquad P = \frac{|V_{Th}|^2 R_L}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}$$

$$\frac{\partial P}{\partial X_L} = \frac{-|V_{Th}|^2 2R_L(X_L + X_{Th})}{[(R_L + R_{Th})^2 + (X_L + X_{Th})^2]^2}$$

$\partial P / \partial X_L$ is zero when

$$X_L = -X_{Th}$$

Similarly



$$\frac{\partial P}{\partial R_L} = \frac{|V_{Th}|^2 [(R_L + R_{Th})^2 + (X_L + X_{Th})^2 - 2R_L(R_L + R_{Th})]}{[(R_L + R_{Th})^2 + (X_L + X_{Th})^2]^2}$$

$\partial P / \partial R_L$ is zero when $R_L = \sqrt{R_{Th}^2 + (X_L + X_{Th})^2}$ $\rightarrow$ **=0 (since $X_L = -X_{th}$)**

$R_L = R_{Th}$

$\rightarrow$ **$Z_L = Z_{Th}^*$**

$Z_L = R_{Th} - jX_{Th}$

Example 10.11 Finding Max Power Transfer in a circuit with an Ideal Transfo.

Finding Max Power Transfer in a circuit.

- a) The value of R_L
- b) What is the maximum average power delivered to R_L

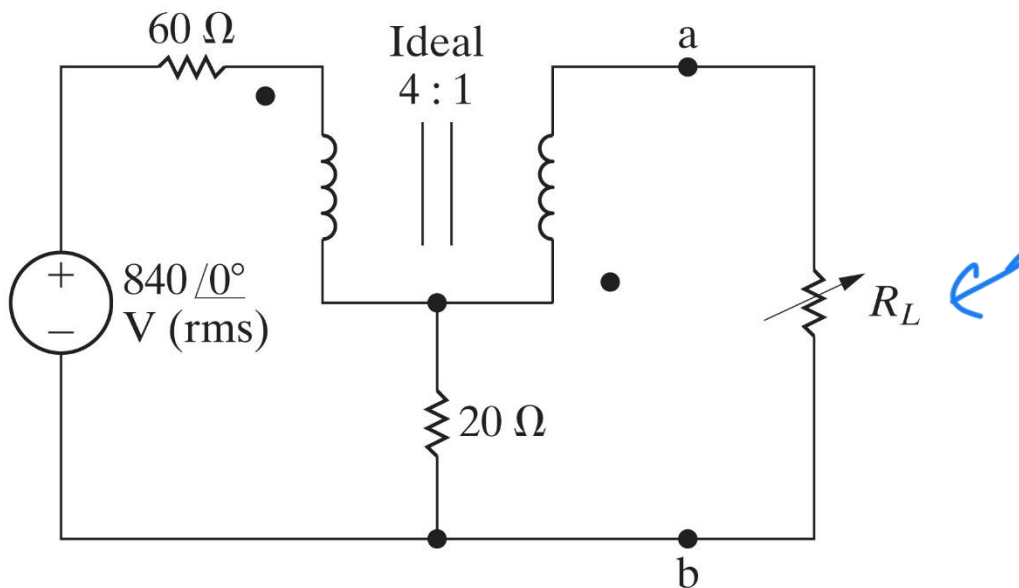


Figure: 10-24Ex10.11

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$$-840 + 60i_1 + N_1 + 20i_1 \neq 0$$

$$\Rightarrow \boxed{20i_1 + V_1 = 840} \quad \text{--- (1)}$$

$$\frac{V_1}{V_2} = -\frac{N_1}{N_2} \quad , \quad \frac{C_2}{C_1} = -\frac{N_1}{N_2}$$

$$N_1 = -\frac{N_1}{N_2} V_2$$

$$\Rightarrow \boxed{V_1 = -4 V_2} \rightarrow \textcircled{2}$$

$$C_2 = -\frac{N_1}{N_2} C_1$$

$$\Rightarrow \boxed{C_2 = -4 C_1} \Rightarrow \boxed{C_1 = 0}$$

$$\rightarrow \boxed{V_1 = 840 \text{ V}}$$

$$V_2 = \frac{-V_1}{4} = -\frac{840}{4}$$

$$\boxed{V_2 = -210 \text{ V}}$$

$$\rightarrow \textcircled{-V_{0,0}} + V_2 = 0$$

$$V_{o.c} = V_Z = \boxed{-210 \text{ V}}$$

$$Z_{Th} = \frac{V_{o.c}}{\underline{I_{s.c}}}$$



$$\begin{aligned}
 V_1 &= -4V_2 \\
 I_2 &= -4C_1
 \end{aligned}
 \rightarrow \begin{matrix} 3 \\ 4 \end{matrix}$$

$$-840 + 60C_1 + V_1 + 20(I_1 - I_2) = 0$$

$$80C_1 - 20C_2 + V_1 = 840 \rightarrow 1$$

$$-V_2 + 20(I_2 - I_1) = 0$$

$$\Rightarrow \boxed{-20 C_1 + 20 C_2 - \sqrt{2} = 0} \quad (2)$$

$$\begin{aligned} \Rightarrow 80 C_1 + 20(-4i) - 4\sqrt{2} &= 840 \\ -20 C_1 + 20(-4i) - \sqrt{2} &= 0 \end{aligned}$$

$$\begin{aligned} \Rightarrow 160 C_1 - 4\sqrt{2} &= 840 \\ \sim 100 C_1 - \sqrt{2} &= 0 \end{aligned}$$

mode $\rightarrow$ S-1

$$I_1 = 1.5 \text{ A} \quad V_2 = -150$$

$$I_2 = -4 I_1 = -4 \times 1.5$$

$$I_{sc} = -6 \text{ A}$$

$$Z_{in} = \frac{V_{oc}}{I_{sc}} = \frac{150}{-6} = -25 \Omega$$

Example 10.11 Finding Max Power Transfer in a circuit with an Ideal Transfo.

a) The value of $R_L = \frac{V_{Th}}{I_{SC}} = \frac{-210}{-6} = 35\Omega$

b) The maximum average power delivered to R_L is $P_{max} = \left(\frac{-210}{70}\right)^2 \times 35 = 315W$

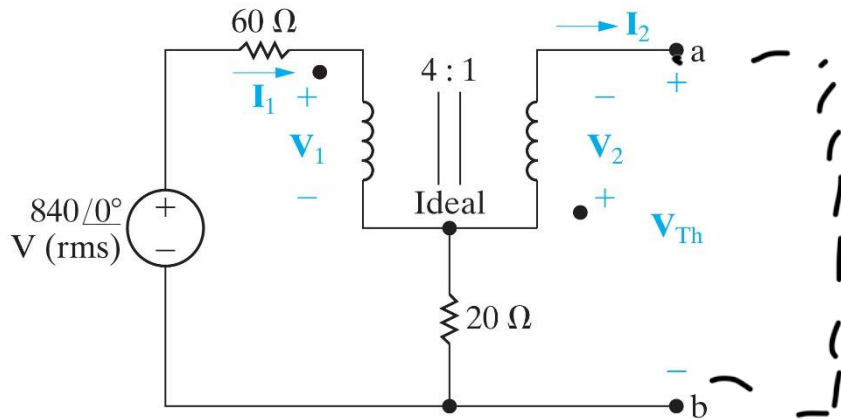


Figure: 10-25Ex10.11

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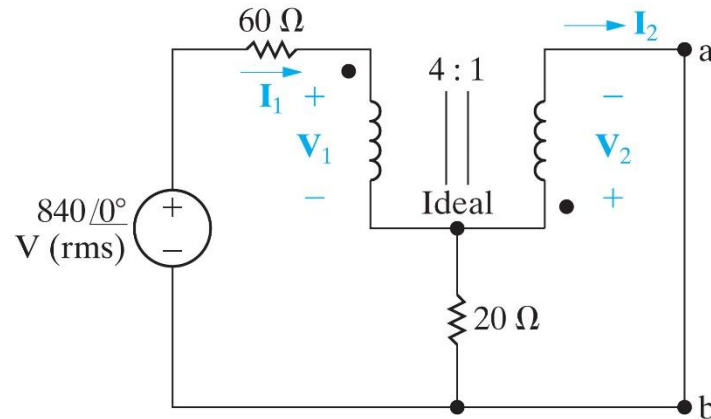


Figure: 10-26Ex10.11

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$$P_{max} = \frac{V_{Th}^2}{4 \times R_{Th}} = \frac{(-210)^2}{4 \times 35} = 315W$$