



MECH 213 Mechanics of Materials

Week 2:

Equilibrium of a Deformable Body

(From Statics to Internal Forces)

Why This Topic Matters?!



“It bridges the gap between calculating external forces and analyzing internal material responses”

- Statics tells us **what forces act externally**.
- Mechanics of Materials tells us **what happens inside the material** upon the applications of external forces.
- Every stress, strain, bending, or torsion analysis **starts here**.
- For a structure to be safe and standing, it **must satisfy the equilibrium equations**: $\Sigma F = 0$ and $\Sigma M = 0$.
- Failure to maintain this equilibrium state leads to **catastrophic outcomes like collapse or rupture**.
- Knowing how a body maintains equilibrium while deforming **allows engineers to predict when it will transition from elastic strain (reversible) to plastic (permanent deformation)** and eventually to failure.

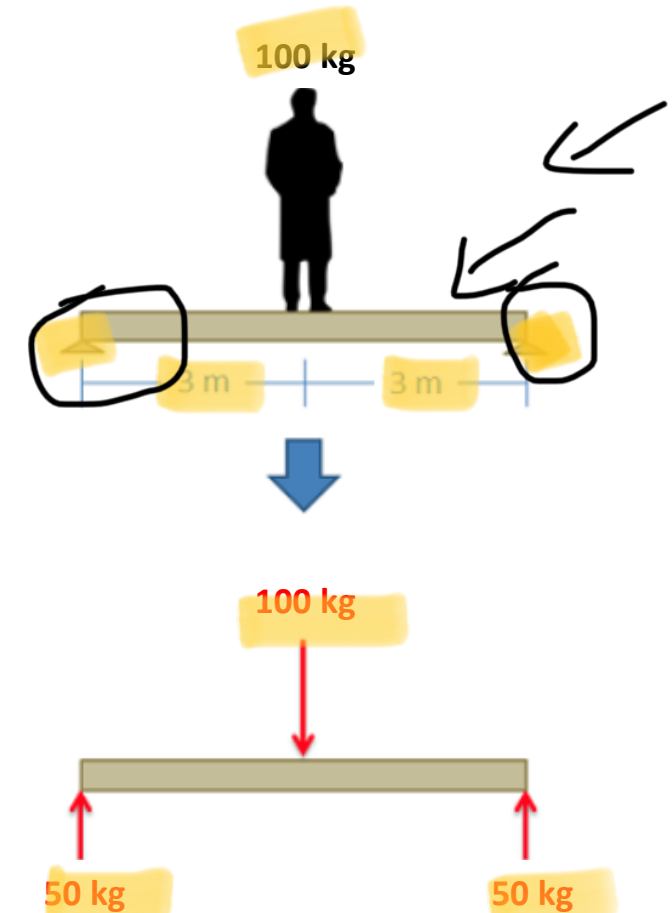


Fig.1.1: Visual Example of Equilibrium

Source: Engineering Mechanics, Jacob Moore, et al.

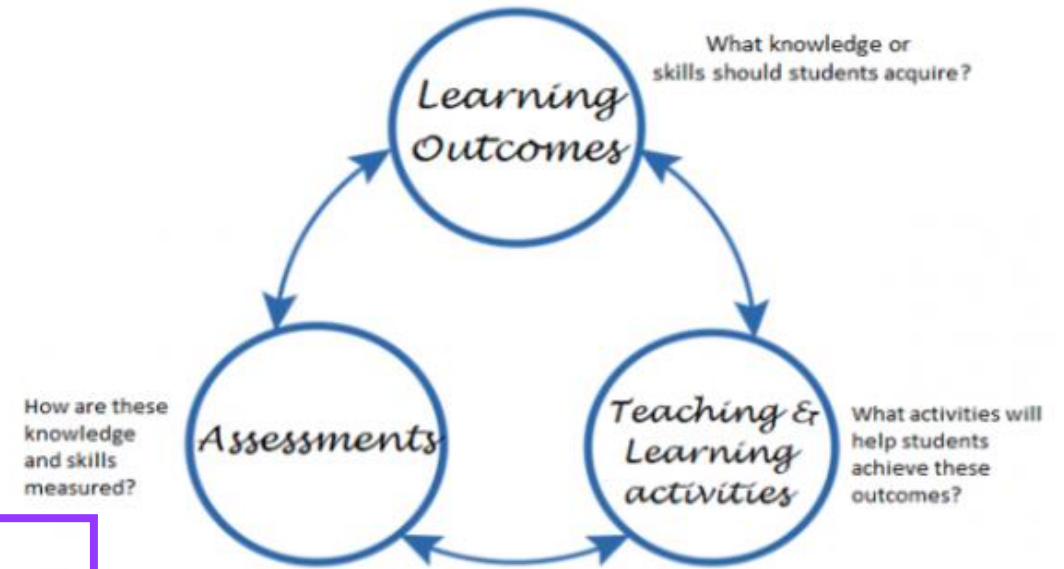


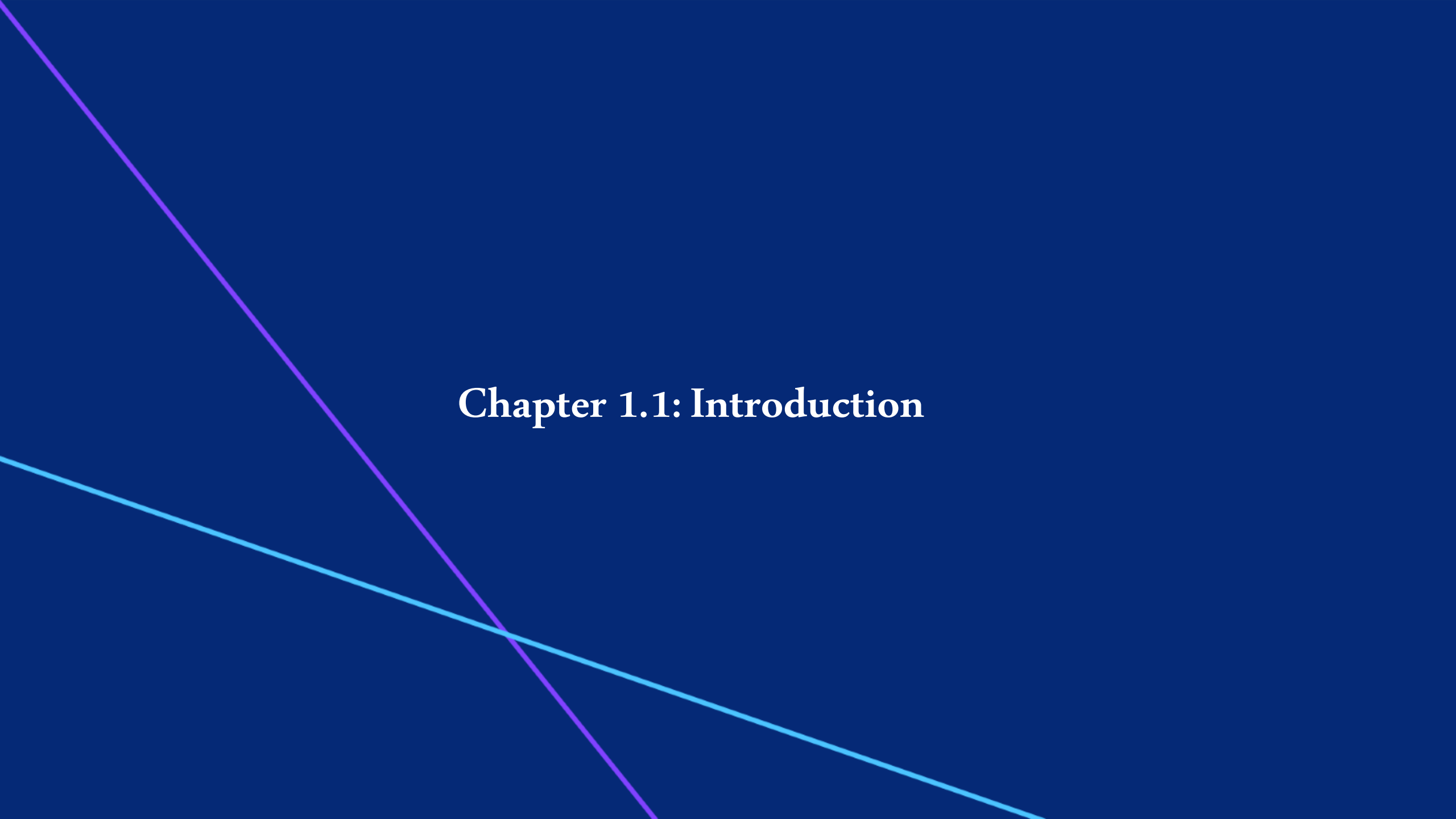
1. Distinguish external loads and internal resultants.

2. Draw a correct Free-Body Diagram (FBD).

3. Apply equilibrium equations to a deformable body.

4. Identify internal forces and moments at a section





Chapter 1.1: Introduction



Definition of Mechanics of Materials

Mechanics of Materials studies:

1. Internal effects of **stress** and **strain**.
2. Relationship between:
 - **External loads**
 - **Internal forces**
 - **Material response**
3. Stress $\rightarrow$ **Strength** $\rightarrow$???
4. **Strain** $\rightarrow$??? *stiffness*
5. **Importance:** Essential for designing machines and structures, as many engineering formulas and design rules derived from this principles.

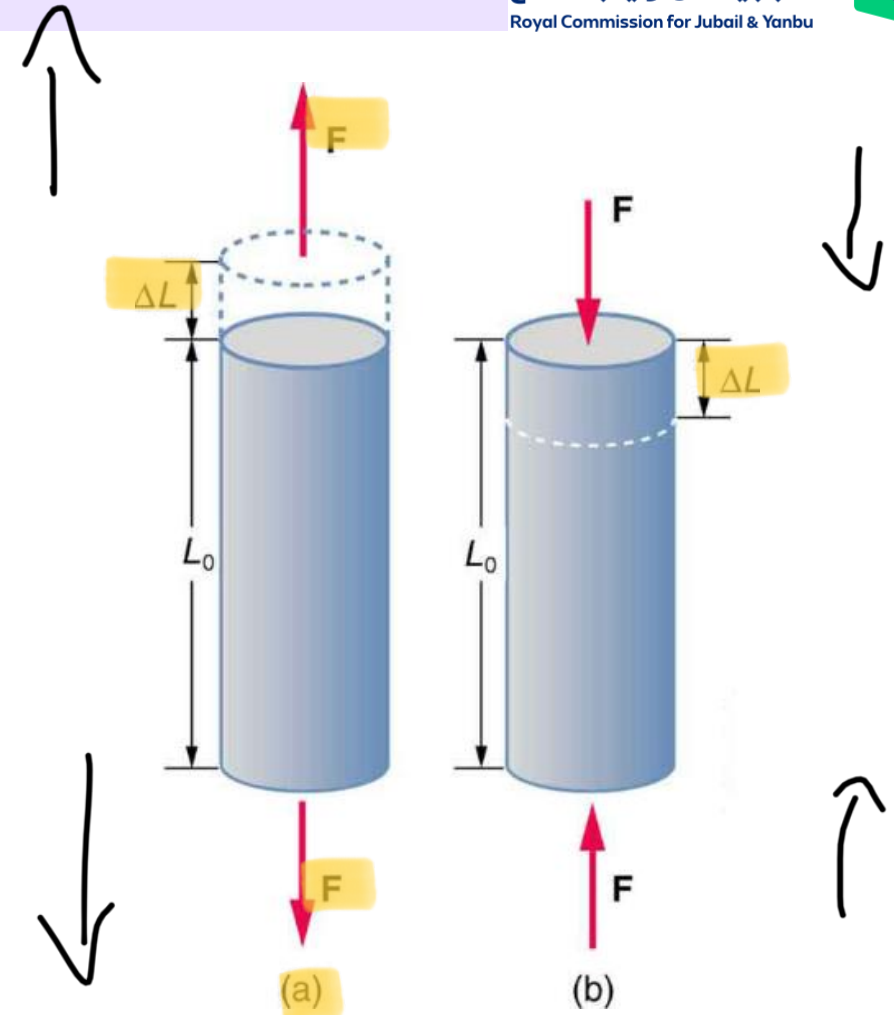


Fig 1.2: External Forces (a: **tensile** – b: **compression**) & Beam Deformation

Key Transition Concept



Statics **vs** Mechanics of Materials (MoM)

Statics	Mechanics of Materials
• External forces	• Internal forces
• Rigid body	• Deformable body
• Equilibrium only	• Equilibrium + Material's behavior

Statics → foundation

MoM builds on it

Basic Definitions (Used in Equilibrium of a Deformable Body)

- Equilibrium →
- Scalar →
- Vector →
- Force →
- Moment →
- Couple →

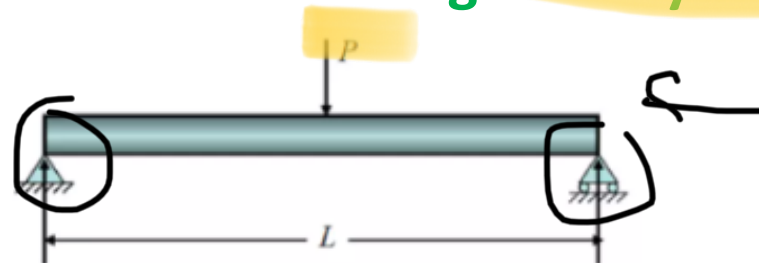
Chapter 1.2: Equilibrium of a Deformable Body

Types of Loads



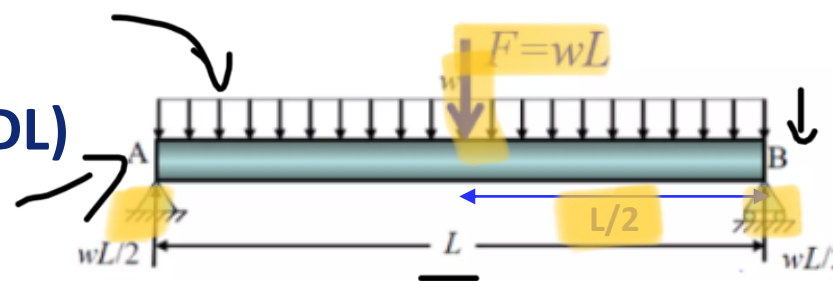
1. Surface Forces/Surface Loads → forces acting directly on the surface of a body

a. Concentrated Load → ?



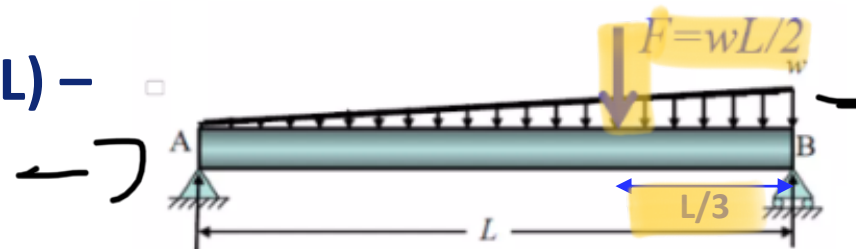
b. Distributed Load → ?

i. Uniformly Distributed Load (UDL)



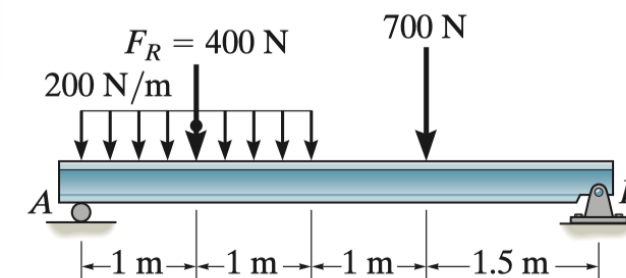
$F = F_R$ = resultant force
 w = load intensity

ii. Uniformly Varying Load (UDL) – Triangular Load



Magnitude of F_R = total area of loading shape
Location of F_R = centroid of that shape

2. Body Forces → Weight of a Body (effect of gravitation)





Surface force(s) develop at the supports or point of contact between bodies.

- No translation of body in given direction(s) → support reaction(s) developed in that direction.
- Rotation is prevented → a couple moment exerted on the body. ↩

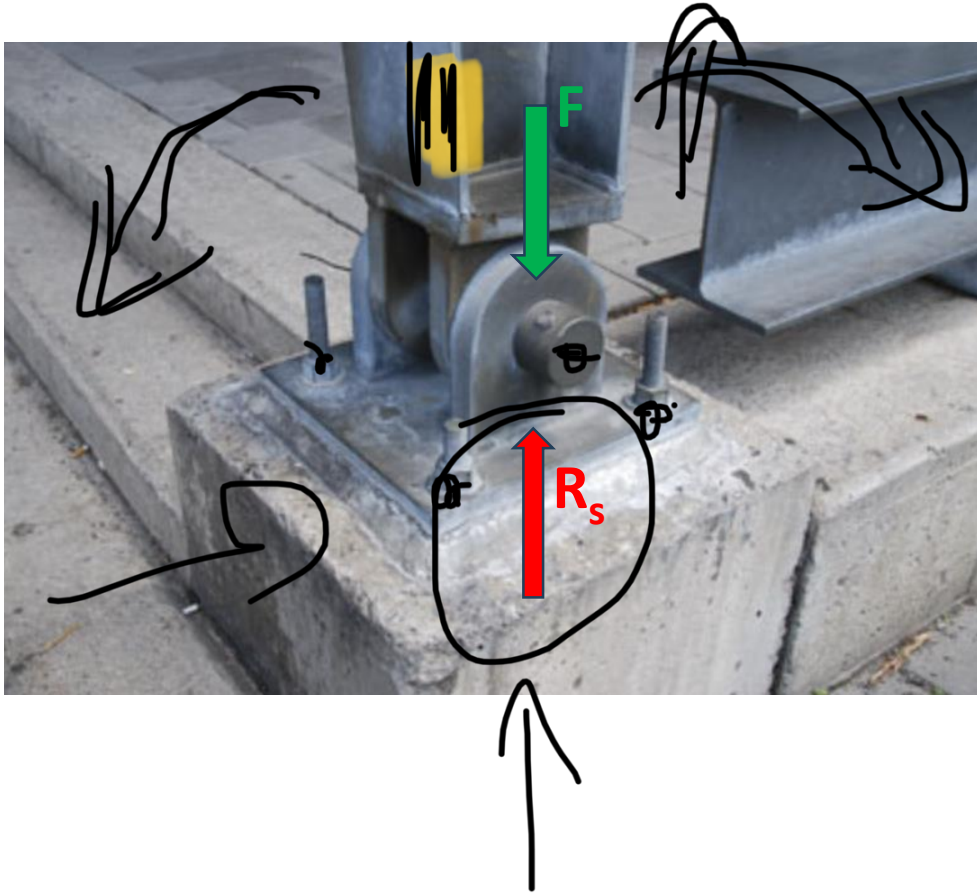
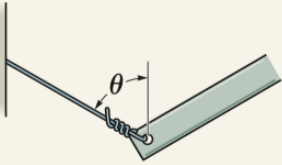
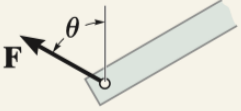
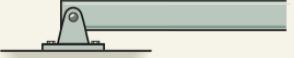
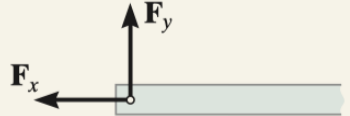
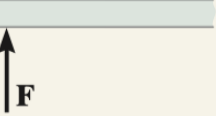
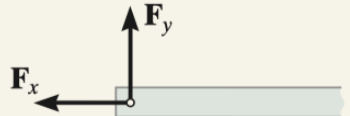
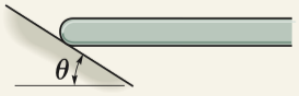
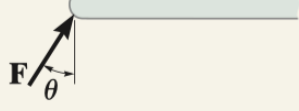
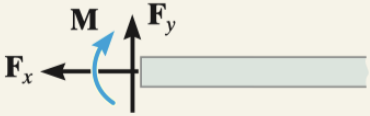
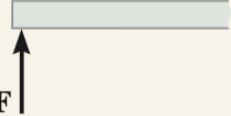
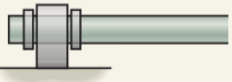
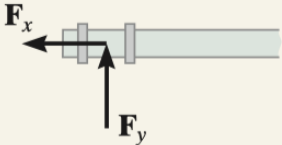


Fig.1.3: Base – Pin Connection, to enable free rotation at its connection. This support exerts a force on a member, but no moment.



Support Reactions

Types of Connection and Reaction:

TABLE 1-1			
Type of connection	Reaction	Type of connection	Reaction
 Cable	 One unknown: F	 External pin	 Two unknowns: F_x, F_y
 Roller	 One unknown: F	 Internal pin	 Two unknowns: F_x, F_y
 Smooth support	 One unknown: F	 Fixed support	 Three unknowns: F_x, F_y, M
 Journal bearing	 One unknown: F	 Thrust bearing	 Two unknowns: F_x, F_y

Equations of Equilibrium

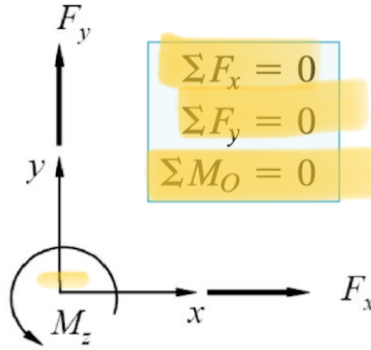
Balance of Forces → to prevent the body from translating or having accelerated motion.

Balance of Moments → to prevent the body from rotating.

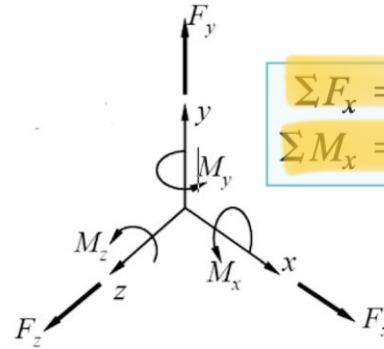
$$\Sigma \mathbf{F} = 0$$

$$\Sigma \mathbf{M}_O = 0$$

Sum of all forces acting on body & sum of moments about any point 'O' must be zero.



2-D positive axis system



3-D positive axis system (Right-Hand)

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0$$

$$\Sigma M_x = 0 \quad \Sigma M_y = 0 \quad \Sigma M_z = 0$$

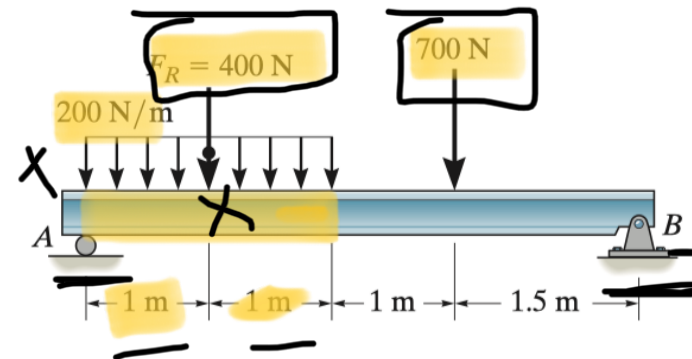
Free-body Diagram - FBD

Before solving → **DRAW FBD**

A correct FBD must show:

- All external forces
- All reaction forces
- Correct dimensions
- Clear coordinate system

Most mistakes come from bad FBDs, not equations



FBD

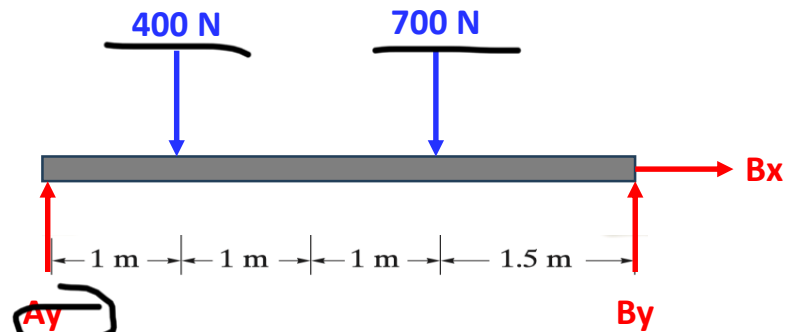


Fig.1.4: Drawing FBD



Internal Resultant Loadings

Method of Section → to determine the resultant loadings that act within a body.

“We cut to see what the material is doing internally.”

Steps:

- Cut the body at the point of interest.
- Isolate one segment.
- Replace internal effects with resultants.
- Apply equilibrium

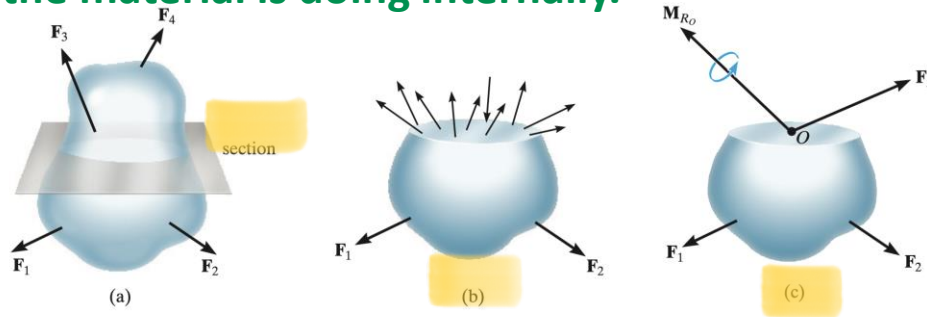


Fig.1.5: Method of Section

At a section, internal resultants are

- Normal Force (**N**) – axial
- Shear Force (**V**) – transverse
- Bending Moment (**M**)
- Torsional Moment (**T**)

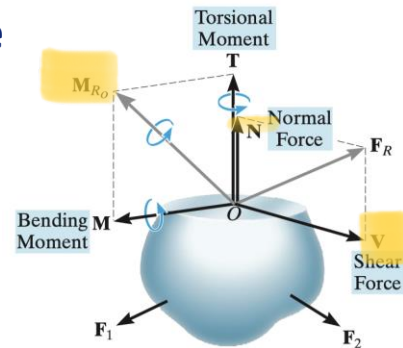


Fig.1.6: Four Different Types of Resultant Loadings

Coplanar Loading – All forces & moments lie within the same two-dimensional plane

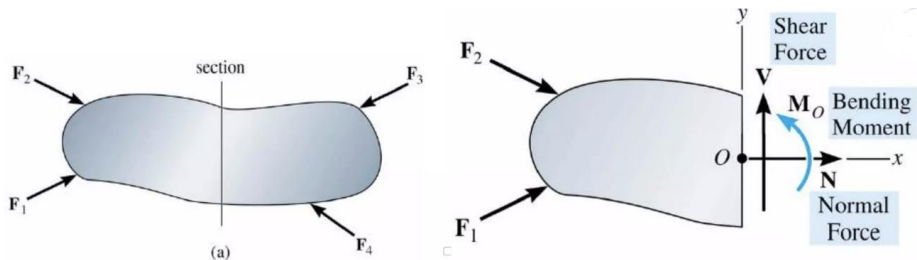


Fig.1.7: Coplanar Loading

For coplanar systems :

- Only **N**, **V**, **M** exist
- Torsion appears later (Week 9)

Procedure for Analysis “Standard Problem-Solving Procedure.”



Support Reactions.

- When the body is sectioned, decide which segment of the body is to be considered. If the segment has a support or connection to another body, then *before* the body is sectioned, it will be necessary to determine the reactions acting on the chosen segment. To do this, draw the free-body diagram of the *entire body* and then apply the necessary equations of equilibrium to obtain these reactions.



Free-Body Diagram.

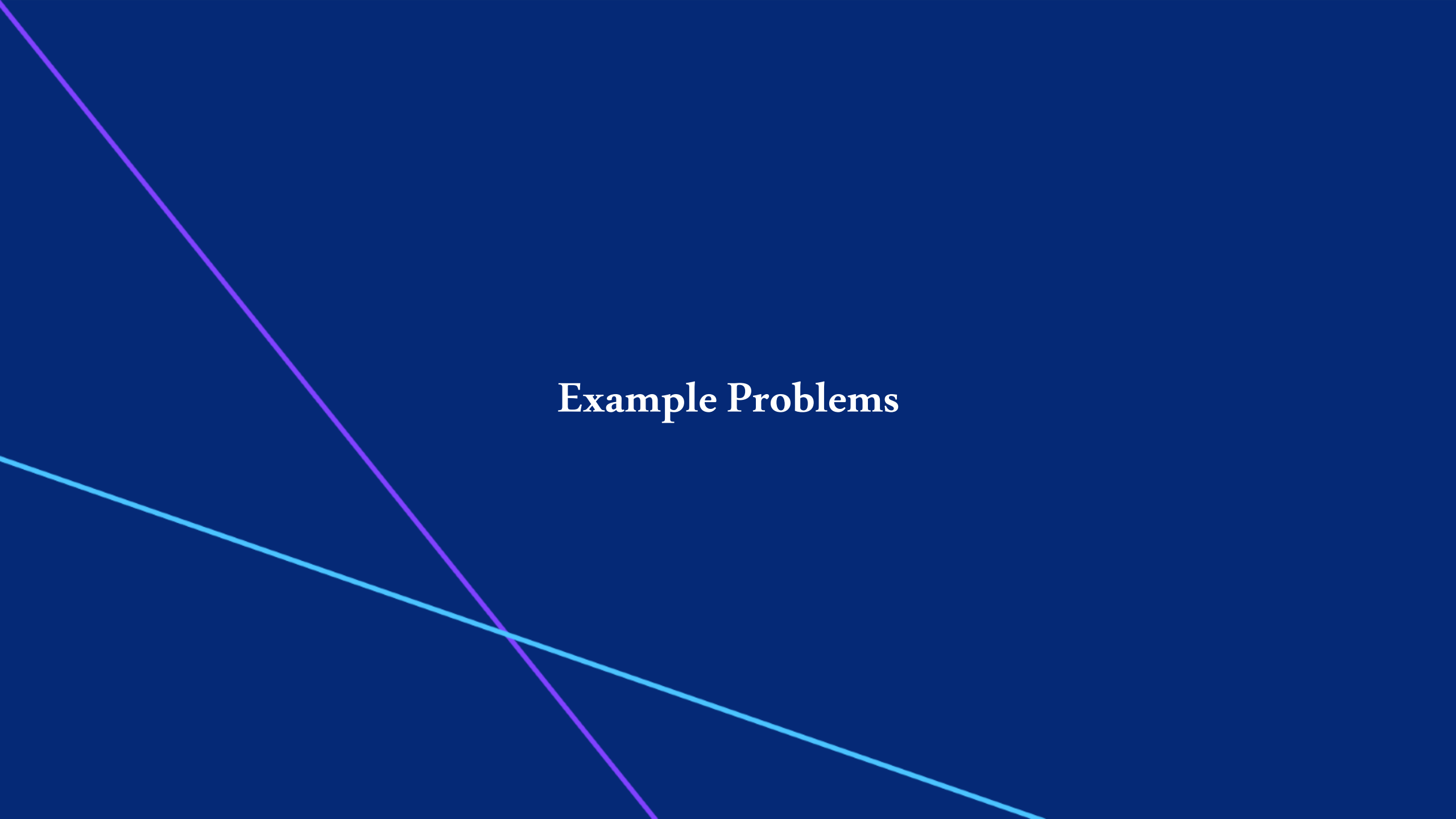
- Keep all external distributed loadings, couple moments, torques, and forces in their *exact locations*, before passing the section through the body at the point where the resultant internal loadings are to be determined.
- Draw a free-body diagram of one of the “cut” segments and indicate the unknown resultants N , V , M , and T at the section. These resultants are normally placed at the point representing the geometric center or *centroid* of the sectioned area.
- If the member is subjected to a *coplanar* system of forces, only N , V , and M act at the centroid.
- Establish the x , y , z coordinate axes with origin at the centroid and show the resultant internal loadings acting along the axes.



- Determine support reactions (if needed)
- Draw FBD
- Apply equilibrium equations
- Interpret signs

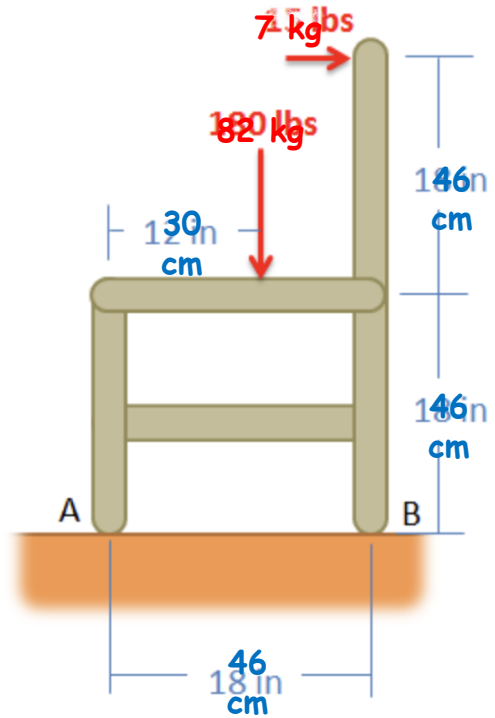
Equations of Equilibrium.

- Moments should be summed at the section, about each of the coordinate axes where the resultants act. Doing this eliminates the unknown forces N and V and allows a direct solution for M and T .
- If the solution of the equilibrium equations yields a negative value for a resultant, the *directional sense* of the resultant is *opposite* to that shown on the free-body diagram.

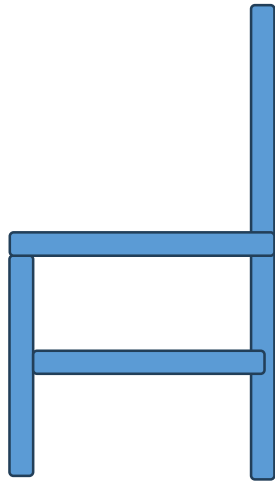


Example Problems

Simple Example of FBD & Equilibrium Equations



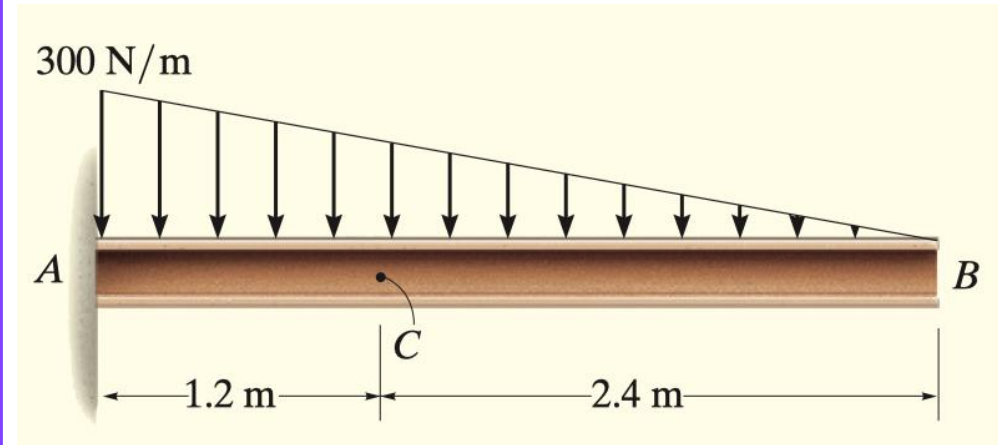
While sitting in a chair, a student exerts the forces as shown in the diagram below. Determine all forces acting on the chair at points A and B. (Assume A is frictionless and B is a rough surface).



Example 1.1 (p.29)

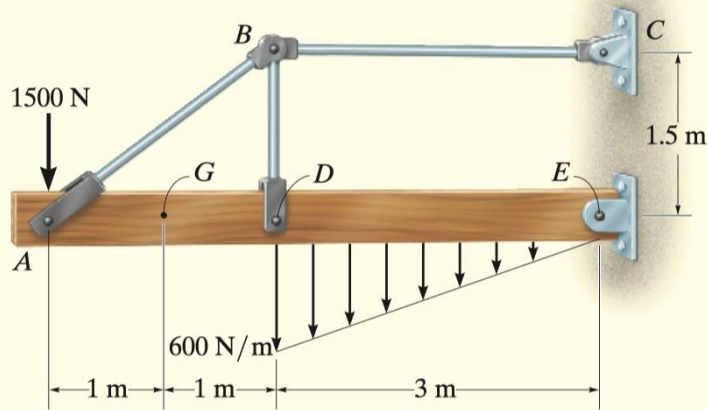


Determine the resultant internal loadings acting on the cross section at **C** of the beam shown in figure below.

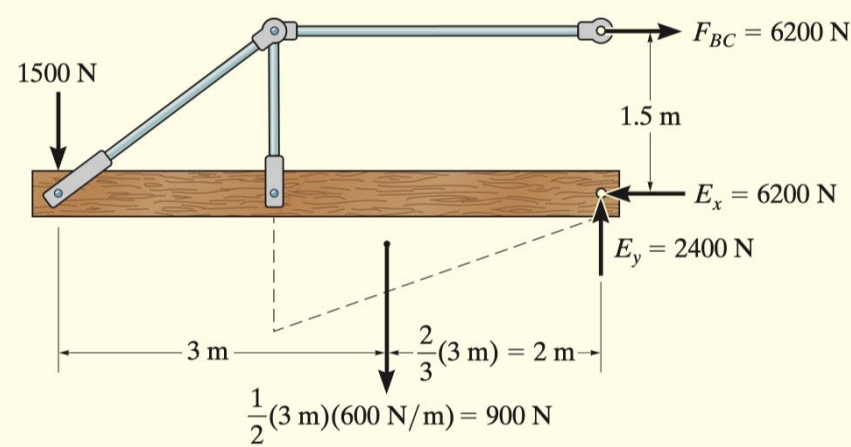




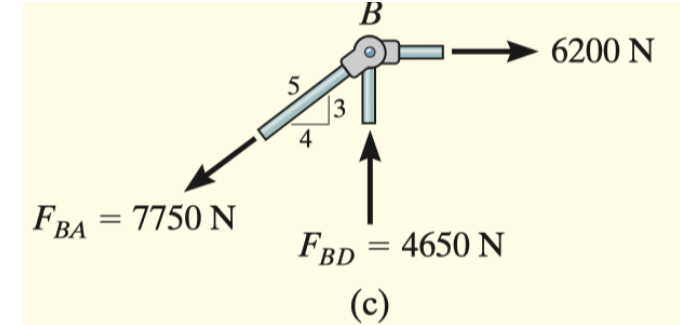
Example 1.3 (p.31)



(a)



(b)

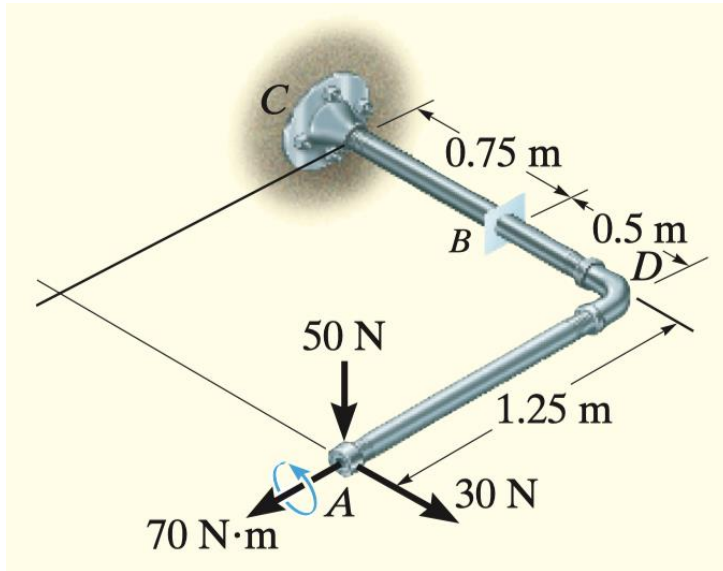


(c)

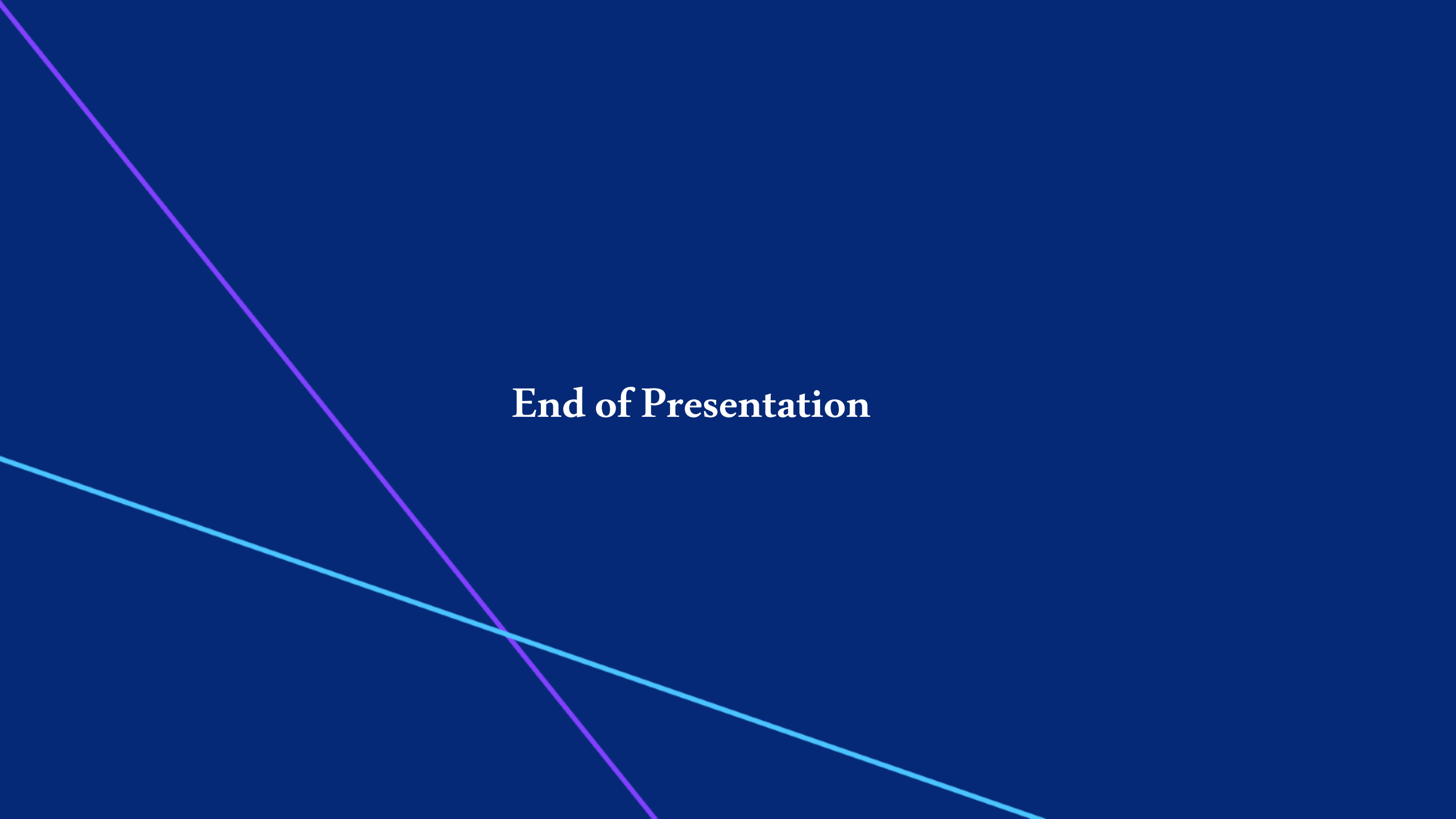
Each joint is pin connected

- Calculate the resultant internal loadings acting on the cross section at G.
- Verify the magnitude reactions at points C and E.
- Verify the magnitude of forces FBA and FBD.

Example 1.4 (p.32)



Calculate the resultant internal loadings acting on the cross section at **B**. Point **A** is subjected to a vertical force of **50 N**, a horizontal force of **30 N**, and a couple moment of **70 N.m**. Neglect the pipe's mass.



End of Presentation