جامعة الأميرة
نورة بنت عبدالرحمن
كلية الهندسة

Signals and Systems
Topic 2- Signals and systems overview

Course Code: ECE 270

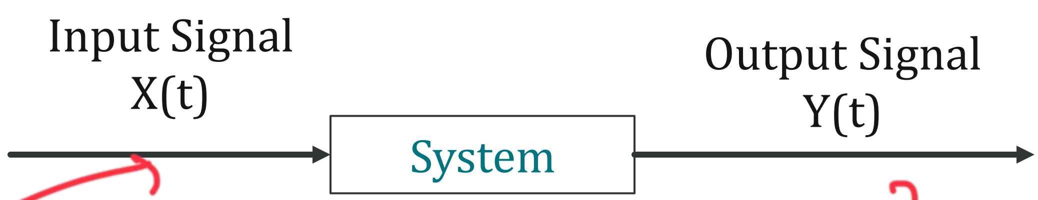
Summary

- What is Signal and System?
- What is Signal?
- Continuous- Digital- Descrete Signals
- Basic Types of Signals
- Representation of Discrete Time Signal (DTS)
- Classification of Signals
- Basic Operations on Signals

$+ \; - \; \times \; \div$

What is Signal and System?

- The topics of signals and systems as related to engineering.
- These topics involve the *modeling* of physical signals by mathematical functions,
- The *modeling* of physical systems by mathematical equations, and the solutions of the equations when excited by the functions.



What is Signal?

- Signal is representation of physical quantity (Sound, temperature, intensity, Pressure, etc..,), which varies with respect to time or space or independent or dependent variable.

$f(x) = x^2+5$

or

- Signal is single valued function which carries information by means of Amplitude, Frequency and Phase.

Example: voice signal, video signal, signals on telephone wires etc.

قمتها

ترددها

متشابهة

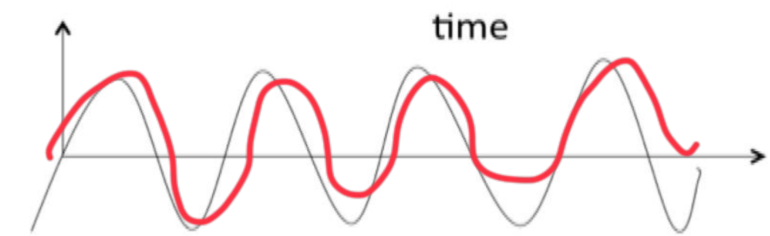
Sin function

What is Signal?

Sin

cos

$$\underline{X}(t) = A\,Sin(\omega t + \phi) = A\,Sin(2\pi f t + \phi)$$

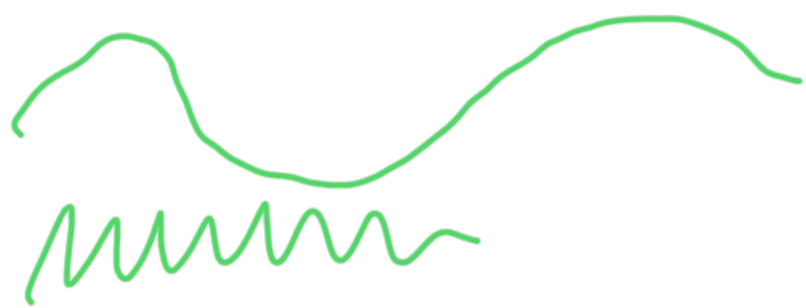


Where A = Peak Amplitude

f = Frequency = $\frac{1}{T\,(Time\ Period)}$

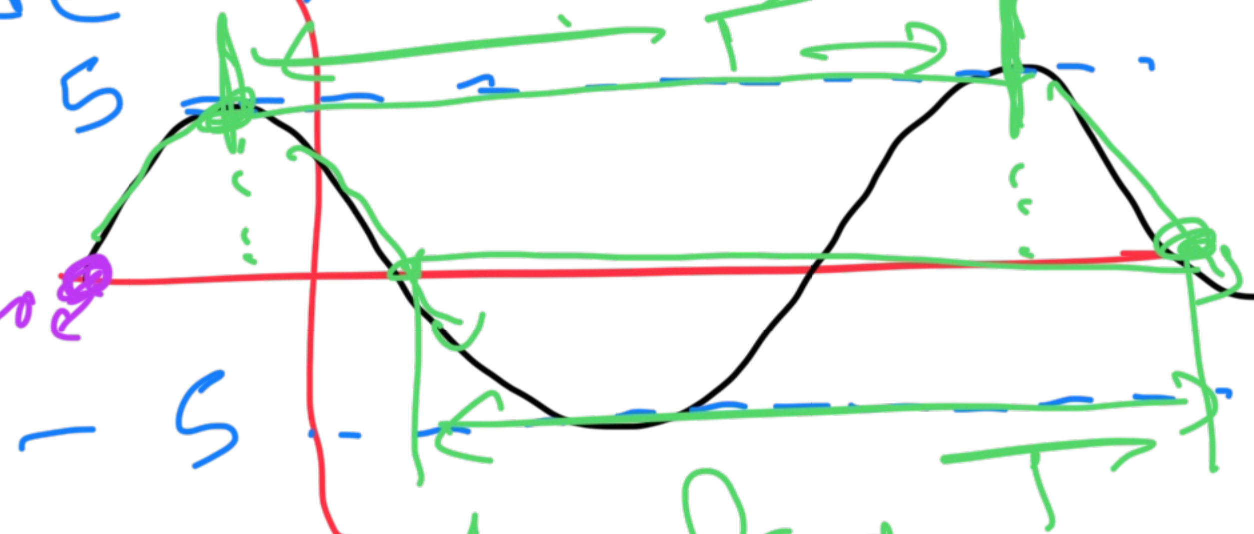
ϕ = Phase angle

Angular Frequency $\omega = 2\pi f$ (Linear Frequency)



$$x(t) = A \sin(\omega t + \phi)$$

Amplitude → $A = 5$



ϕ زاوية البداية

$\omega = 2\pi f$ → Angular Freq

f = freq " التردد "

$$f = \frac{1}{T}$$

T : period = متى تتكرر الدالة مرة أخرى؟!

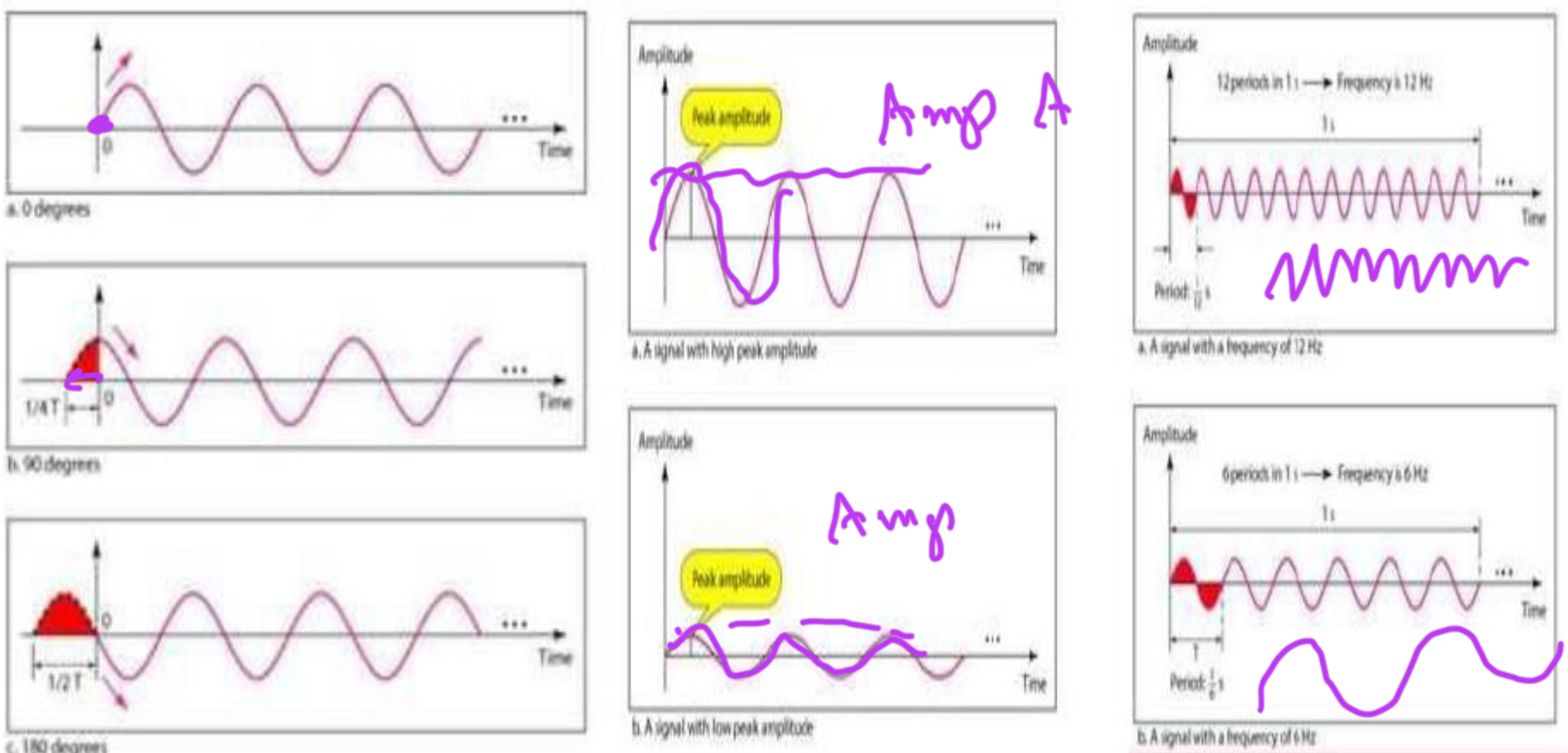
ϕ : phase angle = متى تبدأ الدالة

البداية القمة التردد

What is Signal?

Signal with different Phases, Amplitudes and Frequencies

ϕ



freq

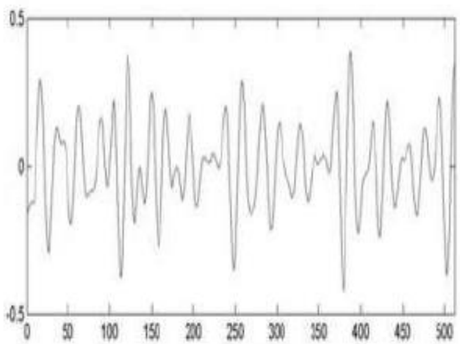
* مشتاحة عدد المتغيرات *

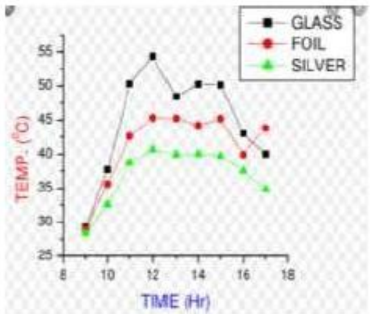
Classification of Signals

Types of Signals with respect to no. of variables or dimensions

- One Dimensional or 1-D Signal: If the signal is function of only one variable or If Signal value varies with respect to only one variable then it is called "One Dimensional or 1-D Signal"

  Examples: Audio Signal, Biomedical Signals, temperature Signal etc.., in which signal is function "time"





Signal Dimension

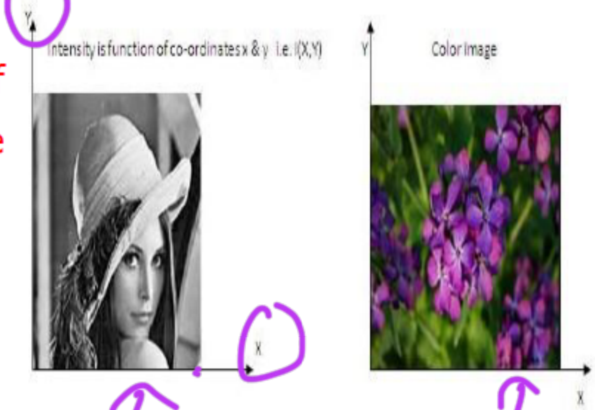
- **Two Dimensional or 2-D Signal:** If the signal is function of two variable or If Signal value varies with respect to two variable then it is called "Two Dimensional or 2-D Signal"

  Examples: Image Signal in which intensity is function of two spatial co-ordinates "X" & "Y" i,.e I (X,Y)

3D

- **Three Dimensional or 3-D Signal:** If the signal is function of three variable or If Signal value varies with respect to three variable then it is called "Three Dimensional or 3-D Signal"

  Examples: Video Signal in which intensity is function of two spatial co-ordinates "X" & "Y" and also time "t" i.e v(x,y,t)



A → B

3D

Continuous- Digital- Discrete Signals

من ناحية الاتصال/ الانقطاع

متصل

Types of Signal with respect to nature of the signal

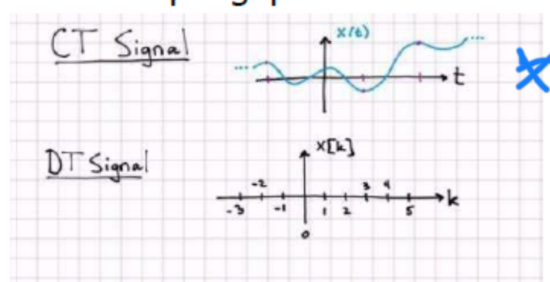
Continuous Time Signal (CTS) or Analog Signal :

If the signal values continuously varies with respect to time then it is called "Continuous Time Signal (CTS) or Analog Signal ". It contains infinite set of values and it is represented as shown below.

Digital Signal: If the signal contains only two values then it is called "Digital Signal".

Discrete Time Signal (DTS): متقطع

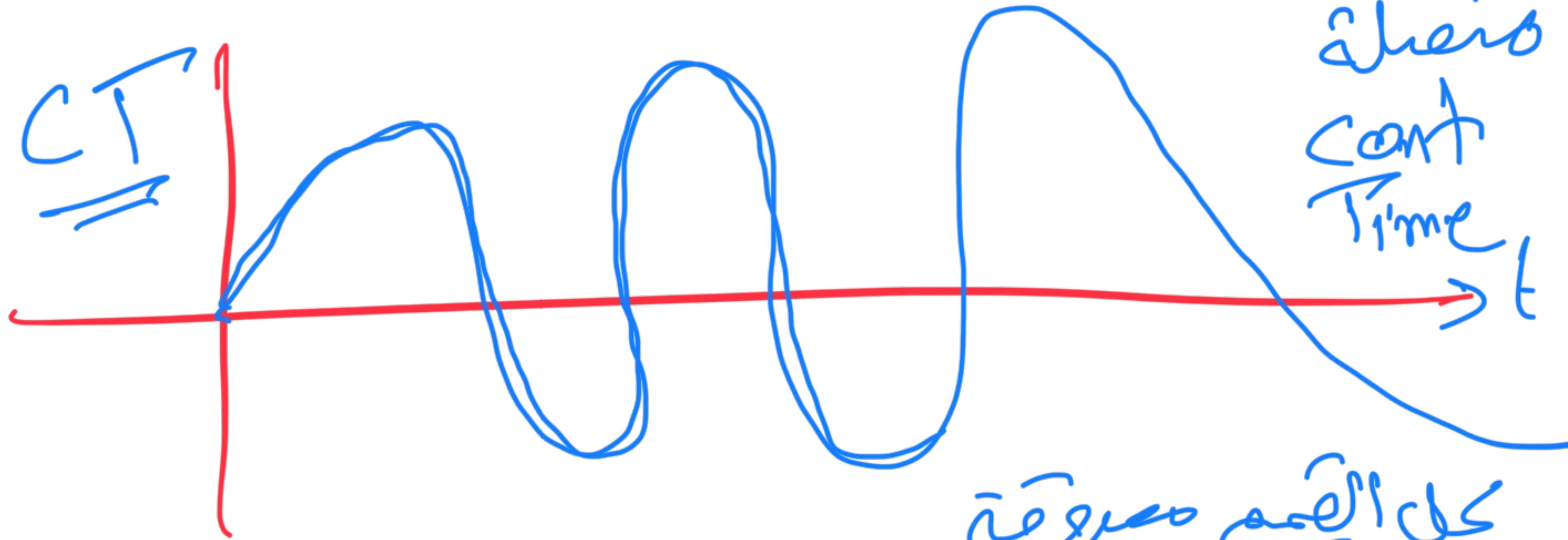
If signal contain discrete set of values with respect to time then it is called "Discrete Time Signal (DTS)". It contains finite set of values. Sampling process converts Continuous time signal in to Discrete time signal.



CT

متصلة

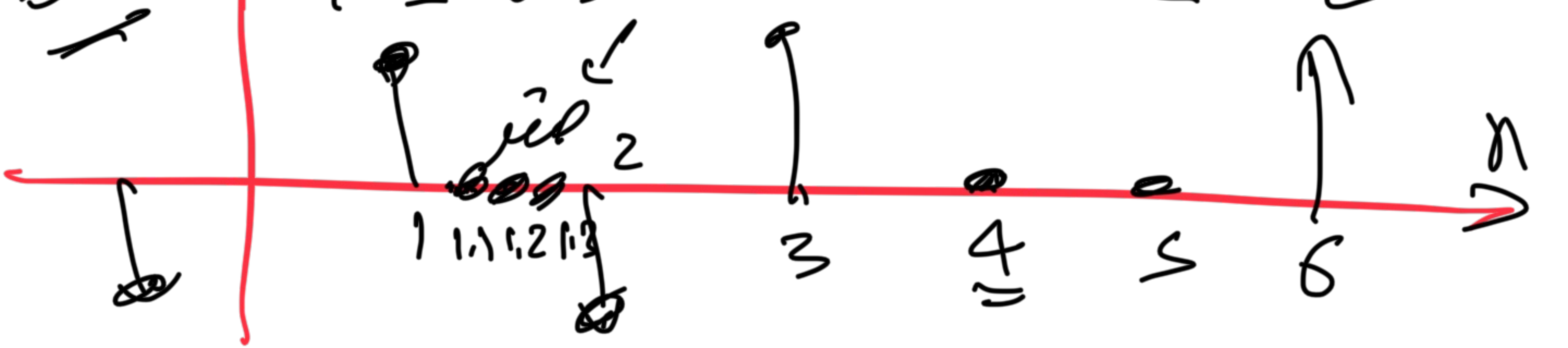
Cont Time



كل القيم معروفة

DT

متقطع "القيم معينة عند وقت معين



$f(x) = x^2$ * يمكنني حساب القواطع

$\hookrightarrow x=1,\quad x=1.1\quad,\quad x=1.2,\quad x=2$

DT $\quad n=0,\quad n=1 \quad n=2 \quad \dots$

قيم صحيحة ⟸ لا تحتوي كسور

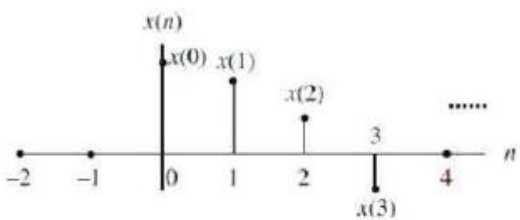
$n=1.3 \quad n=2.5$ قيم فيها فواصل

كل طول ⟸ صفر.

Continuous- Digital- Discrete Signals

Representation of Discrete Time Signal (DTS)

A discrete-time signal $x(n)$ is a function of an integer variable n. In the DS processor, the signal is represented by the discrete encoded values with a finite precision.



$$x(n) = \begin{cases} 1, & for\ n = 1,3 \\ 4, & for\ n = 2 \\ 0, & elsewhere \end{cases}$$

n	…	−2	−1	0	1	2	3	4	5	…
$x(n)$	…	0	0	0	1	4	1	0	0	…

Functional representation

Tabular representation

Graphical representation of a discrete-time signal $x(n)$

$x(n) = \{\dots, 0, \mathbf{0}, 1, 4, 1, 0, 0, \dots\}$ $\quad$ *infinite − duration signal*

$x(n) = \{\mathbf{0}, -2, 1, 4, -1,\}$ $\quad$ *finite − duration signal*

Sequence representation (bold or arrow for origin n=0)

Mathematically a discrete-time signal $x(n)$ can be determined by

$$x(n) = x(t)|_{t=nT} = x(nT)$$

غامق

سهم

n = 0

Ex $x[n] = \begin{cases} 1, & n = 1, 3 \\ 4, & n = 2 \\ 0, & \text{elsewhere} \end{cases}$

$n = 1 \longrightarrow x[1] = 1$

$n = 2 \longrightarrow x[2] = 4$

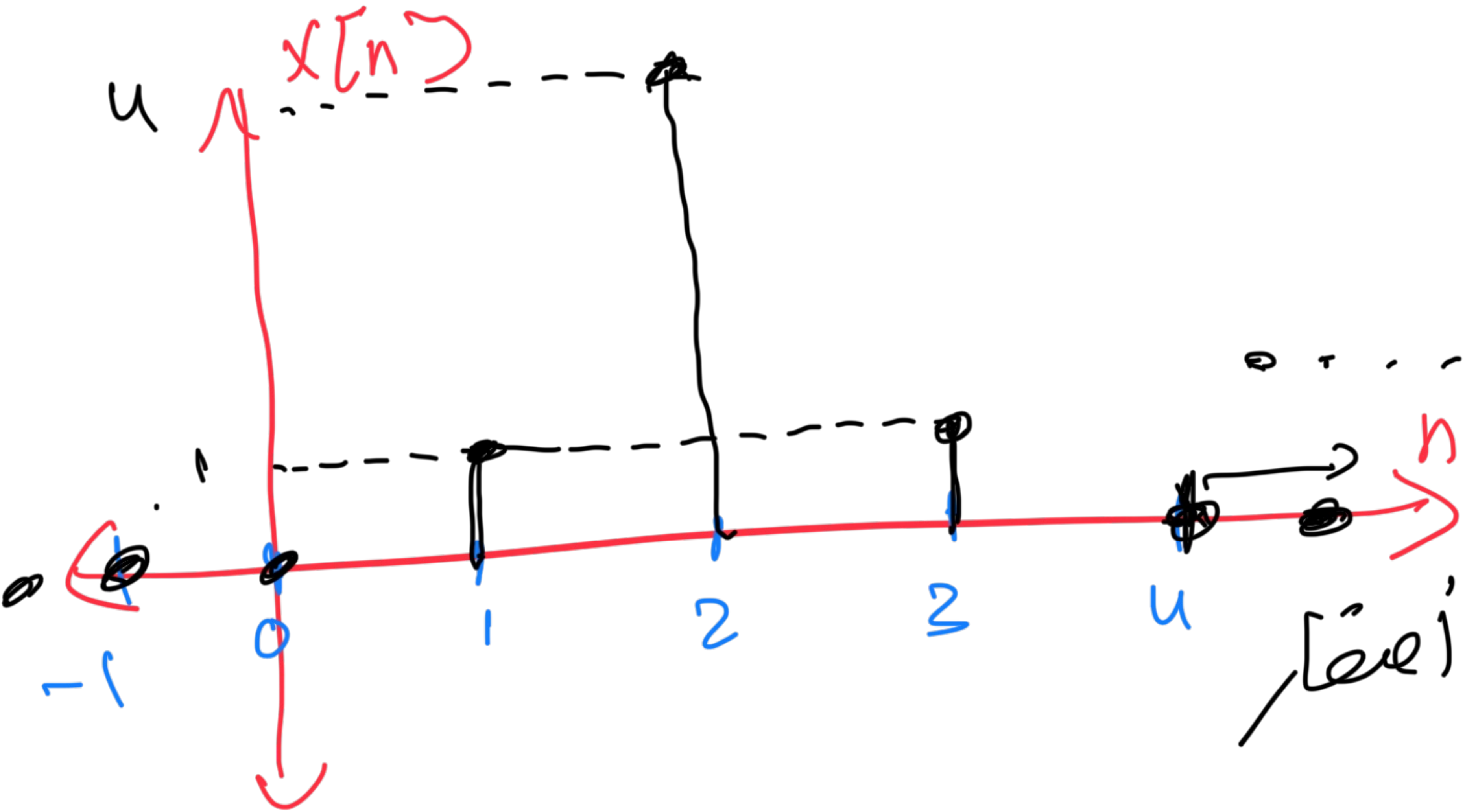
$n = 3 \longrightarrow x[3] = 1 \longleftarrow$

$n = 0 \longrightarrow x[0] = 0$

$n = 10 \longrightarrow x[10] = 0$

$n = 0$

$x[n] = \{ \underline{0}, \underset{n=1}{1}, \underset{n=2}{4}, \underset{n=3}{1}, 0, 0, 0, \ldots \}$



x[n]
4
1
n
-1
0
1
2
3
4

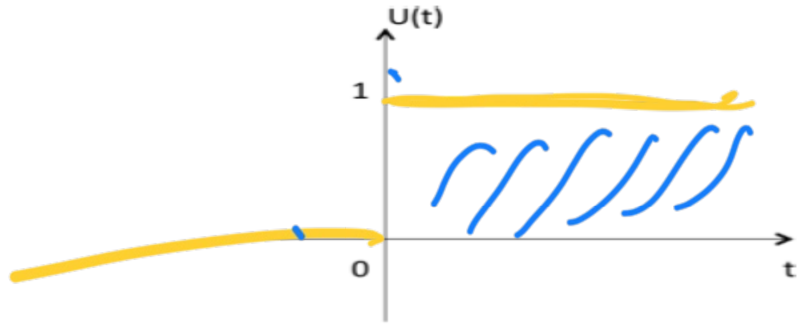
أنواع الدالة

Basic Types of Signals

1

- **Unit Step Function**

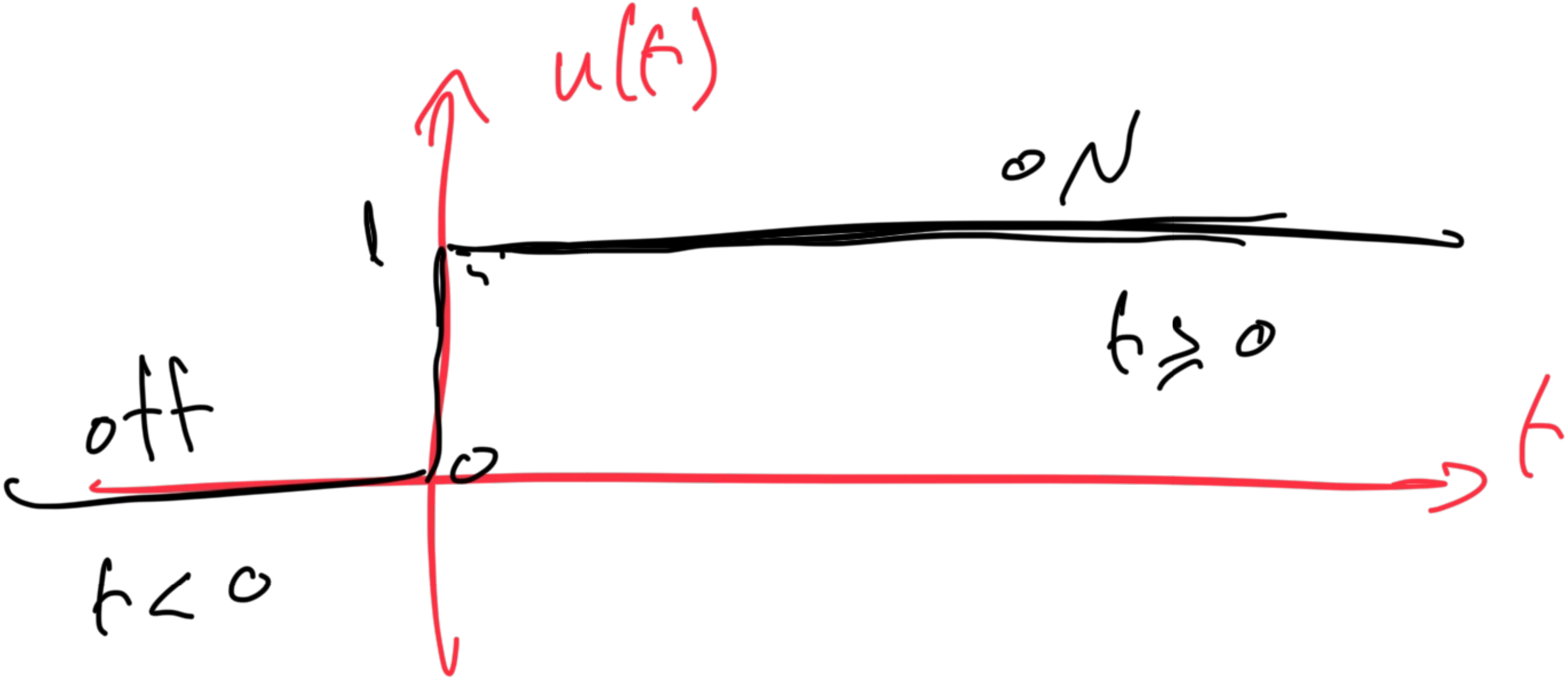
Unit step function is denoted by u(t). It is defined as $u(t) = 1$ when $t \geq 0$
and 0 when $t < 0$



$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$

- It is used as best test signal.
- Area under unit step function is unity.

circuits u(t) on/off switch

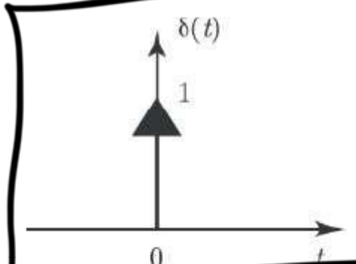


u(t)
oN
1
t ≥ 0
off
0
t
t < 0

Basic Types of Signals

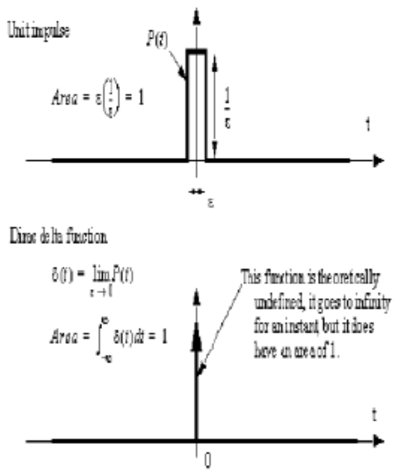
➢Unit Impulse Function

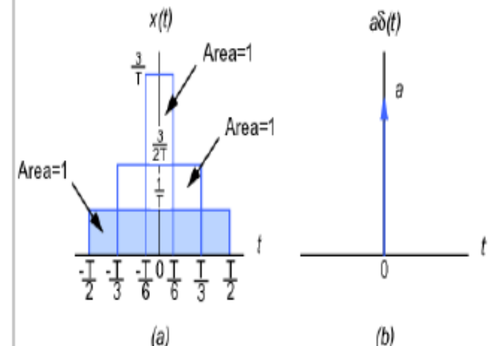
Impulse function is denoted by δ(t). and it is defined as $\delta(t) = \{\, 0; \quad t \neq 0$
$\infty; \quad t = 0 \,\}$



$$\int_{-\infty}^{\infty} \delta(t)dt = u(t)$$

$$\delta(t) = \frac{du(t)}{dt}$$





$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ \infty, & t = 0 \end{cases}$$

تشتغل عند قيمة واحدة فقط $t = 0$

وماعدا ذلك على طول صفر،

$\delta(t)$ $\qquad$ $u(t)$

$$\delta(t) = \frac{\partial u(t)}{\partial t}$$

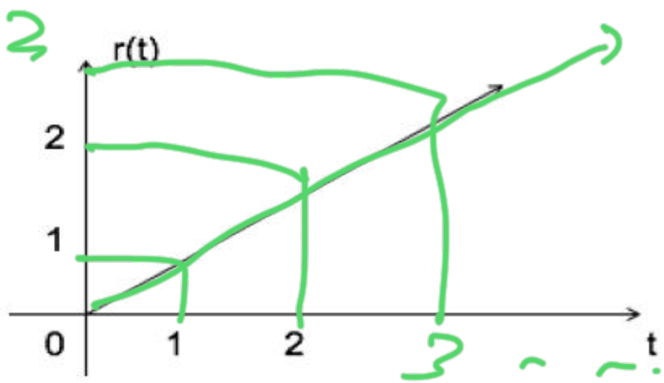
$$\int_{-\infty}^{\infty} \delta(t)\, \partial t = u(t)$$

$\neq 0$

Basic Types of Signals

3 Ramp Signal

Ramp signal is denoted by r(t), and it is defined as $r(t) = \begin{cases} t & t \geqslant 0 \\ 0 & t < 0 \end{cases}$



$$\int u(t) = \int 1 = t = r(t)$$

$$u(t) = \frac{dr(t)}{dt}$$

Area under unit ramp is unity.

r(t)

r(t)

t

$t \geq 0$

t

$$u(t) = \frac{\partial r(t)}{\partial t}$$

$$\int u(t)\,\partial t = r(t)$$

$$\boxed{r(t) = t\,u(t)}$$

Dados

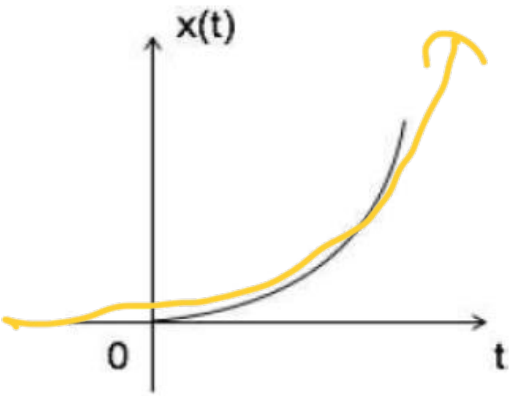
âreto

Basic Types of Signals

قوس

4 **Parabolic Signal**

Parabolic signal can be defined as $x(t) = \boxed{\begin{cases} t^2/2 & t \geqslant 0 \\ 0 & t < 0 \end{cases}}$



$$\iint u(t)dt = \int r(t)dt = \int tdt = \frac{t^2}{2} = parabolicsignal$$

$$\Rightarrow u(t) = \frac{d^2x(t)}{dt^2}$$

$$\Rightarrow r(t) = \frac{dx(t)}{dt}$$

$x(t) \longrightarrow$ parabolic

$$x(t) \underset{\int \partial t}{\overset{\partial/\partial t}{\rightleftarrows}} r(t) \underset{\int \partial t}{\overset{\partial/\partial t}{\rightleftarrows}} u(t) \underset{\int \partial t}{\overset{\partial/\partial t}{\rightleftarrows}} \delta(t)$$

$$x(t) = \int r(t) \quad , \quad r(t) = \frac{\partial x(t)}{\partial t}$$

$$r(t) = \int u(t) \quad , \quad u(t) = \frac{\partial r(t)}{\partial t}$$

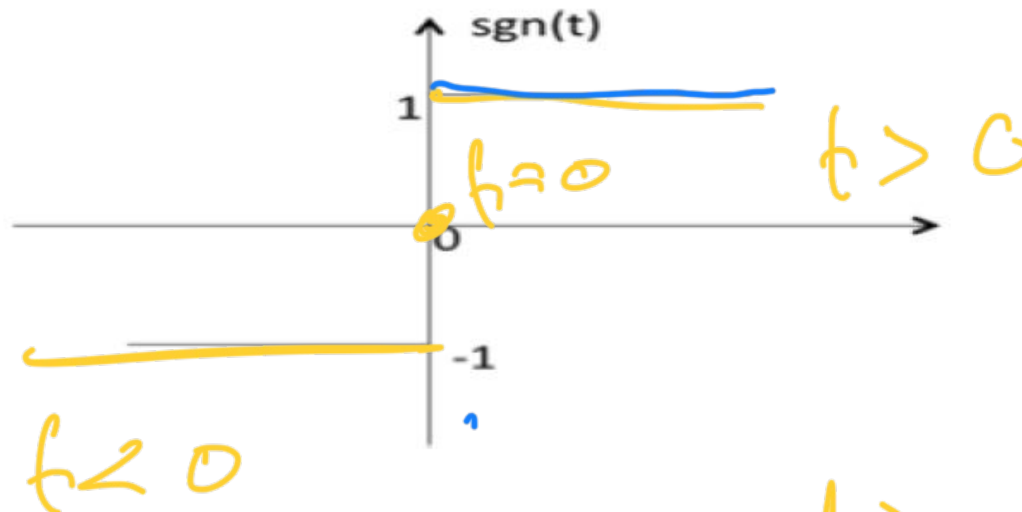
$$v(t) = \int \delta(t) \partial t \quad , \quad \delta(t) = \frac{\partial u(t)}{\partial t}$$

Basic Types of Signals

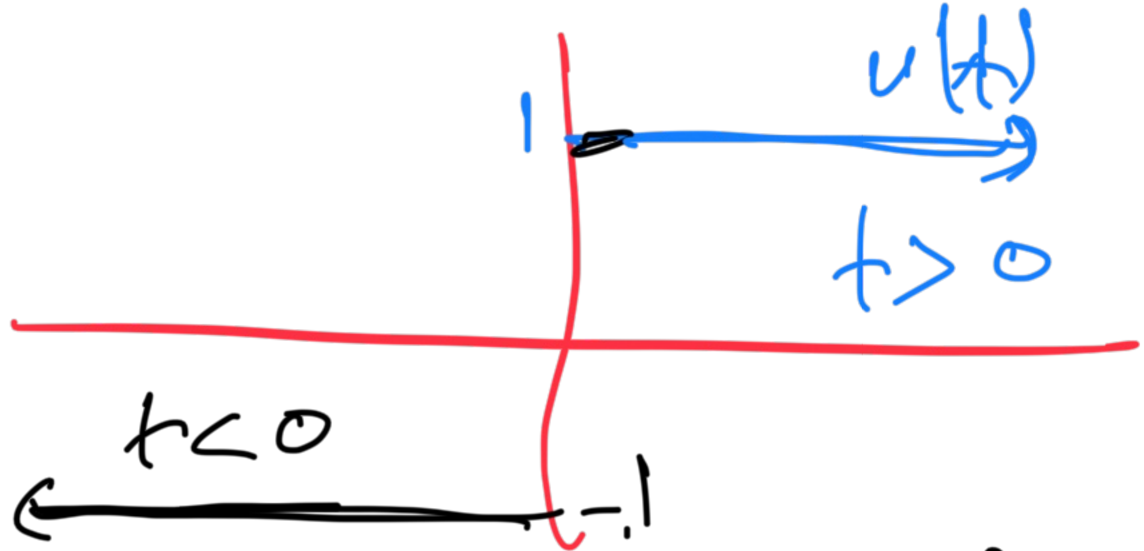
S

Signum Function

Signum function is denoted as sgn(t). It is defined as $\text{sgn}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$



sgn → r(t) مشتقة



$$\operatorname{sgn}(t) = u(t) - u(-t)$$

$$\operatorname{sgn}(t) = 2u(t) - 1$$

Basic Types of Signals

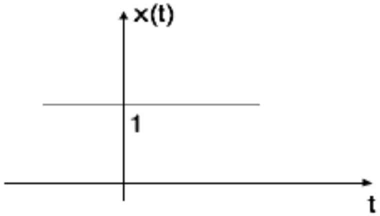
6

- **Exponential Signal**

Exponential signal is in the form of $x(t) = e^{at}$

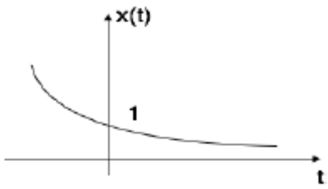
The shape of exponential can be defined by a.

Case i: if $a = 0 \rightarrow x(t) = e^{0} = 1$



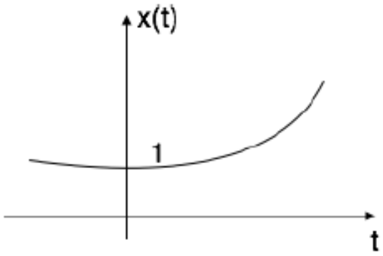
Case ii: if $a < 0$ i.e. -ve then $x(t) = e^{-at}$,

The shape is called decaying exponential.



Case iii: if $a > 0$ i.e. +ve then $x(t) = e^{at}$,

The shape is called raising exponential.

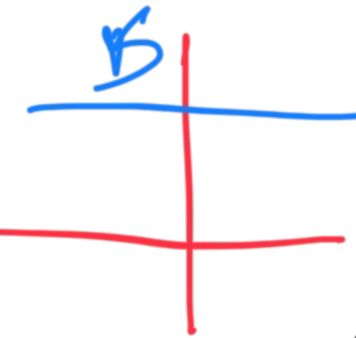


$x(t) = 5e^{at}$

No exp

$a = 0 \longrightarrow x(t) = e^{0} = 1$

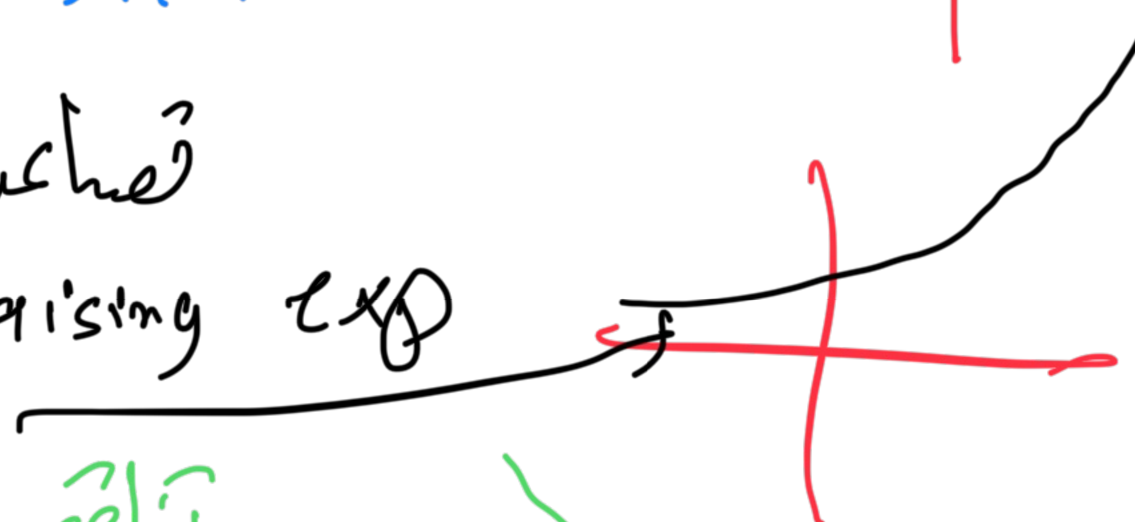
$x(t) = 5$



تصاعدي

$a > 0 \longrightarrow$ raising exp

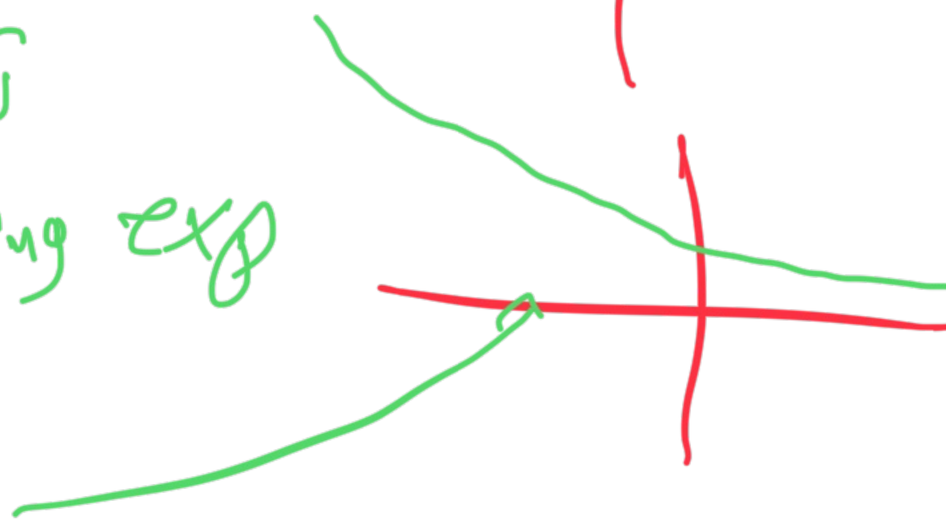
$x(t) = e^{3t}$



متناقص

$a < 0 \longrightarrow$ decaying exp

$x(t) = e^{-5t}$



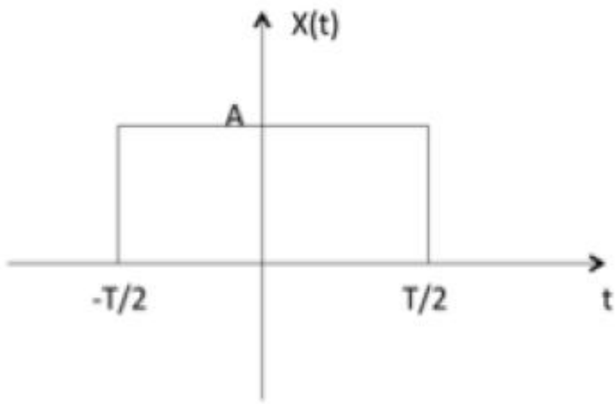
Basic Types of Signals

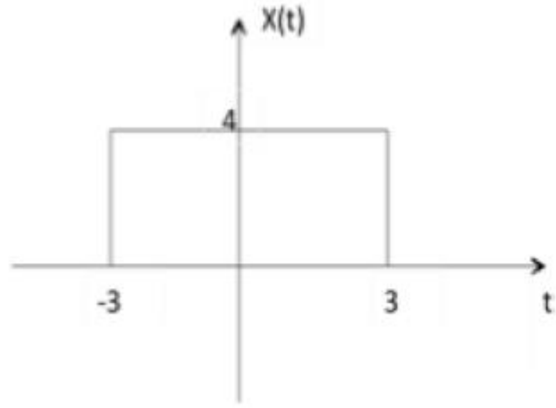
7 Rectangular Signal

Let it be denoted as x(t) and it is defined as

$$x(t) = A\,rect\left[\frac{r}{T}\right]$$

ex: $4\,rect\left[\frac{r}{6}\right]$



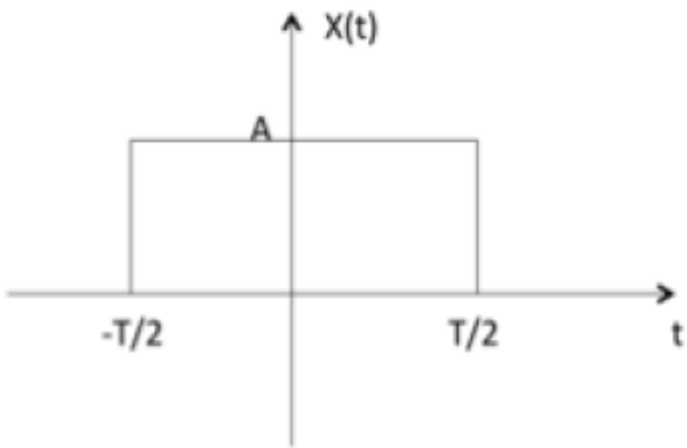


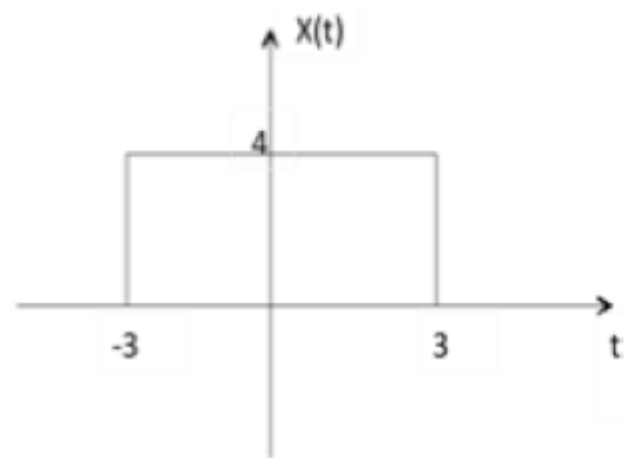
Basic Types of Signals

Rectangular Signal

Let it be denoted as x(t) and it is defined as

$$x(t) = A\, rect\left[\frac{r}{T}\right] \qquad \text{ex}: 4\, rect\left[\frac{r}{6}\right]$$





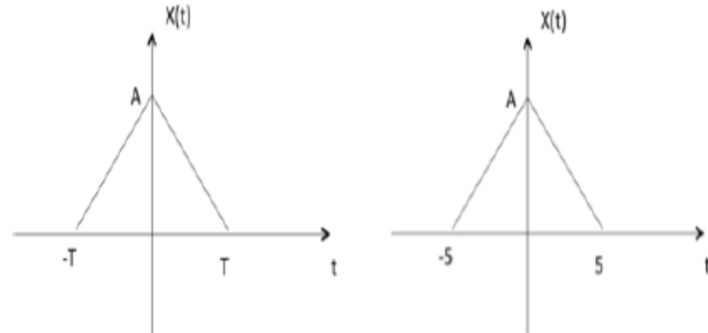
Basic Types of Signals

Triangular Signal

Let it be denoted as x(t),

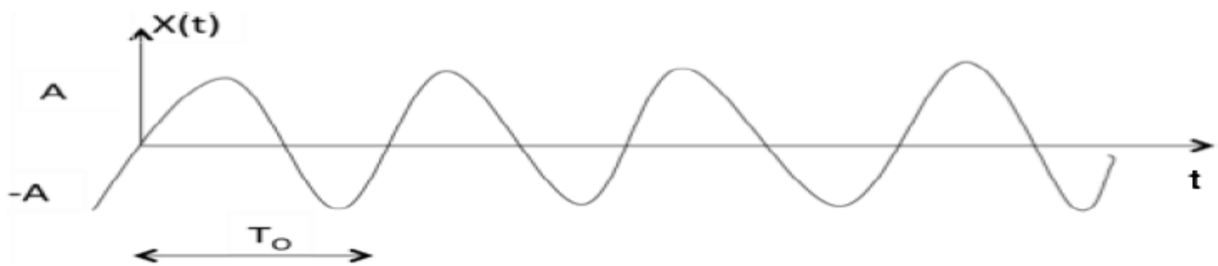
$$x(t) = A\left[1 - \frac{|t|}{T}\right]$$

$$\text{ex}: x(t) = A\left[1 - \frac{|t|}{5}\right]$$



Sinusoidal Signal

Sinusoidal signal is in the form of x(t) = A cos($w0 \pm \phi$) or A sin($w0 \pm \phi$)



Where T0 = 2π/$w0$

Classification of Signals

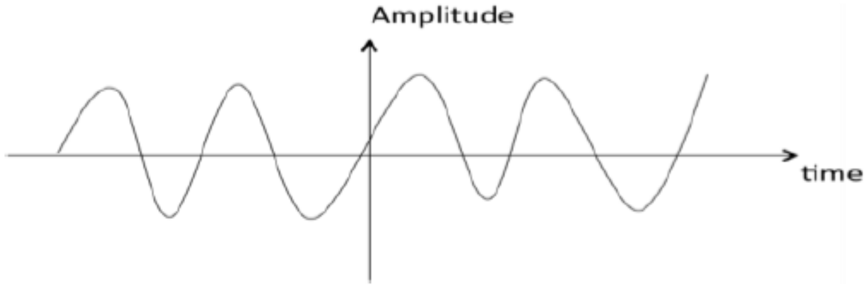
Signals are classified into the following categories:

- Continuous Time and Discrete Time Signals
- Deterministic and Non-deterministic Signals
- Even and Odd Signals
- Periodic and Aperiodic Signals
- Energy and Power Signals
- Real and Imaginary Signals

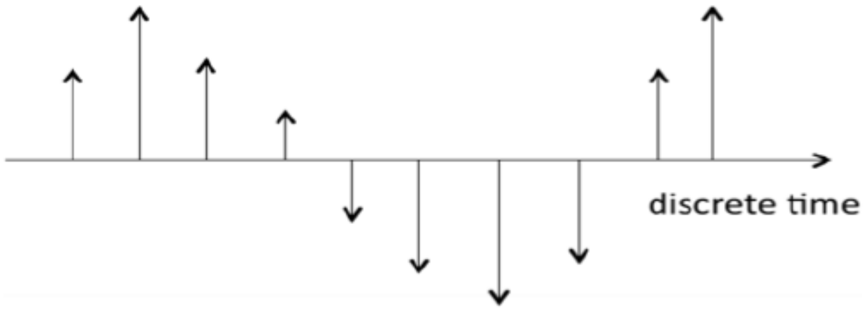
Classification of Signals

- **Continuous Time and Discrete Time Signals**

A signal is said to be continuous when it is defined for all instants of time.



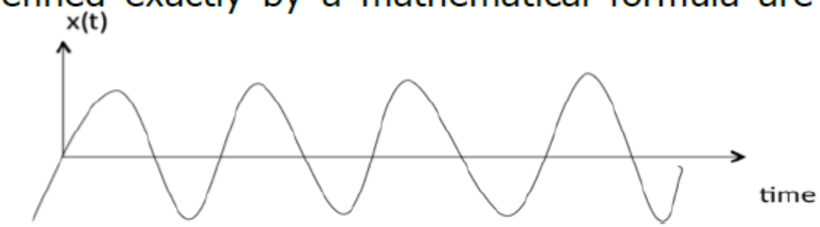
A signal is said to be discrete when it is defined at only discrete instants of time.



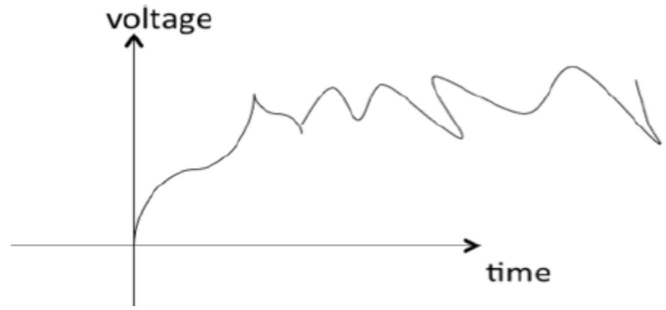
Classification of Signals

Deterministic and Non-deterministic Signals

A signal is said to be **deterministic** if there is no uncertainty with respect to its value at any instant of time. Or, signals which can be defined exactly by a mathematical formula are known as deterministic signals.



A signal is said to be **non-deterministic** if there is uncertainty with respect to its value at some instant of time. Non-deterministic signals are random in nature hence they are called **random signals**. Random signals cannot be described by a mathematical equation.
They are modelled in probabilistic terms.



Classification of Signals

Even and Odd signals

A signal is said to be **even** when it satisfies the condition $x(t) = x(-t)$

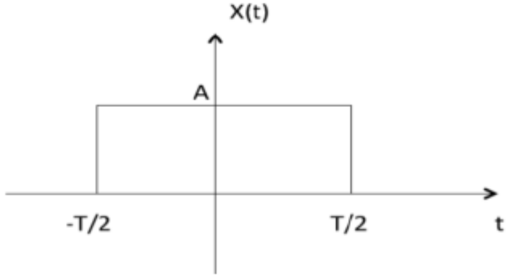
Example 1: t^2, t^4… cost etc.

Let $x(t) = t^2$

$x(-t) = (-t)^2 = t^2 = x(t)$

$\therefore$ t^2 is even function

Example 2: As shown in the following diagram, rectangle function $x(t) = x(-t)$ so it is also even function.



A signal is said to be **odd** when it satisfies the condition $x(t) = -x(-t)$

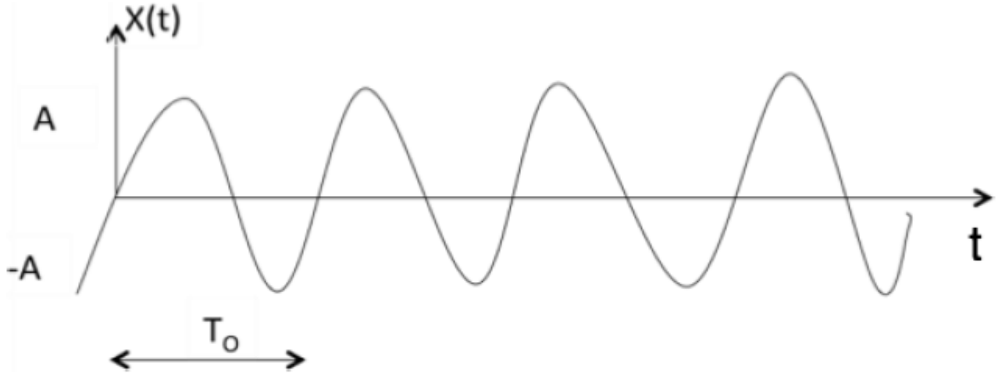
Classification of Signals

Periodic and Aperiodic Signals

A signal is said to be **periodic** if it satisfies the condition x(t) = x(t + T) or x(n) = x(n + N).

Where, T = fundamental time period,

1/T = f = fundamental frequency.



The above signal will repeat for every time interval T0 hence it is periodic with period T0.

Classification of Signals

Energy and Power Signals

A signal is said to be energy signal when it has finite energy.

$$\text{Energy } E = \int_{-\infty}^{\infty} x^2\,(t)dt$$

A signal is said to be power signal when it has finite power.

$$\text{Power } P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)dt$$

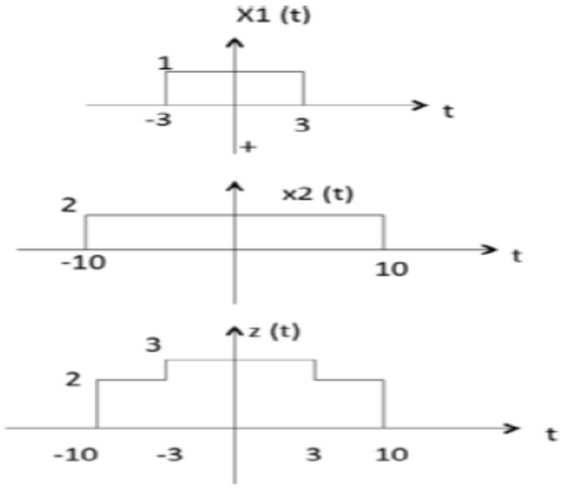
NOTE:A signal cannot be both, energy and power simultaneously. Also, a signal may be neither energy nor power signal.

Power of energy signal = 0 and Energy of power signal = ∞

Classification of Signals

Addition

Addition of two signals is nothing but addition of their corresponding amplitudes. This can be best explained by using the following example:



As seen from the previous diagram,

$-10 < t < -3$ amplitude of $z(t) = x1(t) + x2(t) = 0 + 2 = 2$

$-3 < t < 3$ amplitude of $z(t) = x1(t) + x2(t) = 1 + 2 = 3$

$3 < t < 10$ amplitude of $z(t) = x1(t) + x2(t) = 0 + 2 = 2$

Classification of Signals

Real and Imaginary Signals

A signal is said to be **real** when it satisfies the condition $x(t) = x^*(t)$

A signal is said to be **odd** when it satisfies the condition $x(t) = -x^*(t)$

Example:

If $x(t)= 3$ then $x^*(t)=3^*=3$, here $x(t)$ is a real signal.

If $x(t)= 3j$ then $x^*(t)=3j^* = -3j = -x(t)$, hence $x(t)$ is a odd signal.

Note: For a real signal, imaginary part should be zero. Similarly for an imaginary signal, real part should be zero.

Basic Operations on Signals

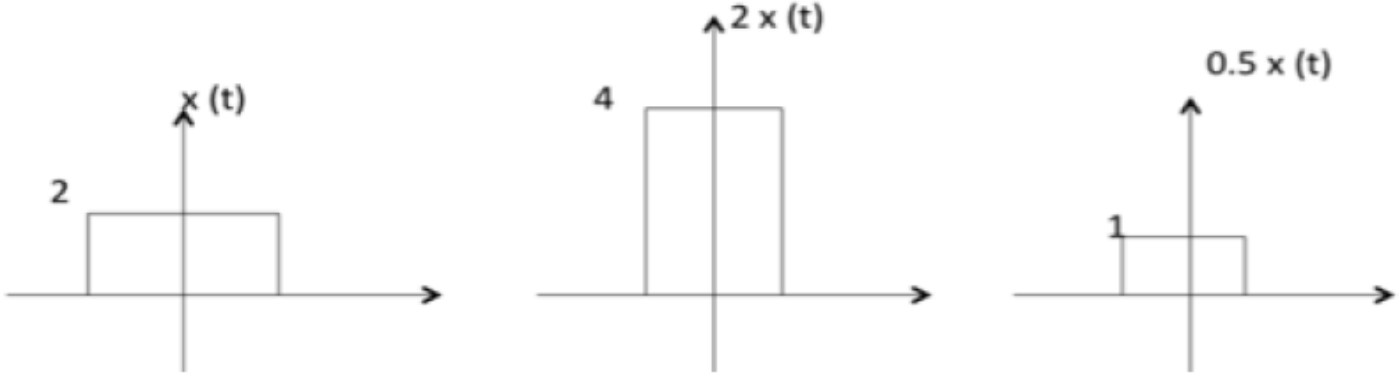
There are two variable parameters in general:

- Amplitude
- Time

The following operation can be performed with amplitude:

Amplitude Scaling

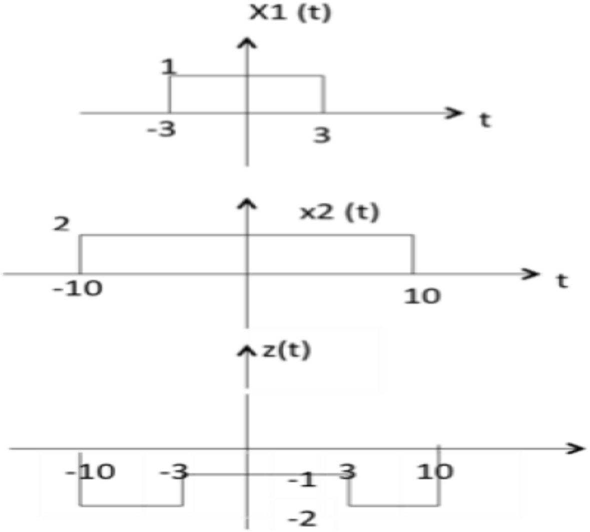
C x(t) is a amplitude scaled version of x(t) whose amplitude is scaled by a factor C.



Basic Operations on Signals

Subtraction

subtraction of two signals is nothing but subtraction of their corresponding amplitudes. This can be best explained by the following example:



As seen from the diagram above,

$-10 < t < -3$ amplitude of z (t) = x1(t) - x2(t) = 0 - 2 = -2

$-3 < t < 3$ amplitude of z (t) = x1(t) - x2(t) = 1 - 2 = -1

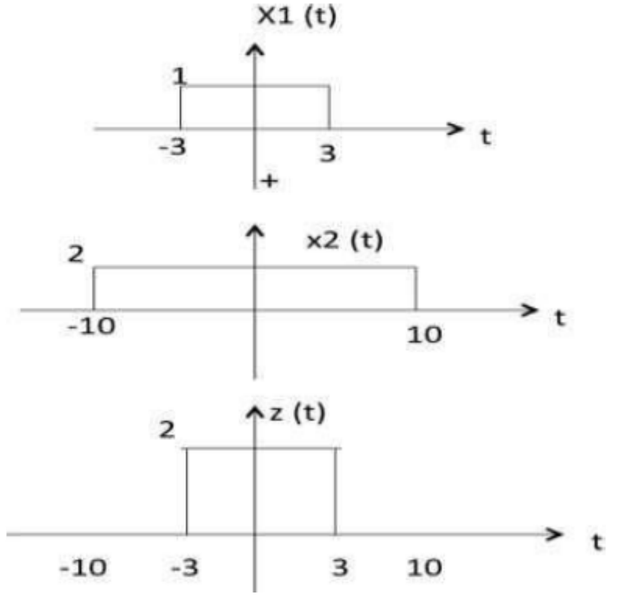
$3 < t < 10$ amplitude of z (t) = x1(t) - x2(t) = 0 - 2 = -2

Basic Operations on Signals

Multiplication

Multiplication of two signals is nothing but multiplication of their corresponding amplitudes.

This can be best explained by the following example:



As seen from the diagram above,

-10 < t < -3 amplitude of z (t) = x1(t) ×x2(t) = 0 ×2 = 0

-3 < t < 3 amplitude of z (t) = x1(t) - x2(t) = 1 ×2 = 2

3 < t < 10 amplitude of z (t) = x1(t) - x2(t) = 0 × 2 = 0

Basic Operations on Signals

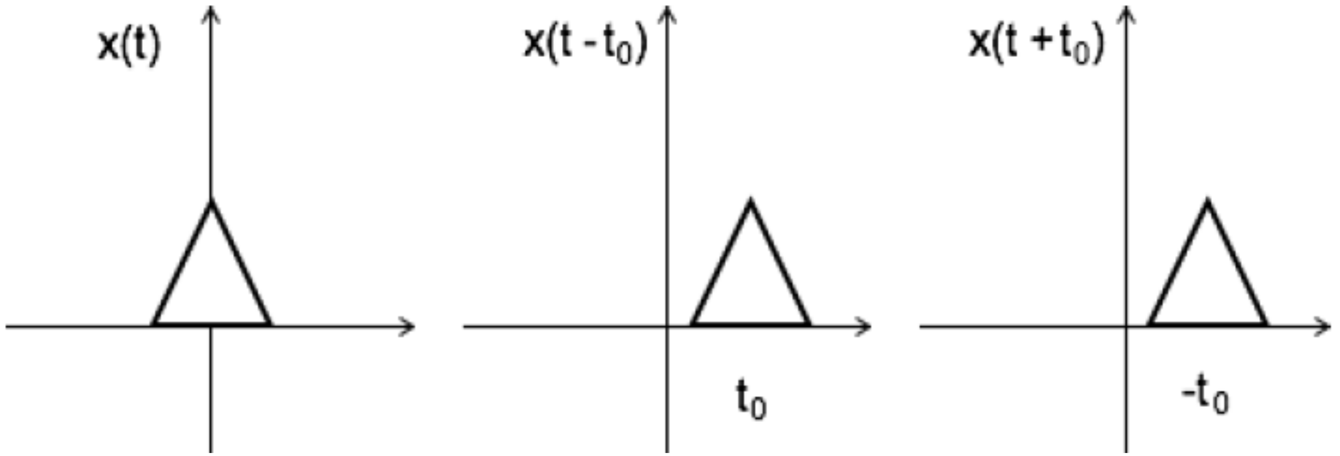
The following operations can be performed with time:

Time Shifting

$x(t \pm t0)$ is time shifted version of the signal $x(t)$.

$x\ (t + t0) \rightarrow$ negative shift

$x\ (t - t0) \rightarrow$ positive shift



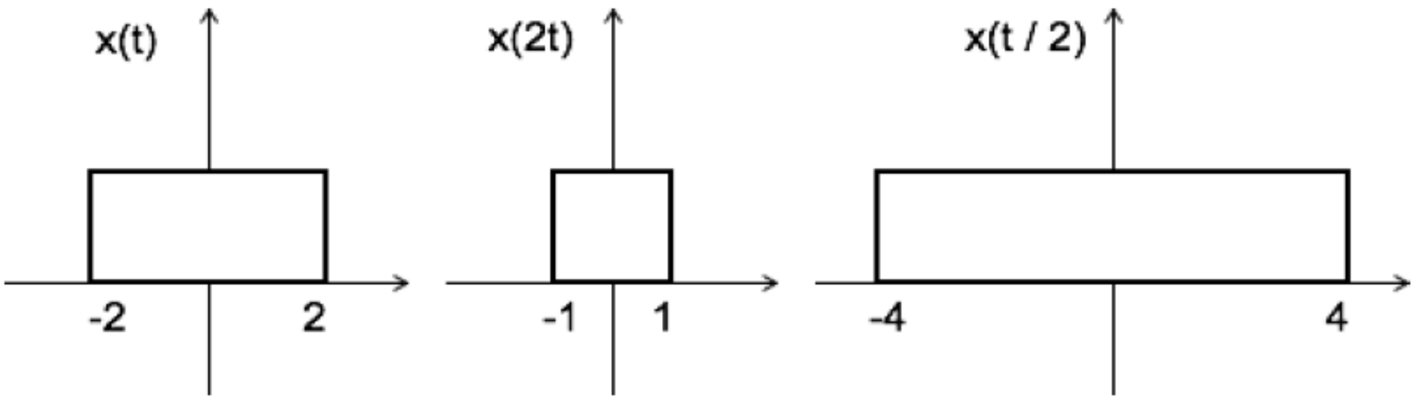
Basic Operations on Signals

Time Scaling

x(At) is time scaled version of the signal x(t). where A is always positive.

$|A| > 1 \rightarrow$ Compression of the signal

$|A| < 1 \rightarrow$ Expansion of the signal

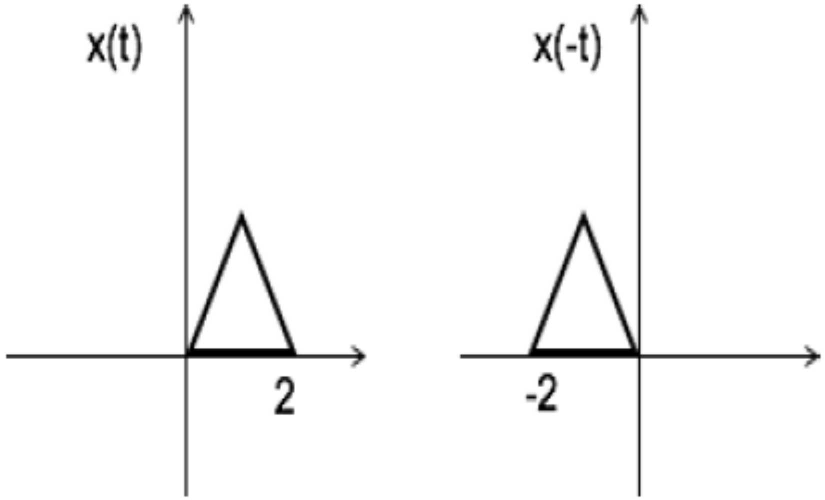


Note: u(at) = u(t) time scaling is not applicable for unit step function.

Basic Operations on Signals

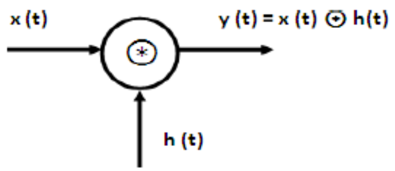
Time Reversal

x(-t) is the time reversal of the signal x(t).



Basic Operations on Signals

Convolution: Convolution between two continuous time signals can be written as



$$y(t) = x(t) \circledast h(t) = \int_{\tau=-\infty}^{\tau=+\infty} x(\tau)h(t-\tau)\,d\tau = \int_{\tau=-\infty}^{\tau=+\infty} h(\tau)x(t-\tau)\,d\tau$$

The following operations are required to compute convolution

1. Time reversal
2. Time Shifting (Delay & Advance)
3. Signal Multiplication
4. Integration

Note: If two signals are finite duration then Graphical Method is used and Else Function Method is employed to compute Convolution