Topic 1

AC Networks Analysis

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part 1

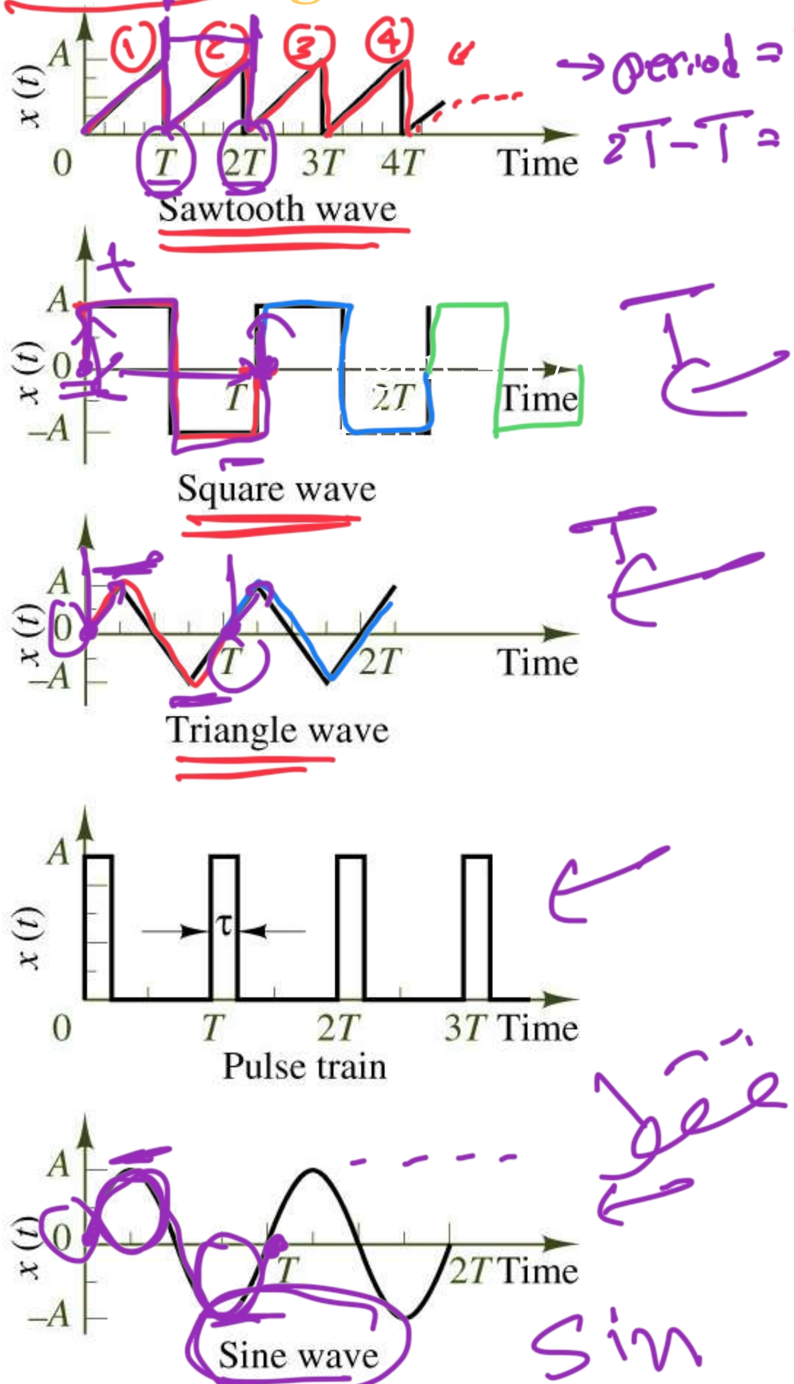
Learning Objectives

1. AC Sources
2. RMS Values
3. Complex Numbers and Phasors
4. Impedance of Passive Elements

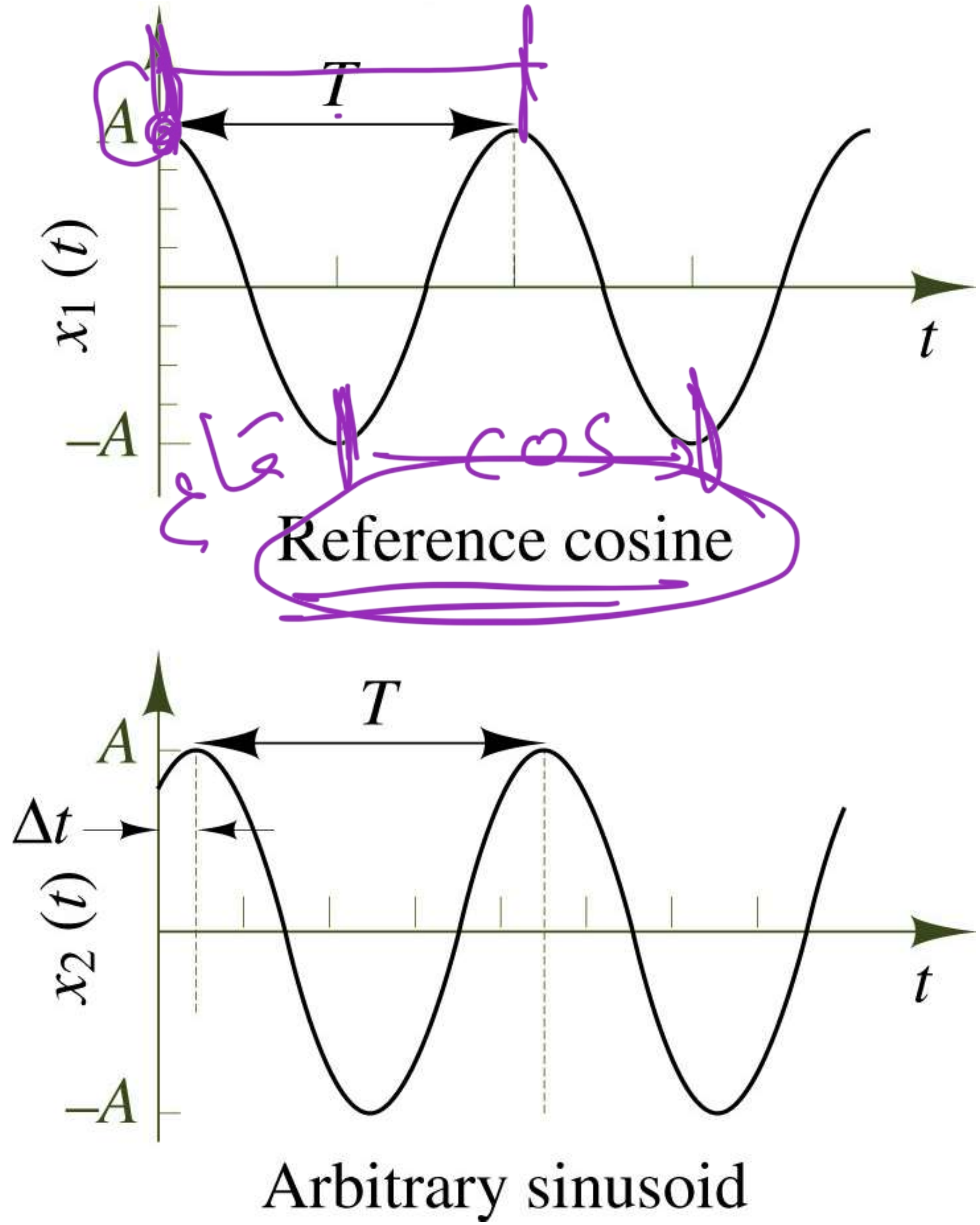
Periodic → دوره‌ای ← تکرار شونده

↳ period → کل که تکرار

Periodic signal waveforms



Sinusoidal waveforms



AC Circuits

An alternating current, as the name implies, goes through a series of different values both positive and negative in a period of time T, after which it continuously repeats this same series of values in a cycle manner. Another definition of alternating current is “ **An alternating current is a periodic current, the average value of which over a period is zero”**.

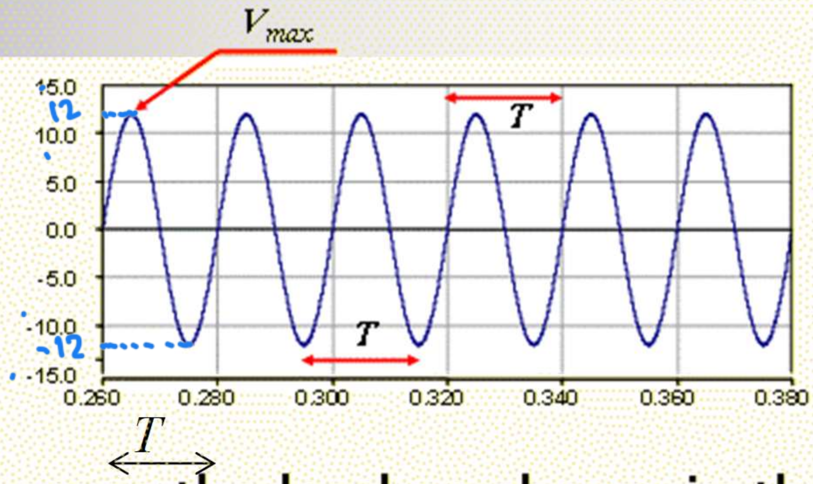
sin | cos

The term alternating indicates only that waveform alternates between two prescribed levels in a set time sequence. **The pattern of particular interest is the sinusoidal ac waveform for voltage** . Since this type of signal is encountered in the vast majority of instances, the abbreviated phrases ac voltage or ac current are commonly applied without confusion. For the other pattern, the descriptive term is always present, but frequently the ac abbreviation is dropped, resulting in the designation **square-wave** or **triangular** waveforms.

One of the important reasons for concentrating on the sinusoidal ac voltage is that it is the voltage generated by utilities throughout the world &its application throughout electrical, electronic, communication and industrial systems.

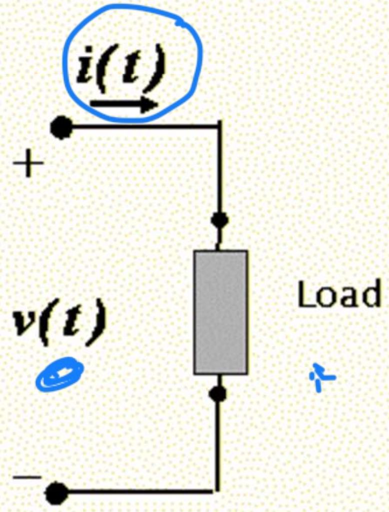
A sinusoidal wave:

A sinusoid is a signal that has the form of the sine or cosine function.



Consider the sinusoidal voltage waveform across the load as shown in the Figure,

$$v(t) = V_{max} \sin \omega t$$



V_{max} : *the amplitude of the sinusoid*

ω : *the angular frequency in radians/s*

$$\omega = 2\pi f \quad rad/s$$

f: frequency Hz

$$f = 1/T \quad Hz$$

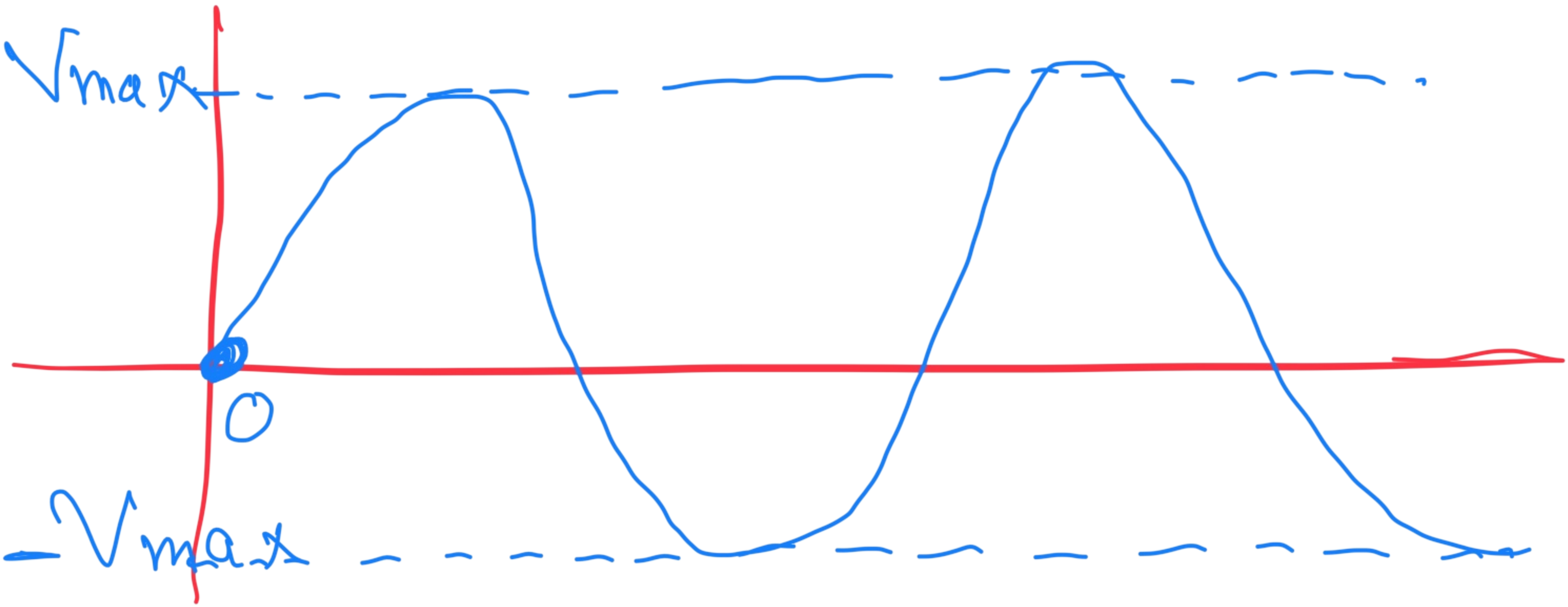
T : *the period in seconds* s

Since the voltage across the load is sinusoidal, then the current waveform is:

$$i(t) = I_{max} \sin(\omega t + \phi)$$

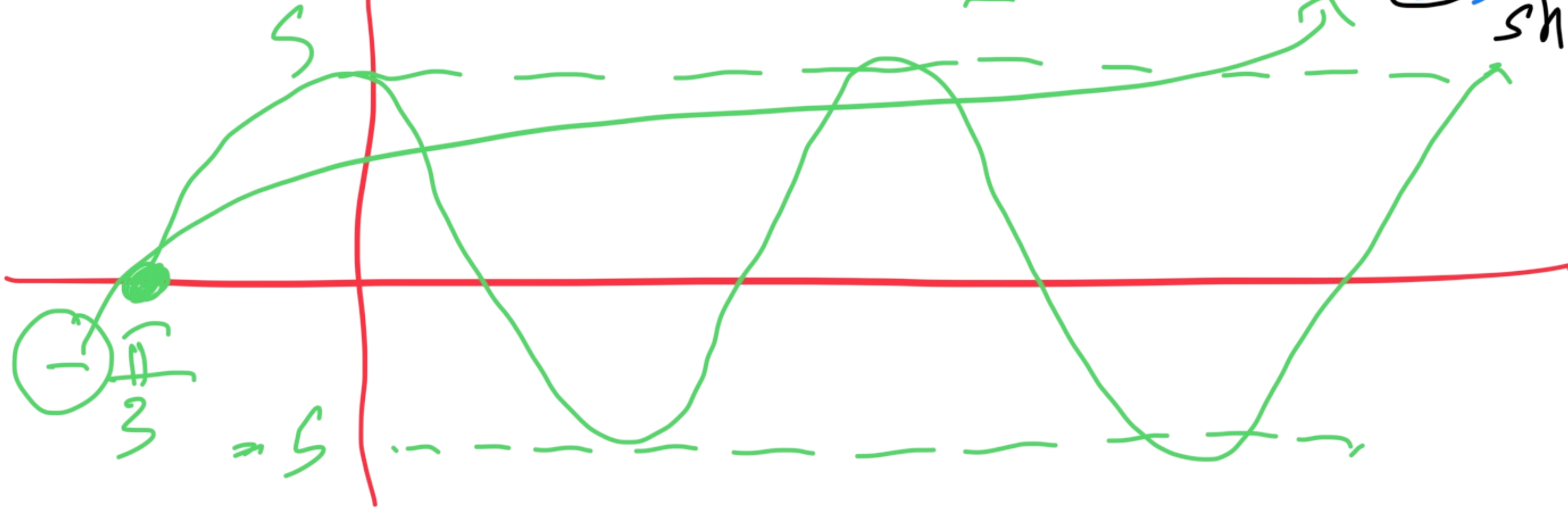
ϕ : *is the phase shift between the current and voltage*

$\sin(\omega t)$

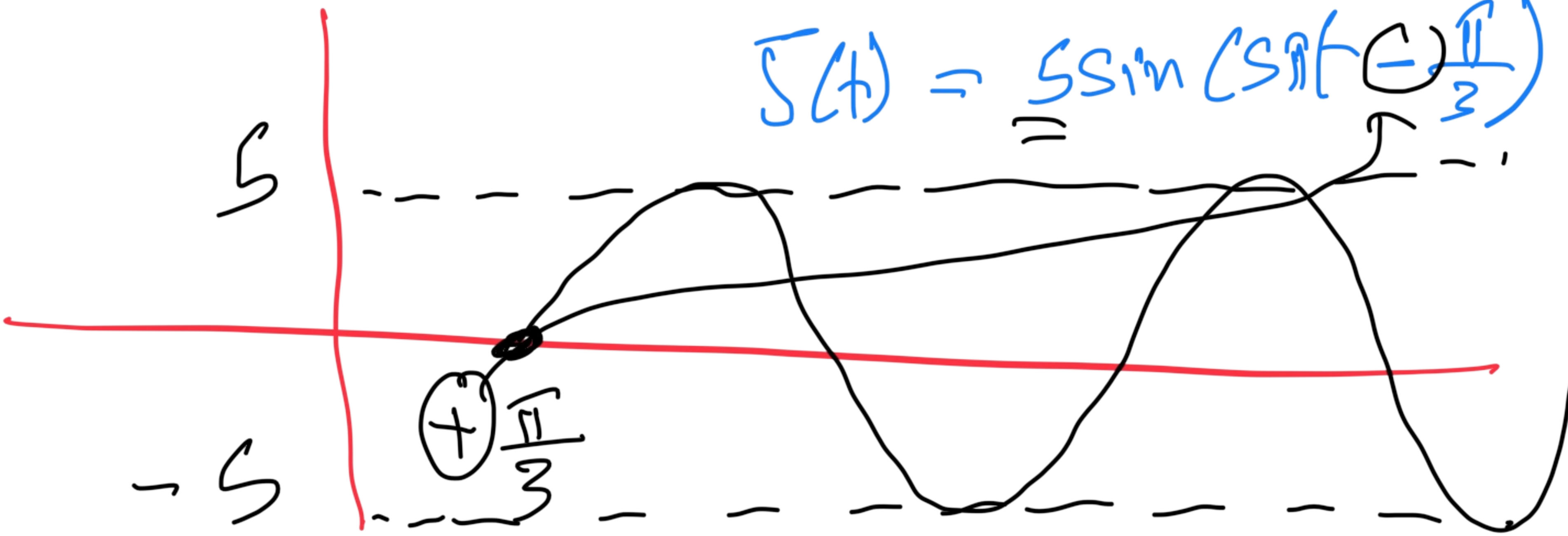


phase shift: هنشوف منين تبدأ الرسمة؟

$y(t) = \underline{5}\sin(5\pi t + \boxed{\frac{\pi}{3}})$ phase shift



$y(t) = \underline{5}\sin(5\pi t - \frac{\pi}{3})$



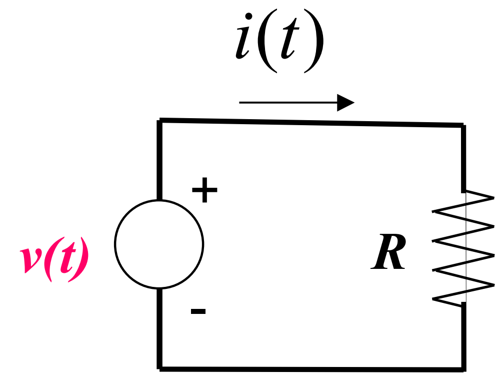
Root Mean Square (RMS) or Effective Values

T: period

The average & root mean square value (or effective value) of a periodic function f(t), that can be a voltage or current, are defined as

$$f_{average} = \frac{1}{T}\int_0^T f(t)\,dt$$

$$f_{rms} = \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt} = F_{eff}$$



Note that this effective value is found by first determining the square of the function, then computing the average or mean value, and finally taking the square root. Thus in reading this mathematical equation, we are determining the root mean square, which we abbreviate as RMS.

rms
eff

Example: Compute the rms value of the waveform $i(t) = I_m \cos(\omega t - \theta)$

$$\omega = \frac{2\pi}{T}$$

Solution:

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2(\omega t - \theta)dt} = \frac{I_m}{\sqrt{2}}$$

Do this integration and verify this

Example: Compute the rms value of the waveform $v = V_m \sin \omega t$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} = \sqrt{\frac{1}{T}\int_0^T V^2 \sin^2 \omega t dt} = \frac{V_{\max}}{\sqrt{2}}$$

$\sqrt{2}$

For a sinusoidal signal, the rms value is equal to the peak divided by square root of 2

Or $$V_{rms} = \frac{V_{\max}}{\sqrt{2}}$$

sin
cos

$$I(t) = I_m \cos(\omega t - \theta)$$

$$V(t) = V_m \sin(\omega t)$$

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}} \qquad I_{rms} = \frac{I_m}{\sqrt{2}}}$$

قانون خاص

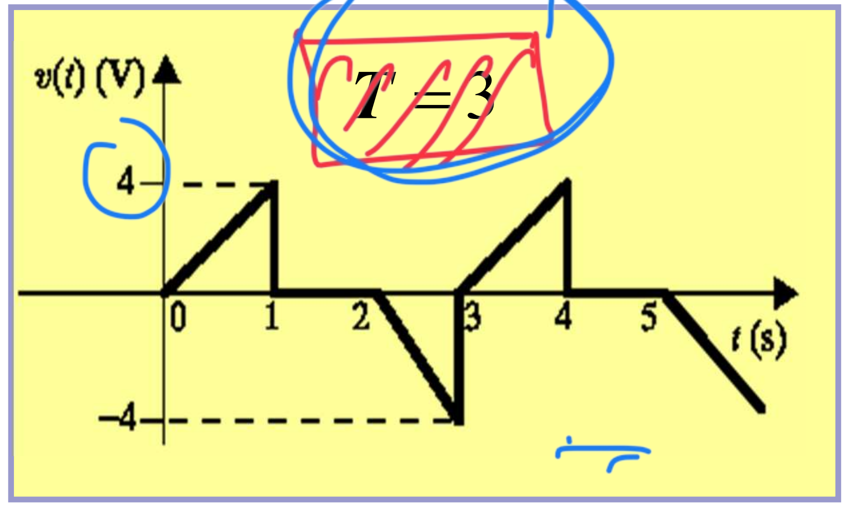
لـ cos

sin

حفظ

Example: Compute the rms value of the voltage waveform

$$v(t) = \begin{cases} 4t & 0 < t \le 1 \\ 0 & 1 < t \le 2 \\ -4(t-2) & 2 < t \le 3 \end{cases}$$

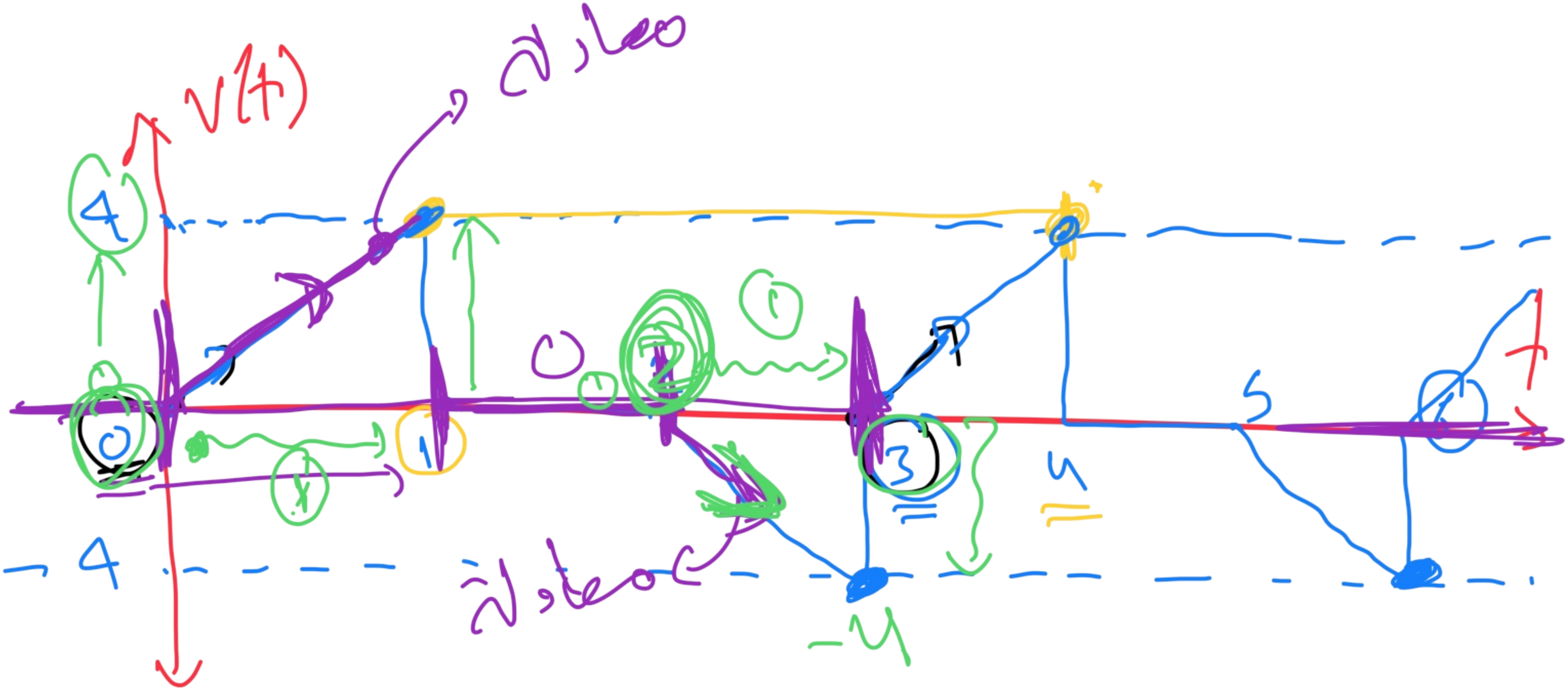


Solution:

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} = \sqrt{\frac{1}{3}\left[\int_0^1 (4t)^2 dt + \int_1^2 (0)^2 dt + \int_2^3 (-4(t-2))^2 dt\right]}$$

$$= \sqrt{\frac{1}{3}\int_0^1 16t^2 dt + \int_2^3 (16t^2 - 64t + 64)\ dt}$$

$$= \sqrt{\frac{1}{3}10.66} = 1.89\text{ V}$$



Period : $T = 4 - 1 = 3$

$T = 3 - 0 = 3$

$T = 6 - 3 = 3$

$$v(t) = \begin{cases} 4(t-0) = 4t & 0 \le t \le 1 \\ 0 & 1 \le t \le 2 \\ -4(t-2) & 2 \le t \le 3 \end{cases}$$

معادلة الخط المستقيم :-

$= m(t -$ نقطة البداية $) +$

$m \longrightarrow$ كل خطوة على محور الـ t طلع أو نزل

$$V(t) = \sqrt{\frac{1}{T}\int_0^T v^2\,dt} \qquad \boxed{T = 3}$$

$$V(t) = \sqrt{\frac{1}{3}\left[\int_0^1 (4t)^2 + \cancel{\int_1^2 (0)^2} + \int_2^3 \left(-4(t-2)\right)^2\right]}$$

$$V(t) = \sqrt{\frac{1}{3}\left[\int_0^1 (4t)^2 + \int_2^3 \left(-4(t-2)\right)^2\right]} = \frac{4\sqrt{2}}{3}$$

$$= 1.8856$$

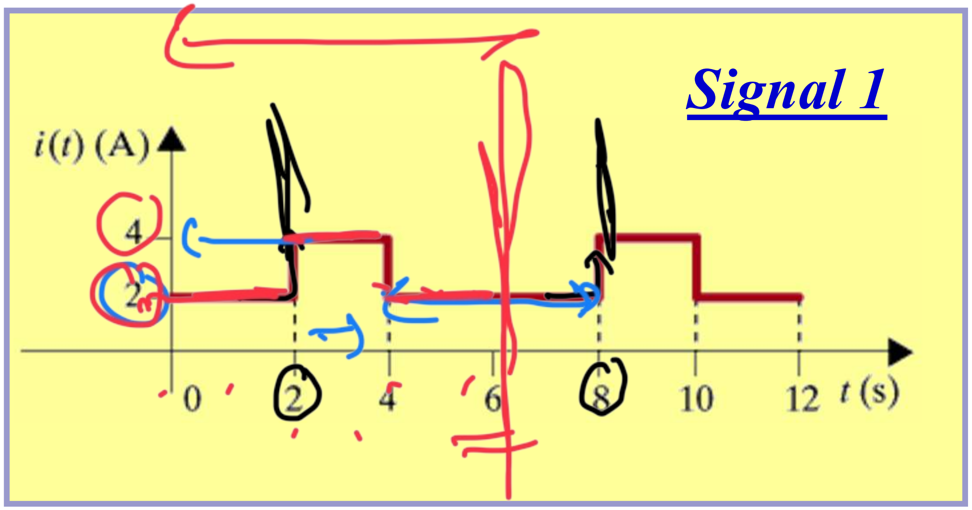
EXAMPLE:

$2^2 \to 4$ $\quad$ $4^2 \to 16$

Determine the RMS value for the following waveforms.

SOLUTION:

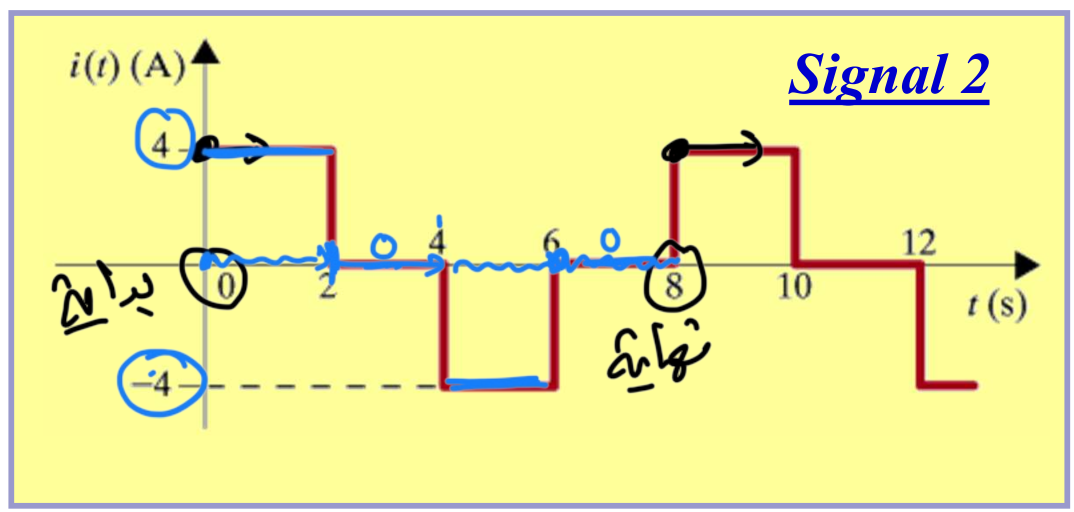
$T \to 6$



$$I_{rms} = \sqrt{\frac{1}{6}\left[\int_0^2 4dt + \int_2^4 16dt + \int_4^6 4dt\right]}$$

$$I_{rms} = \sqrt{\frac{8+32+8}{6}} = \sqrt{8}$$

$$I_{rms} = \sqrt{\frac{1}{8}\left[\int_0^2 16dt + \int_4^6 16dt\right]} = \sqrt{8}$$



Signal 1

period $T = 8-2 = 6$

$$i(t) = \begin{cases} 4 & 2 \le t \le 4 \\ 2 & 4 \le t \le 8 \end{cases}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2} = \sqrt{\frac{1}{6}\left[\int_2^4 4^2 + \int_4^8 2^2\right]}$$

$$\boxed{I_{rms} = 2\sqrt{2}} = \sqrt{8}$$

Signal 2

Period: $T = 8 - 0 = 8$

$$i(t) = \begin{cases} 4 & 0 \le t \le 2 \\ 0 & 2 \le t \le 4 \\ -4 & 4 \le t \le 6 \\ 0 & 6 \le t \le 8 \end{cases}$$

$$I_{rms} = \sqrt{\frac{1}{8}\left[\int_0^2 4^2 + \int_2^4 0^2 + \int_4^6 (-4)^2 + \int_6^8 0^2\right]}$$

$$I_{rms} = 2\sqrt{2} = \sqrt{8}$$

Phasor Domain

- Phasor is a complex number representation in terms of amplitude & phase of a sinusoidal wave shape
- Engineers use phasors to simplify solution of circuitry involving sinusoidal voltages & currents.