Topic 1

AC Networks Analysis

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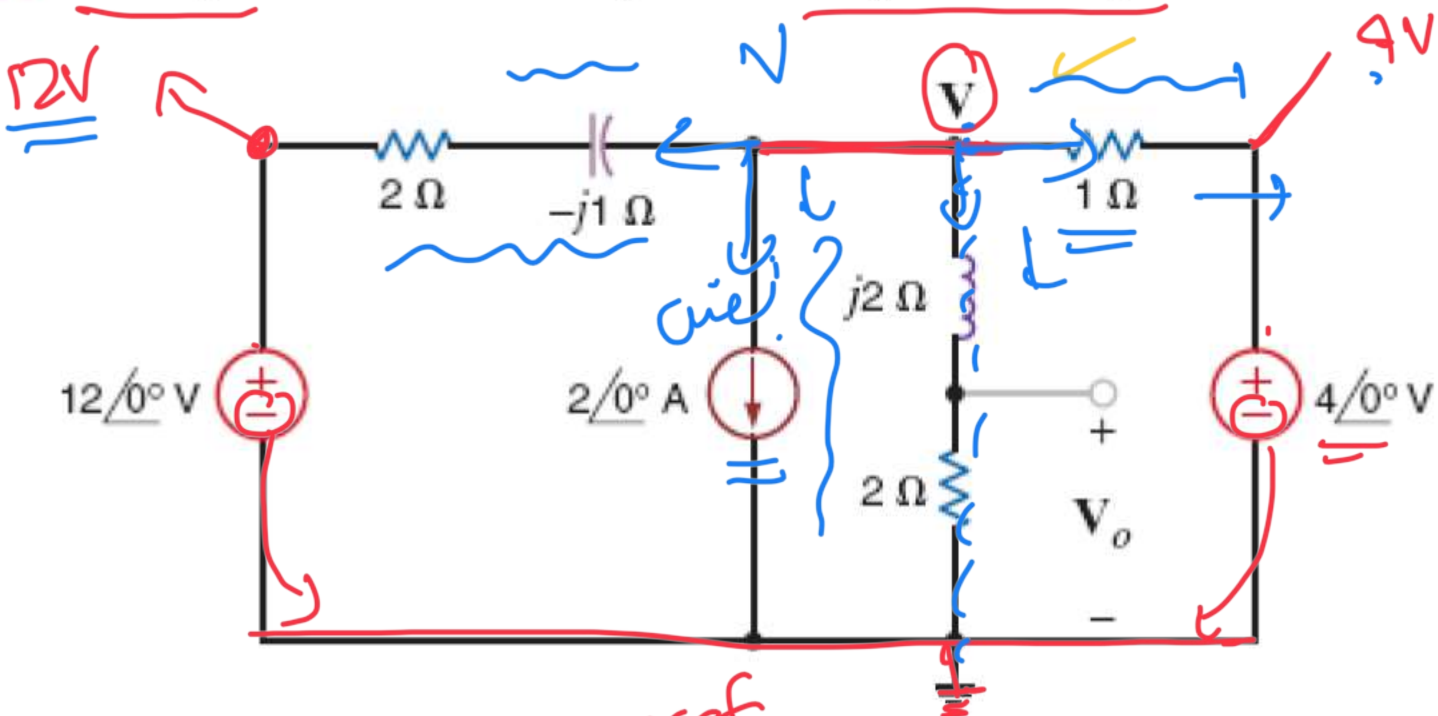
part 4 last

node → نقطة تربط بين عنصرين أو أكثر

8.68 Find $\mathbf{V}_o$ in the network in Fig. P8.68 using nodal analysis.



ref = 0V

Figure P8.68

node V

$$\frac{V-12}{2-j} + 2 + \frac{V-0}{2+2j} + \frac{V-4}{1} = 0$$

$$V\left[\frac{1}{2-j}+\frac{1}{2+2j}+\frac{1}{1}\right] = -2+\frac{4}{1}+\frac{12}{2-j}$$

$$V\left[\frac{33}{20}-j\frac{1}{20}\right] = \frac{34}{5}+j\frac{12}{5}$$

$$\frac{V\cancel{\left[\frac{33}{20}-j\frac{1}{20}\right]}}{\cancel{\frac{33}{20}-j\frac{1}{20}}} = \frac{\frac{34}{5}+j\frac{12}{5}}{\frac{33}{20}-j\frac{1}{20}}$$

node ← نقطة تربط بين عنصرين أو أكثر

8.68 Find $\mathbf{V}_o$ in the network in Fig. P8.68 using nodal analysis.

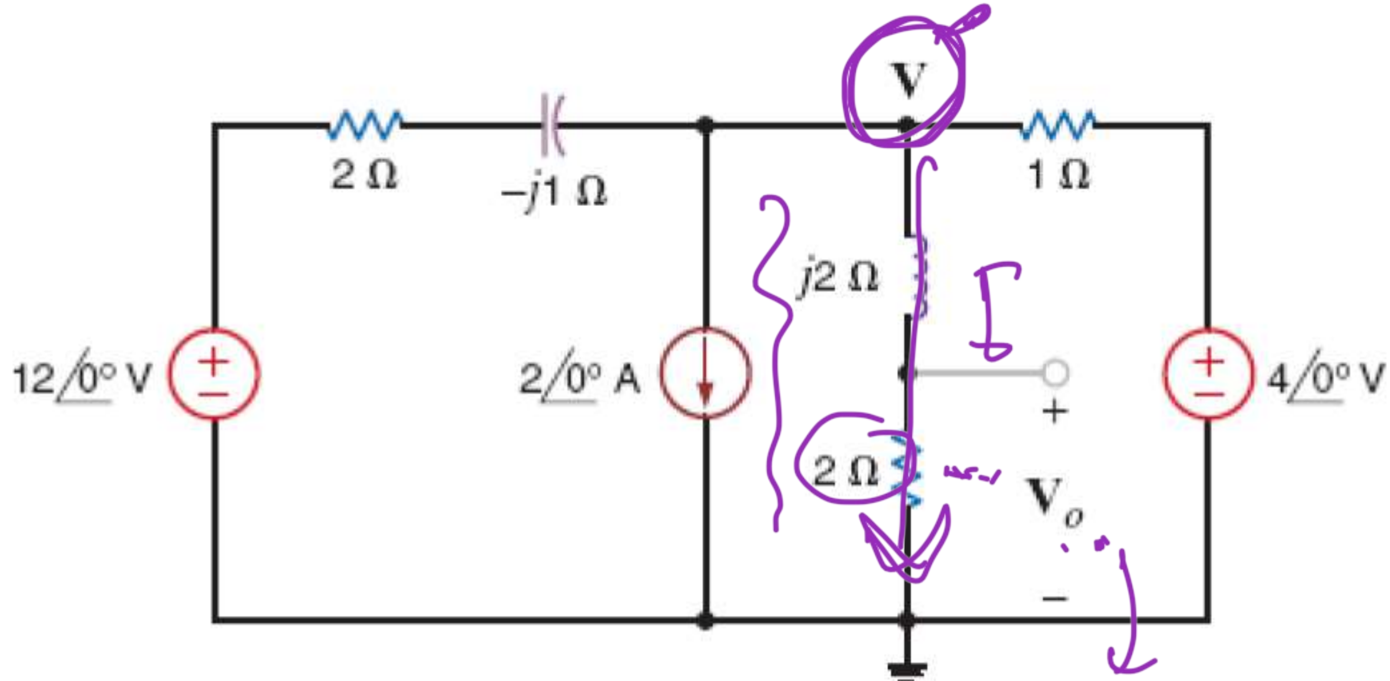


Figure P8.68

$V_o = \underline{\underline{I \times R}}$

$$\boxed{V = 4.3684 \angle 21.176 \quad V}$$

$$I = \frac{V - 0}{2 + j2} = \frac{4.3684 \angle 21.176}{2 + j2}$$

$$I = 1.544 \angle -23.824 \text{ A}$$

$$V_o = 2 * 1.544 \angle -23.824$$

$$\boxed{V_o = 3.1 \angle -23.8^\circ \quad V}$$

node → نقطة تربط بين عنصرين أو أكثر

8.68 Find $\mathbf{V}_o$ in the network in Fig. P8.68 using nodal analysis.

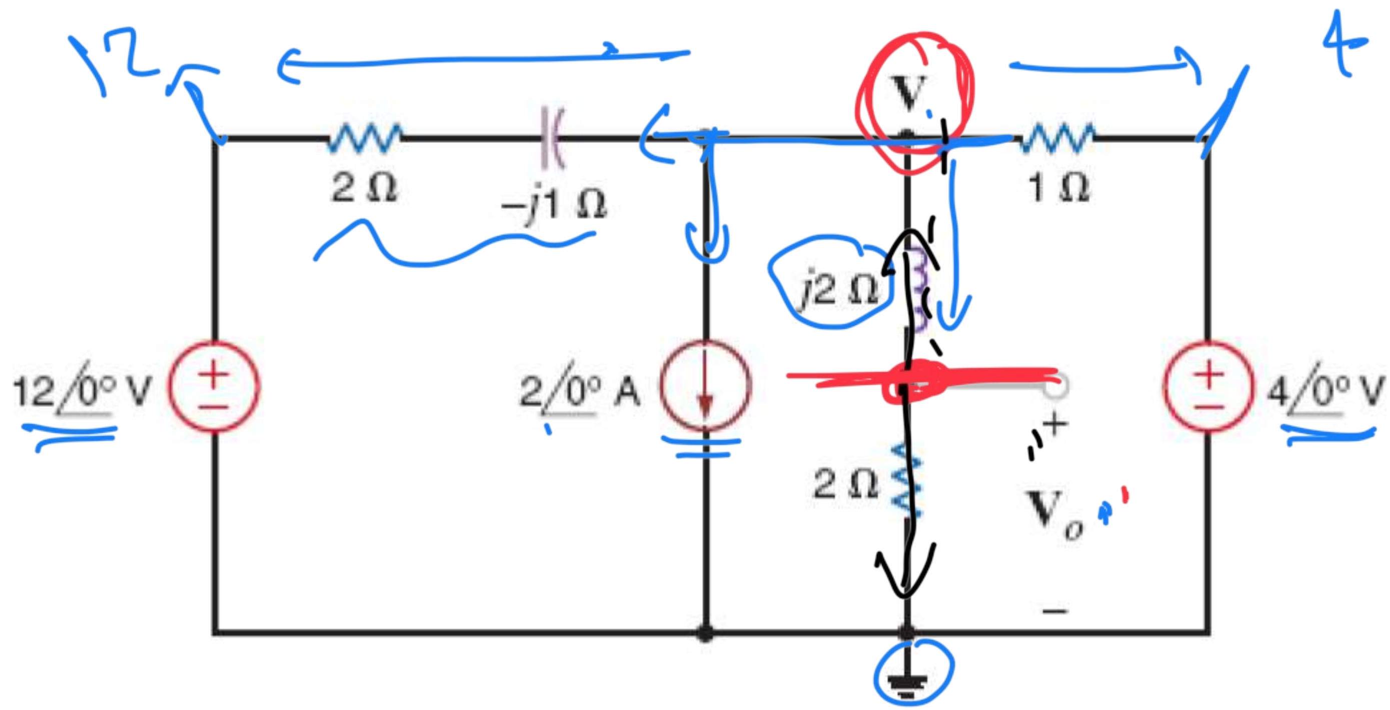


Figure P8.68

0V

node V

$\frac{1}{j2} = -j\frac{1}{2}$

$$\frac{V-12}{2-j} + 2 + \frac{V-V_o}{j2} + \frac{V-4}{1} = 0$$

$$\boxed{V\left[\frac{7}{5}-j\frac{3}{10}\right] + V_o\left[j\frac{1}{2}\right] = 6.8+j2.4}$$

A B E

node Vo

$$\frac{V_o-V}{j2} + \frac{V_o-0}{2} = 0$$

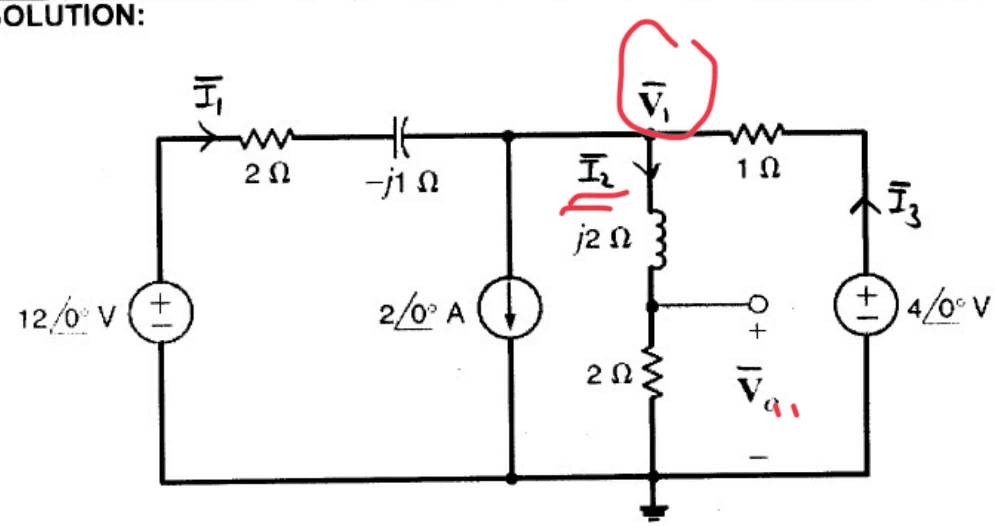
$$\Rightarrow \boxed{V\left[j\frac{1}{2}\right] + V_o\left[\frac{1}{2}-j\frac{1}{2}\right] = 0}$$

C D F

$$V = \frac{ED - BF}{AD - BC} = 4.368 \angle 21.176 \text{ V}$$

$$V_O = \frac{AF - EC}{AD - BC} = 3.1 \angle -23.8 \text{ V}$$

SOLUTION:



KCL at ①: $\bar{I}_1 + \bar{I}_3 = 2\angle 0^\circ + \bar{I}_2$

$$\frac{12\angle 0^\circ - \bar{V}_1}{2 - j1} + \frac{4\angle 0^\circ - \bar{V}_1}{1} = 2\angle 0^\circ + \frac{\bar{V}_1}{2+j2}$$

$$\frac{\bar{V}_1}{2+j2} + \frac{\bar{V}_1}{2-j1} + \bar{V}_1 = \frac{12\angle 0^\circ}{2-j1} + 4\angle 0^\circ - 2\angle 0^\circ$$

$$\bar{V}_1 = \frac{7.21\angle 19.44^\circ}{1.65\angle -1.74^\circ}$$

$\checkmark$

$$\boxed{\bar{V}_1 = 4.37\angle 21.2^\circ \text{ V}}$$

$$\bar{I}_2 = \frac{\bar{V}_1}{2+j2} = \frac{4.37\angle 21.2^\circ}{2+j2}$$

$$\bar{I}_2 = 1.55\angle -23.8^\circ \text{ A}$$

$$\bar{V}_o = 2(1.55\angle -23.8^\circ)$$

$$\boxed{\bar{V}_o = 3.1\angle -23.8^\circ \text{ V}}$$

8.39 Find $v_o(t)$ and $i_o(t)$ in the network in Fig. P8.39.

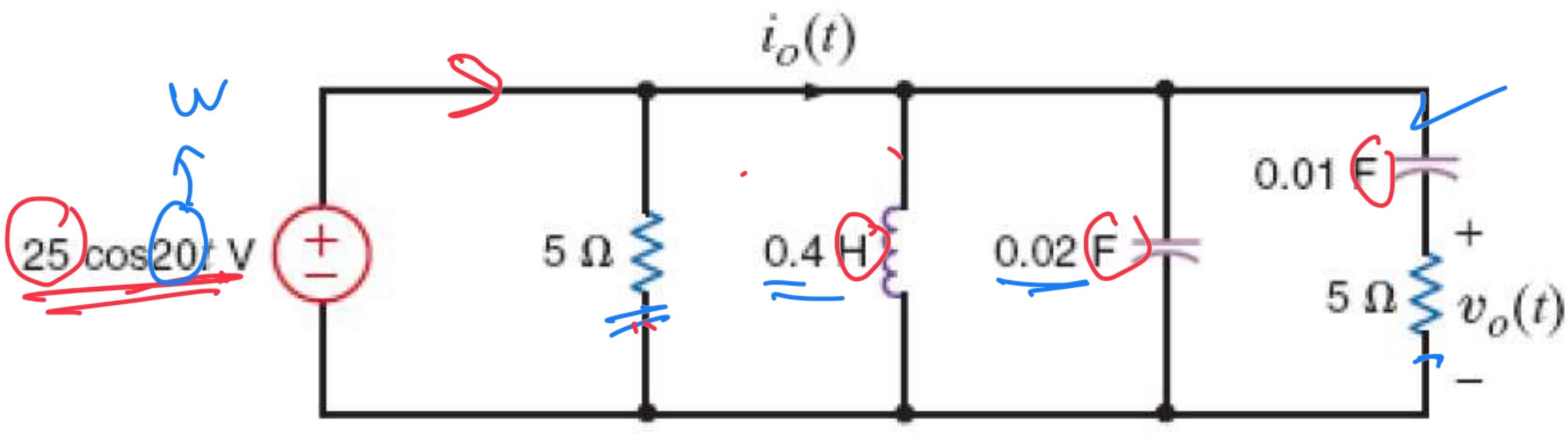


Figure P8.39

Time domain

سؤال كويز

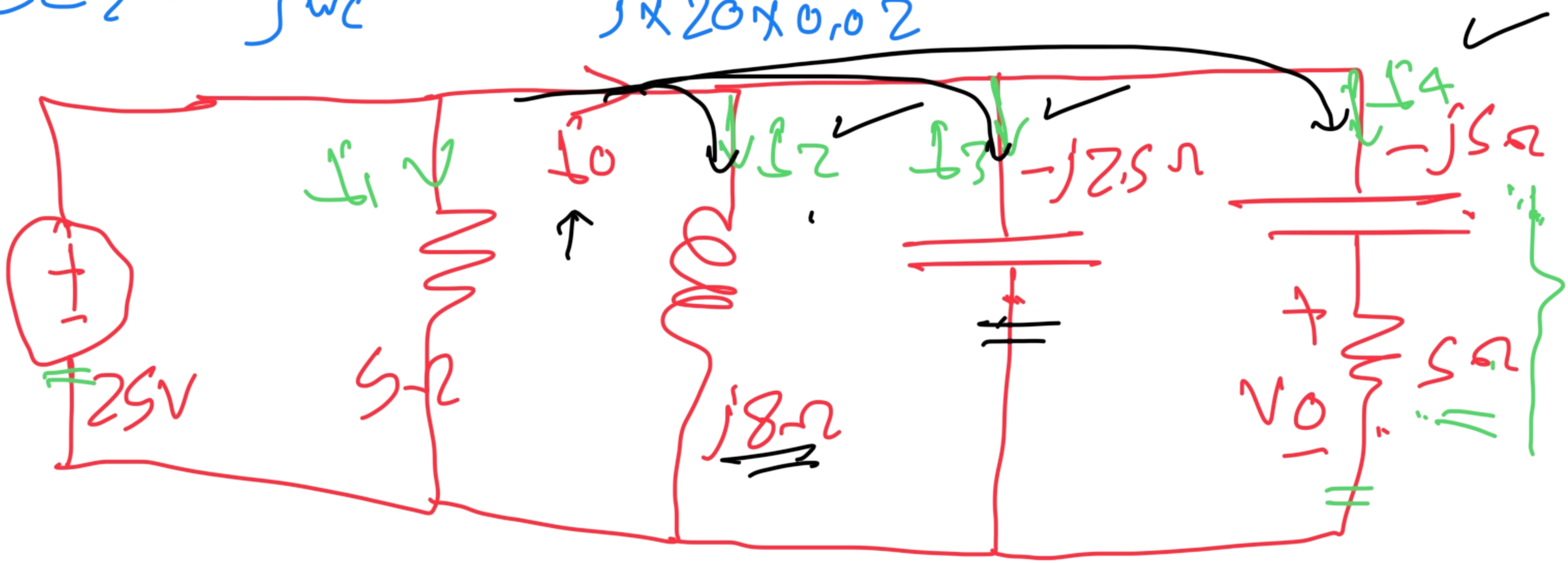
$$\omega = 20 \text{ rad/s}$$

$$Z_R = R = 5\,\Omega$$

$$Z_L = j\omega L = j \times 20 \times 0.4 = j8\,\Omega$$

$$Z_{C_1} = \frac{1}{j\omega C} = \frac{1}{j \times 20 \times 0.01} = -j5\,\Omega$$

$$Z_{C_2} = \frac{1}{j\omega C} = \frac{1}{j \times 20 \times 0.02} = -j2.5\,\Omega$$



Parallel

$V_o = I_4 * R$

$I_4 = \frac{V}{Z_2} = \frac{25}{5-j5} = 2.5+j2.5\,A$

$V_o = (2.5+j2.5) * 5 = 12.5+j12.5\,V$

$$\boxed{V_o = 17.678 \angle 45\ V}$$

$I_o = I_2 + I_3 + I_4$

$I_3 = \frac{V}{Z} = \frac{25}{-j2.5} = j10\,A$

$$I_2 = \frac{V}{Z} = \frac{25}{j8} = -j3{,}125 \text{ A}$$

$$I_0 = -j3{,}125 + j10 + 2{,}5 + j2{,}5 \text{ A}$$

$$\boxed{I_0 = 2{,}5 + j9{,}375 \text{ A} = 9{,}703 \angle 75{,}1 \text{ A}}$$

$$v_0(t) = 17{,}678 \cos(20t + 45) \text{ V}$$

$$i_0(t) = 9{,}703 \cos(20t + 75{,}1) \text{ A}$$

8.39 Find $v_o(t)$ and $i_o(t)$ in the network in Fig. P8.39.

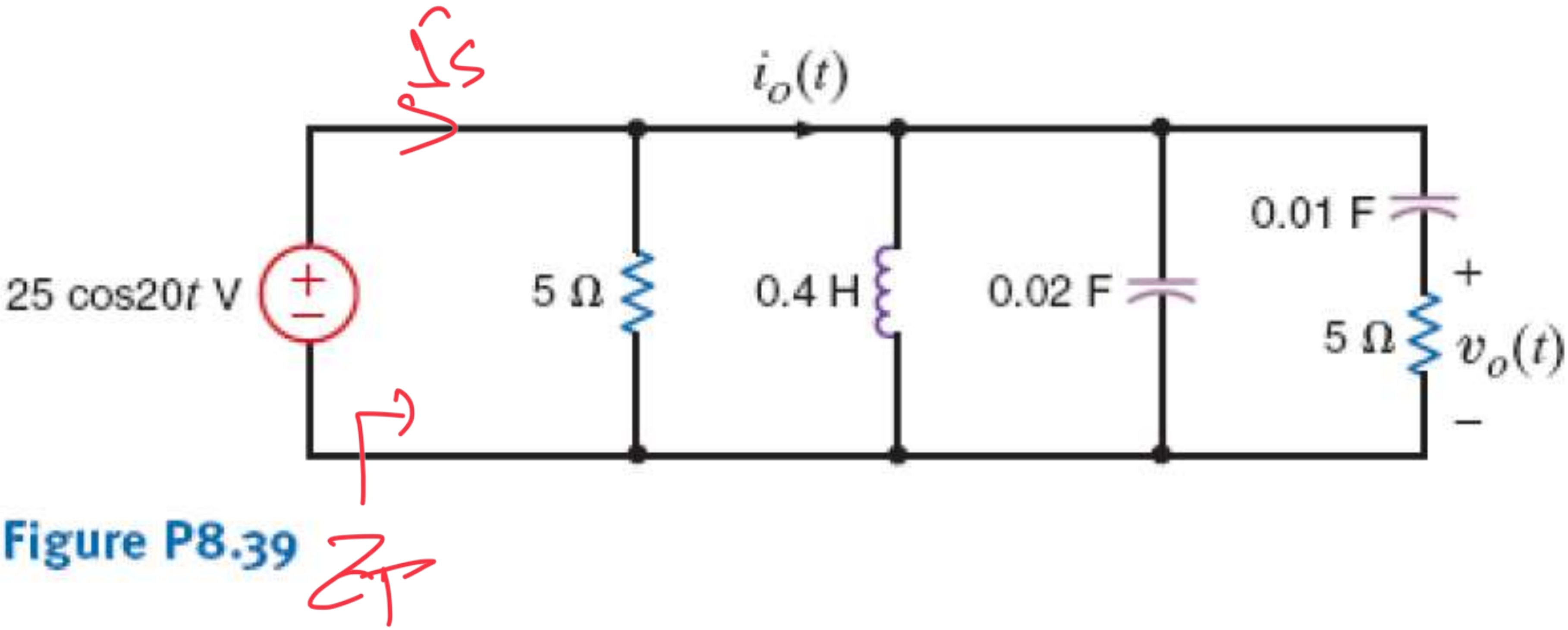


Figure P8.39

سؤال الكويز

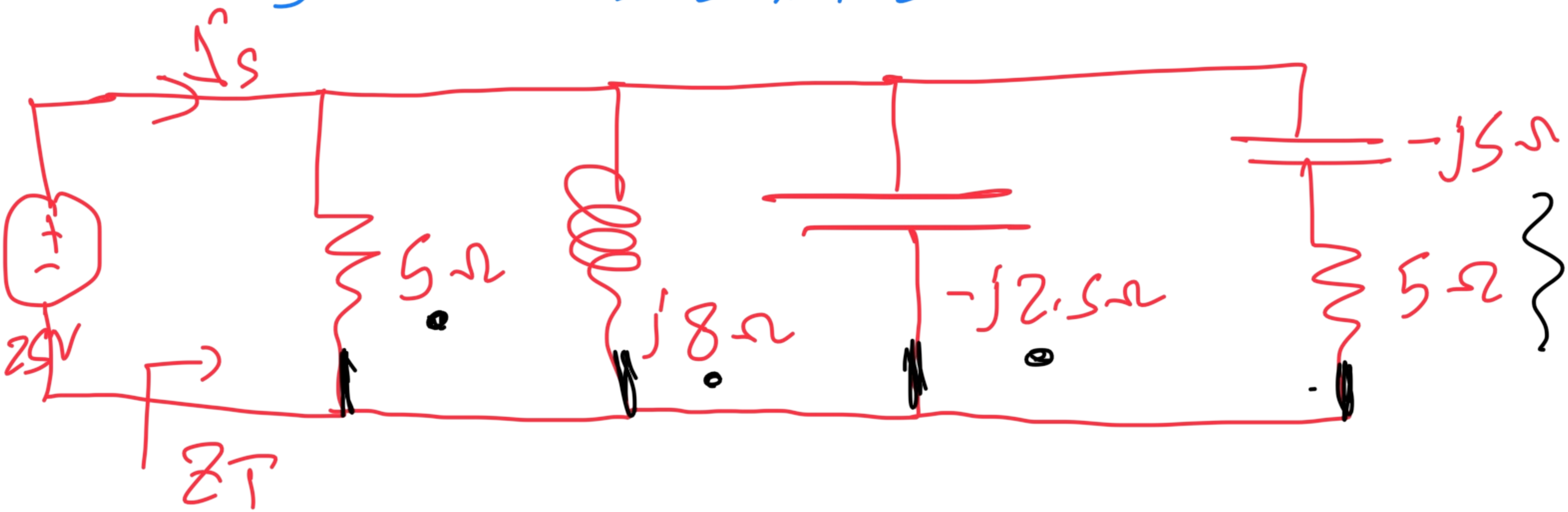
$w = 20 \text{ rad/s}$

$Z_R = R = 5\,\Omega$

$Z_L = jwL = j \times 20 \times 0.4 = j8\,\Omega$

$Z_{C_1} = \dfrac{1}{jwC} = \dfrac{1}{j \times 20 \times 0.01} = -j5\,\Omega$

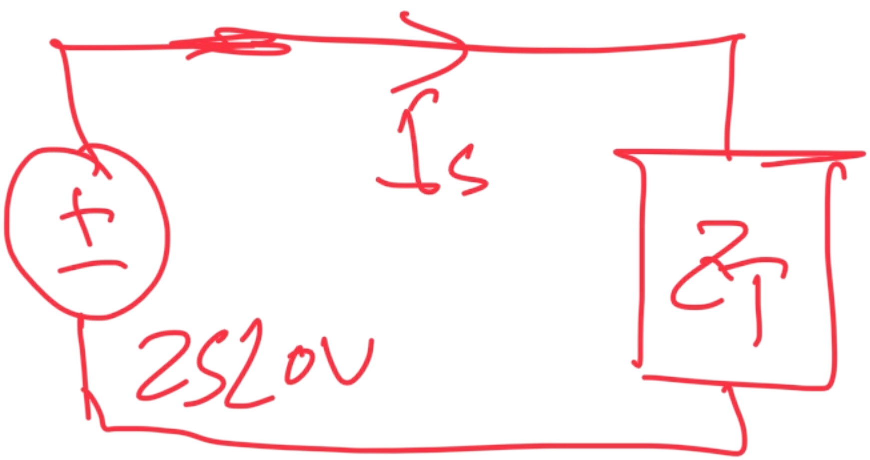
$Z_{C_2} = \dfrac{1}{jwC} = \dfrac{1}{j \times 20 \times 0.02} = -j2.5\,\Omega$



Parallel

$$Z_T = \frac{1}{\frac{1}{5} + \frac{1}{j8} + \frac{1}{-j2.5} + \frac{1}{5-j5}}$$

$$Z_T = 2.082 \angle -51.34^\circ \ \Omega$$



$$I_s = \frac{V}{Z_T} = \frac{25}{2.082\angle{-51.34}}$$

$$I_s = 7.5 + j\,9.375\ A$$

$$I_s = 12.01\angle{51.34}\ A$$

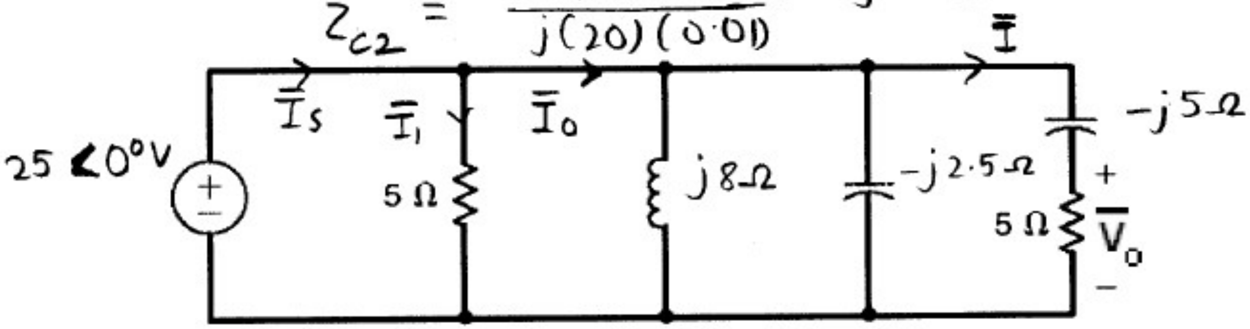
$$i_s(t) = 12.01\cos(20t + 51.34)\ A$$

SOLUTION:

$$\bar{Z}_L = j(20)(0.4) = j8\,\Omega$$

$$\bar{Z}_{c1} = \frac{1}{j(20)(0.02)} = -j2.5\,\Omega$$

$$\bar{Z}_{c2} = \frac{1}{j(20)(0.01)} = -j5\,\Omega$$



$$\bar{I} = \frac{25\angle 0^\circ}{5-j5} = 3.54\angle 45^\circ\,\text{A}$$

$$\bar{V}_o = (3.54\angle 45^\circ)(5)$$

$$\bar{V}_o = 17.68\angle 45^\circ\ \text{V}$$

$$V_o(t) = 17.68\cos(20t + 45^\circ)\ \text{V}$$

$$\bar{I}_1 = \frac{25\angle 0^\circ}{5} = 5\angle 0^\circ\text{A}$$

$$\bar{Z}_{eq} = [(5-j5)\,||\,-j2.5]\,||\,j8\,||\,5$$

$$\bar{Z}_{eq} = \left[\frac{(5-j5)(-j2.5)}{5-j5-j2.5}\right] \parallel j8 \parallel 5$$

$$\bar{Z}_{eq} = (1.96\angle -78.7^\circ \parallel j8) \parallel 5$$

$$\bar{Z}_{eq} = 2.57\angle -75.08^\circ \parallel 5$$

$$\boxed{\bar{Z}_{eq} = 2.08\angle -51.4^\circ\,\Omega}$$

$$\boxed{\bar{I}_s = \frac{25\angle 0^\circ}{2.08\angle -51.4^\circ} = 12.02\angle 51.4^\circ \text{ A}}$$

KCL: $\bar{I}_s = \bar{I}_1 + \bar{I}_0$

$$\bar{I}_0 = \bar{I}_s - \bar{I}_1$$

$$\bar{I}_0 = 12.02\angle 51.4^\circ - 5\angle 0^\circ$$

$$\boxed{\begin{aligned}\bar{I}_0 &= 9.72\angle 75.1^\circ \text{ A}\\ i_o(t) &= 9.72\cos(20t + 75.1^\circ) \text{ A}\end{aligned}}$$

8.79 Using loop analysis and MATLAB, find $\mathbf{I}_o$ in the network in Fig. P8.79.

mesh analysis

phasor Domain

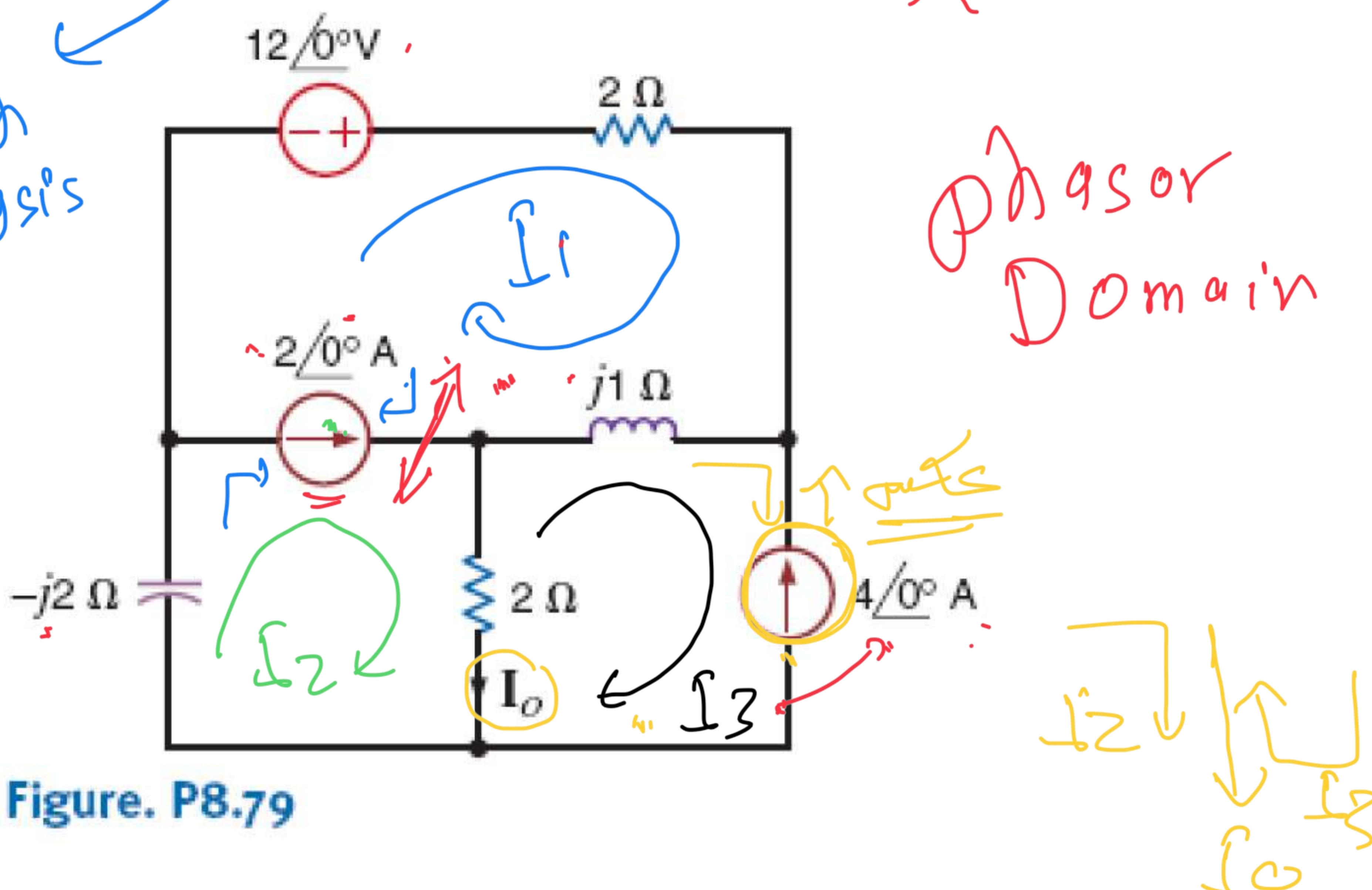


Figure. P8.79

Note

$I_0 = \boxed{I_2} - I_3 \longrightarrow (1)$

$I_3 = -4A \longrightarrow (2)$

(3) $\Longrightarrow$ super mesh / super loop $\Rightarrow$ 2 eq

$\hookrightarrow I_2 - I_1 = 2 \longrightarrow (1)$

loop 1 + 2

$$-j2\, I_2 - 12 + 2 I_1 + j1(I_1 - I_3) + 2(I_2 - I_3) = 0$$

$I_3 = -4$

$$I_1(2+j) + I_2(2-j2) = 12 + jI_3 + 2I_3$$

$12 + j x - 4 + 2x - 4$

$$\boxed{I_1(2+j) + I_2(2-j2) = 4 - j4} \rightarrow 1$$

$$\boxed{I_1(-1) + I_2(1) = 2} \rightarrow 2$$

A B E

C D F

$$x_1 = \frac{ED - BF}{AD - BC}$$

$$\boxed{x_1 = 0A}$$

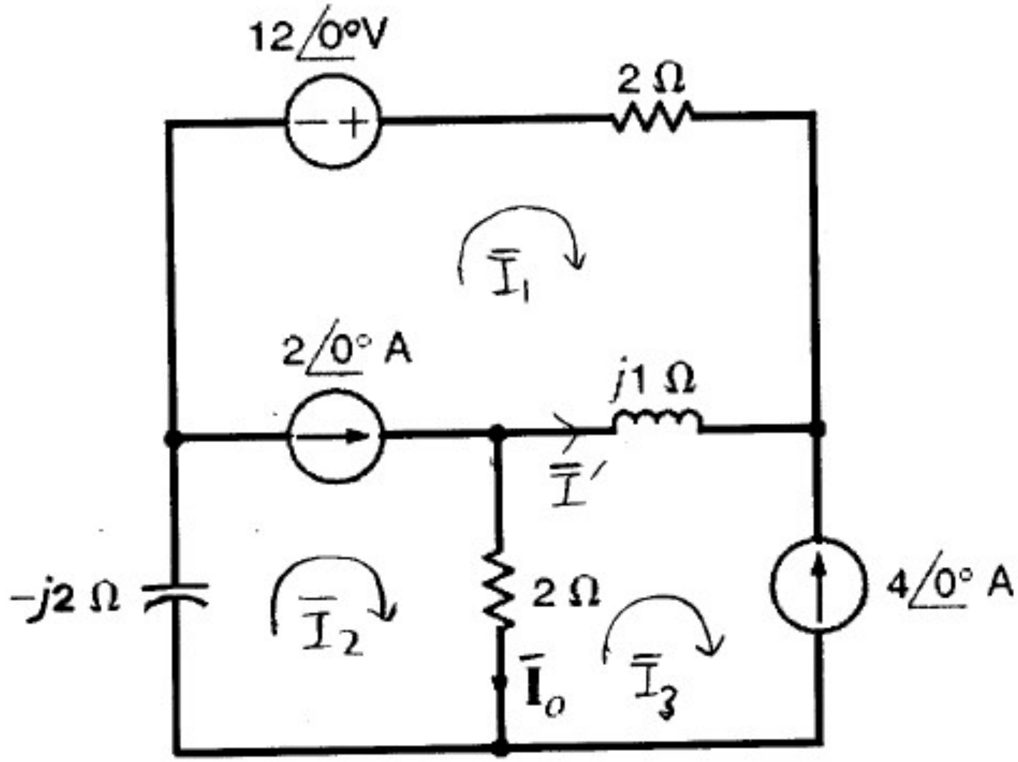
$$I_2 = \frac{AF - EC}{AD - BC} = \underline{\underline{2A}} = 2\angle 0\ A$$

$$I_0 = I_2 - I_3 = 2 - (-4)$$

$$I_0 = 2 + 4 = 6A$$

$$\boxed{I_0 = 6\angle 0\ A}$$

SOLUTION:



KCL : $\bar{I}_0 + \bar{I}_3 = \bar{I}_2$

$$\bar{I}_0 = \bar{I}_2 - \bar{I}_3$$

KCL: $\bar{I}' + \bar{I}_0 = 2\angle 0^\circ$

$$\bar{I}' = 2\angle 0^\circ - \bar{I}_0$$

KVL: $2\bar{I}_1 + j1(-\bar{I}') + 2\bar{I}_0 - j2\bar{I}_2 = 12\angle 0^\circ$

$$2\bar{I}_1 - j1(2\angle 0^\circ - \bar{I}_0) + 2(\bar{I}_2 - \bar{I}_3) - j2\bar{I}_2 = 12\angle 0^\circ$$

$$2\bar{I}_1 + (2 - j2)\bar{I}_2 - 2\bar{I}_3 + j1(\bar{I}_2 - \bar{I}_3) = 12\angle 0^\circ + j1(2\angle 0^\circ)$$

$$2\bar{I}_1 + (2 - j1)\bar{I}_2 + (-2 - j1)\bar{I}_3 = 12\angle 0^\circ + j1(2\angle 0^\circ)$$

$$\bar{I}_3 = -4\angle 0^\circ = 4\angle 180^\circ \text{A}$$

$$2\bar{I}_1 + (2 - j1)\bar{I}_2 + (-2 - j1)(4\angle 180^\circ) = 12.17\angle 9.46^\circ$$

$$2\bar{I}_1 + (2 - j1)\bar{I}_2 = 4.48\angle -26.54^\circ$$

$$\text{KCL}: \bar{I}_1 + 2\angle 0^\circ = \bar{I}_2$$

$$\bar{I}_1 - \bar{I}_2 = 2\angle 180^\circ$$

$$2\bar{I}_1 + (2-j1)\,\bar{I}_2 = 4.48\angle -26.54^\circ$$

$$\bar{I}_1 - \bar{I}_2 = 2\angle 180^\circ$$

$$\boxed{\bar{I}_1 = 0\ A}$$

$$\bar{I}_2 = 2\angle 1.43^\circ A$$

$$\bar{I}_3 = 4\angle 180^\circ\ A$$

$$\bar{I}_0 = \bar{I}_2 - \bar{I}_3$$

$$\bar{I}_0 = 2\angle 1.4^\circ - 4\angle 180^\circ$$

$$\bar{I}_0 = 6\angle 0^\circ A$$

MATLAB Solution:

```
Z = [2 2-i ; 1 -1]
V = [4-2i ; -2]
I = inv(Z)*V
I2 = I(2)
I3 = -4
I0 = I2-I3
```

Answer:

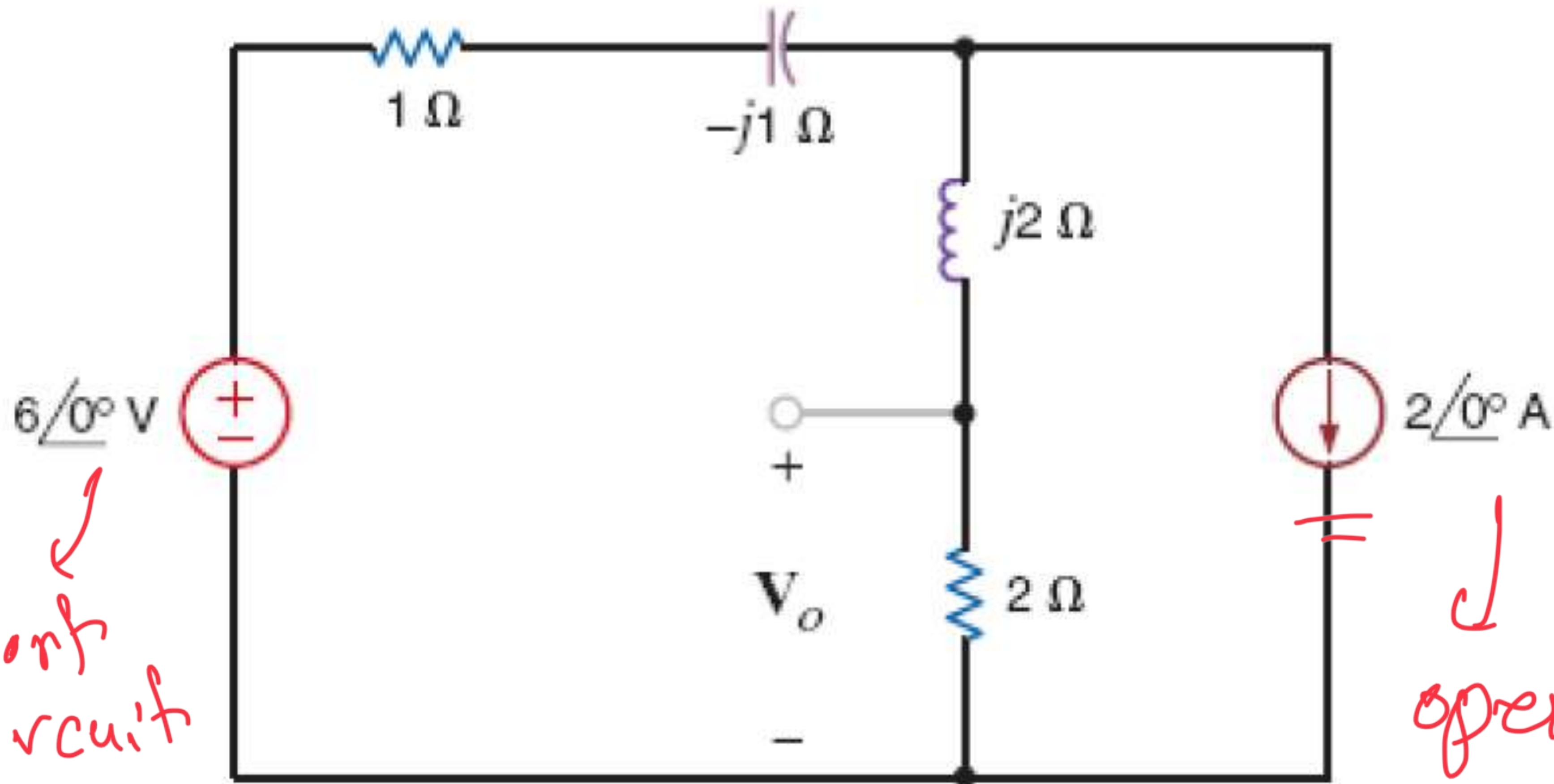
```
Z =
    2.0000        2.0000-1.0000i
    1.0000       -1.0000

V =
    4.0000  - 2.0000i
   -2.0000

I =
   -0.0000
    2.0000

I2 =
      2
I3 =
     -4
I0 =
     6
```

8.91 Using superposition, find $\mathbf{V}_o$ in the circuit in Fig. P8.91.

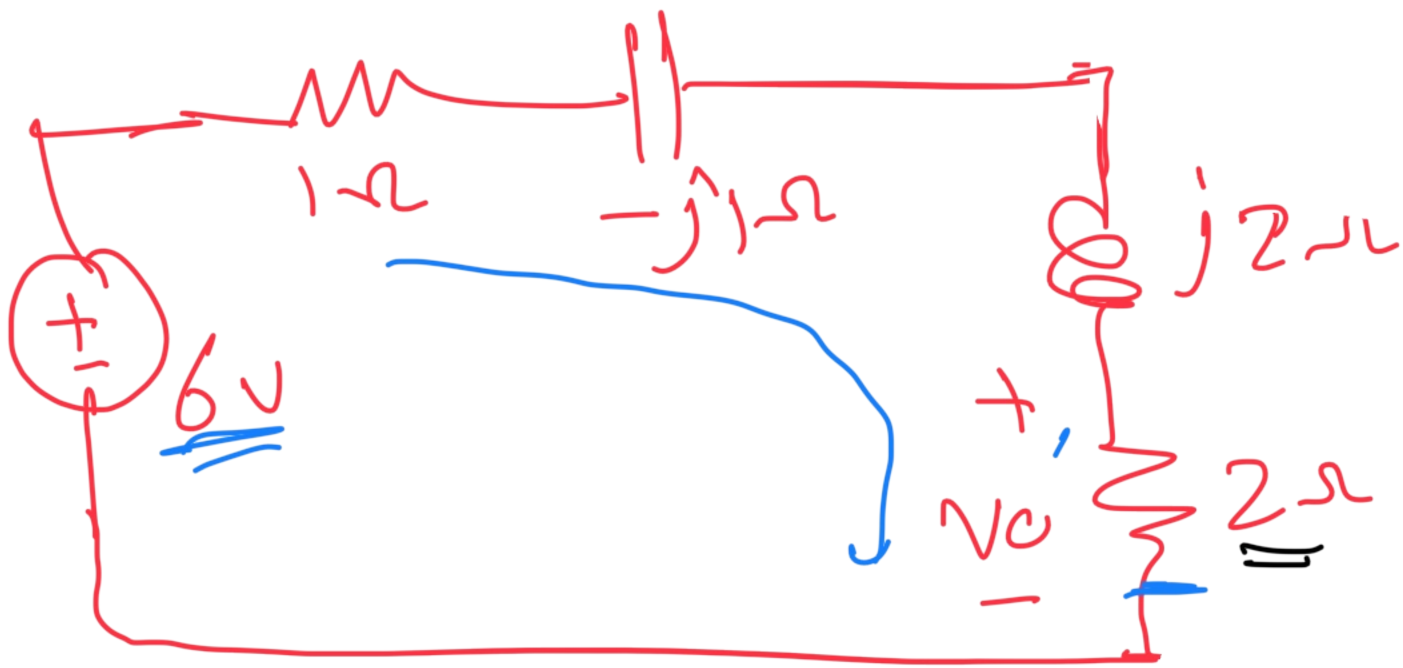


short circuit
"سلك"

open circuit
"شيله"

Figure P8.91

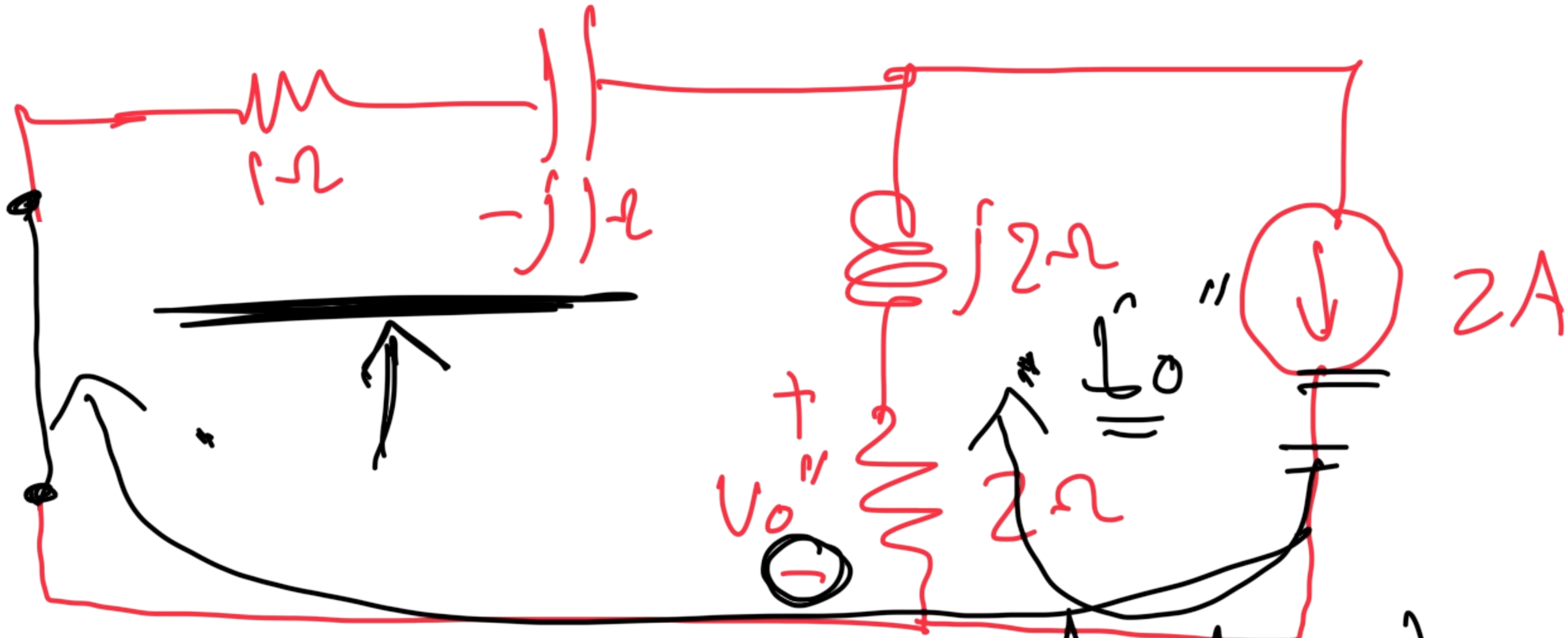
[1] Turn off 2A and keep 6V



series $V_o' = \dfrac{2}{1 - j1 + j2 + 2} * 6$

$V_o' = 3.6 - j1.2\,V = 3.79 \angle -18.43\,V$

[2] Turn off 6V and keep 2A



parallel $\Longrightarrow$ current divider

$$I_o'' = \frac{1-j1}{1-j+j2+2} * 2$$

$$I_o'' = 0,4 - j0,8\ A$$

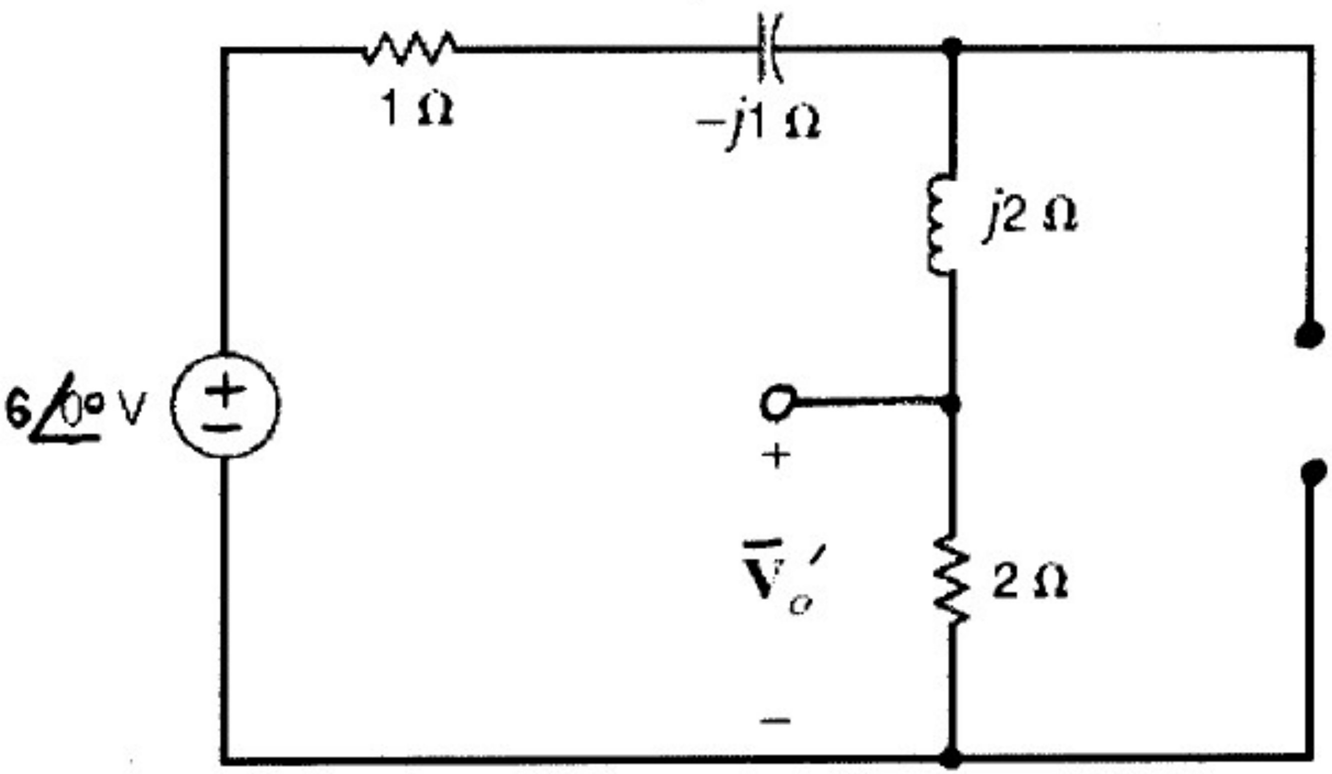
$$V_o'' = - I_o'' * R = -(0,4 - j0,8) * 2$$

$$V_o'' = -0,8 + j1,6\ V = 1.79 \angle 116.565\ V$$

$$V_o = V_o' + V_o'' = 3,6 - j1,2 + -0,8 + j1,6$$

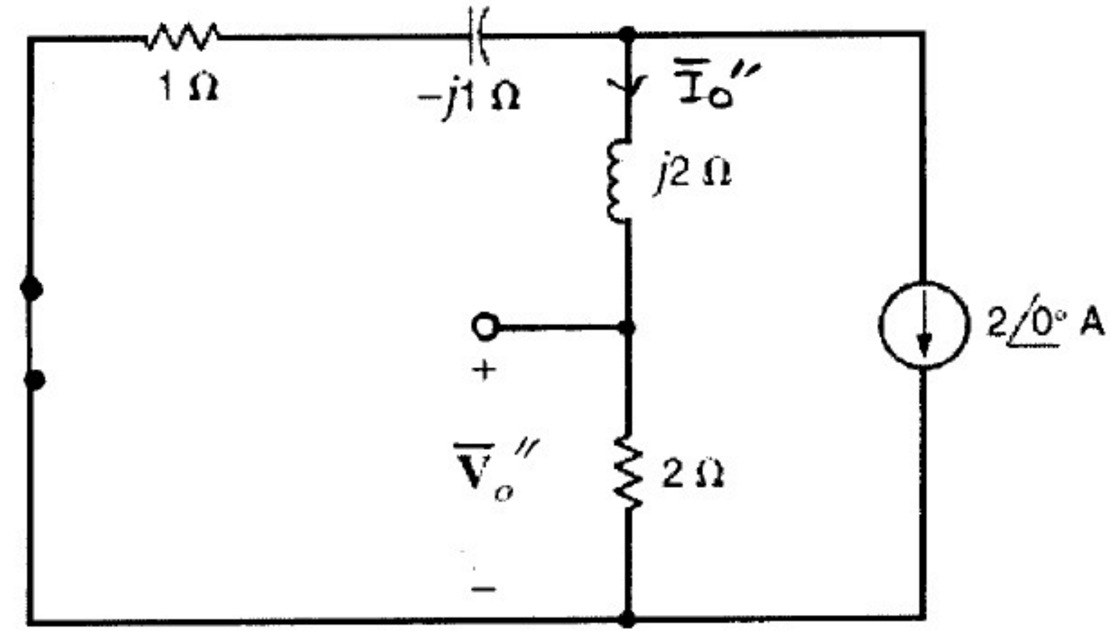
$$\boxed{V_o = 2,8 + j0,4 = 2,828 \angle 8,13\ V}$$

SOLUTION:



$$\bar{V}_o' = \left(\frac{2}{2+j2+1-j1}\right)(6\angle 0^\circ)$$

$$\bar{V}_o' = 3.79 \angle -18.43^\circ \text{ V}$$



$$\bar{I}_0'' = \left(\frac{1-j1}{1-j1+2+j2}\right)\left(-2\angle 0^\circ\right)$$

$$\bar{I}_0'' = 0.89\angle 116.57^\circ \text{ A}$$

$$\bar{V}_0'' = 2(0.89\angle 116.57^\circ \text{ A})$$

$$\boxed{\bar{V}_0'' = 1.79\angle 116.57^\circ \text{ V}}$$

$$\bar{V}_0 = \bar{V}_0' + \bar{V}_0'' = 3.79\angle -18.43^\circ + 1.79\angle 116.57^\circ$$

$$\bar{V}_0 = 2.82\angle 8.2^\circ \text{V}$$

8.104 Apply Thévenin's theorem twice to find $\mathbf{V}_o$ in the circuit in Fig. P8.104.

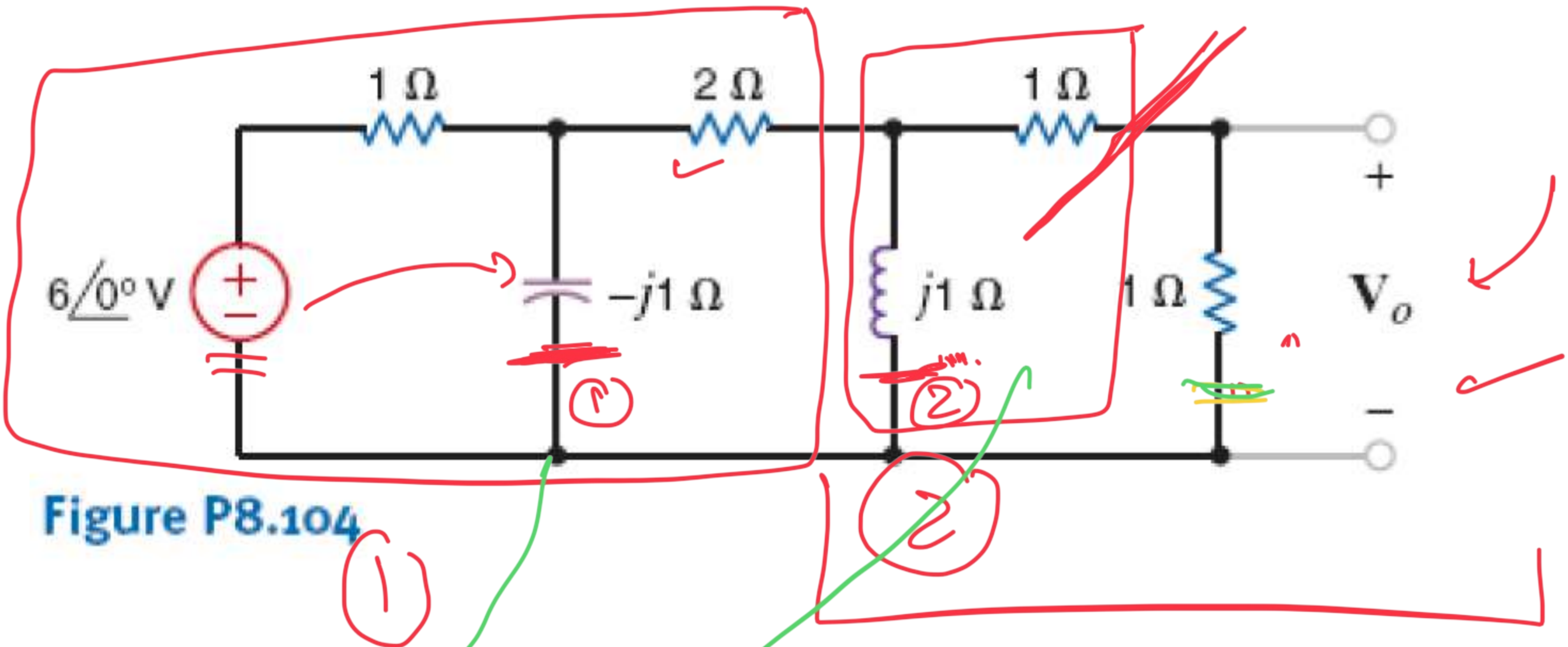


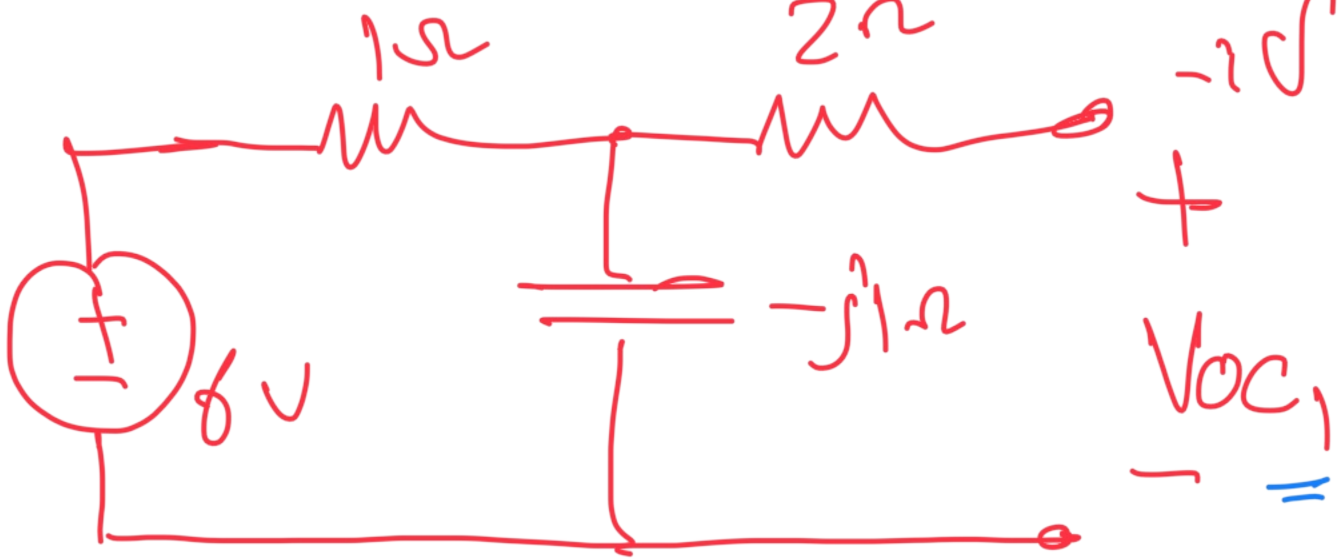
Figure P8.104

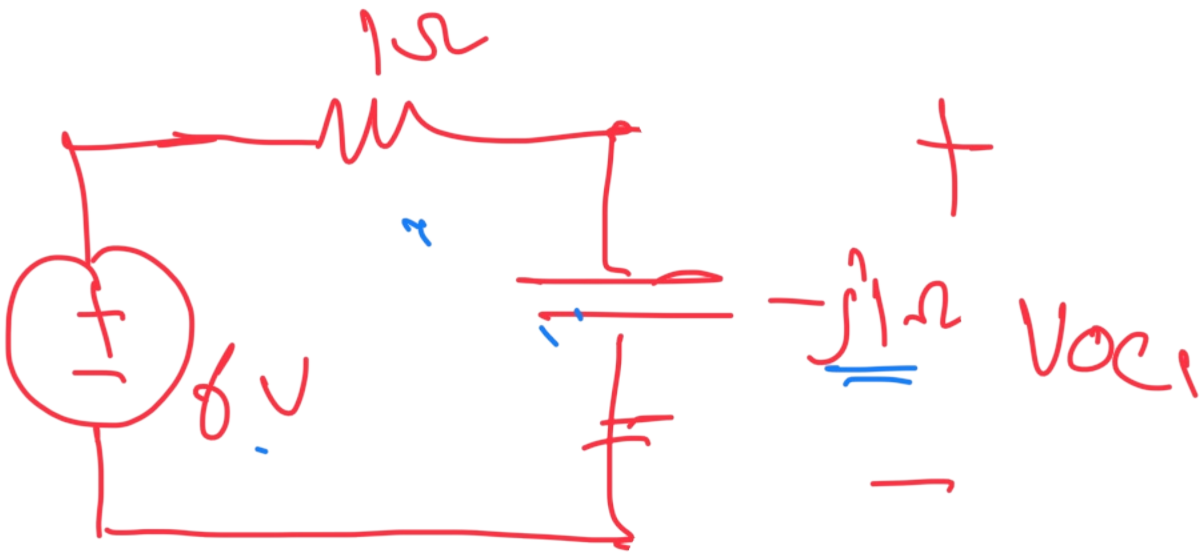
Vo.c.c

Zth

Thev

المرحلة الأولى:



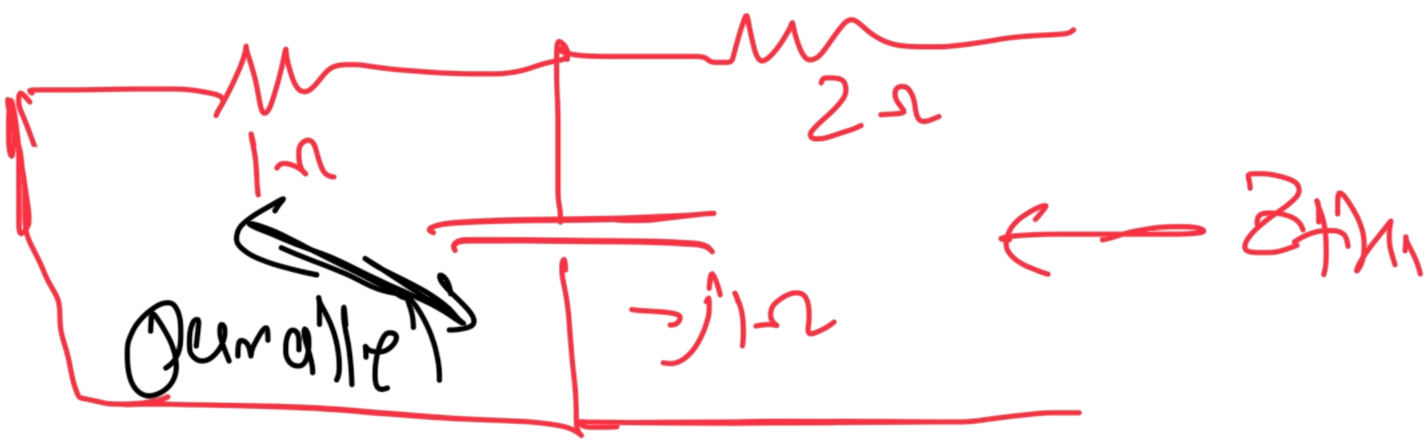


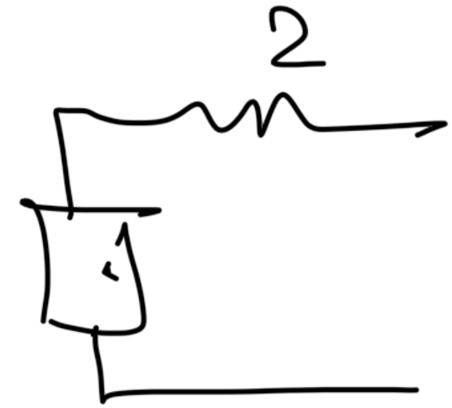
open

series $\Longrightarrow$ voltage divider

$$V_{oc1} = \frac{-j1}{1-j1} * 6 = 3-j3 = 3\sqrt{2}\angle -45\ V$$

To find Z_{th}: Turn off all ind source

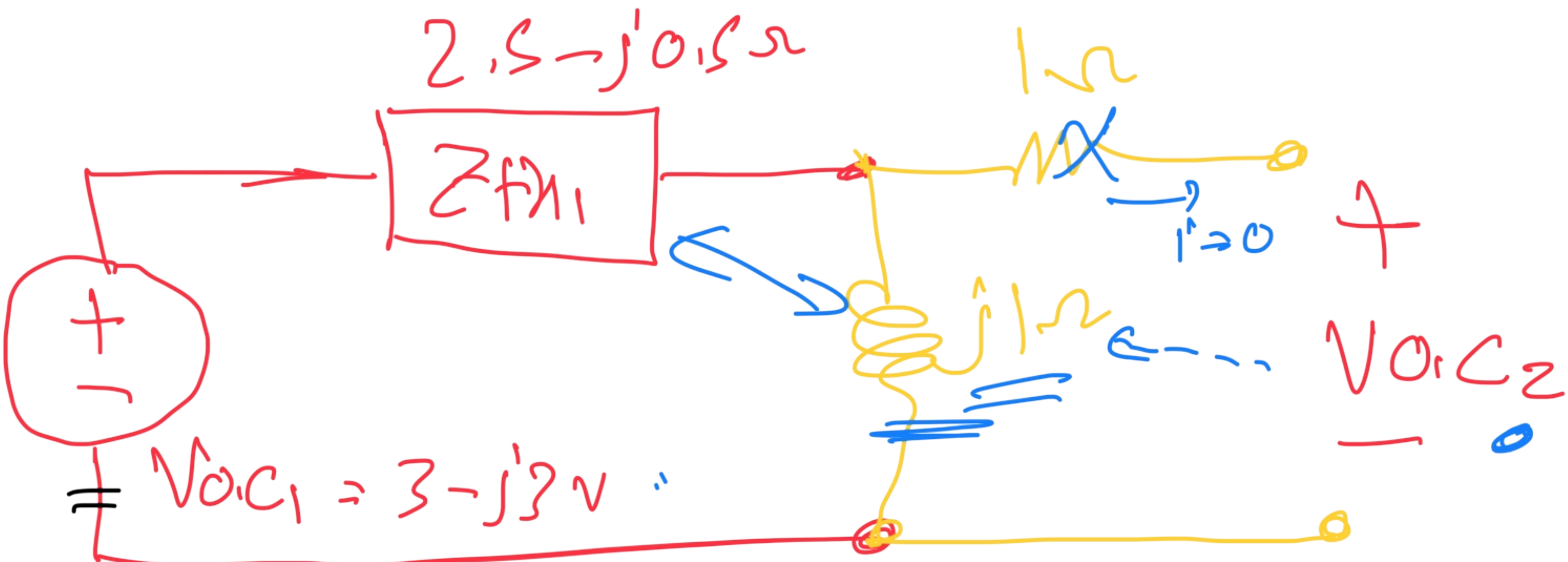




$$Z_{th_1} = \frac{1}{\frac{1}{1} + \frac{1}{-j1}} + 2$$

$$Z_{th_1} = 2.5 - j0.5 = 2.55 \angle -11.31\ \Omega$$

المرحلة الثانية :-

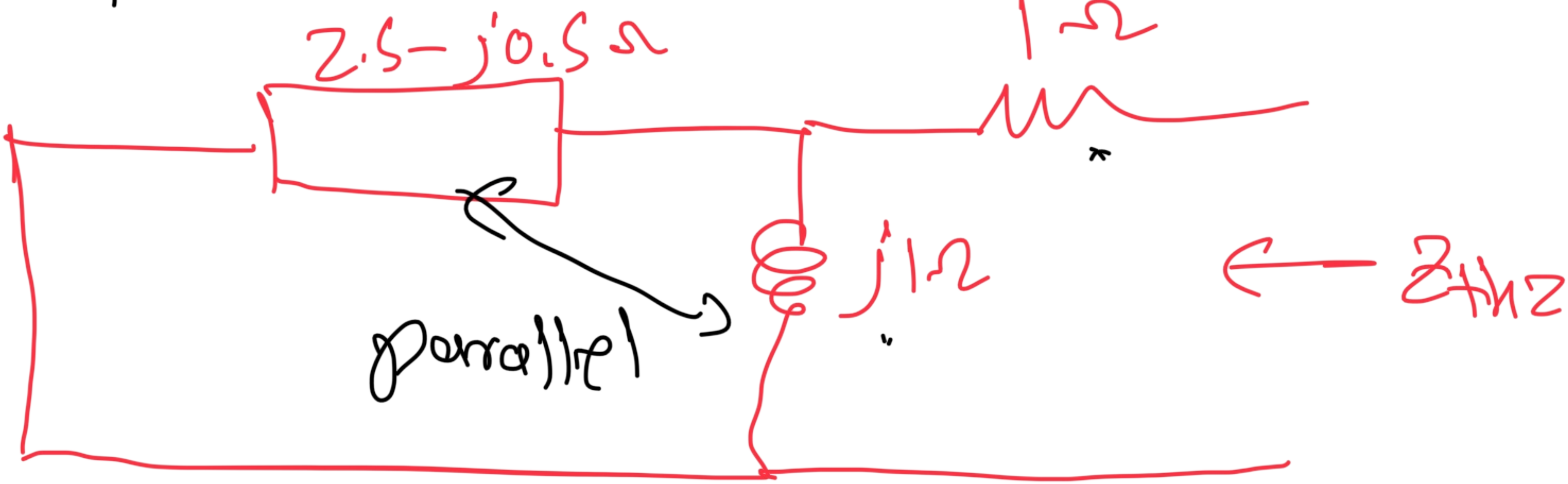


series ⟹ voltage divider

$$V_{o.c_2} = \frac{j1}{2.5 - j0.5 + j1} \times (3 - j3)$$

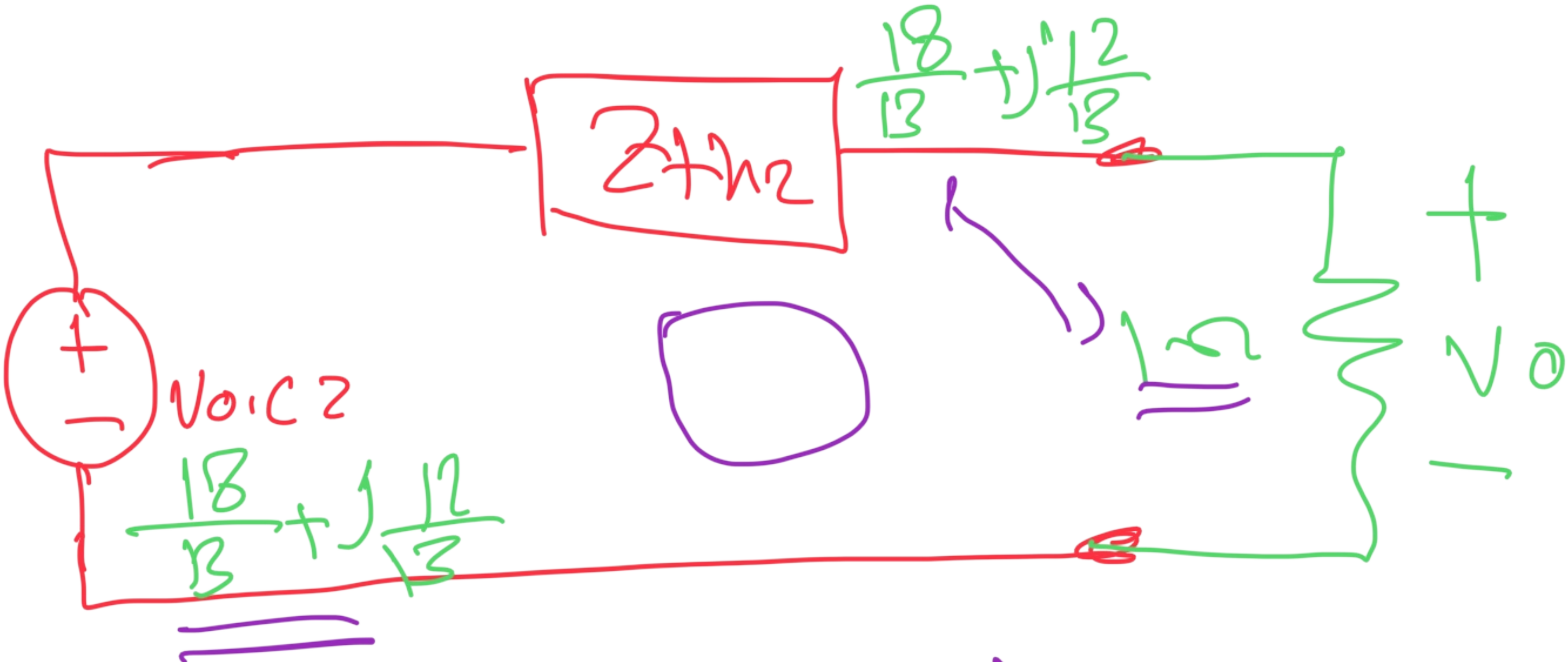
$$V_{O.C\,2} = \frac{18}{13} + j\frac{12}{13}\ V = 1.66\angle 33.7\ V$$

$Z_{th\,2}$



$$Z_{th\,2} = 1 + \frac{1}{\frac{1}{j1} + \frac{1}{2.5 - j0.5}}$$

$$Z_{th2} = \frac{18}{13} + j\frac{12}{13}\,\Omega = 1.66\angle 33.7\,\Omega$$

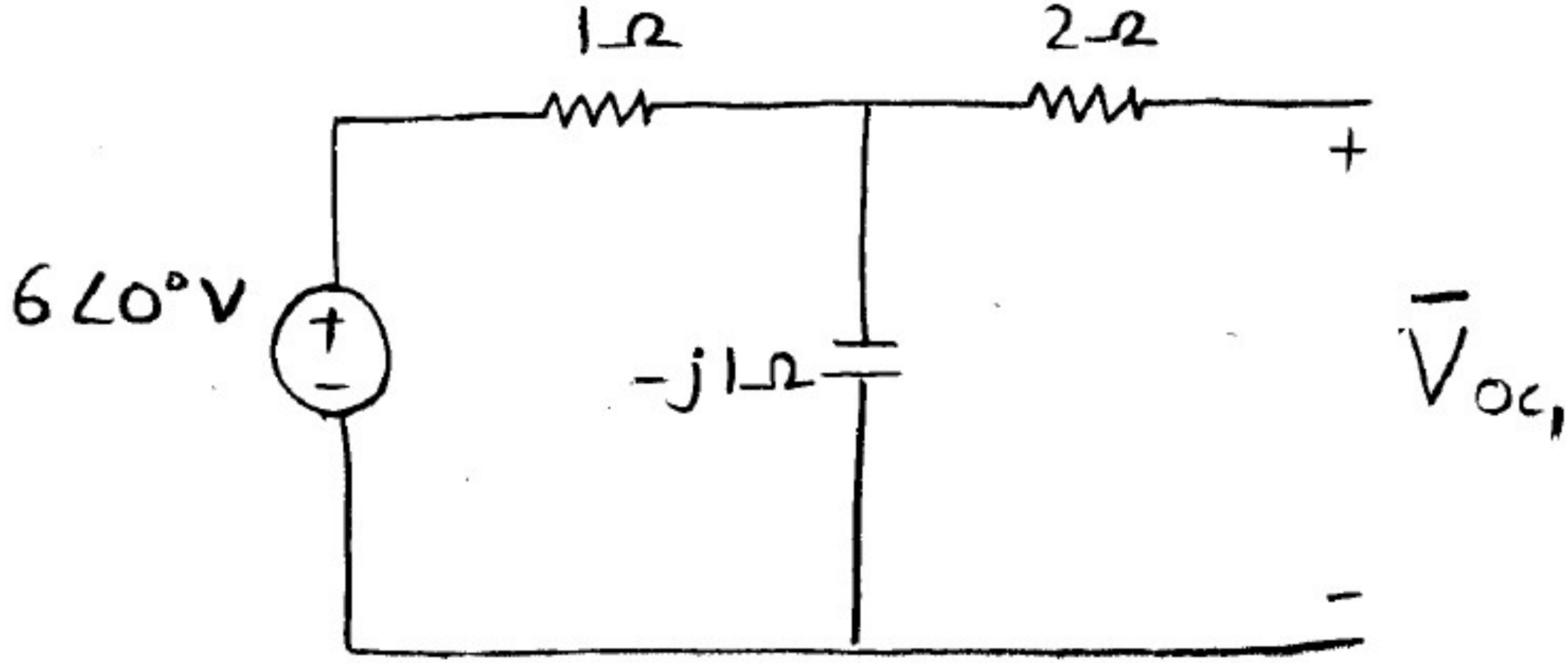


Series $\Longrightarrow$ Voltage divider

$$V_o = \frac{1}{1 + \frac{18}{13} + j\frac{12}{13}} \times \left(\frac{18}{13} + j\frac{12}{13} \right)$$

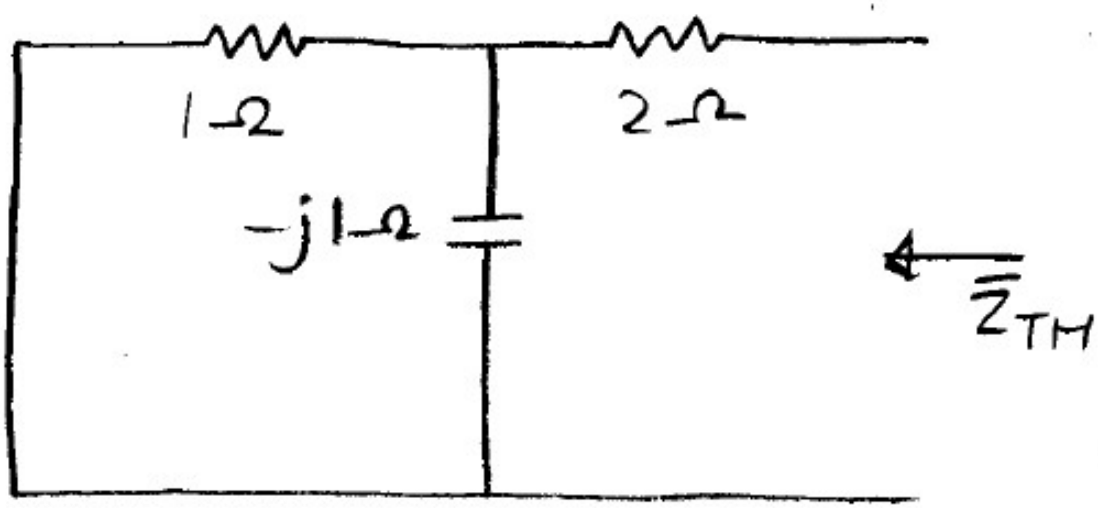
$$\Rightarrow \boxed{\begin{array}{l} V_o = \frac{54}{85} + j\frac{12}{85}\ V \\ \\ V_o = 0{,}6508 \angle 12{,}5288\ V \end{array}}$$

SOLUTION:



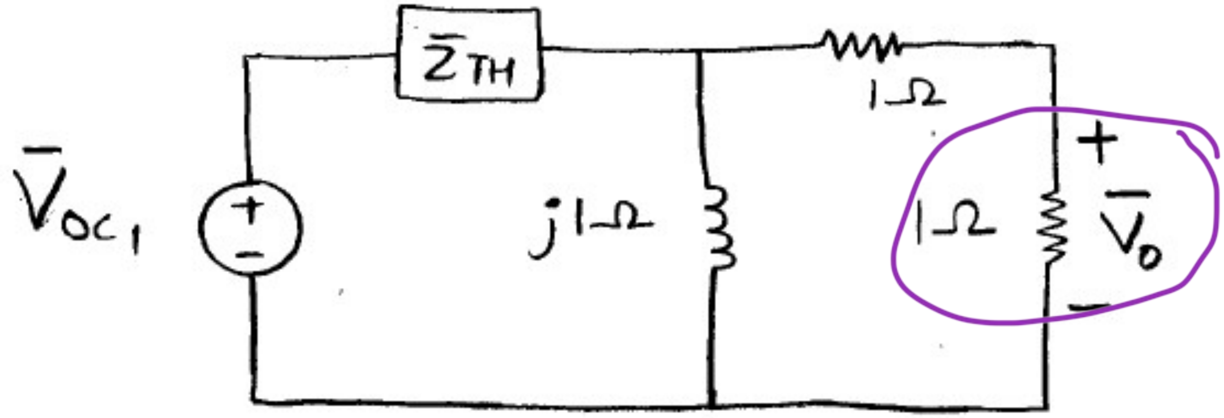
$$\bar{V}_{oc_1} = \left(\frac{-j1}{1-j1}\right)(6\angle 0^\circ)$$

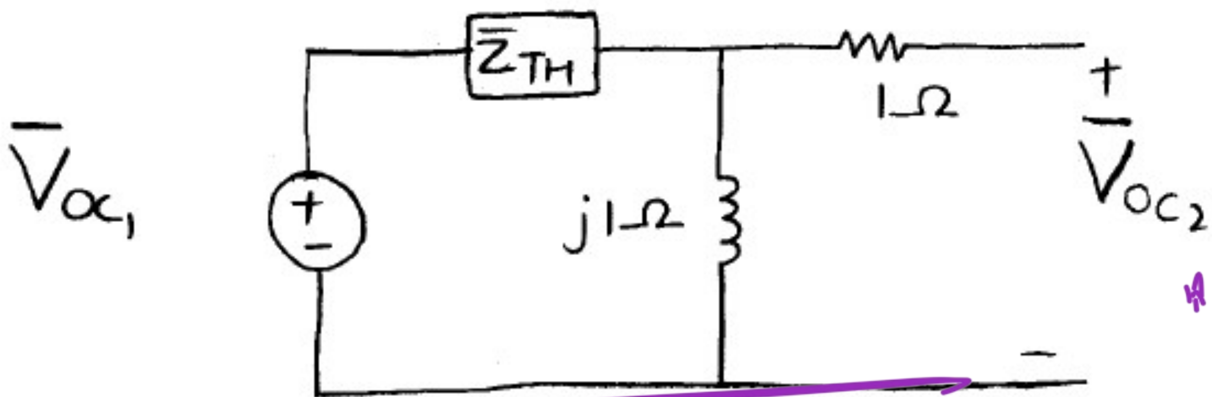
$$\bar{V}_{oc_1} = 4.24\angle -45^\circ\ \text{V}$$



$$\bar{Z}_{TH} = 2 + (1 \parallel -j1) = 2 + \frac{1(-j1)}{1-j1}$$

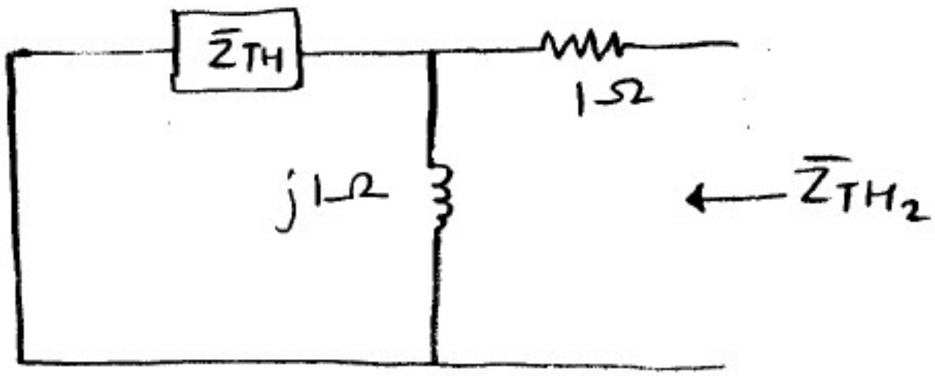
$$\bar{Z}_{TH} = 2.55 \angle -11.31^\circ \ \Omega$$





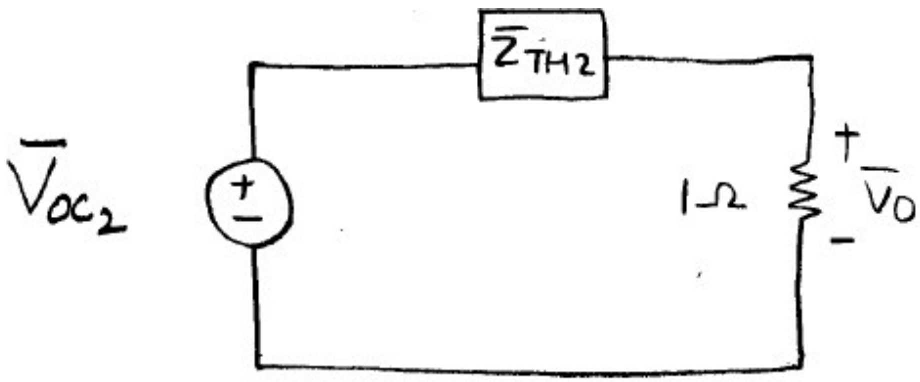
$$\bar{V}_{oc_2} = \left(\frac{j1}{j1 + 2.55\angle -11.31^\circ}\right)(4.24\angle -45^\circ)$$

$$\bar{V}_{oc_2} = 1.66\angle 33.7^\circ$$



$$\bar{Z}_{TH_2} = 1 + (Z_{TH_1} \parallel j1)$$

$$\bar{Z}_{TH_2} = 1 + \frac{(2.55\angle -11.31^\circ)(j1)}{2.55\angle -11.31^\circ + j1} = 1.66\angle 33.67^\circ\,\Omega$$



$$\bar{V}_O = \left(\frac{1}{1 + 1.66\angle 33.67^\circ}\right)(1.66\angle 33.67^\circ)$$

$$\boxed{\bar{V}_O = 0.65\angle 12.54^\circ\ \text{V}}$$