Topic 1

# AC Networks Analysis

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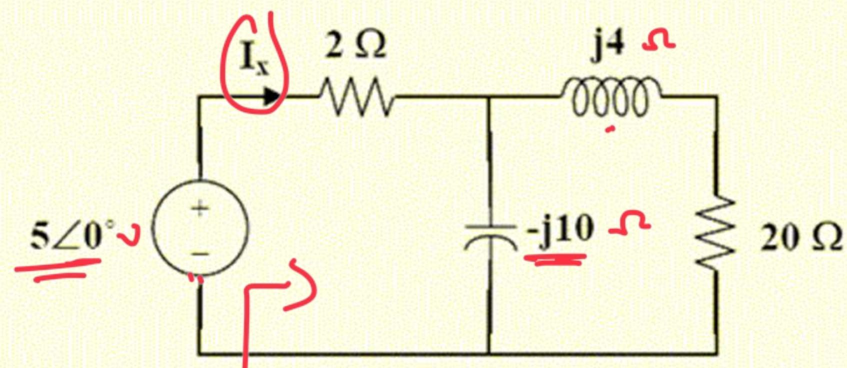
Part 3

**Example:**

For the circuit shown in the Figure, determine:

1. The total impedance
2. The current $I_x$




$Z_T$ phasor domain

**Solution:**

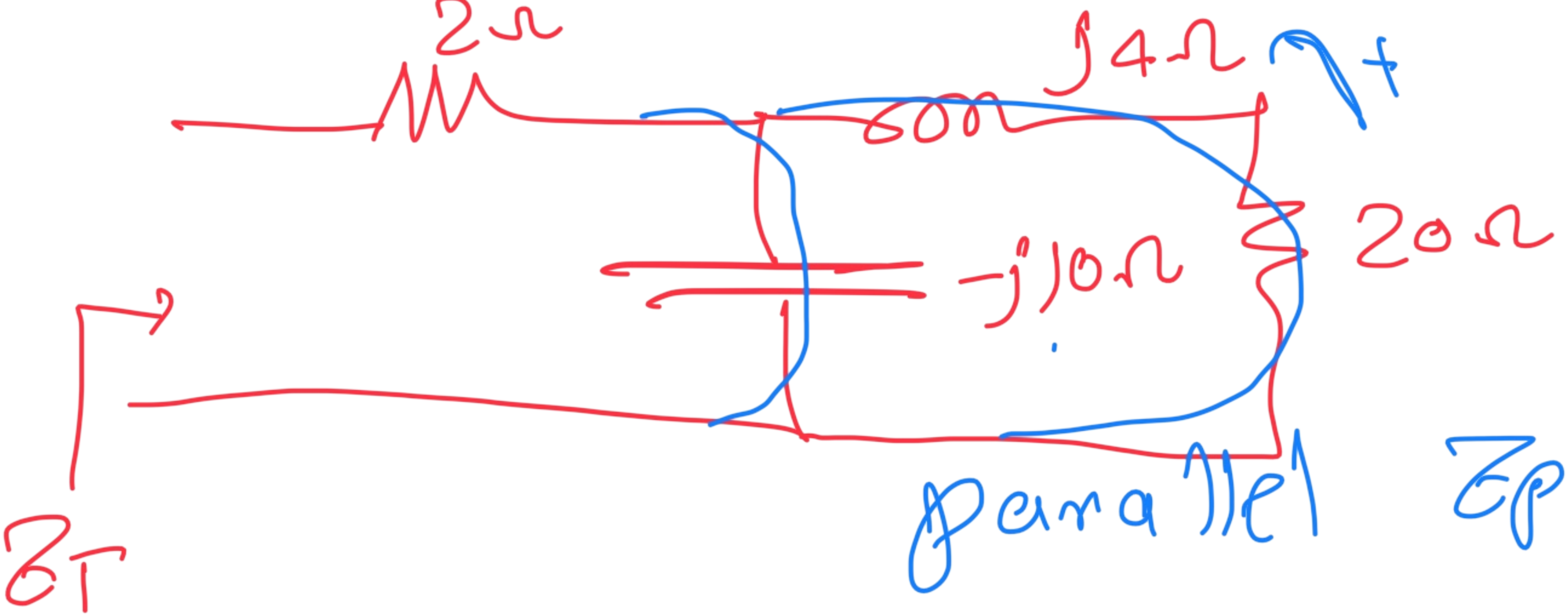
$$I_X = \frac{V}{Z_T}$$

$$Z_T = 2 + \left(\frac{(-j10)(20+j4)}{(-j10)+(20+j4)}\right) = 4.588 - j8.626\ \Omega$$

6.587

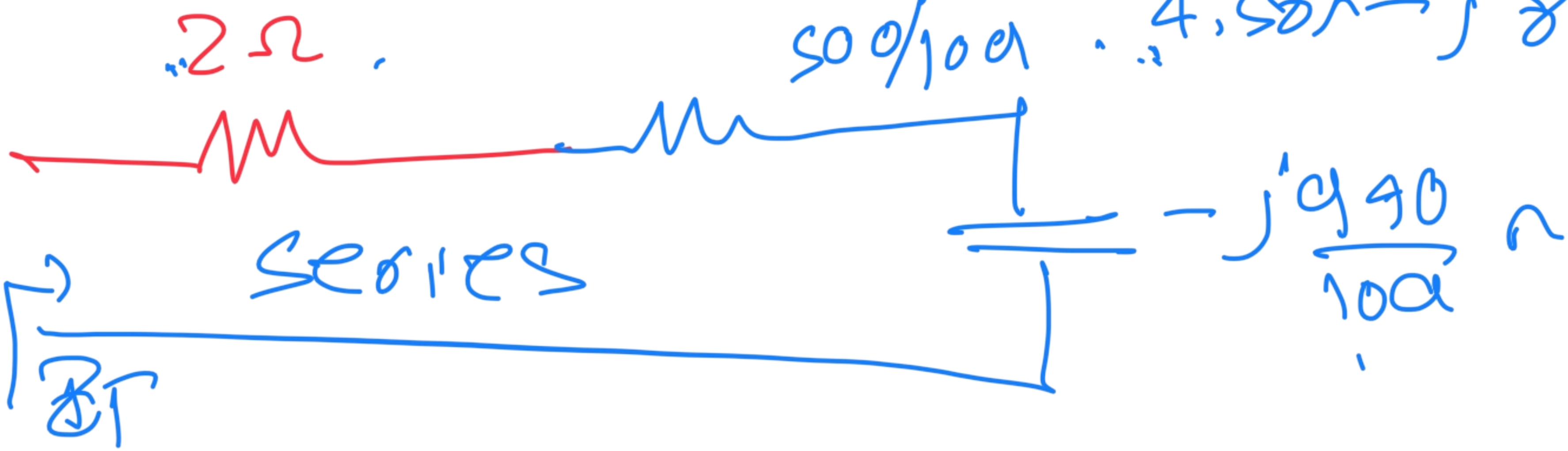
$$I_X = \frac{5}{2+\frac{-j10(20+j4)}{-j10+20+j4}} = \frac{5}{2+4.588-j8.626} = \frac{5}{10.854\angle -52.63^\circ} = 0.4607\angle 52.63^\circ$$

$i_s(t) =$ **0.4607cos(2000t +52.63°) A**



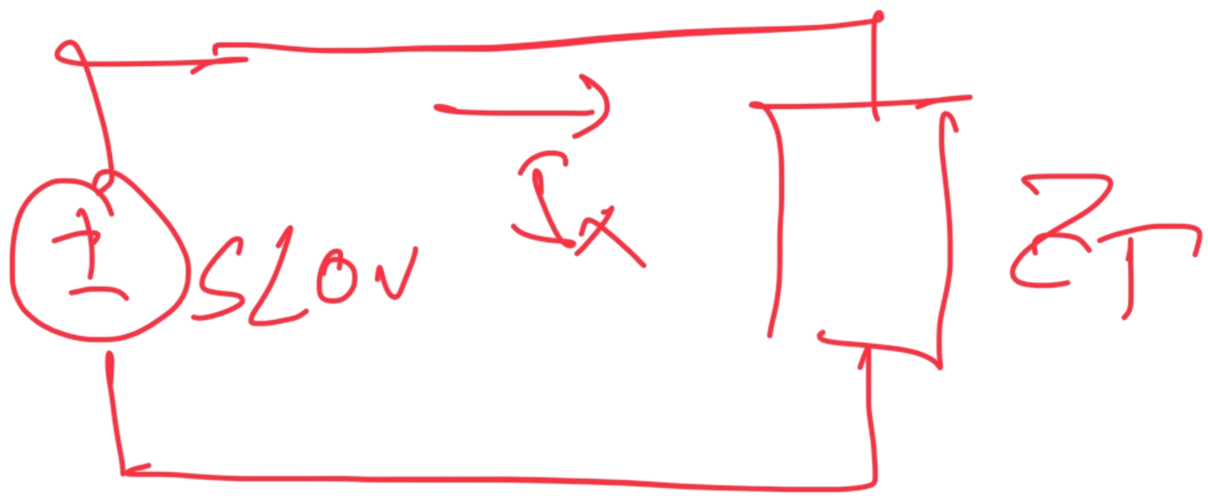

$$Z_p = \frac{1}{\frac{1}{-j10} + \frac{1}{20+j4}} \Rightarrow \frac{500}{109} - j\frac{990}{109}\,\Omega$$

$$4{,}587 - j\,8{,}624$$



$$Z_T = Z + \frac{500}{109} - j\frac{940}{109}$$

$$Z_T = 6.587 - j8.624\ \Omega$$




$$I_x = \frac{V}{Z_T} = \frac{5\angle 0}{6.587 - j8.624}$$

$$I_x = \underline{\underline{0{,}4607}} \angle \underline{52.63^\circ}$$ صيغة قطبية

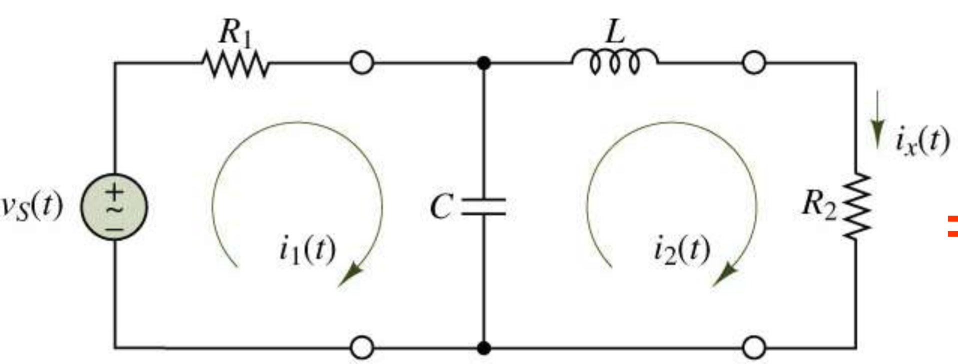
polar

$$w = \underline{\underline{2000}} \ \text{rad/s}$$

$$\boxed{i_x(t) = 0{,}4607 \cos(2000t + 52.63) \ A}$$

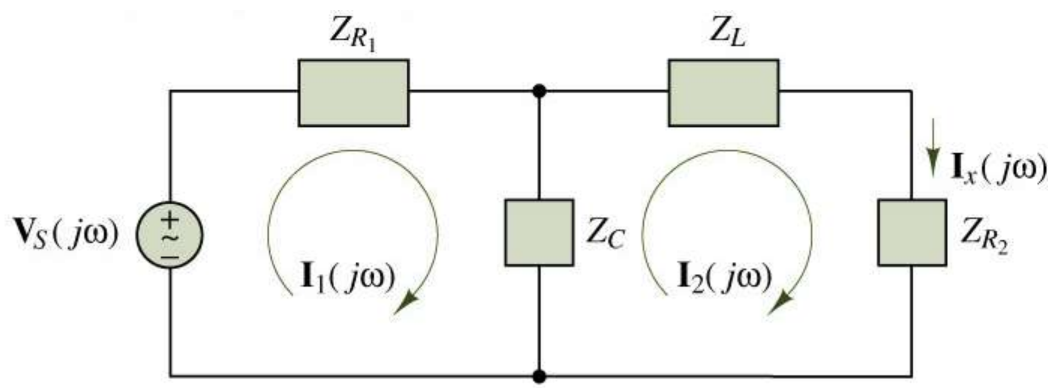
## Analysis of AC Circuits

- Substitute for every network with its impedance equivalent in phasor domain
- Express currents & voltages in phasor domain.
- Use same approach as DC network (Nodal analysis, Mesh Analysis or superposition) to solve for desired component




A sample circuit
for AC analysis

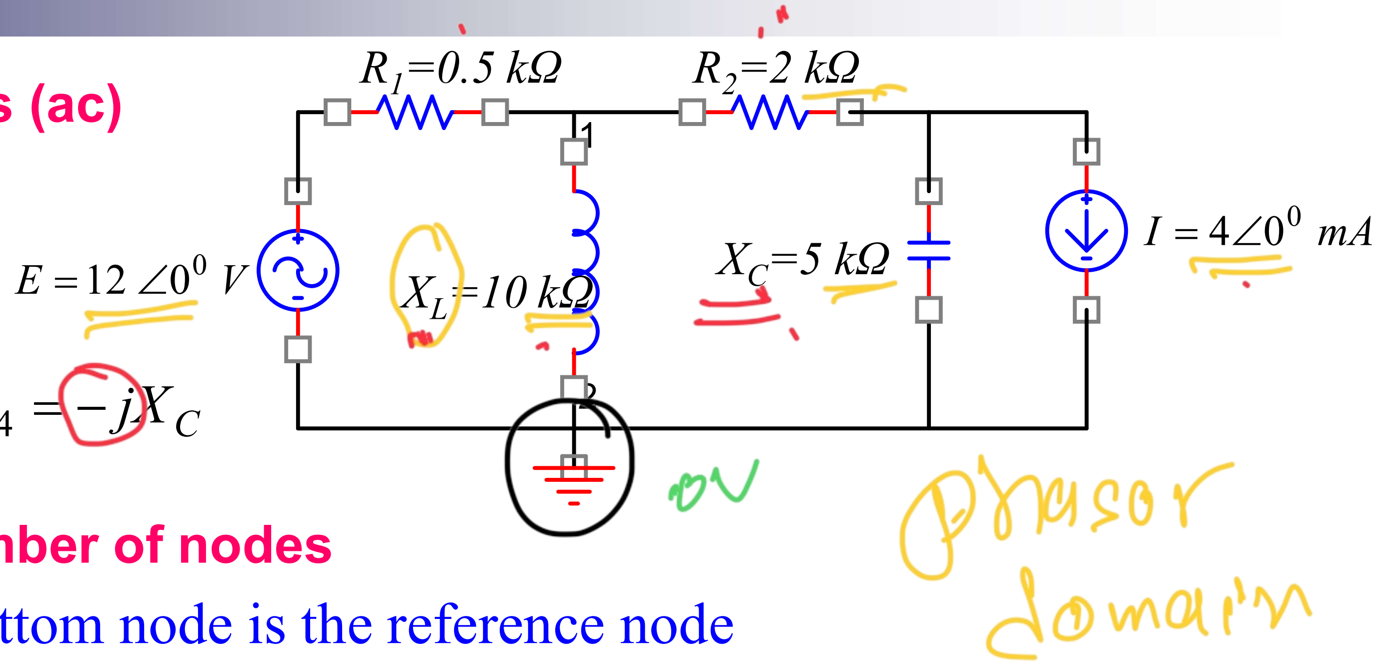
==>




The same circuit
in phasor form

حول كل شي لل phasor

## Example: Nodal analysis (ac)



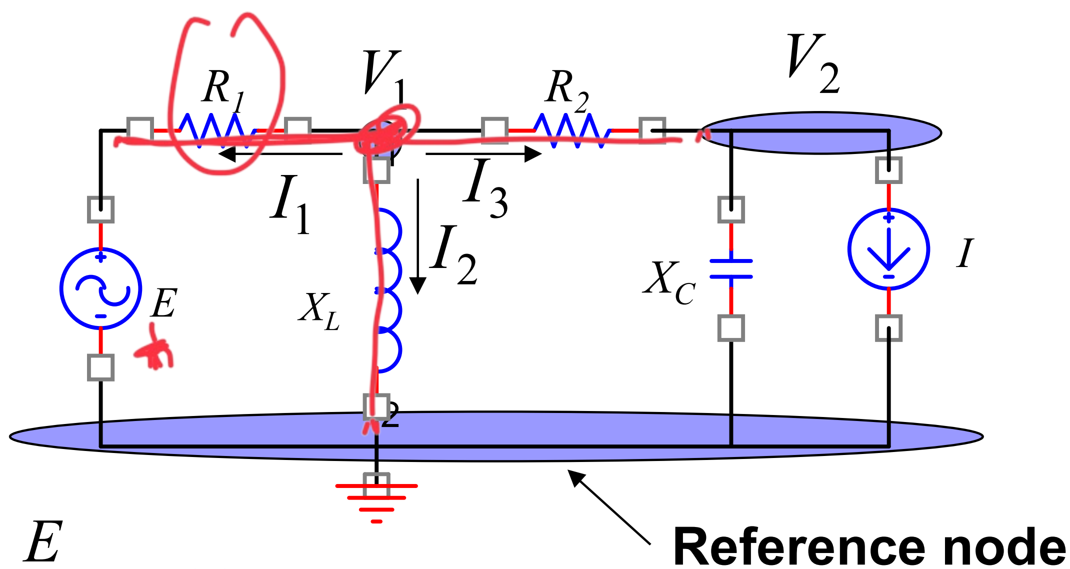

For simplification let
$Z_1 = R_1, Z_2 = jX_L, Z_3 = R_2, Z_4 = -jX_C$

**Step 1: Determine the number of nodes**

3 nodes in this circuit. Bottom node is the reference node

**Step 2: Apply KCL at all nodes except reference node**

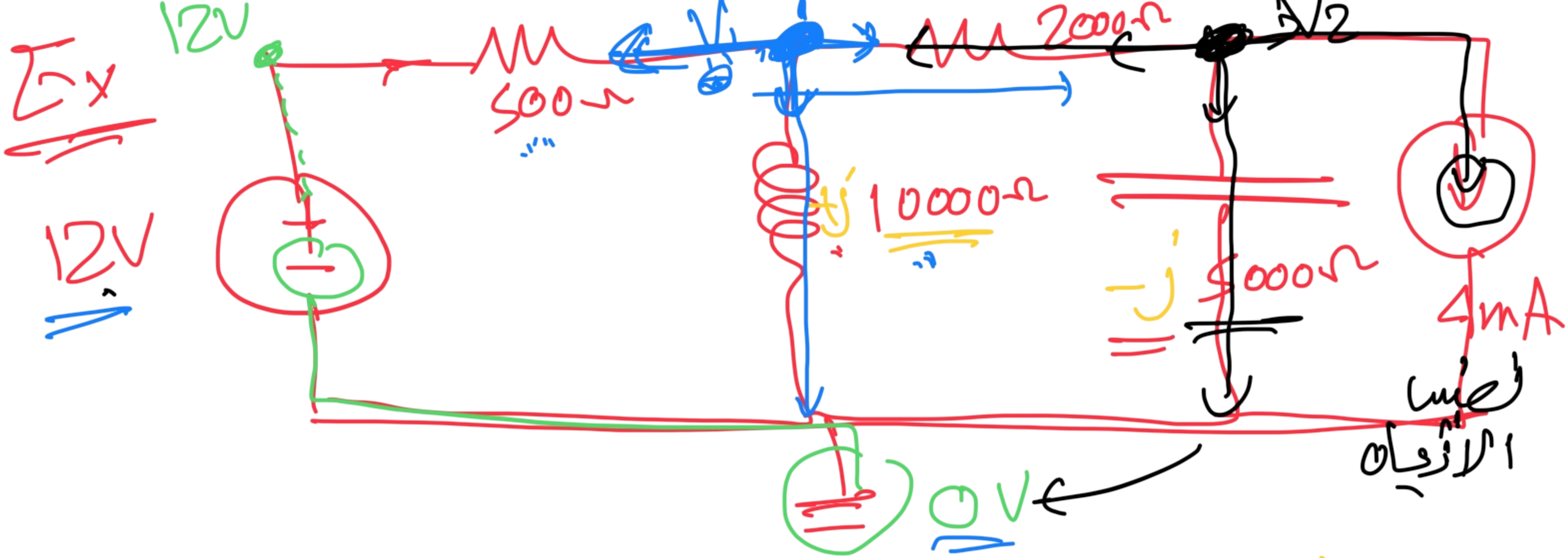
Node V1:




$$0 = I_1 + I_2 + I_3$$

$$\frac{V_1 - E}{Z_1} + \frac{V_1}{Z_2} + \frac{V_1 - V_2}{Z_3} = 0$$

$$V_1\left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right] - V_2\left[\frac{1}{Z_3}\right] = \frac{E}{Z_1} \qquad (1)$$




$$Z_L = j\omega L = +jX_L \rightarrow X_L = \omega L$$

$$Z_C = \frac{1}{j\omega C} = \frac{-j1}{\omega C} = -jX_C \Rightarrow X_C = \frac{1}{\omega C}$$

Nodal Analysis    KCL    $I = \frac{V}{Z}$

L د

اشارات الكل خارج من نود

node $V_1$

$$\frac{V_1 - 12}{500} + \frac{V_1 - 0}{j10000} + \frac{V_1 - V_2}{2000} = 0$$

$$V_1\left[\frac{1}{500} + \frac{1}{j10000} + \frac{1}{2000}\right] + V_2\left[\frac{-1}{2000}\right] = \frac{12}{500}$$

$$\boxed{V_1\left[\frac{1}{400} - j\frac{1}{10000}\right] + V_2\left[\frac{-1}{2000}\right] = \frac{12}{500}} \quad (1)$$

A    B    E

## node $V_2$

$$\frac{V_2 - V_1}{2000} + \frac{V_2 - 0}{-j5000} + 4\times10^{-3} = 0$$

$$V_1\left[\frac{-1}{2000}\right] + V_2\left[\frac{1}{2000} + \frac{1}{-j5000}\right] = -4\times10^{-3}$$

$$\boxed{V_1\left[\frac{-1}{2000}\right] + V_2\left[\frac{1}{2000} + j\frac{1}{5000}\right] = -4\times10^{-3}}$$

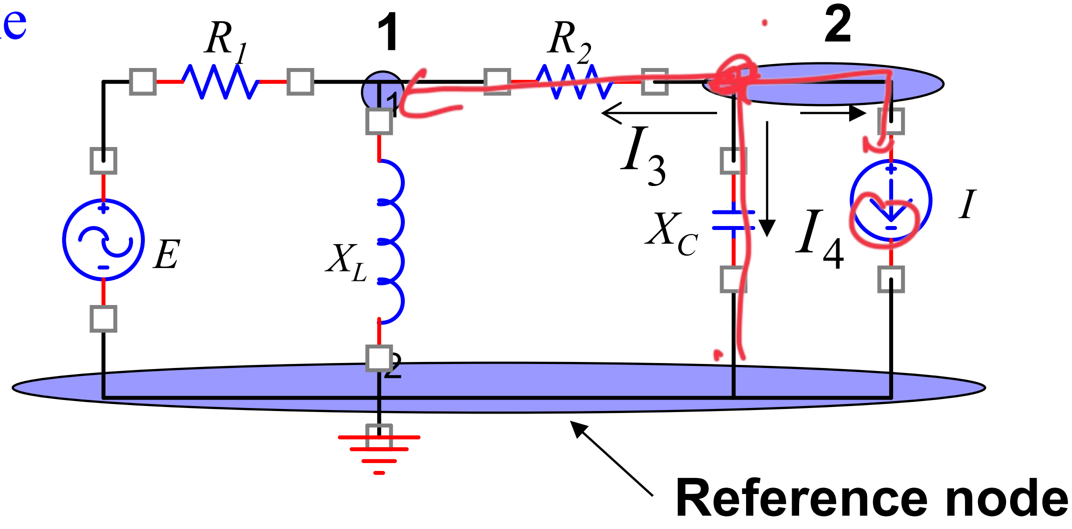
C D F

2

$$V_1 = \frac{ED - BF}{AD - BC} = 9.95 \angle 1.835^\circ \text{ V}$$

$$V_2 = \frac{AF - EC}{AD - BC} = 1.83 \angle -12.496^\circ \text{ V}$$

Applying KCL at node 2 (note the direction of currents in node 2)




$$0 = I_3 + I_4 + I$$

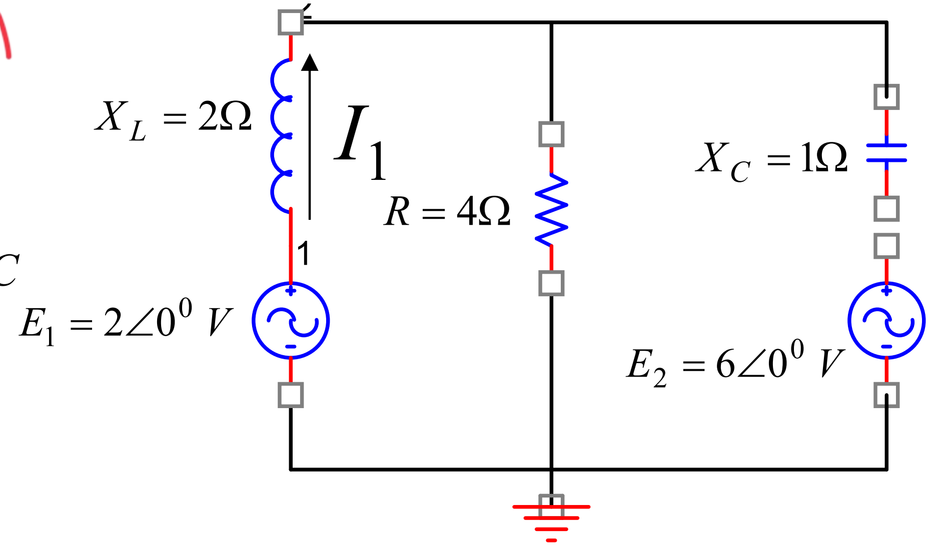
$$\frac{V_2 - V_1}{Z_3} + \frac{V_2}{Z_4} + I = 0$$

$$V_2\left[\frac{1}{Z_3} + \frac{1}{Z_4}\right] - V_1\left[\frac{1}{Z_3}\right] = -I \qquad (2)$$

Solve equation (1) and (2) and find $V_1$ and $V_2$, and then find the node currents

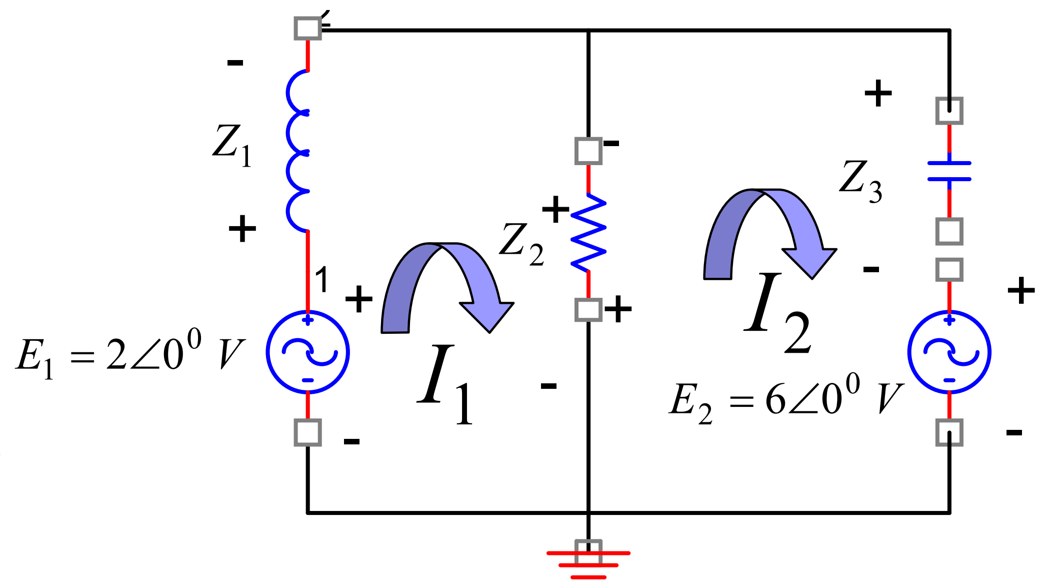
## Example: Loop analysis (ac)

Using loop analysis find the current $I_1$




$$Z_1 = jX_L, \quad Z_2 = R = 4\Omega, \quad Z_3 = -jX_C$$

**Apply KVL around each closed loop**



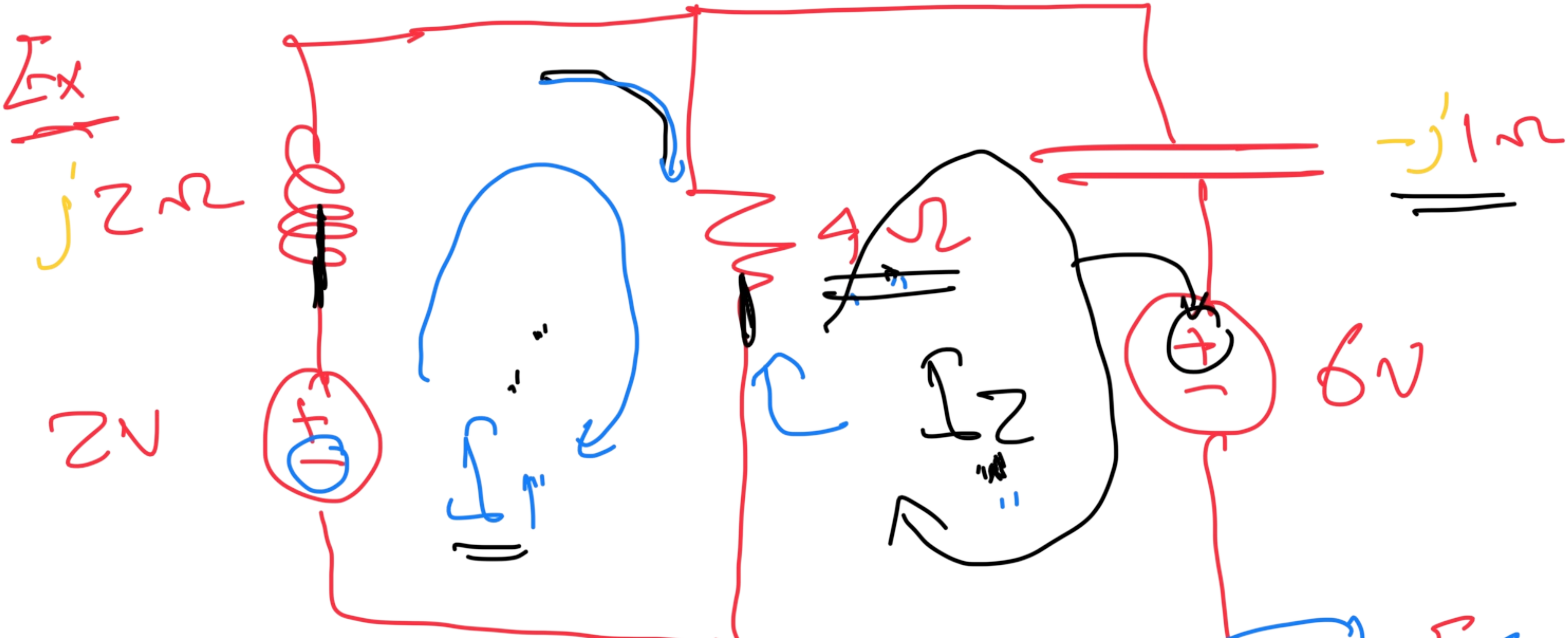

$$Loop\ 1: \quad +E_1 - I_1 Z_1 - Z_2(I_1 - I_2) = 0$$

$$Loop\ 2: \quad -Z_2(I_2 - I_1) - I_2 Z_3 - E_2 = 0$$

Solve above equations to find $I_1$ and $I_2$

$$\boxed{I_1 = 3.61\angle -236.30^0}$$

$3,61 \angle 123.7^0$




mesh 1: $(4+j2)\,I_1 + I_2(-4) = 2$ → ①

mesh 2: $I_1(-4) + (4-j)\,I_2 = -6$ ②

$$I_1 = \frac{ED - BF}{AD - BC} = 3.6056 \angle 123.69^{\circ} \text{ A}$$

$$I_2 = \frac{AF - EC}{AD - BC} = 4.4721 \angle 153.43^{\circ} \text{ A}$$

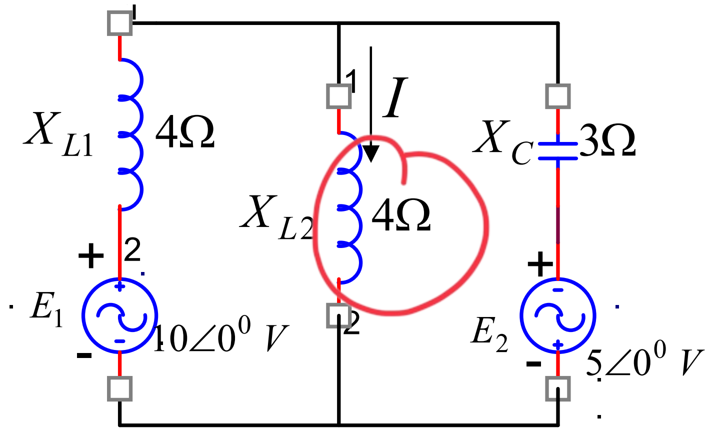
$-180 \leq \theta \leq 180$ complex غي علع

**Example (ac): Find the current I through the 4-Ω reactance ($X_{L2}$) for the network below using superposition theorem.**

$$Z_1 = jX_{L1} = j4\Omega$$

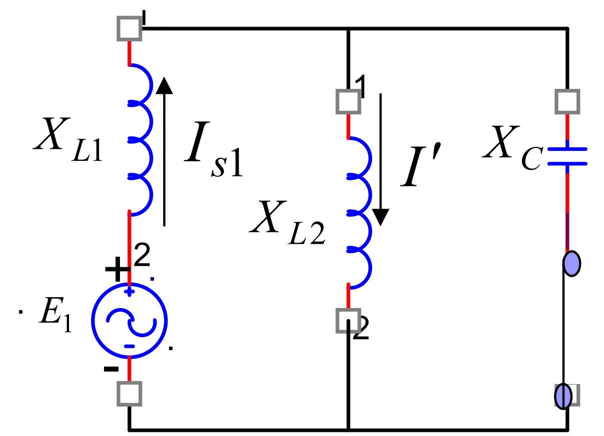
$$Z_2 + jX_{L2} = j4\Omega$$

$$Z_3 = -jX_C = -j3\Omega$$



(a)

Considering the effect of voltage source $E_1$ (fig. b)



(b)

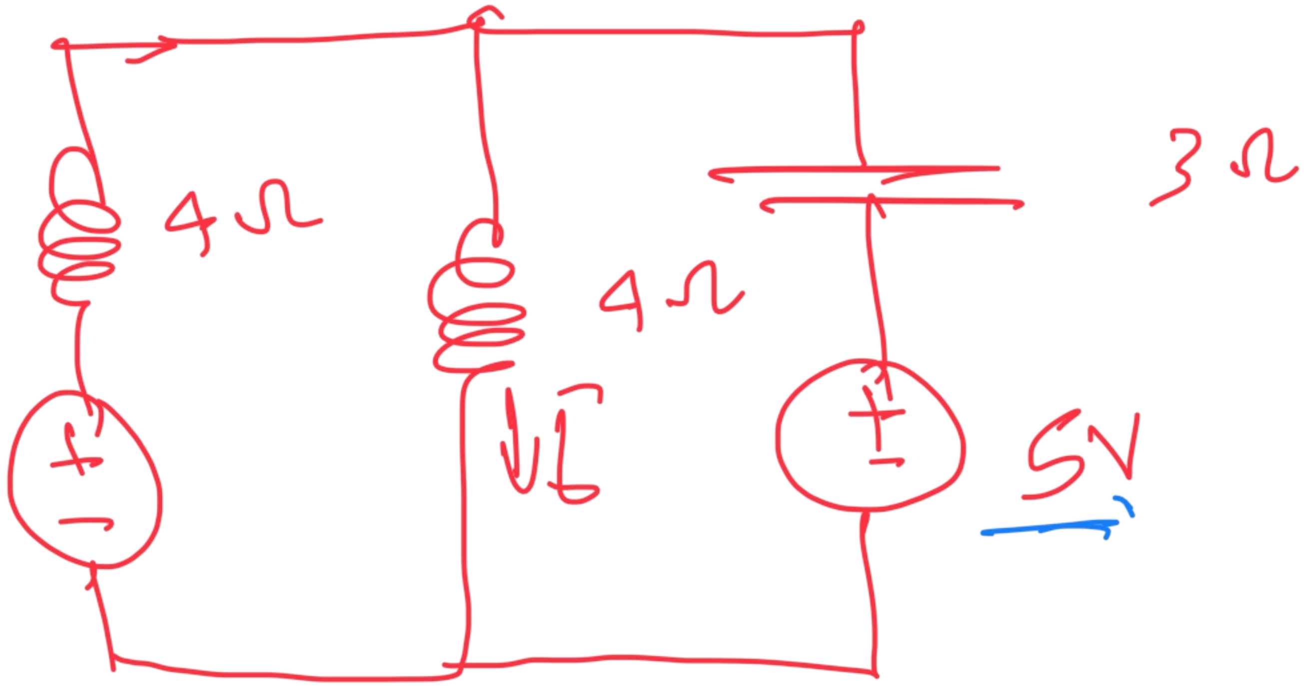
$$Z_2 \| Z_3 = \frac{Z_2 Z_3}{Z_2 + Z_3} = \frac{(j4)(-j3)}{j4 - j3} = \frac{12}{j} = \frac{12}{j} \times \frac{j}{j} = -j12\ \Omega$$

$$I_{s1} = \frac{E_1}{Z_2 \| Z_3 + Z_1} = \frac{10\angle 0^0}{-j12 + j4} = \frac{10\angle 0^0}{-j8} = \frac{10\angle 0^0}{8\angle -90^0}$$

$$= 1.25\angle 0 - (-90^0) = 1.25\angle 90^0 = j1.25$$

$$I' = I_{s1} \times \frac{Z_3}{Z_2 + Z_3} = \frac{(-j3)(j1.25)}{j4 - j3} = \frac{3.75}{j1} = \frac{3.75\angle 0^0}{1\angle 90^0} = 3.75\angle -90^0\ A$$

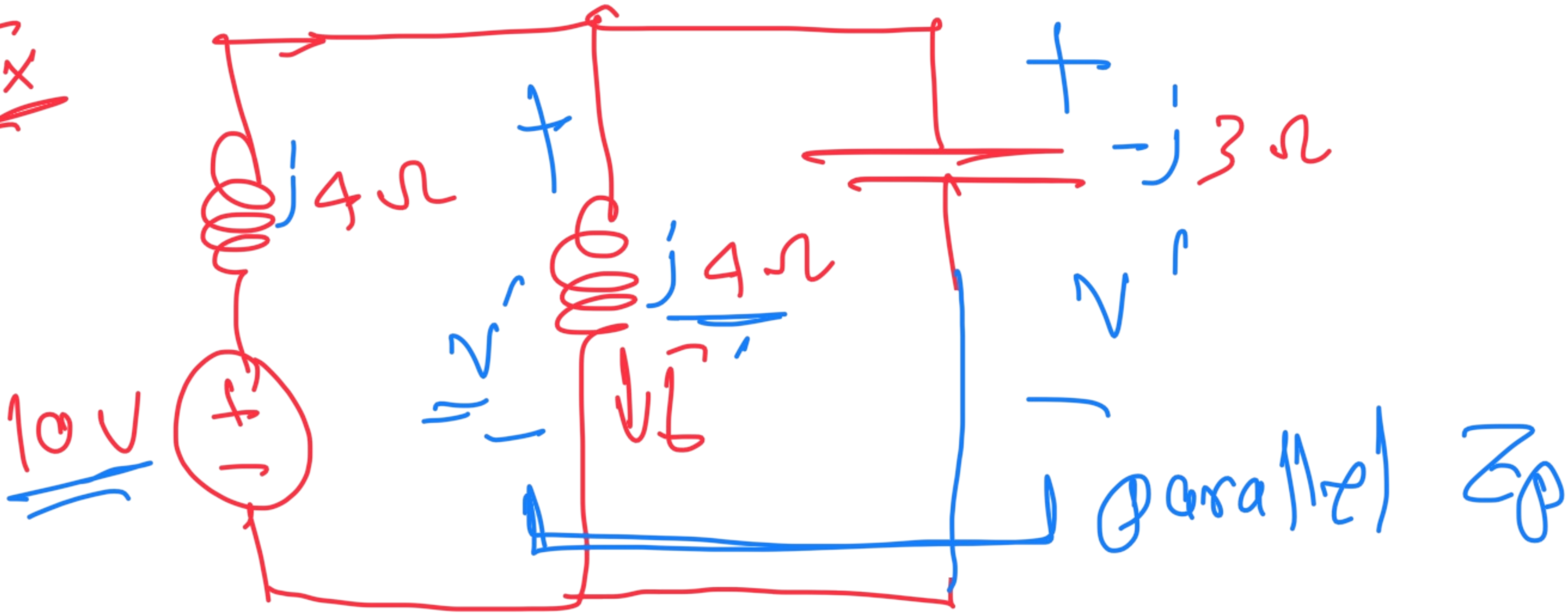
Ex




Super position :            sNw

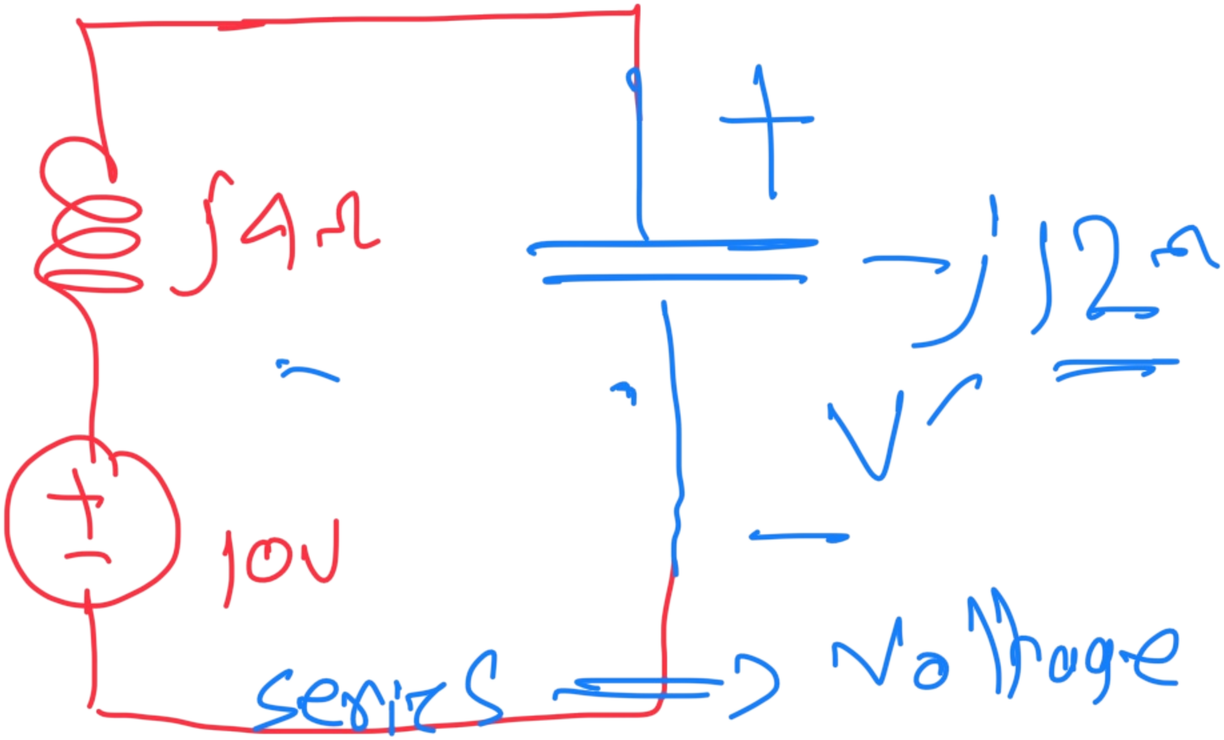
- → Turn off voltage ⇒ 0 V ⇒ short circuit
- → Turn off current ⇒ 0A ⇒ open circuit

Ex




Turn off 5V and keep 10V

$$Z_p = \frac{1}{\frac{1}{j4} + \frac{1}{-j3}} = -j12\Omega$$

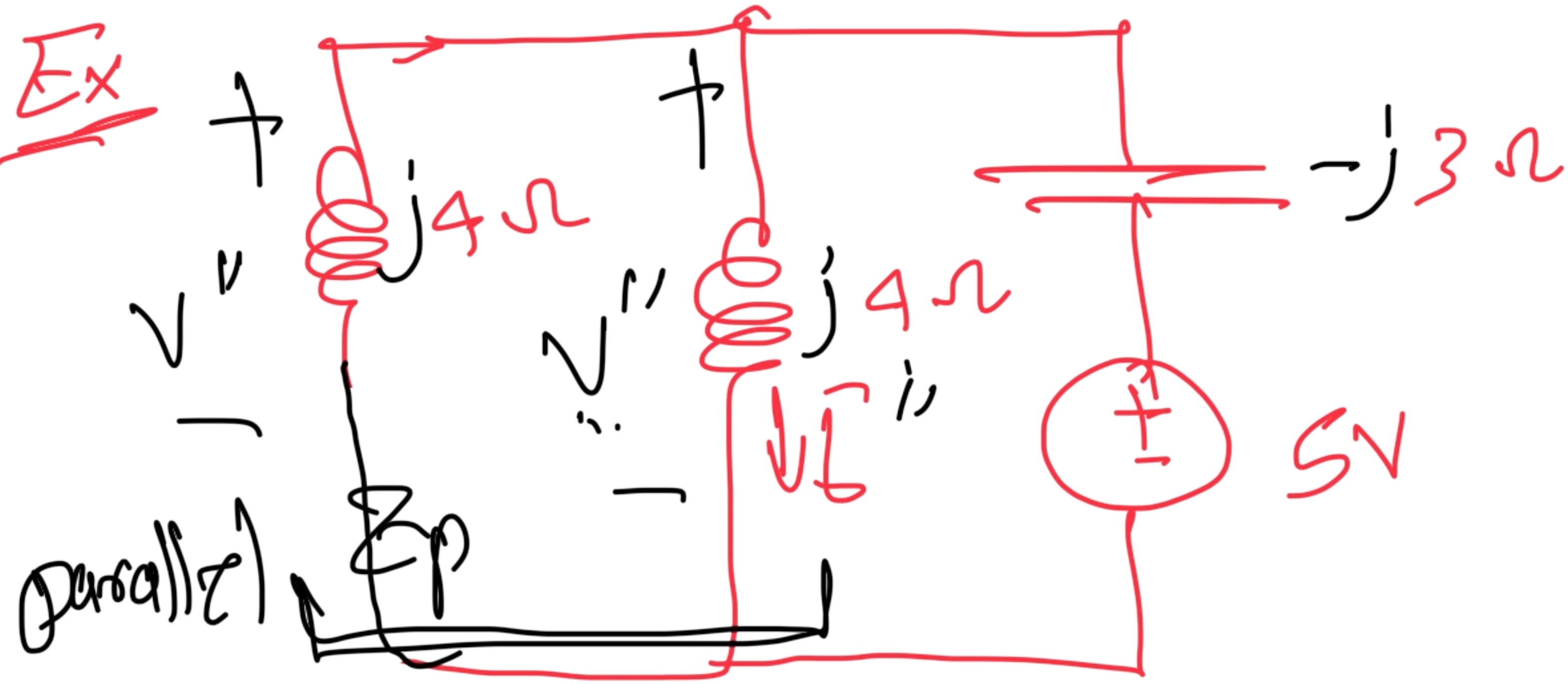



series ⇒ voltage divider

$$\Rightarrow V' = \frac{-j12}{-j12+j4} \times 10 = 15\,V$$

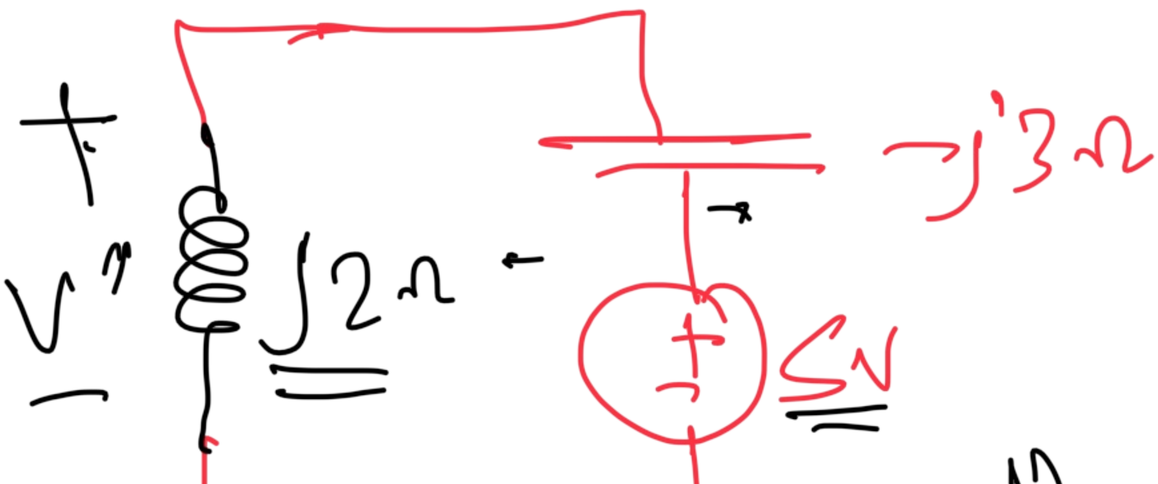
$$I' = \frac{V'}{Z} = \frac{15}{j4} = \boxed{-j3.75 = 3.75\angle{-90}\,A}$$

<u>Ex</u>




Turn off 10V and keep 5V

$$Z_p = \frac{1}{\frac{1}{j4} + \frac{1}{j4}} = j2\Omega$$




series ⟶ voltage divider

$$V'' = \frac{j2}{j2 - j3} \times 5 = -10\,V$$

$$I'' = \frac{V''}{Z} = \frac{-10}{j4} = \boxed{j2.5\,A = 2.5\angle 90\,A}$$

$$I_{Total} = I' + I''$$

$$I = -j3.75 + j2.5$$

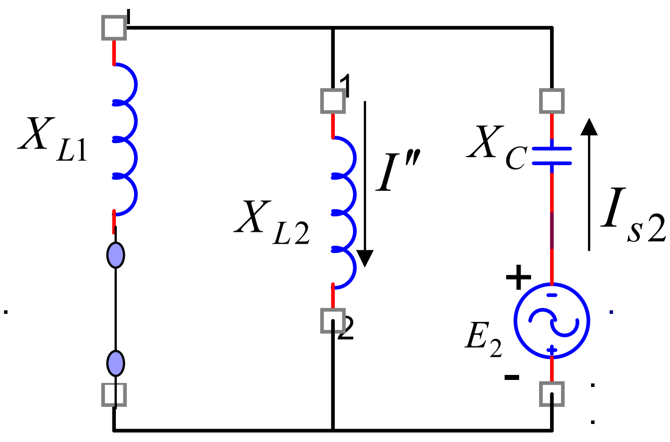
$$\boxed{I = -j1.25 = 1.25\angle{-90}}$$

Considering the effect of voltage source $E_2$ (fig. c)

$$Z_1 \| Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(j4)(j4)}{j4 + j4} = \frac{j^2 16}{j8} = j2\,\Omega$$

$$I_{s2} = \frac{E_2}{Z_1 \| Z_2 + Z_3} = \frac{5\angle 0^0}{j2 - j3} = \frac{5\angle 0^0}{1\angle -90^0} = 5\angle 90^0$$

$$I'' = I_{s2} \times \frac{Z_1}{Z_1 + Z_2} = \boxed{2.5\angle 90^0}$$



(c)

**The resultant current through the 4-Ω reactance.**

$$I = I' + I'' = 3.75\angle -90^0 + 2.5\angle 90^0 = -j3.75 + j2.5 = \boxed{-j1.25 = 1.25\angle -90^0}$$

**8.12** Find the impedance, Z, shown in Fig. P8.12 at a frequency of 60 Hz.

time domain

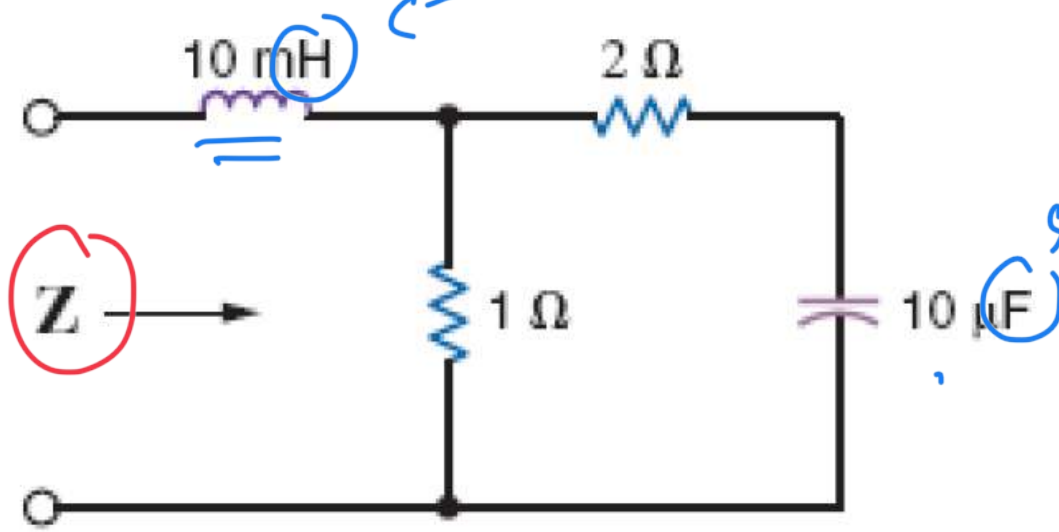
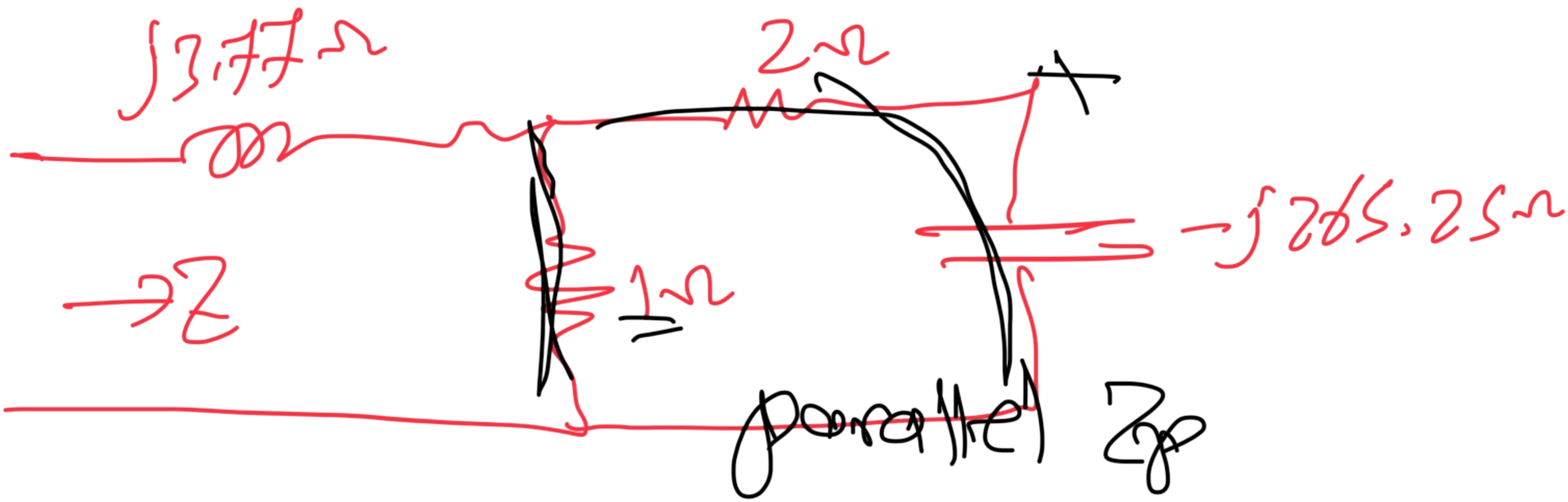



**Figure P8.12**

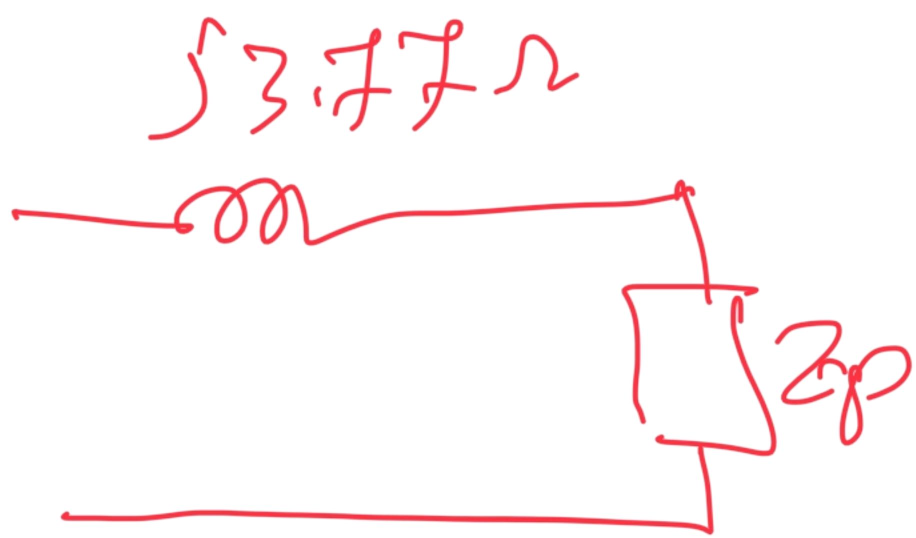
$$\omega = 2\pi f = 2\pi \times 60 = 120\pi \approx 377 \text{ rad/s}$$

$$Z_L = j\omega L = j \times 377 \times 10 \times 10^{-3} = j\,3.77\ \Omega$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \times 377 \times 10 \times 10^{-6}} = -j\,265.25\ \Omega$$



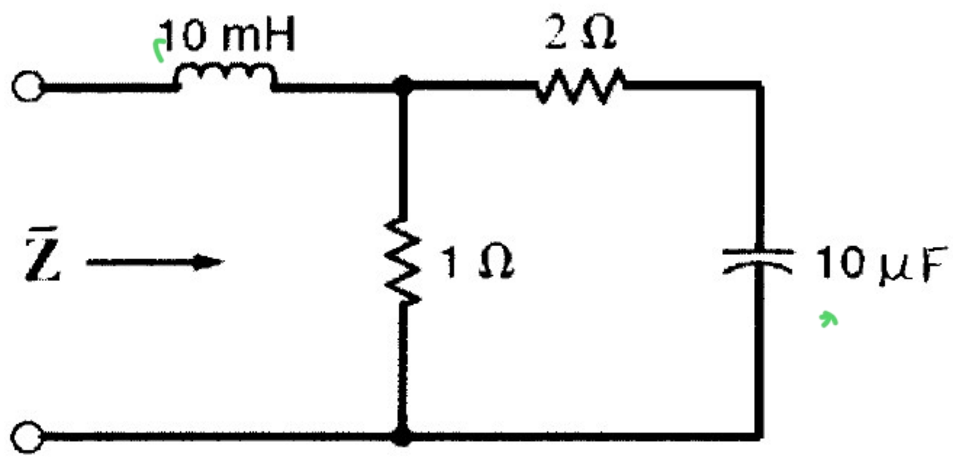
$$Z_p = \frac{1}{\frac{1}{1} + \frac{1}{2 - j265.25}} = 1 - j3.769 \times 10^{-3}\ \Omega$$




$$Z_T = j3.77 + Z_p =$$

$$\boxed{Z_T = 1 + j3.7662\ \Omega}$$

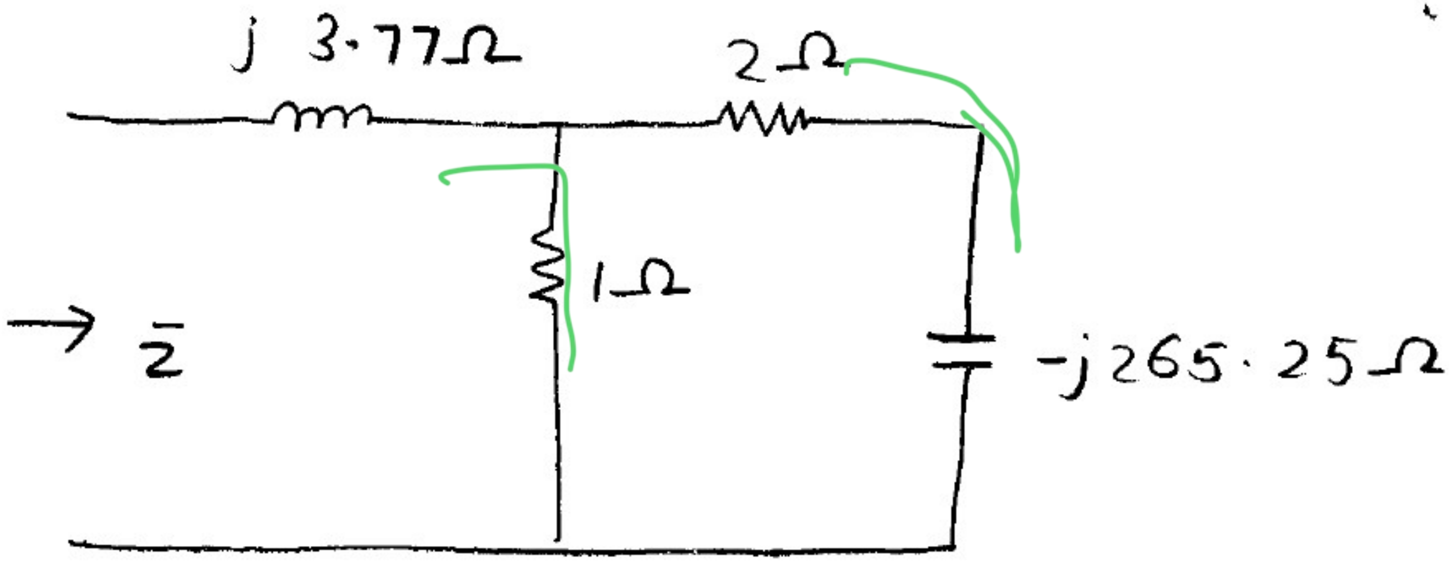
**SOLUTION:**




$$\bar{Z}_L = j(377)(10m) = j\,3.77\,\Omega$$

$$\bar{Z}_C = \frac{1}{j(377)(10\mu)} = -j\,265.25\,\Omega$$

// → parallel




$$\bar{Z} = \left[ 1 \,\|\, [2 - j\,265.25] \right] + j\,3.77$$

**8.36** Find $v_x(t)$ in the circuit in Fig. P8.36.

Time domain

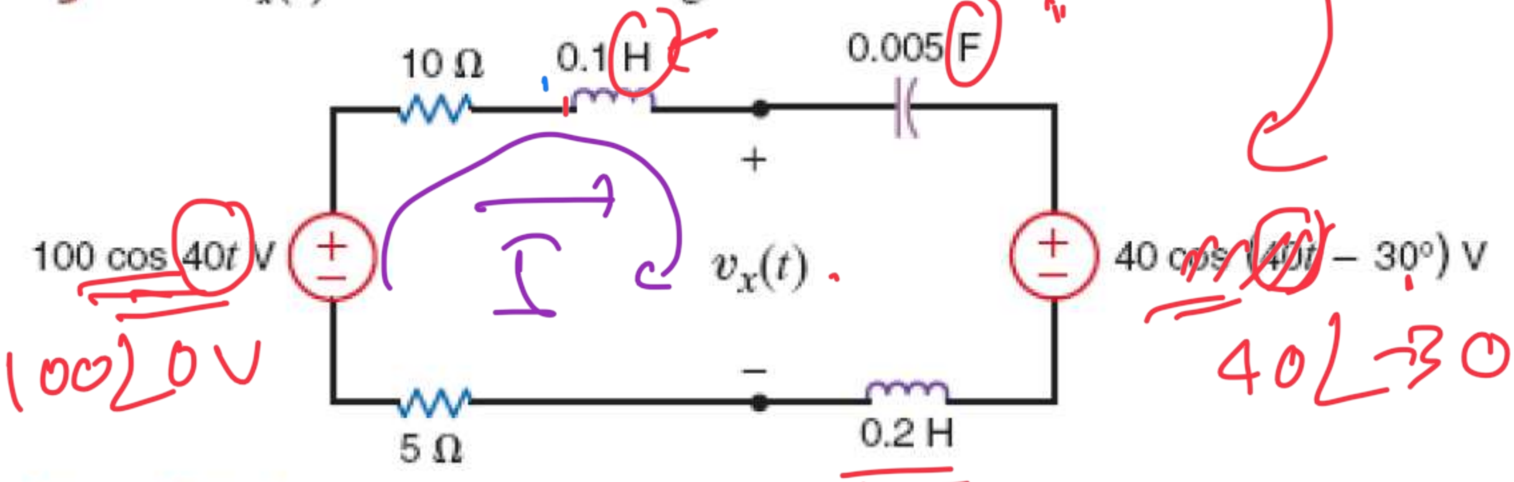



**Figure P8.36**

series

**SOLUTION:**

$$\bar{Z}_{L_1} = j(40)(0.1) = j4\,\Omega$$

$$\bar{Z}_{L_2} = j(40)(0.2) = j8\,\Omega$$

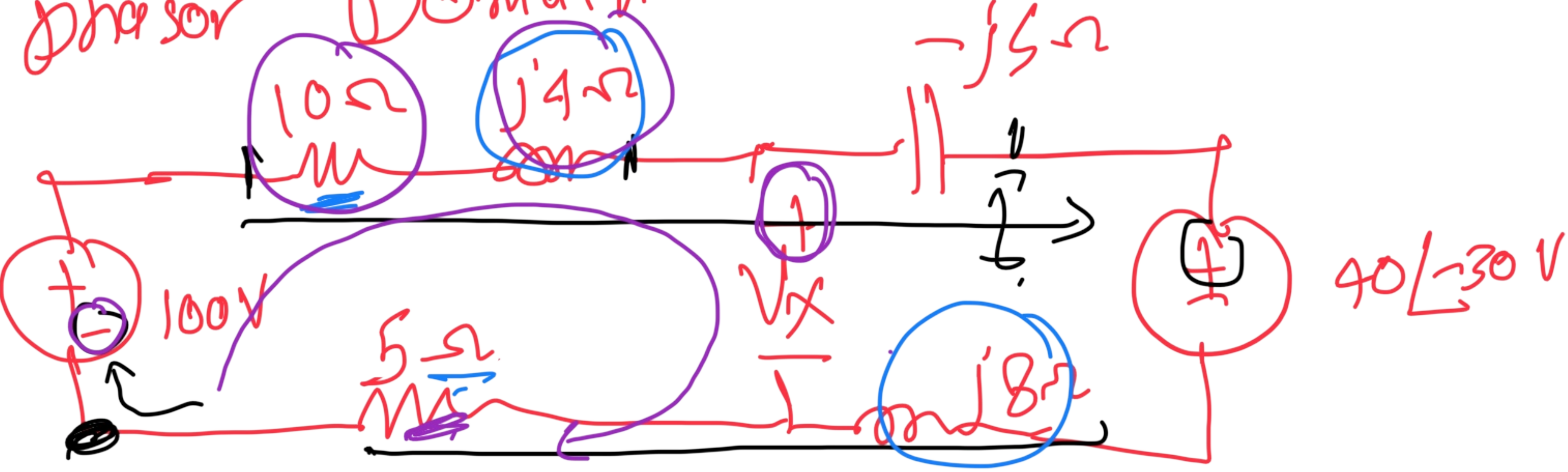
$w = 40 \text{ rad/s}$

$$Z_{L_1} = jwL_1 = j \times 40 \times 0.1 = j4\Omega$$

$$Z_{L_2} = jwL_2 = j \times 40 \times 0.2 = j8\Omega$$

$$Z_C = \frac{1}{jwC} = \frac{1}{j \times 40 \times 0.005} = -j5\Omega$$

Phasor Domain



To find current I using KVL

$$\Sigma V = 0$$

$$-100 + 10I + j4I - j5I + 40\angle{-30} + j8I + 5I = 0$$

$$\frac{\cancel{(15 + j7)}I}{\cancel{15 + j7}} = \frac{100 - 40\angle{-30}}{15 + j7}$$

$$I = \frac{100 - 40\angle{-30}}{15 + j7}$$

$I = 4.13 \angle -8^\circ$ A

To find $V_x$ using Also KVL

$\underline{-100 + 10I + j4I} + \underline{\underline{V_x}} + \underline{5I} = 0$

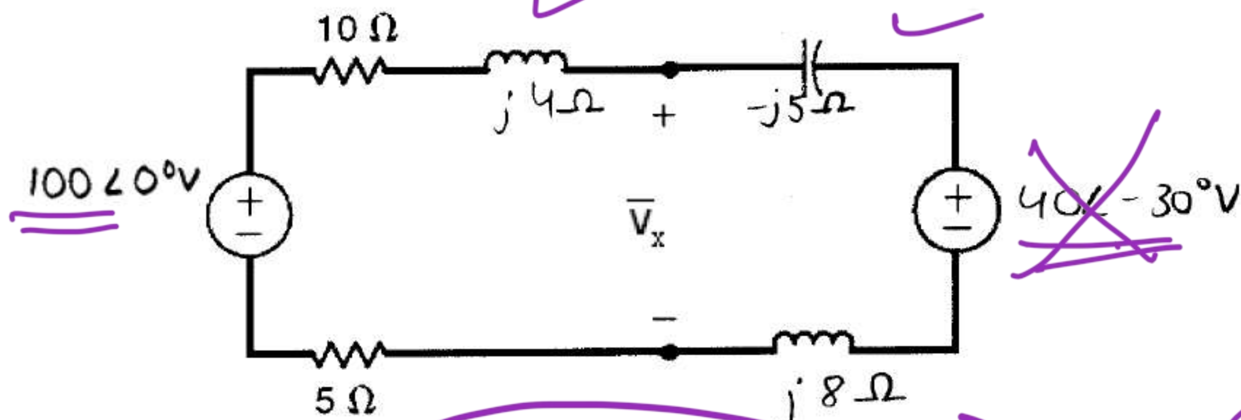
$V_x = 100 + (-15 - j4) I$

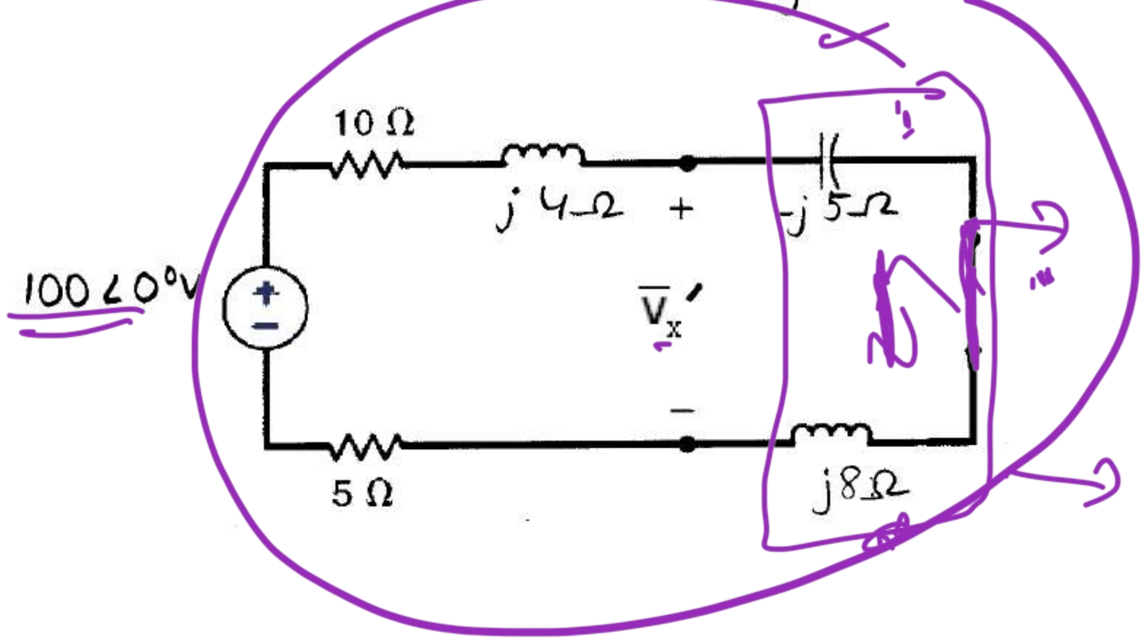
$V_x = 100 + (-15 - j4)(4.13 \angle -8)$

$$\boxed{V_x = 37.18 \angle -12 \text{ V}}$$

$$\boxed{v_x(t) = 37.18\cos(40t - 12)\ \mathrm{V}}$$

$$\bar{Z}_C = \frac{1}{j(40)(0.005)} = -j5\,\Omega$$



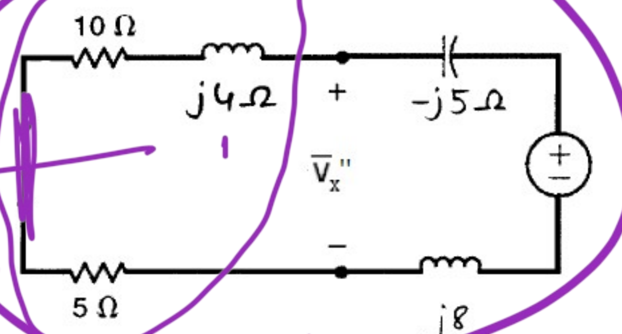




super position

voltage divider

$$\bar{V}_x' = \left(\frac{j8 - j5}{j8 - j5 + 10 + j4 + 5}\right)(100\angle 0^\circ)$$

$$\bar{V}_x' = 18.12\angle 64.98^\circ \text{ V}$$




$$\bar{V}_x'' = \left(\frac{5 + 10 + j4}{5 + 10 + j4 + j8 - j5}\right)(40\angle -30^\circ)$$

Voltage divider

$$\bar{V}_x'' = 37.51\angle -40.1^\circ \text{ V}$$

$$\bar{V}_x = \bar{V}_x' + \bar{V}_x'' = 18.12\angle 64.98^\circ + 37.51\angle -40.1^\circ$$

$$\bar{V}_x = 37.17\angle -12.02^\circ \text{ V}$$

$$\boxed{V_x(t) = 37.17\cos(40t - 12.02^\circ) \text{ V}}$$

**8.68** Find $\mathbf{V}_o$ in the network in Fig. P8.68 using nodal analysis.

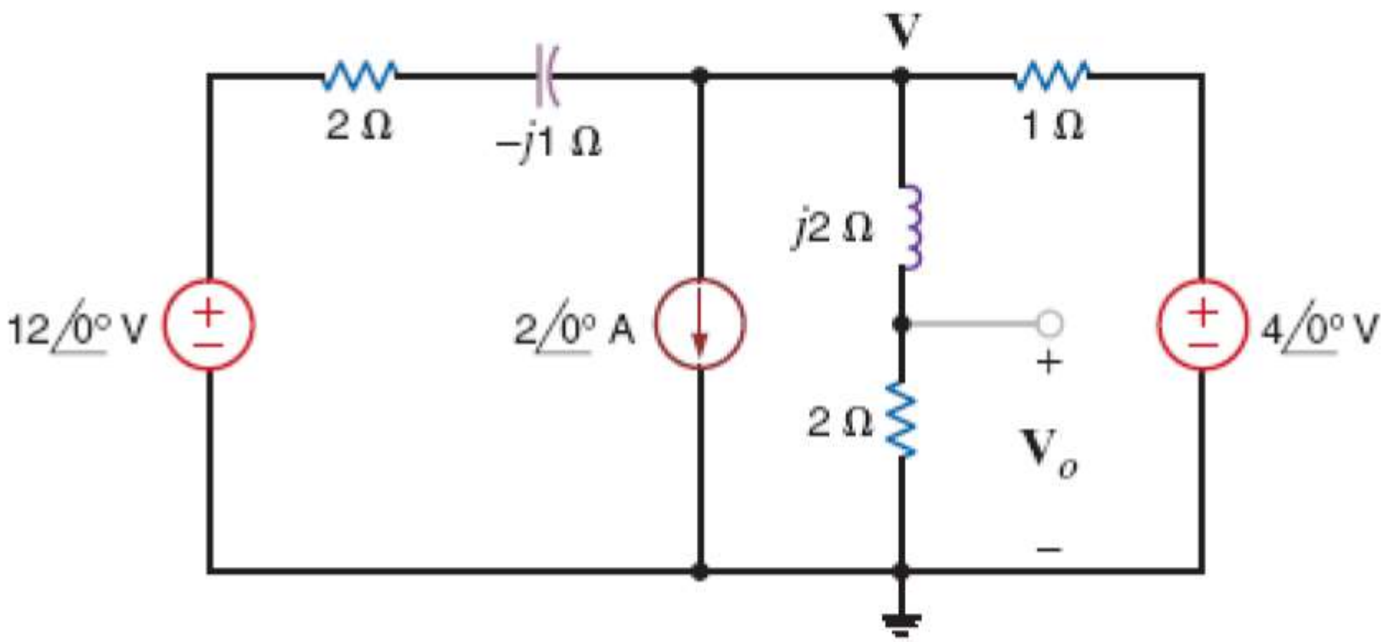



Figure P8.68