Topic 1

AC Networks Analysis

Dr. Ala A. Hussein

Part 2

Phasor Domain

- Phasor is a complex number representation in terms of amplitude & phase of a sinusoidal wave shape
- Engineers use phasors to simplify solution of circuitry involving sinusoidal voltages & currents.

* Time domain : *

$V(t) = 5\cos(30t + 60°)$ R:Ω , L:H , C:F

R $v_R = iR$

C $i_c = C \frac{\partial v}{\partial t} \rightarrow v_c = \frac{1}{C} \int i_c$

L $V_L = L \frac{\partial i}{\partial t} \rightarrow i_L = \frac{1}{L} \int V_L$

* Complex number : *

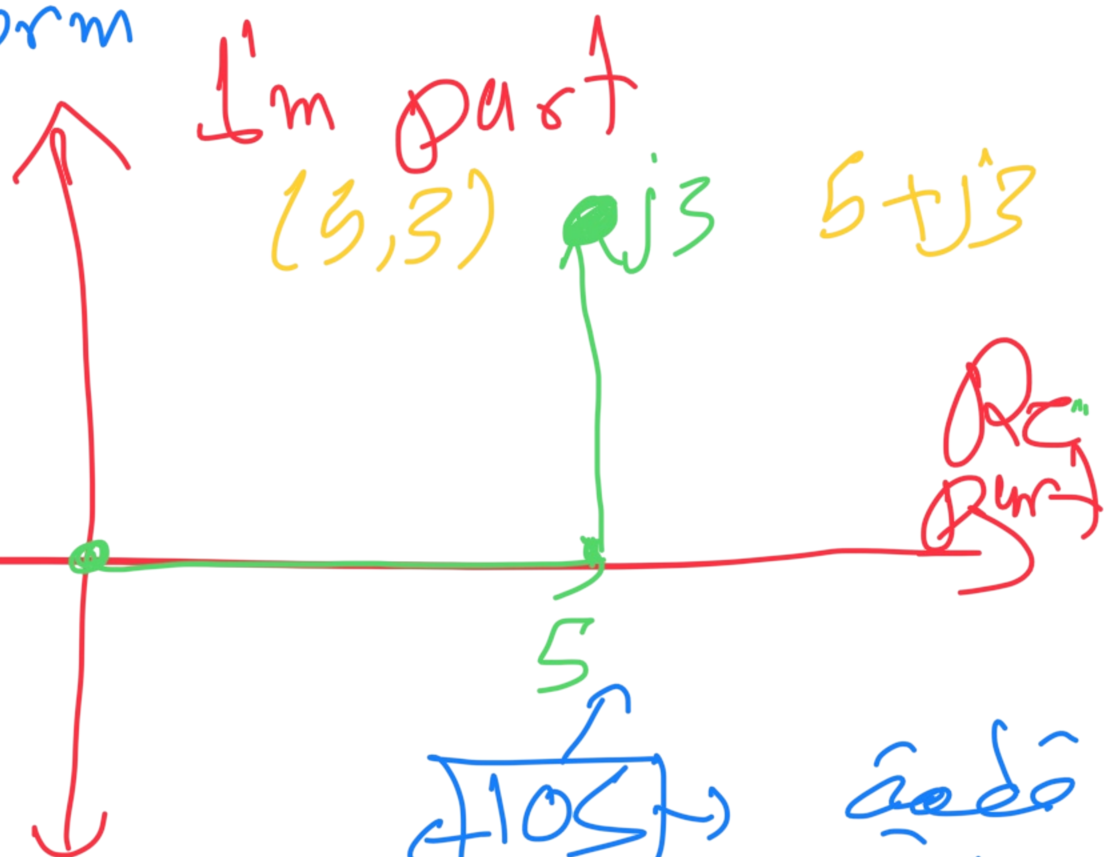
1. ↳ $z = \underline{a} + \underline{j}b$ → imag part تخيلي

Rectangular Form

Real part حقيقي

$j = \sqrt{-1}$

i



$Z = \boxed{5 + j3}$

قطعة واحدة

$\boxed{105}$

قطعة واحدة

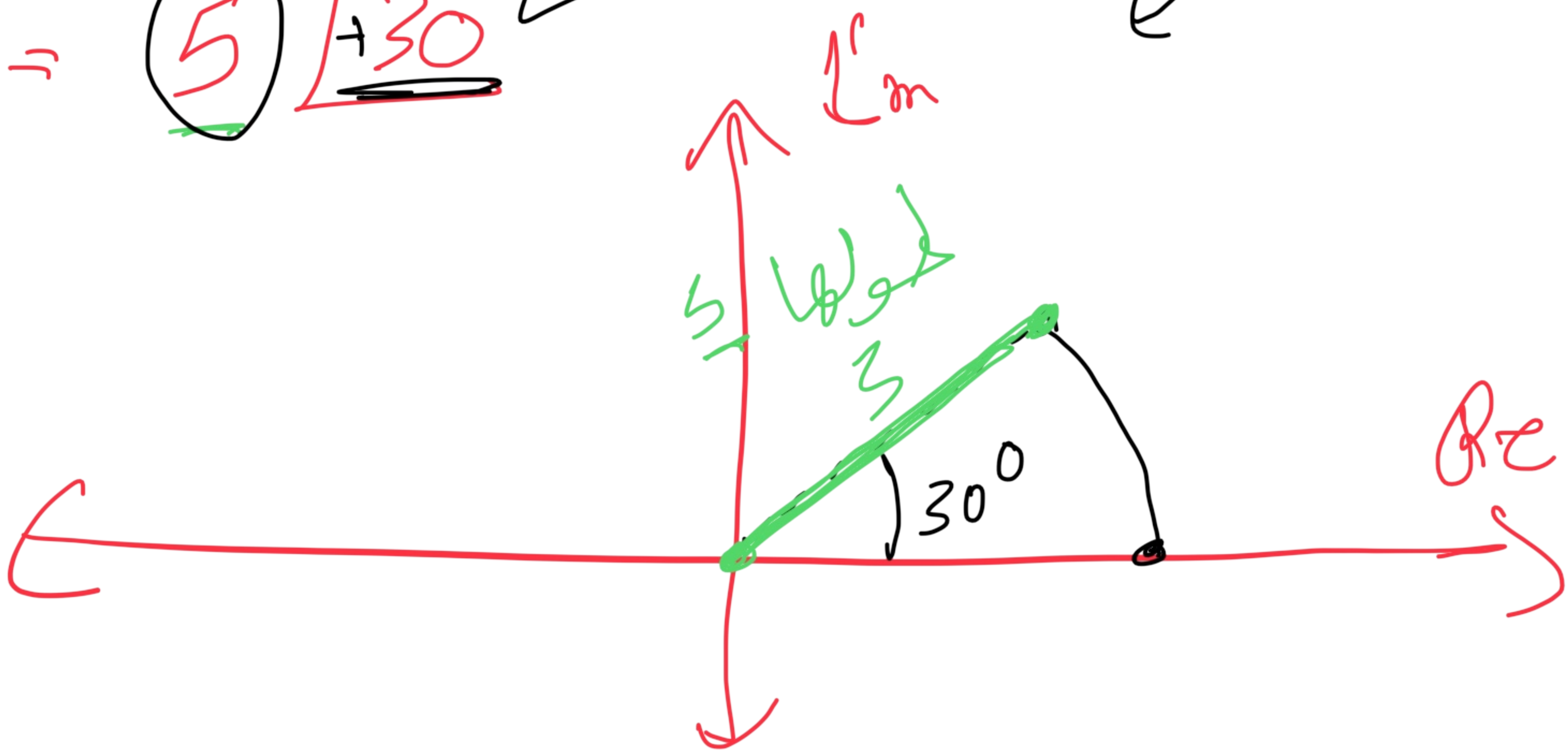
② polar form

$$z = |z| \angle \theta$$

$+$

$-$

$$z = 5 \angle +30$$



$Z = a + jb \longrightarrow$ rect

$Z = |Z| \angle \theta \longrightarrow$ polar

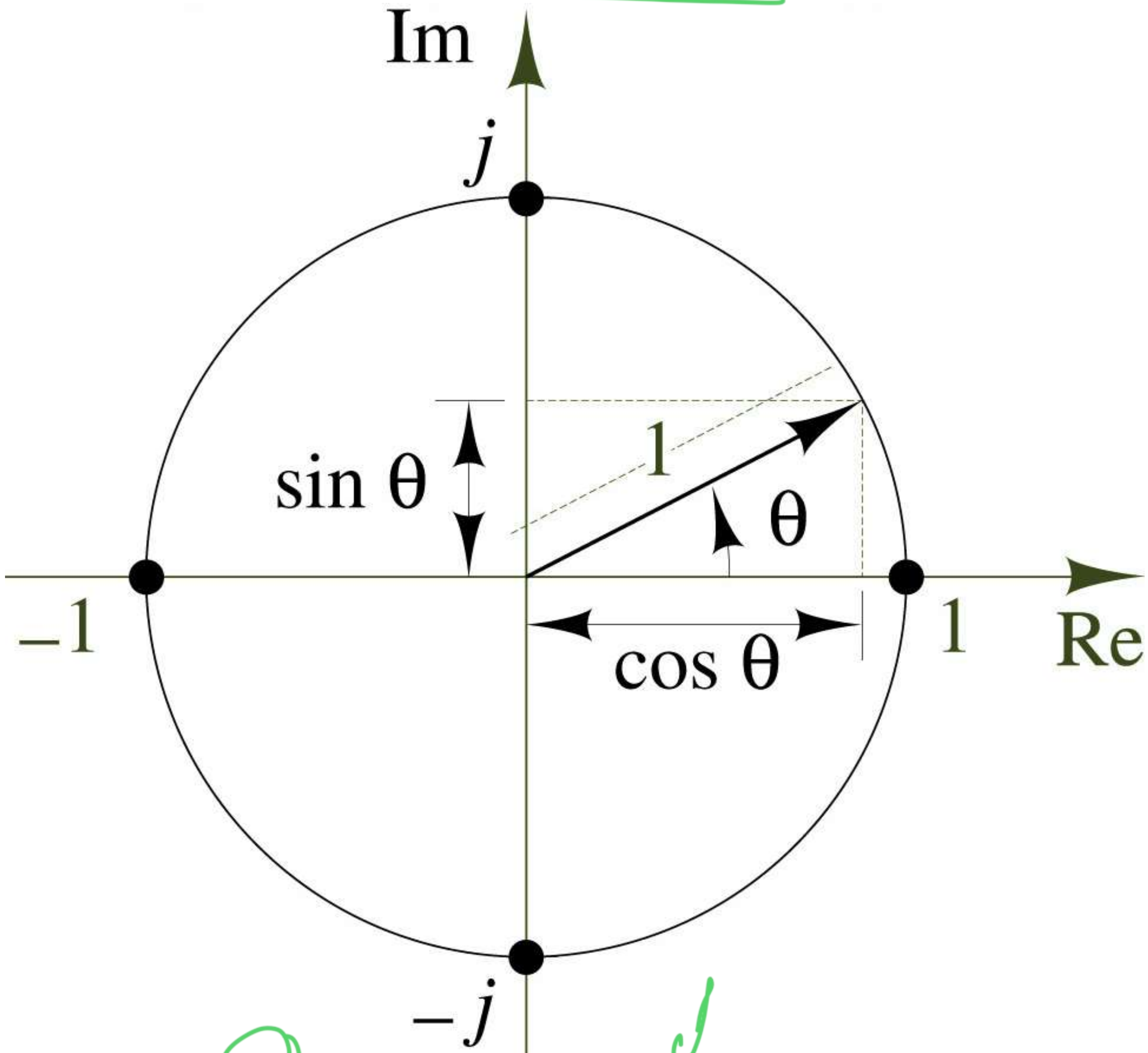
كيف نحول من rect ال polar ؟

$$|Z| = \sqrt{a^2 + b^2} \quad , \quad \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

كيف نحول من polar ال rect ؟

$$a = |Z| \cos\theta \quad , \quad b = |Z| \sin\theta$$

Euler's identity



$$e^{j\theta} = \underline{\cos\theta} + j\,\underline{\sin\theta}$$

→ Imag

real

for Voltage v(t)

$$v(t) = V_M \cos(w\,t + \Theta)$$

Using Euler's formula

$$e^{jx} = \cos(x) + j\sin(x)$$

$$\Rightarrow V_M e^{j(\omega t+\Theta)} = V_M\cos(\omega t+\Theta) + jV_M\sin(\omega t+\Theta)$$

$$\Rightarrow v(t) = V_M\cos(\omega t+\Theta) = \mathrm{Re}\left[V_M e^{j(\omega t+\Theta)}\right]$$

$$= \mathrm{Re}\left[V_M e^{j\omega t} e^{j\Theta}\right]$$

$$= \mathrm{Re}\left[V_M \angle\Theta\, e^{j\omega t}\right]$$

x

$$\Rightarrow v(t) = V e^{j\omega t}$$

where $\underbrace{V = V_M\angle\Theta}_{\text{Phasor form}}$

Dropping $e^{j\omega t}$ since it exists in all terms

Phasor representation :

$$\text{Time domain} \leftrightarrow \text{Phasor form}$$

$$A\cos(\omega t \pm \Theta) \leftrightarrow A\angle \pm \Theta$$

$$+\ A\sin(\omega t \pm \Theta) \leftrightarrow A\angle(\boxed{\pm\Theta - 90^o})$$

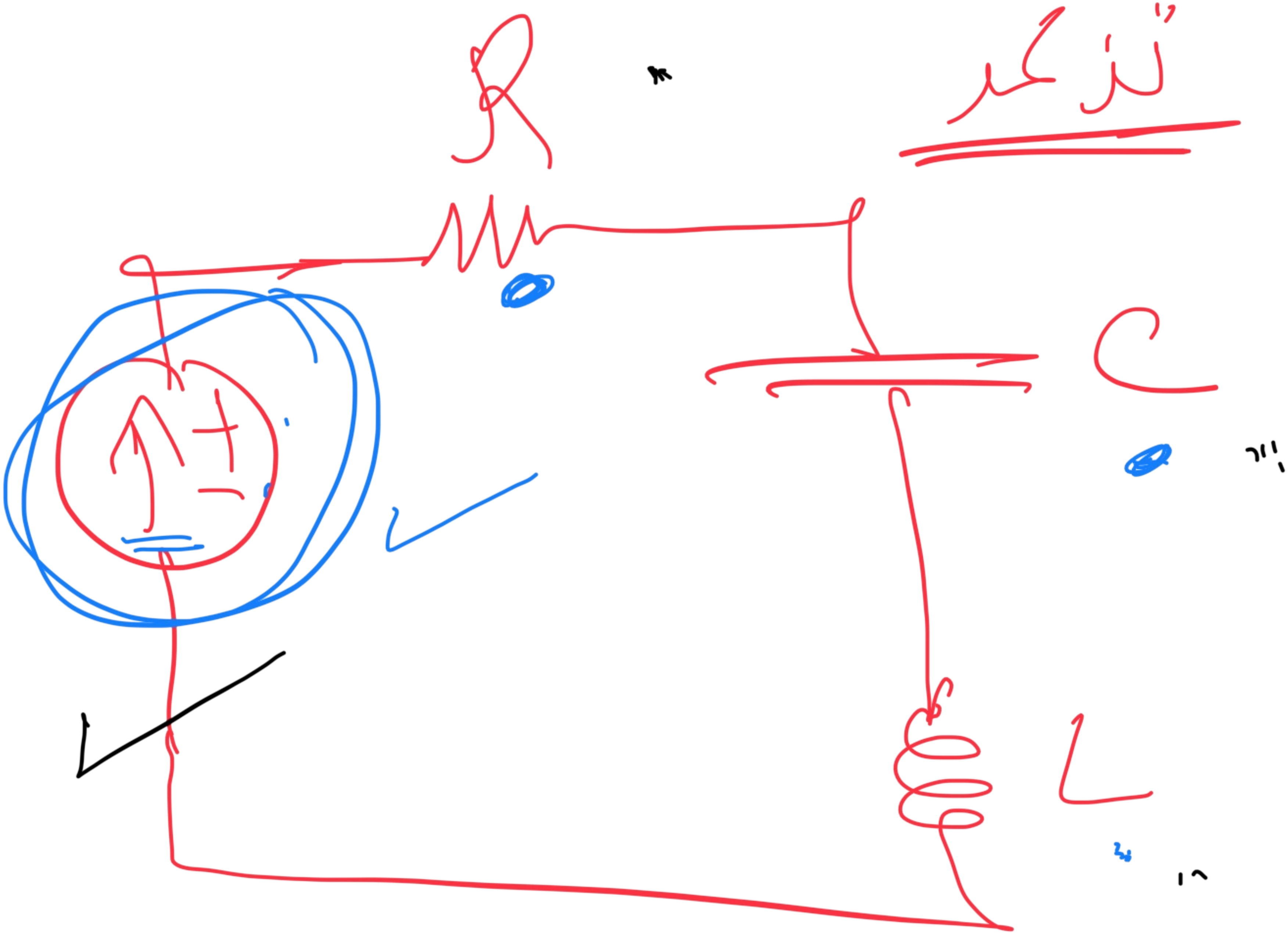
Use Cosine as reference

$+\sin(\omega t+\theta) \longrightarrow \cos(\omega t+\theta-90)$

$-\sin(\omega t+\theta) \longrightarrow \cos(\omega t+\boxed{\theta+90})$

Phasor form

current $\Rightarrow$ cos ✓
voltage sin


R
C
L

Example :

Convert from time domain to phasor form

حول

$$v(t) = 24\cos\left(337t - 45^{\circ}\right)$$

$$i(t) = 12\sin\left(337t + 120^{\circ}\right)$$

$$V = 24\angle(-45^{\circ})\ V$$

$$I = 12\angle(120^{\circ} - 90^{\circ}) = 12\angle 30^{\circ}\ A$$

$$v(t) = 24\ \cancel{\cos(377t}\ -45)$$

$$\boxed{V = 24 \angle -45 \ V}$$

$$i(t) = +12 \sin(377t + 120)$$

حول ال
cos

$$= 12\ \cancel{\cos(377t}\ + (120 - 90))$$

$$\boxed{I = 12 \angle 30 \ A}$$

Example :

Convert from phasor form to time domain

$$V = 16\angle 120^{\circ}\ V$$

$$I = 10\angle(-75^{\circ})$$

$$\text{if}\ \boxed{f = 1\,KHz}$$

$$\omega = 2\pi f = 2\pi(1000) = 2000\pi$$

$$v(t) = 16\cos\left(2000\pi t + 120^{\circ}\right)$$

$$i(t) = 10\cos\left(2000\pi t - 75^{\circ}\right)$$

Phasor V , I حرف كبير

Time v(t) , i(t) حرف صغير

$$V = 16 \angle 120 \ V$$

$$v(t) = 16 \cos(\omega t + 120) \ v$$

$$\omega = 2\pi f$$

$$\omega = 2\pi \times 1000$$

$$\omega = 2000\pi \ \text{rad/s}$$

$$\boxed{v(t) = 16 \cos(2000\pi t + 120) \ V}$$

$$I = 10 \angle -75 \ A$$

$$i(t) = 10 \cos(\omega t - 75) \ A$$

$$\boxed{i(t) = 10 \cos(2000\pi t - 75) \ A}$$

Phasor Domain Representation of Circuit Elements

For Resistors:

cos

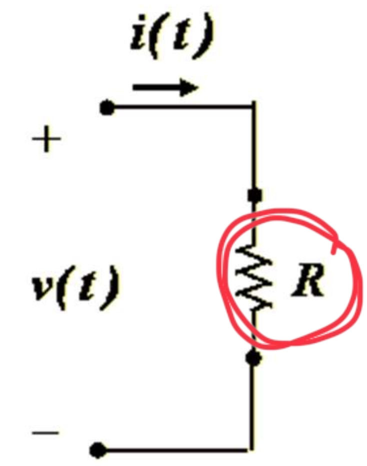
$$i(t) = I_{max}\cos(\omega t + \phi)$$

$$\boxed{v(t) = R * i(t)}$$

Ohm

$$v(t) = RI_{max}\cos(\omega t + \phi)$$

$$v(t) = V_{max}\cos(\omega t + \phi)$$



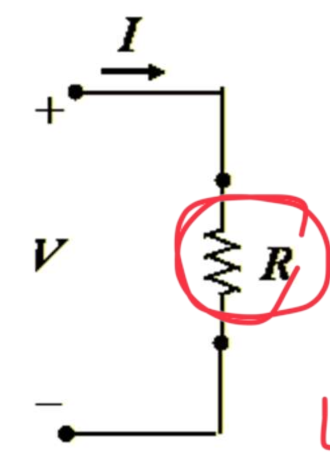
Time-domain

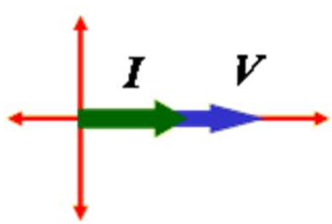
Phasor domain

$$I = I_{max}\angle + \phi^o$$

$$V = V_{max}\angle\phi^o$$

$$R = \frac{V_{max}\angle\phi^o}{I_{max}\angle\phi^o} = \frac{V_{max}}{I_{max}}$$





NOTE: The voltage and the current are in phase.

$$\boxed{Z_R = R}$$

Ω

Phasor domain

Impedance → phasor

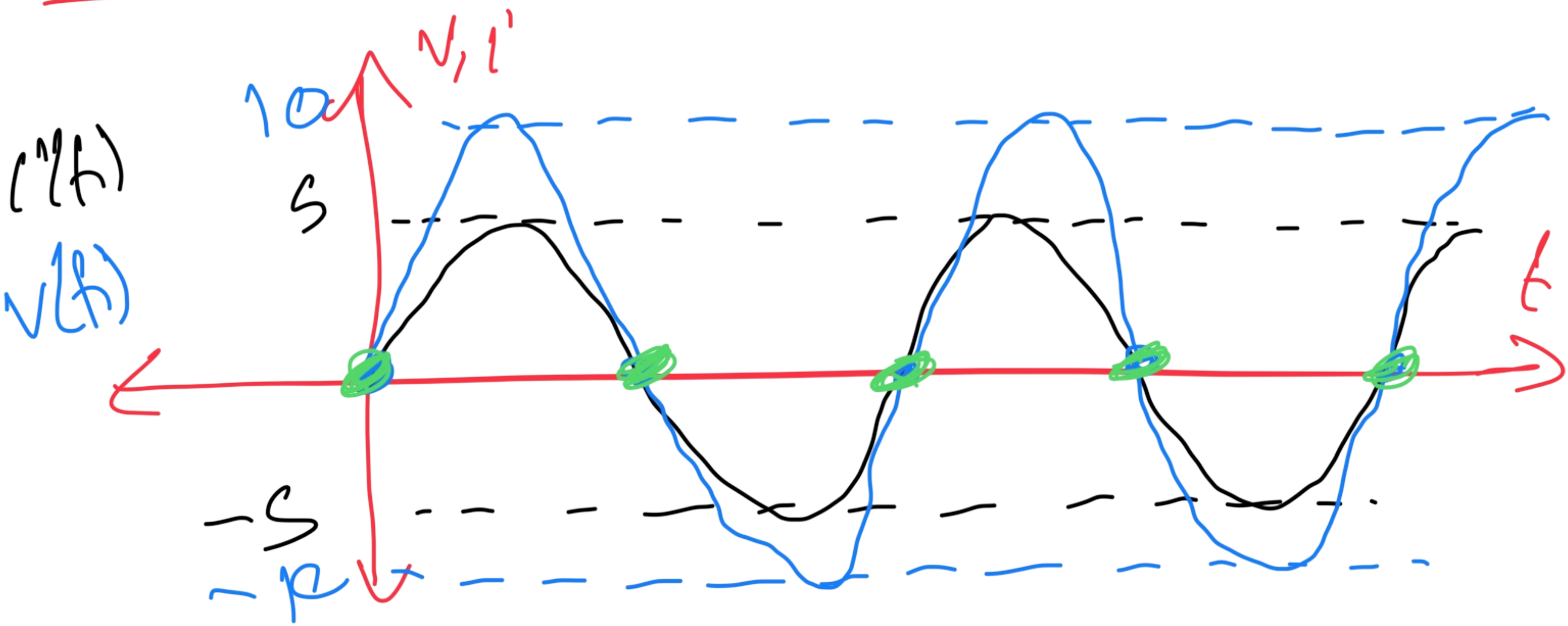
$\boxed{\text{in phase}}$: يمشو مع بعض وانتهو مع بعض

Ex

$$i(t) = \boxed{5\cos(\omega t)\ A}$$

$$R = 2\ \Omega$$

$$\Rightarrow V = iR = 5 \times 2 = \boxed{10\cos(\omega t)\ V}$$



For Inductors:

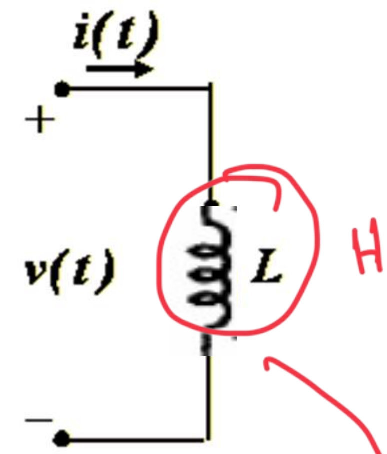
$Z_L = j\omega L$

$$\boxed{i(t) = I_{max}\cos(\omega t + \phi)}$$

$$v(t) = L\frac{di(t)}{d(t)}$$

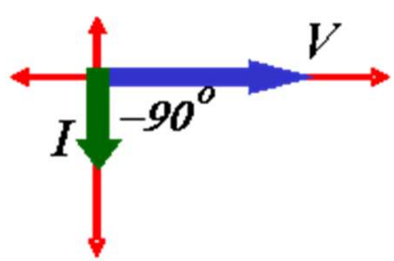
$$v(t) = -\omega L I_{max}\sin(\omega t + \phi)$$

$$v(t) = +\omega L I_{max}\cos(\omega t + \phi + 90)$$



Time-domain

Phasor domain



NOTE: The current lagging the voltage by 90°

$$I = I_{max}\angle +\phi^o$$

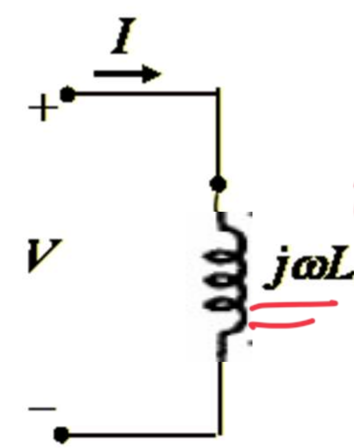
$$V = \omega L I_{max}\angle(\phi + 90)^o$$

$$V = (\omega L\angle 90^o)(I_{max}\angle\phi^o)$$

$$V = j\omega L\, I$$

$$Z_L = \frac{V}{I} = j\omega L$$

$$\boxed{Z_L = j\omega L}\ \Omega$$



Phasor domain Ω

$$V_L = L \frac{di}{dt} \qquad i(t) = I_m \cos(\omega t + \theta)$$

$$V_L = -L \cdot I_m \omega \cdot \sin(\omega t + \theta)$$

$$V_L = - I_m L \omega \sin(\omega t + \theta)$$

$$V_L = + I_m L \omega \cos(\omega t + \underbrace{\theta + 90}$$

$$\left.\begin{array}{l} \theta_V = \theta + 90 \\ \theta_i = \theta \end{array}\right\} \Rightarrow \theta_V > \theta_i$$

lead : القائد / المركز الأول

lag : التابع / المركز الثاني

محمد سابق علي ← محمد : المركز الأول
علي : المركز الثاني

علي متأخر عن محمد ← محمد : مركز الأول
علي : مركز الثاني

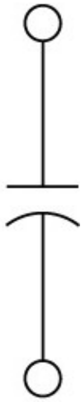
$\theta_v > \theta_i$

↳ Voltage lead current by $\theta_v - \theta_i$

↳ current lag voltage by $\theta_v - \theta_i$

For Capacitors:

$$Z_C = \frac{1}{j\omega C}$$

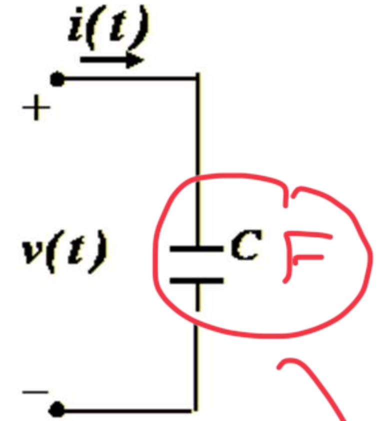


$$v(t) = V_{max}\cos(\omega t + \phi)$$

$$i(t) = C\frac{dv(t)}{d(t)}$$

$$i(t) = -\omega C V_{max}\sin(\omega t + \phi)$$

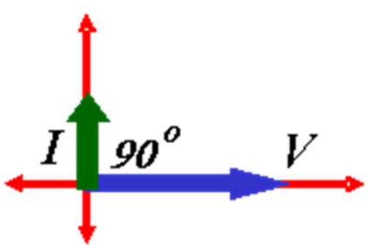
$$i(t) = +\omega C V_{max}\cos(\omega t + \phi + 90)$$



Time-domain

Phasor domain

$\theta_c > \theta_v$



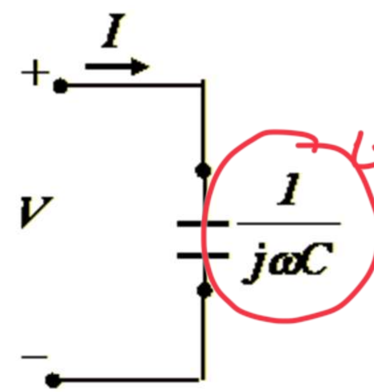
$$V = V_{max}\angle + \phi^o$$

$$I = \omega C V_{max}\angle(\phi + 90)^o$$

$$I = (\omega C\angle 90^o)(V_{max}\angle\phi^o)$$

$$I = j\omega C\, V$$

$$Z_C = \frac{V}{I} = \frac{1}{j\omega C}$$



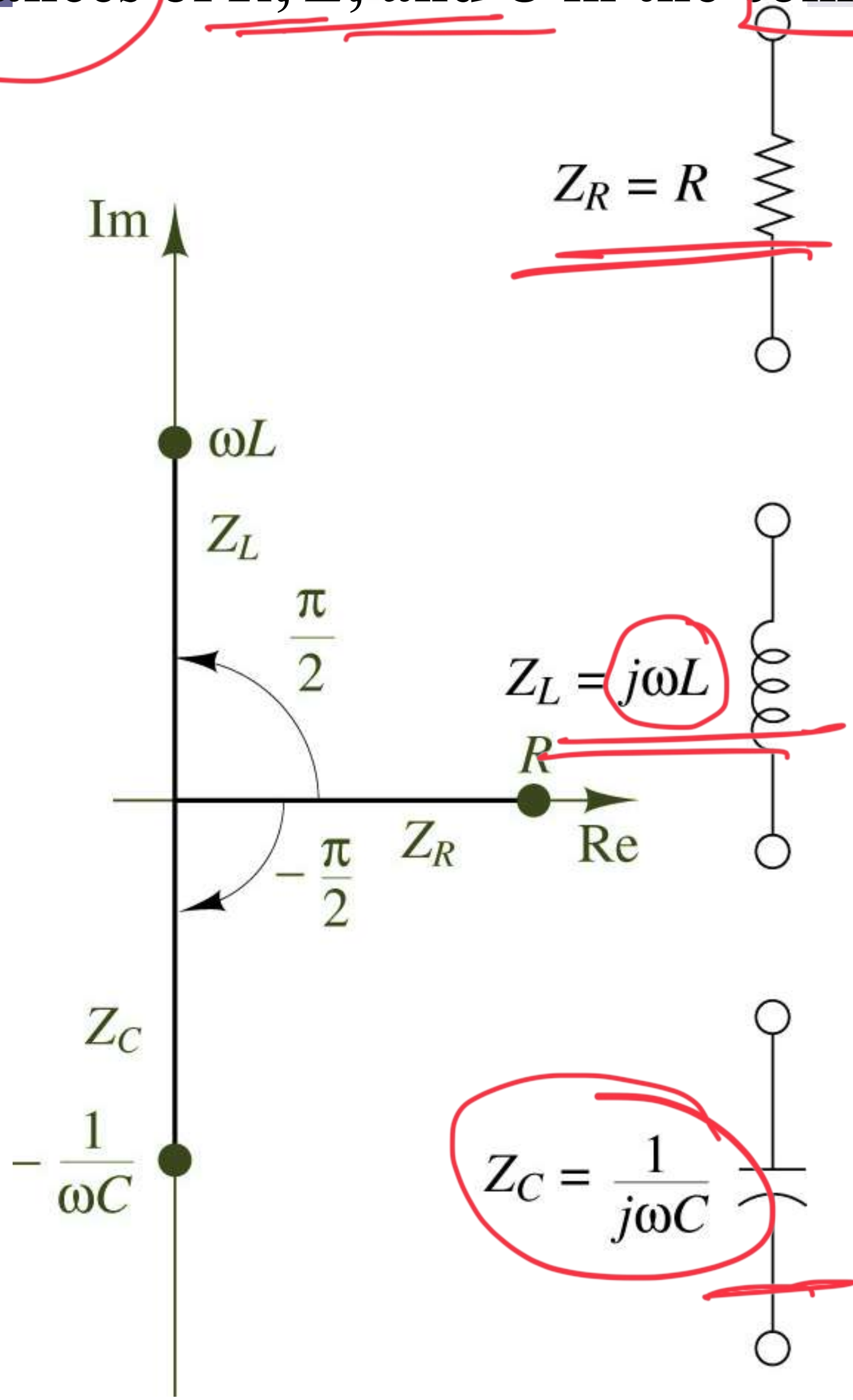
NOTE: The current leading the voltage by 90°

Phasor domain

$$\boxed{Z_C = \frac{1}{jwC} \;\; \Omega}$$

Impedances of R, L, and C in the complex plane

Phasor domain



-: ఇది చాలా ముఖ్యం

Impedance : $Z = \underline{R} \mp j\,\underline{X}$

"Rect / complexplane"

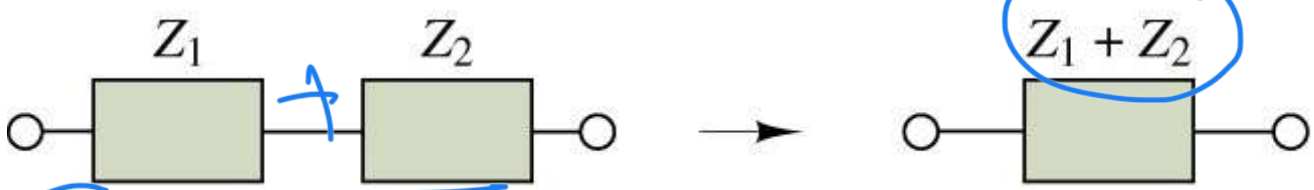
Real part : $R \Longrightarrow$ Resistor ఉంటుంది

Imag part : $X \Longrightarrow$ $+ \Longrightarrow$ Inductor.

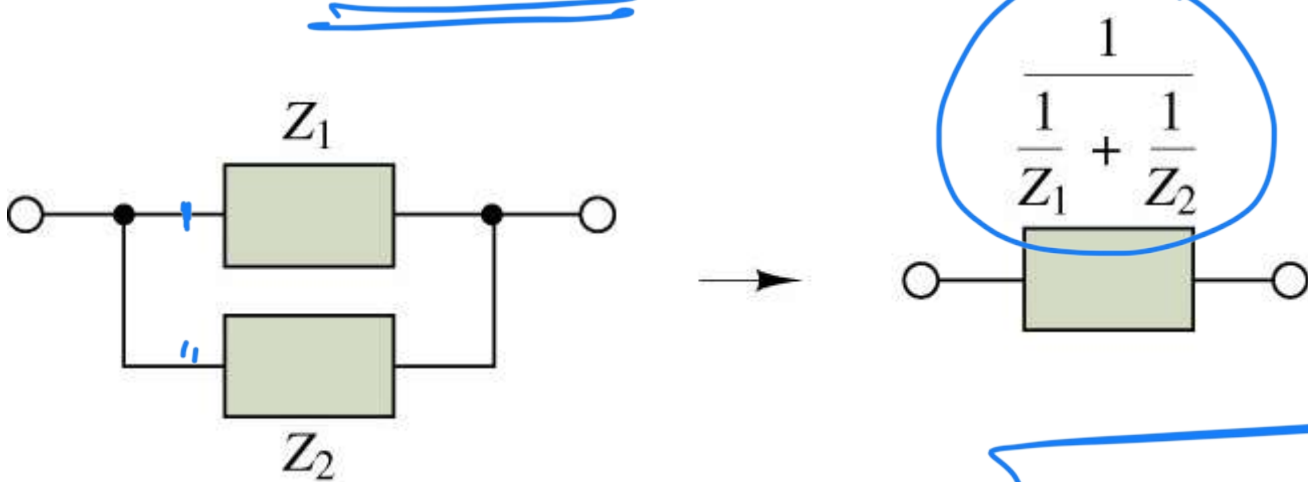
$- \Longrightarrow$ capacitor.

Rules for impedance reduction

Impedances in series add:



Impedances in parallel behave like resistors in parallel:



$$Z = \frac{1}{\frac{1}{Z_1}+\frac{1}{Z_2}} = \boxed{\frac{\text{حاصل ضرب}}{\text{حاصل جمع}}}$$

In summary :

polar

phasor form

$$Z = |Z| \angle \Theta z$$

$$|Z| = \frac{V_m}{I_m}$$

$$\Theta_z = \Theta_v - \Theta_i$$

rect

complex form

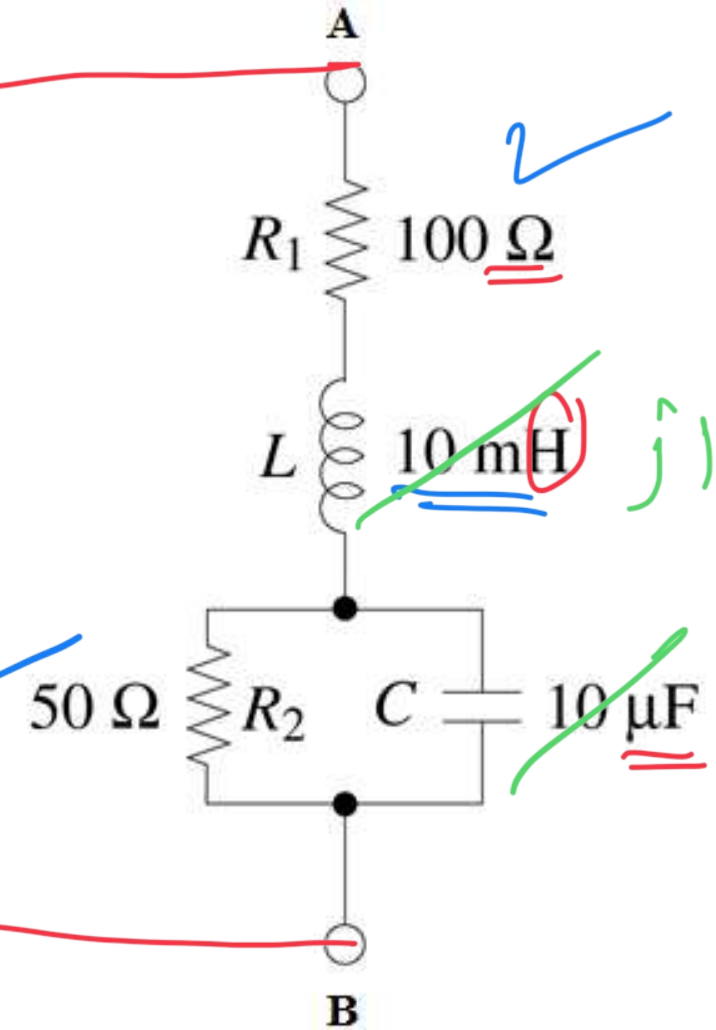
$$Z = R + jX$$

$$|Z| = \sqrt{R^2 + X^2}$$

$$\Theta_z = \tan^{-1}\left(\frac{X}{R}\right)$$

Class Activity:

Find Z_{AB} as a function of frequency.



Time domain

$\omega = 1000$ rad/s

$j10\Omega$

$-j100\Omega$

phasor domain تحول إل

$$Z_R = R$$

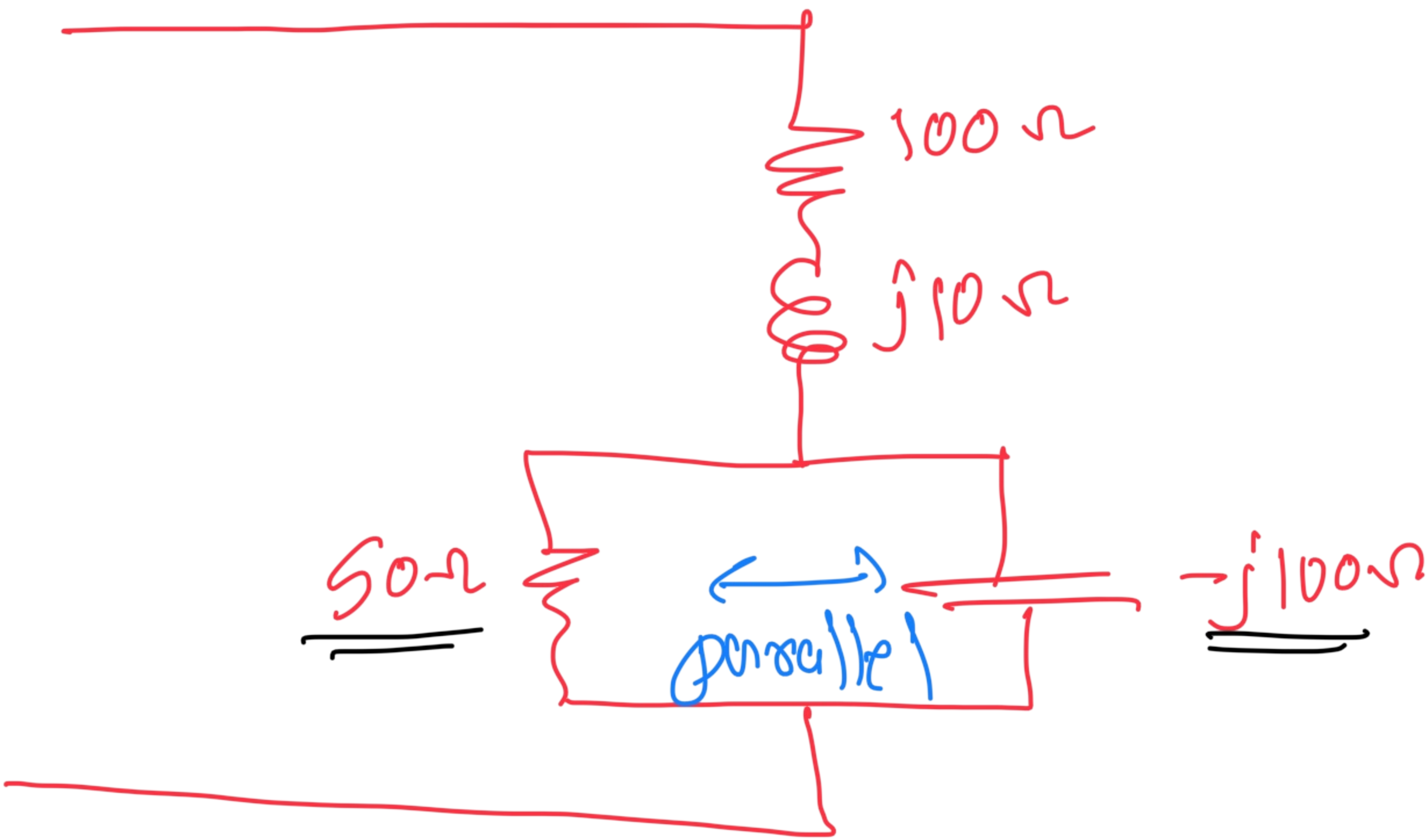
$$Z_L = j\omega L = j \times 1000 \times 10 \times 10^{-3} = +j\,10\,\Omega$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \times 1000 \times 10 \times 10^{-6}} = -j\,100\,\Omega$$

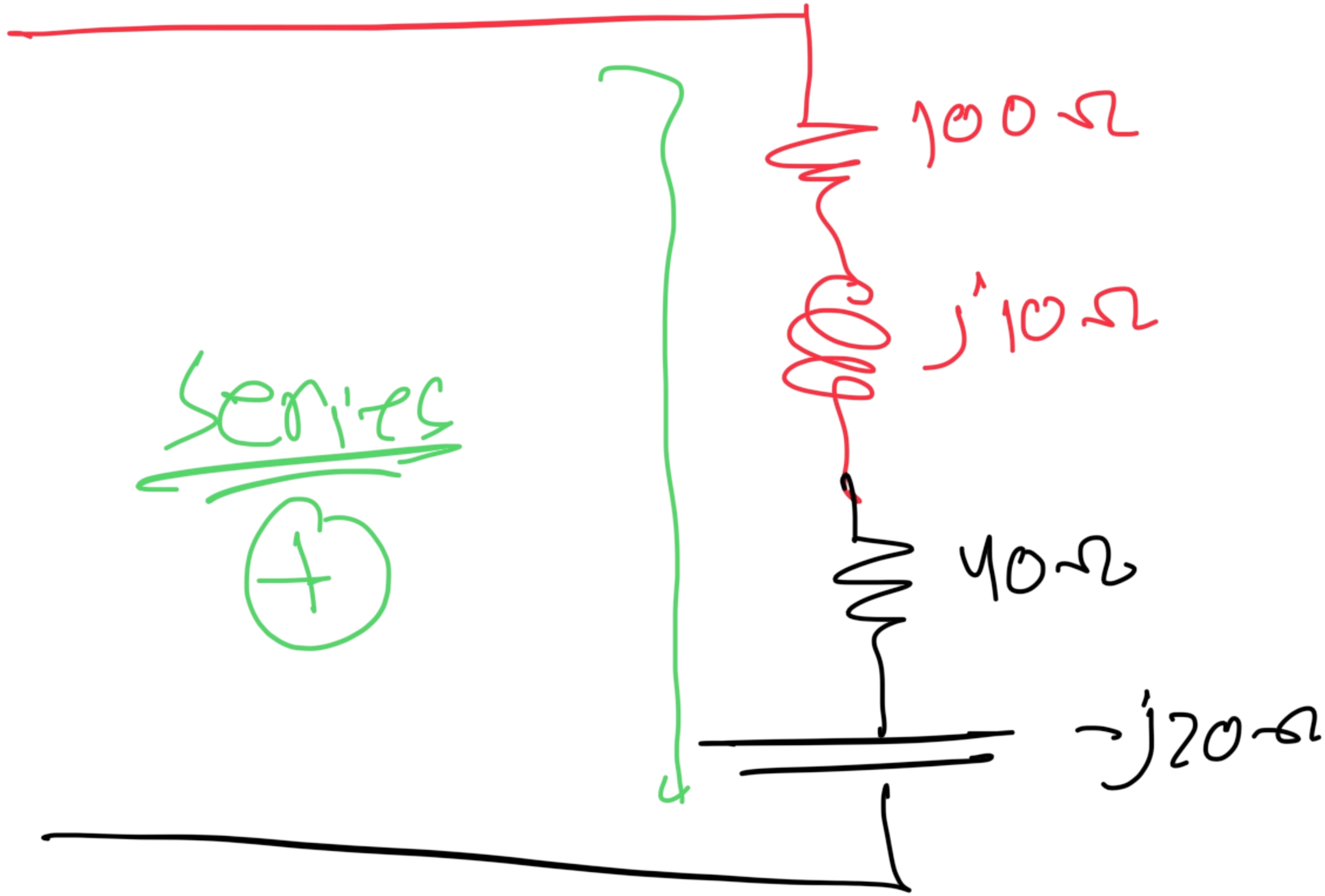
ملاحظة جدا مهم :

$$Z_L \longrightarrow +j$$

$$Z_C \longrightarrow -j$$

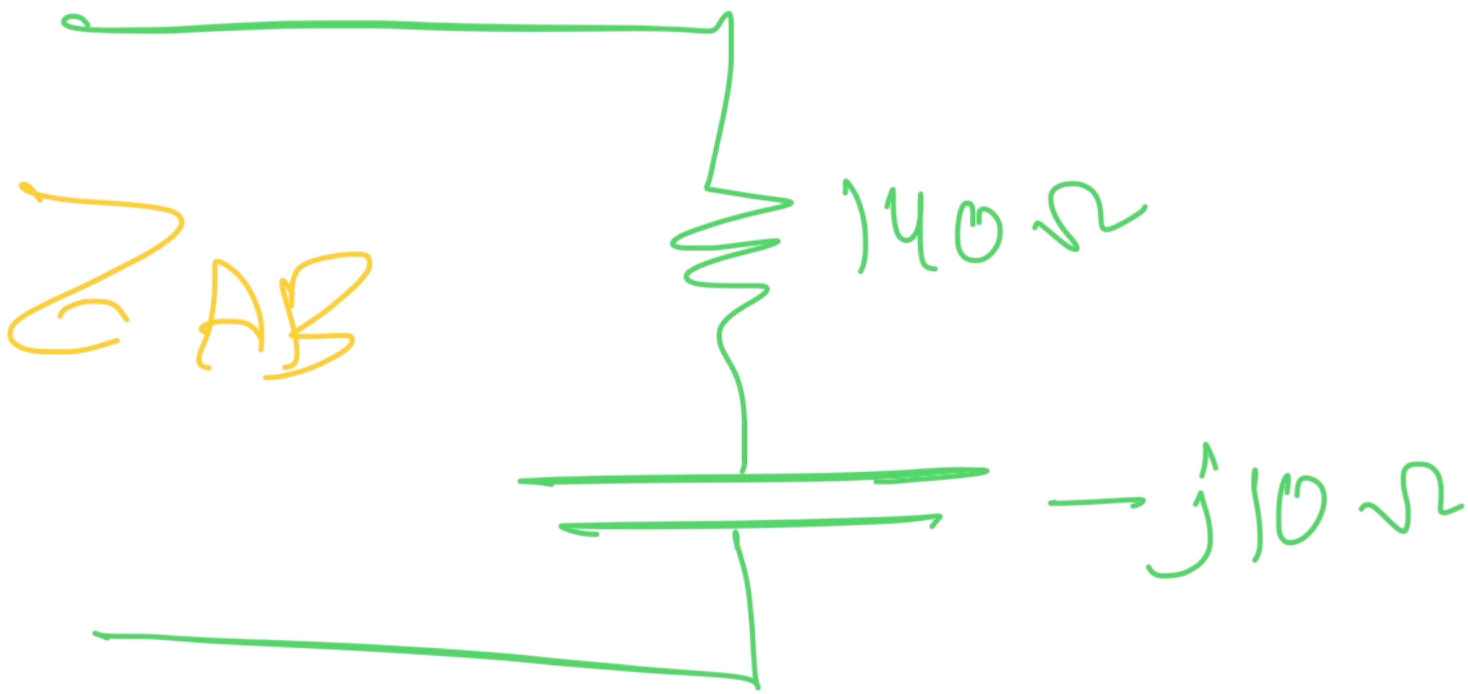


$$\frac{1}{\frac{1}{50}+\frac{1}{-j100}} = \underbrace{40}_{R} - \underbrace{j20}_{C}\,\Omega$$



$$Z_{AB} = 100 + j10 + 40 - j20$$

$$\boxed{Z_{AB} = 140 - j10\ \Omega}$$



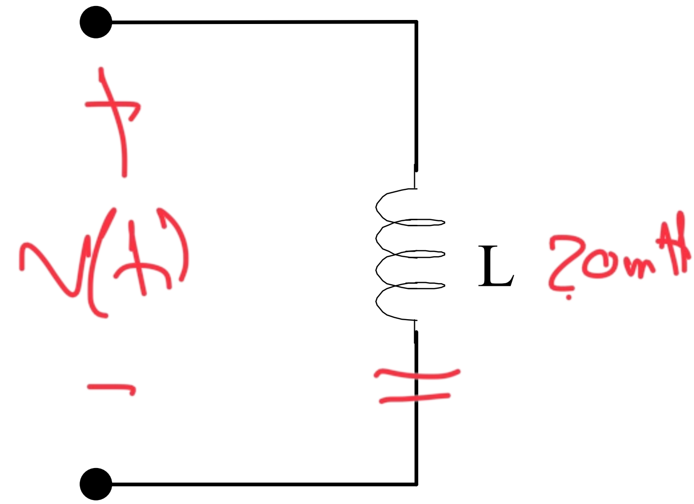
Z_{AB}
$140\,\Omega$
$-j10\,\Omega$

Example :

If the voltage across the 20 mH inductor is

$v(t) = 12 \cos(377t + 20^{o})$ find $i(t)$?

Time domain



$$V = j\omega L I$$

$$12\angle 20^{o} = j(377)(20\ m) I$$

$$I = \frac{12\angle 20^{o}}{(377)(20\ m)\angle 90^{o}} = 1.59\ \angle(-70^{o})$$

$$i(t) = 1.59\ \cos\left(377\ t - 70^{o}\right)$$

$v(t) = 12\cos(377t + 20)$ V

$L = 20\,mH$

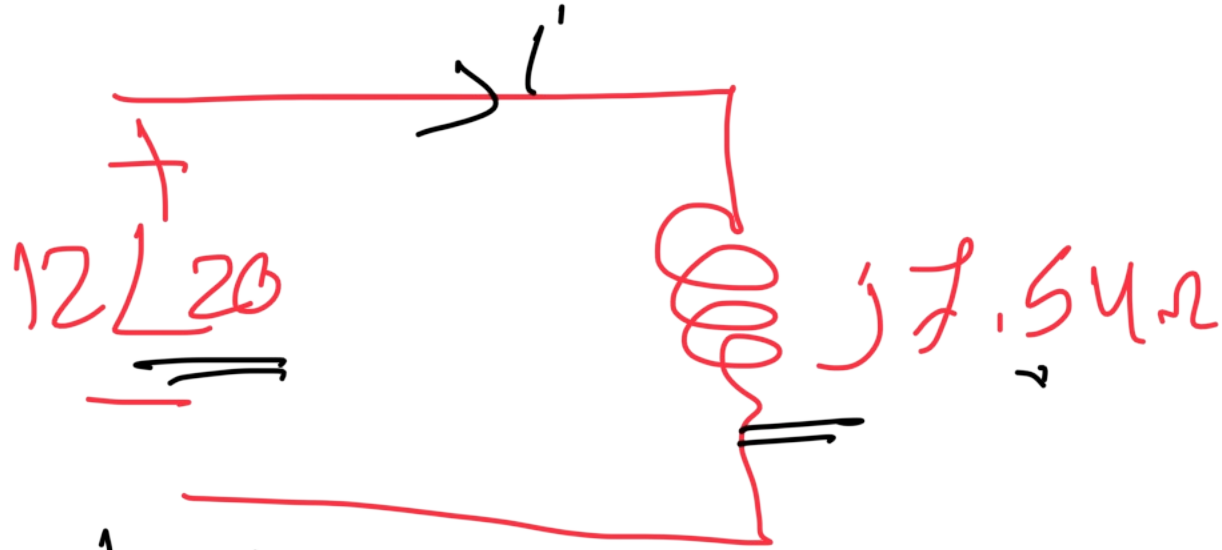
Time domain

Phasor domain

نحول ل

$V = 12\angle 20$ V , $\omega = 377$ rad/s

$Z_L = j\omega L = j \times 377 \times 20 \times 10^{-3} = j7.54\,\Omega$



ohm

$$I = \frac{V}{Z} = \frac{12\angle 20}{j7.54}$$

phasor

$$I = 1.59\angle -70^\circ \text{ A} = \frac{600}{377}\angle -70^\circ \text{ A}$$

$$\boxed{i(t) = 1.59\cos(377t - 70^\circ)\text{ A}}$$

$2 \rightarrow$ غير مرغوب فيها ؟

$$V(t) = 12\cos(377t + 20)$$

$$L = 20\,mH$$

$$V_L(t) = L\frac{di}{dt}$$

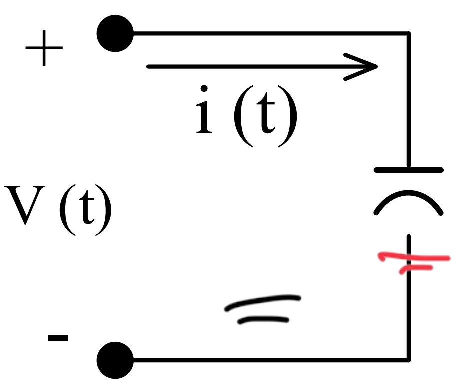
$$i_L(t) = \frac{1}{L}\int V_L(t) = \frac{1}{20\times10^{-3}}\int 12\cos(377t + 20)$$

Example :

If the voltage across the capacitor is

$v(t) = 100\cos(314t + 15^o)$ find $i(t)$??

$C = 100\ \mu F$



$$I = j\omega CV$$

$$= j(314)(100)(100*10^{-6})\angle 15^o$$

$$I = (314)(10^4)(10^{-6})\angle(15^o + 90^o) = 3.14\angle 105^o$$

$$i(t) = 3.14\cos(314t + 105^o)$$

or

$$i = C\frac{dv}{dt} = -(314)(100)(100*10^{-6})\sin(314t + 15^o)$$

$$i(t) = 3.14\cos(314t + 105^o)$$

$$v(t) = 100\cos(314t + 15) \quad , \quad C = 100\,\mu F$$

$$V = 100 \angle 15 \ V \qquad \omega = 314 \ rad/s$$

phasor ببریم

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \times 314 \times 100 \times 10^{-6}} = -j\,\frac{5000}{157}\,\Omega$$

$$= -j\,31.85\,\Omega$$

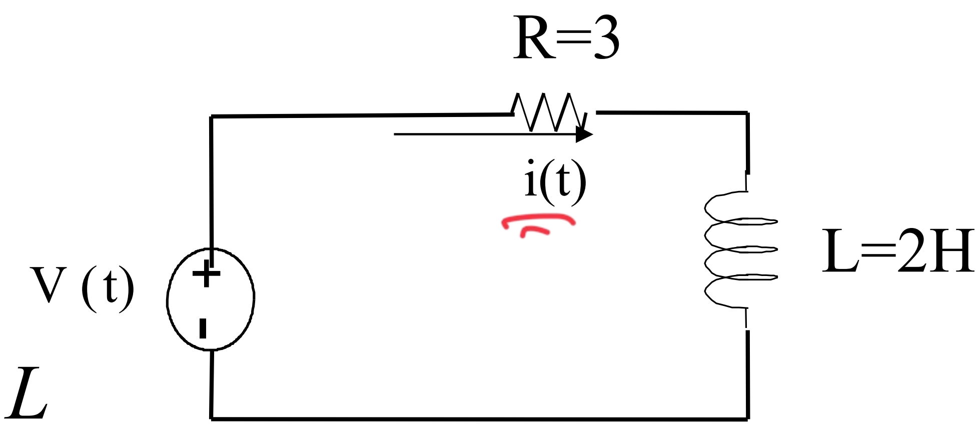
$$I = \frac{V}{Z} = \frac{100 \angle 15}{-j\,31.85} = 3.14 \angle 105 \ A$$

time ببریم

$$\boxed{i(t) = 3.14\cos(314t + 105)\ A}$$

Example :

$$v(t)=10\sin(3t) \Leftrightarrow v=10\angle-90$$



Apply KVL to solve for the current

$$-V+RI+Z_L I=0 \quad where \quad Z_L=j\omega L$$

$$-10\angle-90+3I+j6I=0$$

Solve for I

$$I=\frac{10\angle\text{-}90}{3+j6}=\frac{10\angle\text{-}90}{6.7\angle 63.4}=1.49\angle-153.4$$

in time domain reference is cosine

$$i(t)=1.49\cos(3t-153.43)$$

$$\sin(\phi)=\cos(\phi+90)$$

$$i(t)=1.49\sin(3t-63.43)$$

+90

فلسفة

لا تريدها

$v(t) = +10 \sin(3t)$ $\qquad R = 3\,\Omega \qquad L = 2\,H$

Phasor تحويل

$v(t) = 10 \cos(3t - 90)$

$V = 10\angle -90\ V \quad , \quad \omega = 3\ rad/s$

$Z_R = R = 3\,\Omega$

$Z_L = j\omega L = j \times 3 \times 2 = j6\,\Omega$

Series

$$I = \frac{V}{\Sigma Z} = \frac{10\angle -90}{3+j6}$$

$$I = \frac{2\sqrt{5}}{3} \angle -153.43\ \text{A}$$

$$\frac{2\sqrt{5}}{3} \rightarrow 1.49$$

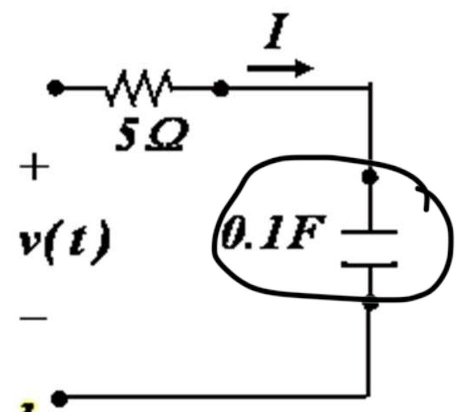
نرجع لل Time

$$\boxed{i(t) = 1.49 \cos(3t - 153.43)\ \text{A}}$$

Example:

Find the voltage across the capacitor.

$$v(t) = 10\cos(4t)$$



Solution:

Express circuit
In phasor domain

$$V = 10\angle 0^o$$

$$Z_T = R + Z_C = R + \frac{1}{j\omega C} = 5 + \frac{1}{j(4)(0.1)}$$

$$Z_T = 5 - j2.5 \;\; \Omega$$

Solve for current

$$I = \frac{V}{Z_T} = \frac{10\angle 0^o}{5 - j2.5}$$

$$I = 1.789\angle 26.57^o \;\; A$$

Solve for
capacitor voltage

$$V_C = IZ_C = (1.789\angle 26.57^o)\left(\frac{1}{j(4)(0.1)}\right)$$

$$V_C = 4.47\angle -63.43^o$$

Express voltage
In time domain

$$v_C(t) = 4.47\cos(4t - 63.43^o)$$

$v(t) = 10\cos(4t)$ $R = 5\,\Omega$ $C = 0.1\,F$

$\omega = 4$ rad/s

$\Rightarrow V = 10\angle 0 = 10V$

$Z_R = R = 5\,\Omega$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \times 4 \times 0.1} = -j2.5\,\Omega$$

Series

$$I = \frac{V}{\Sigma Z} = \frac{10}{5 - j2.5}$$ ①

$$I = 1.789 \angle 26.57 \text{ A}$$

$$V_C = I Z_C = (1.789 \angle 26.57) \times (-j2.5)$$ ②

$$V_C = 4.47 \angle -63.43 \text{ V}$$

Phasor

$$\boxed{V_C(t) = 4.47 \cos(4t - 63.43) \text{ V}}$$

series $\Rightarrow$ voltage divider

$$V_C = \frac{-j2.5}{5-j2.5} \times 10 \quad (1)$$

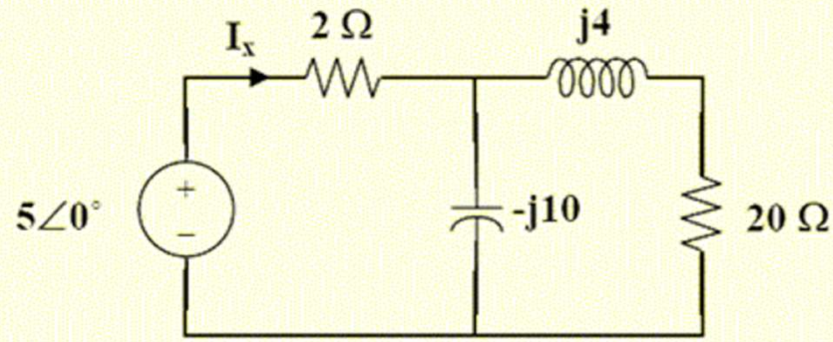
$$V_C = 4.47 \angle -63.43 \text{ V}$$

$$\boxed{v_C(t) = 4.47 \cos(4t - 63.43) \text{ V}}$$

Example:

For the circuit shown in the Figure, determine:

1. The total impedance
2. The current *Ix*



Solution:

$$I_X = \frac{V}{Z_T}$$

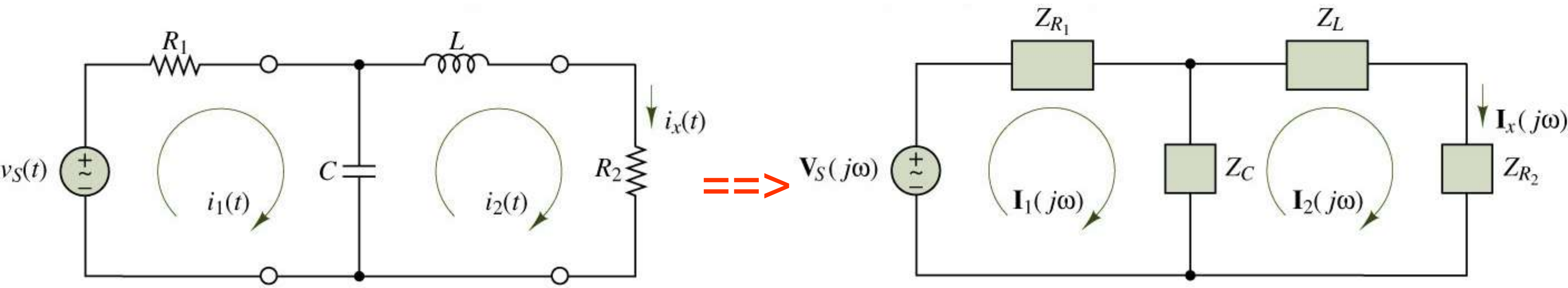
$$Z_T = 2 + (\frac{(-j10)(20+j4)}{(-j10)+(20+j4)} = 4.588 - j8.626\ \Omega$$

$$I_X = \frac{5}{2 + \frac{-j10(20+j4)}{-j10+20+j4}} = \frac{5}{2+4.588-j8.626} = \frac{5}{10.854\angle-52.63^\circ} = 0.4607\angle 52.63^\circ$$

$i_s(t)$ = **0.4607cos(2000t +52.63°) A**

Analysis of AC Circuits

- Substitute for every network with its impedance equivalent in phasor domain
- Express currents & voltages in phasor domain.
- Use same approach as DC network (Nodal analysis, Mesh Analysis or superposition) to solve for desired component



A sample circuit for AC analysis

The same circuit in phasor form