EEE 221: Electromagnetism

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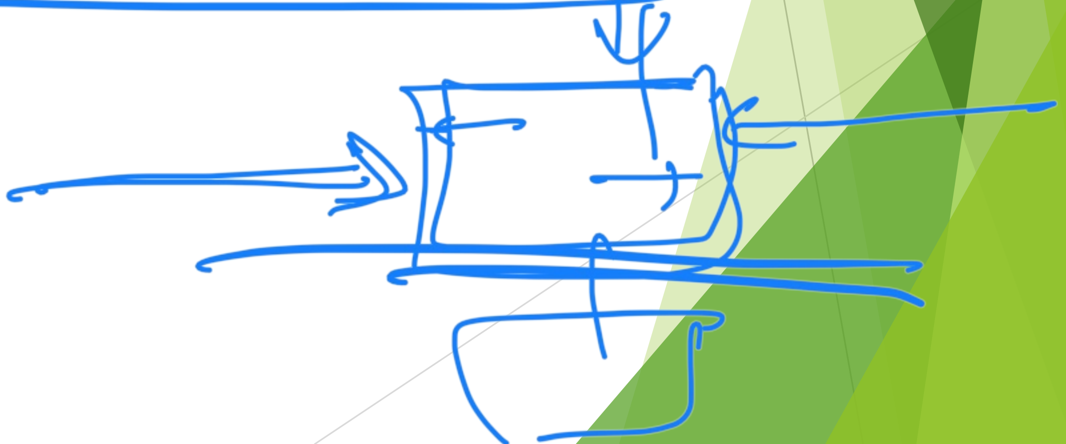
Chapter 1: Vector Algebra



قياسي / متجه

SCALARS AND VECTORS

A **scalar** is a quantity that has only magnitude.

Quantities such as time, mass, distance, temperature, entropy, electric potential, and population are scalars. A vector has not only magnitude, but direction in space.

A **vector** is a quantity that is described by both magnitude and direction.

Vector quantities include velocity, force, momentum, acceleration displacement, and electric field intensity. Another class of physical quantities is called *tensors*, of which scalars and vectors are special cases. For most of the time, we shall be concerned with scalars and vectors.[4]

To distinguish between a scalar and a vector it is customary to represent a vector by a letter with an arrow on top of it, such as $\vec{A}$ and $\vec{B}$, or by a letter in boldface type such as **A** and **B**. A scalar is represented simply by a letter—for example, *A*, *B*, *U*, and *V*.

$\vec{A}$ $\vec{A}$ $\Longrightarrow$ Vectors Arrow

A **B** $\Longrightarrow$ Vectors Bold

A C K $\Rightarrow$ scalar

UNIT VECTOR

متجه الوحدة

A vector **A** has both magnitude and direction. The *magnitude* of **A** is a scalar written as A or $|\mathbf{A}|$. A *unit vector* $\mathbf{a}_A$ along **A** is defined as a vector whose magnitude is unity (i.e., 1) and its direction is along **A**; that is,

$$\boxed{\mathbf{a}_A = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{\mathbf{A}}{A}}$$

Bold

Note that $|\mathbf{a}_A| = 1$. Thus we may write **A** as

$$\boxed{A = \sqrt{A_x^2 + A_y^2 + A_z^2}}$$

Unit vector

$$\mathbf{a}_A = \frac{A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

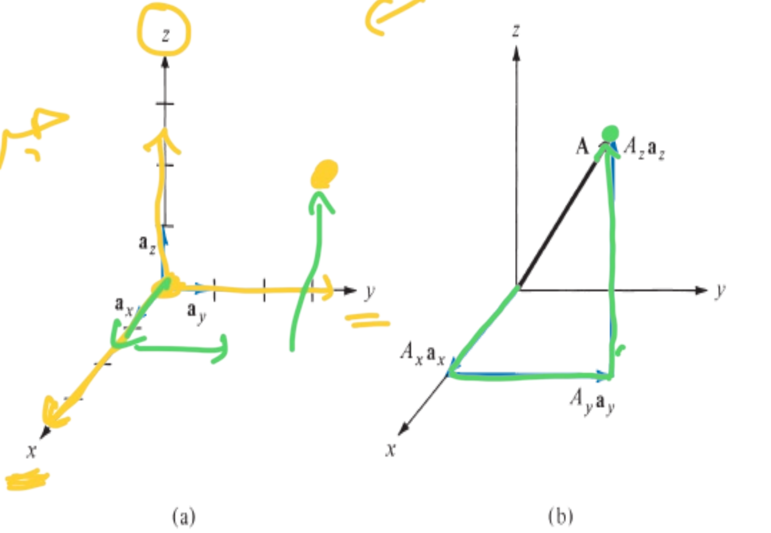


FIGURE 1.1 (a) Unit vectors $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$, (b) components of **A** along $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$.

$\vec{A} = 5i + 7j - 3k \longrightarrow$ vector

$\vec{A} = \langle 5, 7, -3 \rangle \longrightarrow$ vector

$\vec{A} = 5a_x + 7a_y - 3a_z \longrightarrow$ vector

$x \quad y \quad z$

magnitude vector A → 1

A

$$|A| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$= \sqrt{5^2 + 7^2 + (-3)^2}$$

$$25 + 49 + 9$$

$$\sqrt{83} \approx 9.11\ldots$$

$$\boxed{|A| = \sqrt{83}}$$

Unit Vector → "a" متجه الوحدة

= متجه مقداره "mag" يساوي 1

طوله

كمية

$$a_A = \frac{\vec{A}}{|A|}$$

في حال لم يكتب

يساوي (1)

VECTOR ADDITION & SUBTRACTION

Two vectors A and B can be added together to give another vector C; that is,

$$\mathbf{C} = \mathbf{A} + \mathbf{B}$$

The vector addition is carried out component by component. Thus, if $\mathbf{A} = (A_x, A_y, A_z)$ and $\mathbf{B} = (B_x, B_y, B_z)$.

$$\mathbf{C} = (A_x + B_x)\mathbf{a}_x + (A_y + B_y)\mathbf{a}_y + (A_z + B_z)\mathbf{a}_z$$

Vector subtraction is similarly carried out as

$$\mathbf{D} = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$
$$= (A_x - B_x)\mathbf{a}_x + (A_y - B_y)\mathbf{a}_y + (A_z - B_z)\mathbf{a}_z$$

VECTOR ADDITION & SUBTRACTION

Law تبادل	Addition	Multiplication
Commutative	$A + B = B + A$	$kA = Ak$
Associative	$A + (B + C) = (A + B) + C$	$k(\ell A) = (k\ell)A$
Distributive	$k(A + B) = kA + kB$	

تجميع

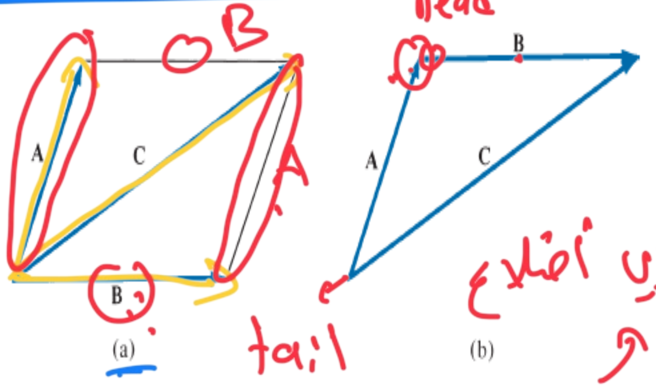


FIGURE 1.2 Vector addition $C = A + B$: (a) parallelogram rule, (b) head-to-tail rule.

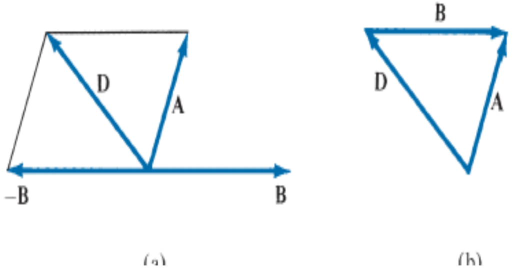


FIGURE 1.3 Vector subtraction $D = A - B$: (a) parallelogram rule, (b) head-to-tail rule.

POSITION AND DISTANCE VECTORS

The **position vector** $\mathbf{r}_P$ (or **radius vector**) of point P is defined as the directed distance from the origin O to P; that is,

البداية من (0,0,0)

$$\mathbf{r}_P = OP = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$$

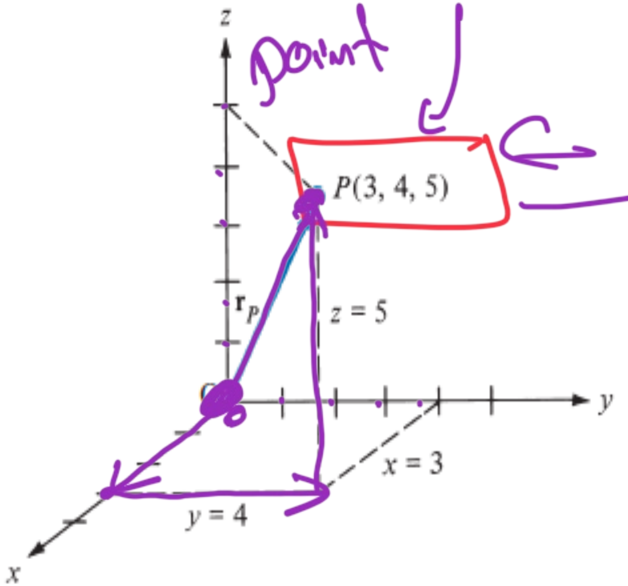
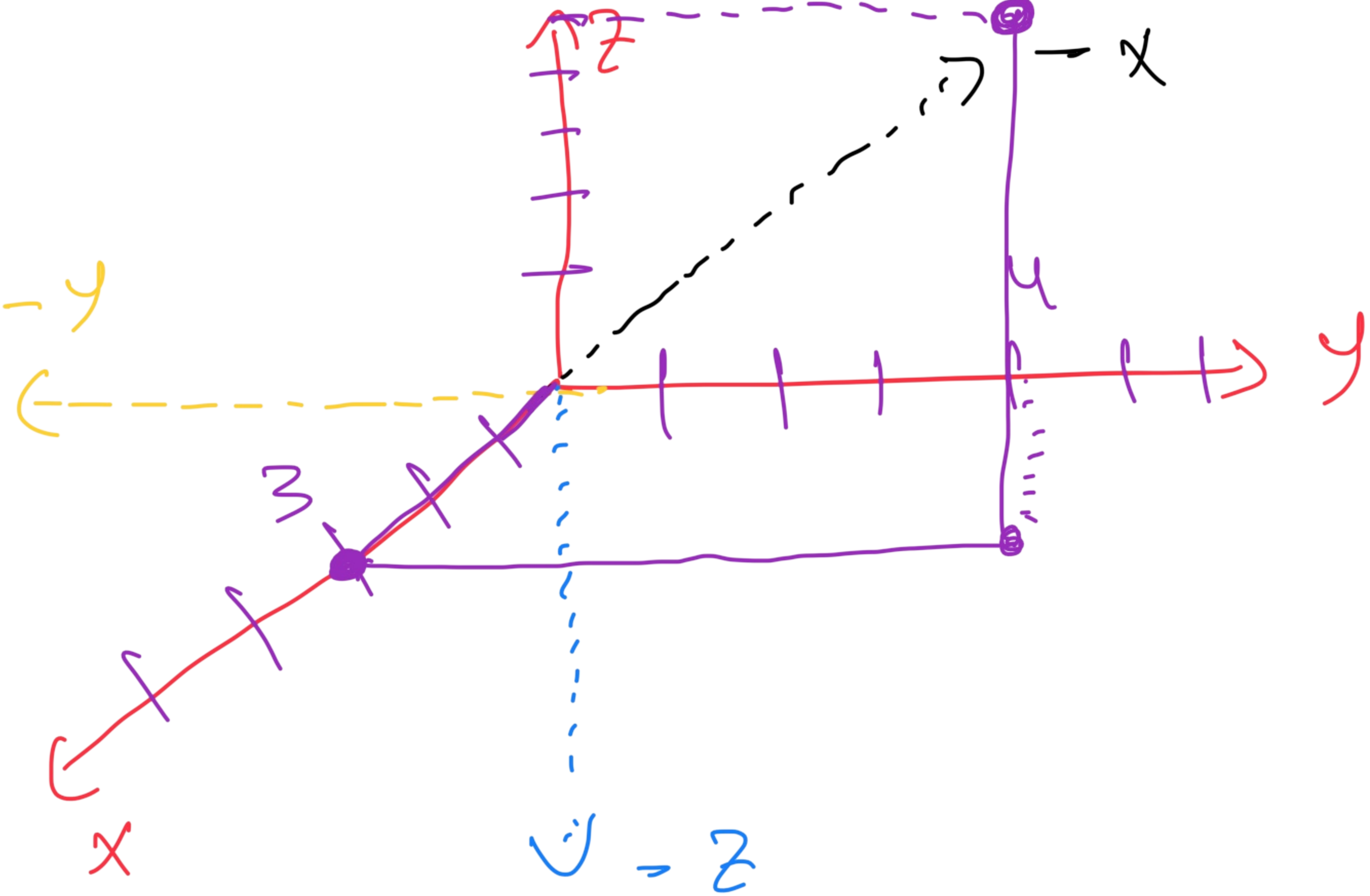


FIGURE 1.4 Illustration of position vector $\mathbf{r}_P = 3\mathbf{a}_x + 4\mathbf{a}_y + 5\mathbf{a}_z$.

vector

(+3, +4, +5)

x y z



z
-x
4
-y
y
3
x
-z

EXAMPLE 1.1 If $\mathbf{A} = 10\mathbf{a}_x - 4\mathbf{a}_y + 6\mathbf{a}_z$ and $\mathbf{B} = 2\mathbf{a}_x + \mathbf{a}_y$, find (a) the component of $\mathbf{A}$ along $\mathbf{a}_y$, (b) the magnitude of $3\mathbf{A} - \mathbf{B}$, (c) a unit vector along $\mathbf{A} + 2\mathbf{B}$.

Solution:

(a) The component of $\mathbf{A}$ along $\mathbf{a}_y$ is $A_y = -4$.

(b) $$\begin{aligned} 3\mathbf{A} - \mathbf{B} &= 3(10, -4, 6) - (2, 1, 0) \\ &= (30, -12, 18) - (2, 1, 0) \\ &= (28, -13, 18) \end{aligned}$$

Hence,

$$\begin{aligned} |3\mathbf{A} - \mathbf{B}| &= \sqrt{28^2 + (-13)^2 + (18)^2} = \sqrt{1277} \\ &= 35.74 \end{aligned}$$

(c) Let $\mathbf{C} = \mathbf{A} + 2\mathbf{B} = (10, -4, 6) + (4, 2, 0) = (14, -2, 6)$.

A unit vector along $\mathbf{C}$ is

$$\mathbf{a}_c = \frac{\mathbf{C}}{|\mathbf{C}|} = \frac{(14, -2, 6)}{\sqrt{14^2 + (-2)^2 + 6^2}}$$

or

$$\mathbf{a}_c = 0.9113\mathbf{a}_x - 0.1302\mathbf{a}_y + 0.3906\mathbf{a}_z$$

Note that $|\mathbf{a}_c| = 1$ as expected.

$$\vec{A} = 10a_x - 4a_y + 6a_z \rightarrow A_y = -4$$

$$B = 2a_x + 1a_y + 0a_z$$

$$3\vec{A} = 30a_x - 12a_y + 18a_z$$

$$3\vec{A} - \vec{B} = 28a_x - 13a_y + 18a_z$$

$$|3\vec{A}-\vec{B}| = \sqrt{28^2 + (-13)^2 + 18^2}$$

$$= \sqrt{1277}$$

$$\vec{A} = 10a_x - 4a_y + 6a_z$$

$$\vec{B} = 2a_x + 1a_y$$

$$2\vec{B} = 4a_x + \boxed{2a_y}$$

$$|\vec{A}+2\vec{B}| = 14a_x - 2a_y + 6a_z$$

$$|\vec{A}+2\vec{B}| = \sqrt{14^2+(-2)^2+6^2}$$

$$= \boxed{2\sqrt{59}}$$

Unit Vector $a_{A+2B} = \frac{\text{Vector}}{\text{mag}}$

$$a_{A+2B} = \frac{14a_x}{2\sqrt{5a}} - \frac{2a_y}{2\sqrt{5a}} + \frac{6a_z}{2\sqrt{5a}}$$

$\downarrow$

$$\vec{a} = 0{,}911\,a_x - 0{,}1302\,a_y + 0{,}3906\,a_z$$

EXAMPLE 1.2

نقاط

Points P and Q are located at $(0, 2, 4)$ and $(-3, 1, 5)$. Calculate

(a) The position of vector $\mathbf{r}_P$

(b) The ~~distance~~ vector from P to Q

(c) The distance between P and Q

(d) A vector parallel to PQ with magnitude of 10

Solution:

(a) $\mathbf{r}_P = 0\mathbf{a}_x + 2\mathbf{a}_y + 4\mathbf{a}_z = 2\mathbf{a}_y + 4\mathbf{a}_z$

(b) $\mathbf{r}_{PQ} = \mathbf{r}_Q - \mathbf{r}_P = (-3, 1, 5) - (0, 2, 4) = (-3, -1, 1)$

or $\mathbf{r}_{PQ} = -3\mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z$

(c) Since $\mathbf{r}_{PQ}$ is the distance vector from P to Q, the distance between P and Q is the magnitude of this vector; that is,

$$d = |\mathbf{r}_{PQ}| = \sqrt{9 + 1 + 1} = 3.317$$

Alternatively:

$$d = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2 + (z_Q - z_P)^2}$$
$$= \sqrt{9 + 1 + 1} = 3.317$$

(d) Let the required vector be $\mathbf{A}$, then

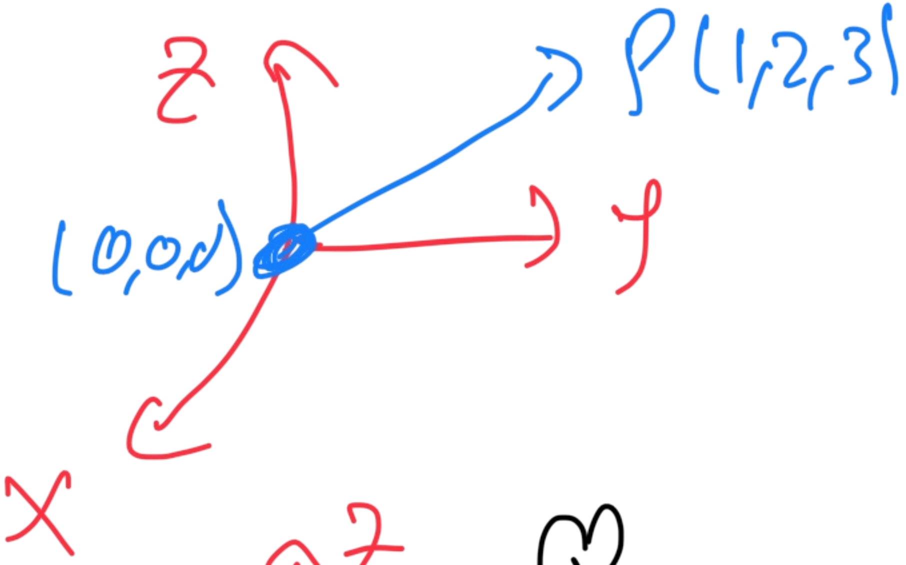
$$\mathbf{A} = A\mathbf{a}_A$$

where $A = 10$ is the magnitude of $\mathbf{A}$. Since $\mathbf{A}$ is parallel to PQ, it must have the same unit vector as $\mathbf{r}_{PQ}$ or $\mathbf{r}_{QP}$. Hence,

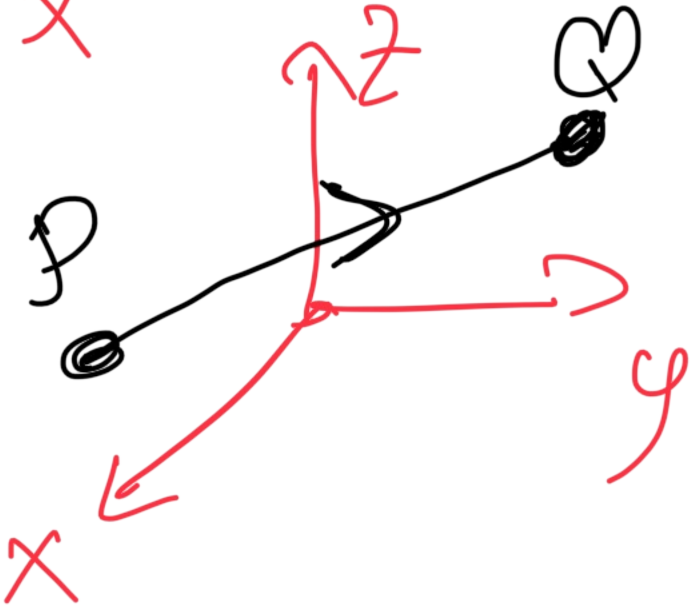
$$\mathbf{a}_A = \pm\frac{\mathbf{r}_{PQ}}{|\mathbf{r}_{PQ}|} = \pm\frac{(-3, -1, 1)}{3.317}$$

and

$$\mathbf{A} = \pm\frac{10(-3, -1, 1)}{3.317} = \pm(-9.045\mathbf{a}_x - 3.015\mathbf{a}_y + 3.015\mathbf{a}_z)$$



$\Longrightarrow$ position vector



$\Longrightarrow$ distance vector

$P \longrightarrow Q$ "PQ"

$P(0, 2, 4) \qquad Q(-3, 1, 5)$

1. $r_P = 0a_x + 2a_y + 4a_z$

2. $P \to Q = \overrightarrow{PQ} = Q - P$

$\overrightarrow{PQ} = -3a_x - 1a_y + 1a_z$

$\therefore$

$\boxed{3}$ distance $\Rightarrow$ mag $\rightarrow$ مقدار

مسافة

لها d

معينة

$$|d| = |\vec{PQ}| = \sqrt{(-3)^2 + (-1)^2 + (1)^2}$$

$$d = \sqrt{9+1+1} = \underline{\sqrt{11}} \approx 3,3$$

$\boxed{4}$ Parallel $\Longrightarrow$ موازي

$$\vec{A} = k\,\vec{B}$$

$$\vec{PQ} = -3a_x - 1a_y + 1a_z$$

↳ Unit vector $\Longrightarrow$ mag $= 1$

$$\vec{a}_{pQ} = \frac{-3\,a_x}{\sqrt{11}} - \frac{1\,a_y}{\sqrt{11}} + \frac{1\,a_z}{\sqrt{11}}$$

$\hookrightarrow$ parallel $\rightarrow$ 1 حوله

$$F = \vec{f}\,10\,\vec{a}_{pQ} = \vec{f}\left(\frac{-30}{\sqrt{11}}a_x - \frac{10}{\sqrt{11}}a_y + \frac{10}{\sqrt{11}}a_z\right)$$

$+ \rightarrow$ نفس اتجاه

$- \rightarrow$ عكس اتجاه

VECTOR MULTIPLICATION

When two vectors A and B are multiplied, the result is either a scalar or a vector depending on how they are multiplied. Thus there are two types of vector multiplication:

1. Scalar (or dot) product: $A \cdot B$ → scalar
2. Vector (or cross) product: $A \times B$ → vector

Multiplication of three vectors A, B, and C can result in either:

3. Scalar triple product: $A \cdot (B \times C)$

or

4. Vector triple product: $A \times (B \times C)$

VECTOR MULTIPLICATION (contd)

A. Dot Product

The **dot product** of two vectors A and B, written as A · B, is defined geometrically as the product of the magnitudes of A and B and the cosine of the smaller angle between them when they are drawn tail to tail.

$$\boxed{\mathbf{A}\cdot\mathbf{B} = AB\cos\theta_{AB}}$$

$$\vec{A}\cdot\vec{B} = |A|\,|B|\cos\theta_{AB} \rightarrow ①$$

②

Note that dot product obeys the following:

(i) *Commutative law:*

$$\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$$

(ii) *Distributive law:*

$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{C}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{C}$$

mag

(iii)

$$\mathbf{A}\cdot\mathbf{A} = |\mathbf{A}|^2 = A^2$$

مش ر [illegible]

الضرب

Also note that

$$\mathbf{a}_x\cdot\mathbf{a}_y = \mathbf{a}_y\cdot\mathbf{a}_z = \mathbf{a}_z\cdot\mathbf{a}_x = 0$$

$$\mathbf{a}_x\cdot\mathbf{a}_x = \mathbf{a}_y\cdot\mathbf{a}_y = \mathbf{a}_z\cdot\mathbf{a}_z = 1$$

VECTOR MULTIPLICATION (contd)

B. Cross Product

The **cross product** of two vectors **A** and **B**, written as **A** × **B**, is a vector quantity whose magnitude is the area of the parallelogram formed by **A** and **B** (see Figure 1.7) and is in the direction of advance of a right-handed screw as **A** is turned into **B**.

$$\boxed{\mathbf{A} \times \mathbf{B} = AB \sin \theta_{AB} \mathbf{a}_n}$$

$$\vec{A} \times \vec{B} = |A|\,|B| \sin \theta_{AB}\, \vec{a}_n$$

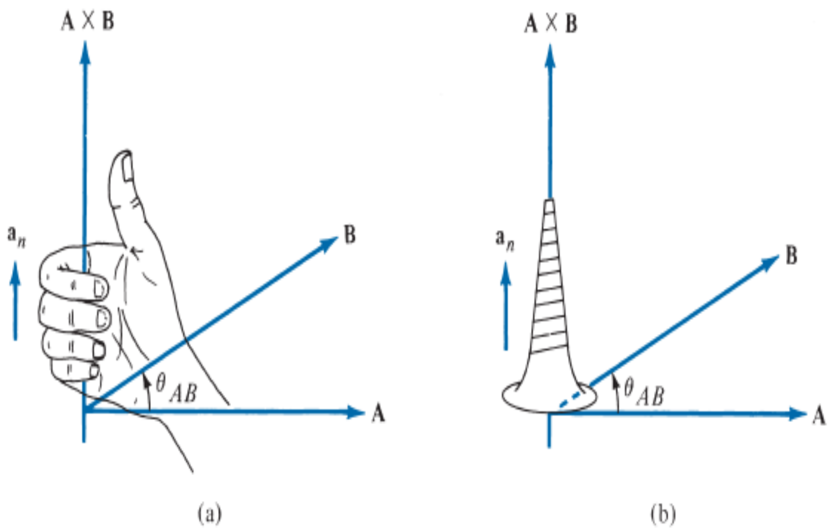
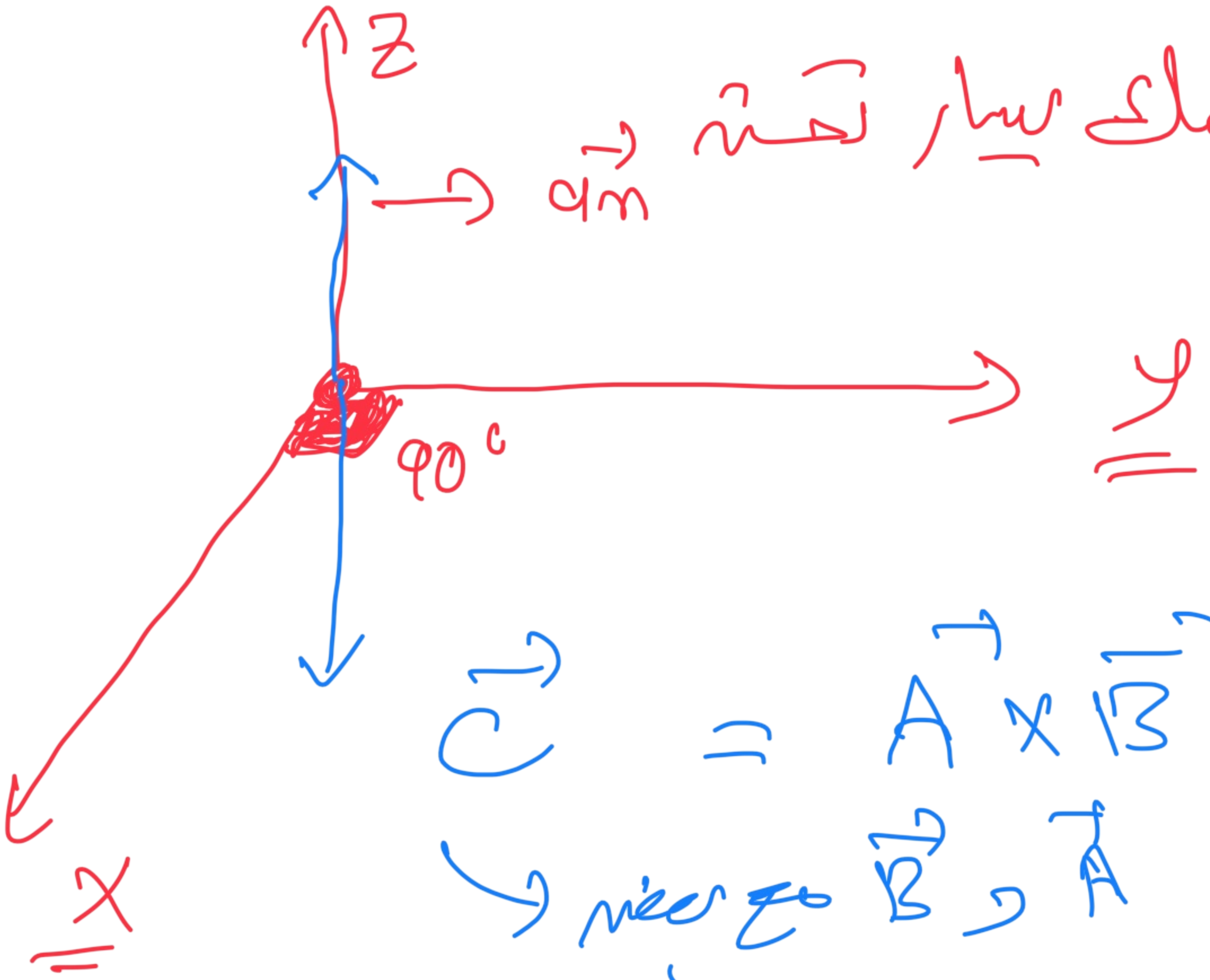


FIGURE 1.8 Direction of **A** × **B** and $\mathbf{a}_n$ using (a) the right-hand rule and (b) the right-handed-screw rule.

زاوية غرضك يسار تحته



$$\vec{C} = \vec{A} \times \vec{B}$$

→ عامودي على $\vec{A}$ و $\vec{B}$ مع بعض

VECTOR MULTIPLICATION (contd)

Note that the cross product has the following basic properties:

(i) It is not commutative:

$$\mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A}$$

It is anticommutative:

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$$

(ii) It is not associative:

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) \neq (\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$$

(iii) It is distributive:

$$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C}$$

(iv) Scaling:

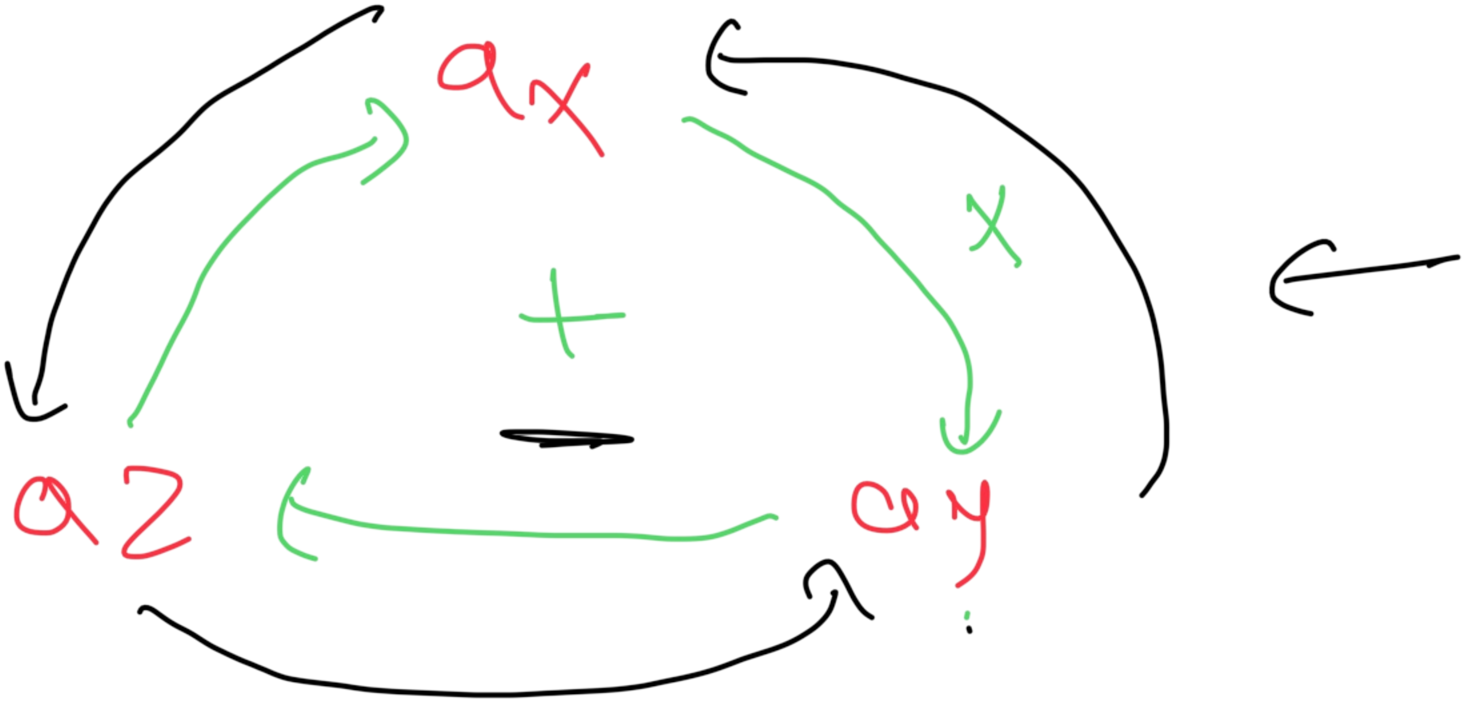
$$k\mathbf{A} \times \mathbf{B} = \mathbf{A} \times k\mathbf{B} = k(\mathbf{A} \times \mathbf{B})$$

(v)

$$\boxed{\mathbf{A} \times \mathbf{A} = 0}$$

Also note that

$$\boxed{\begin{aligned} \mathbf{a}_x \times \mathbf{a}_y &= \mathbf{a}_z \\ \mathbf{a}_y \times \mathbf{a}_z &= \mathbf{a}_x \\ \mathbf{a}_z \times \mathbf{a}_x &= \mathbf{a}_y \end{aligned}}$$



$a_x \times a_y \rightarrow a_z$

+ مع عقارب الساعة

− عكس عقارب الساعة

COMPONENT OF A VECTOR

A direct application of scalar product is its use in determining the projection (or component) of a vector in a given direction. The projection can be scalar or vector. Given a vector **A**, we define the *scalar component* A_B of **A** along vector **B** as [see Figure 1.10(a)]

$$A_B = A\cos\theta_{AB} = |\mathbf{A}||\mathbf{a}_B|\cos\theta_{AB}$$

$$\boxed{A_B = \mathbf{A}\cdot\mathbf{a}_B}$$

$$\boxed{\mathbf{A}_B = A_B\mathbf{a}_B = (\mathbf{A}\cdot\mathbf{a}_B)\mathbf{a}_B}$$

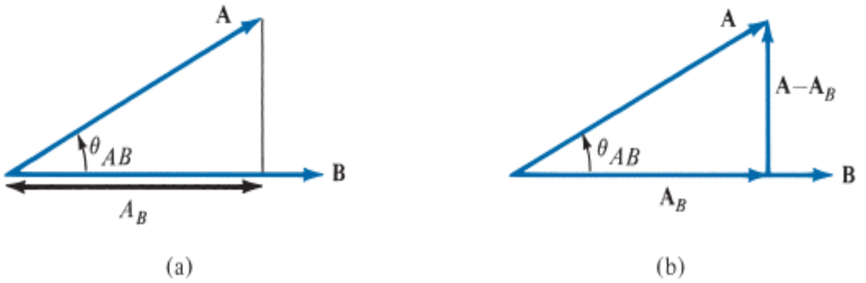


FIGURE 1.10 Components of **A** along **B**: (**a**) scalar component A_B, (**b**) vector component $\mathbf{A}_B$.

EXAMPLE 1.4 Given vectors $\mathbf{A} = 3\mathbf{a}_x + 4\mathbf{a}_y + \mathbf{a}_z$ and $\mathbf{B} = 2\mathbf{a}_y - 5\mathbf{a}_z$, find the angle between **A** and **B**.

Solution:

The angle θ_{AB} can be found by using either dot product or cross product.

$$\begin{aligned}
\mathbf{A} \cdot \mathbf{B} &= (3, 4, 1) \cdot (0, 2, -5) \\
&= 0 + 8 - 5 = 3 \\
|\mathbf{A}| &= \sqrt{3^2 + 4^2 + 1^2} = \sqrt{26} \\
|\mathbf{B}| &= \sqrt{0^2 + 2^2 + (-5)^2} = \sqrt{29}
\end{aligned}$$

$$\cos \theta_{AB} = \frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}||\mathbf{B}|} = \frac{3}{\sqrt{(26)(29)}} = 0.1092$$

$$\theta_{AB} = \cos^{-1} 0.1092 = 83.73^\circ$$

Alternatively:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 3 & 4 & 1 \\ 0 & 2 & -5 \end{vmatrix}$$

$$\begin{aligned}
&= (-20 - 2)\mathbf{a}_x + (0 + 15)\mathbf{a}_y + (6 - 0)\mathbf{a}_z \\
&= (-22, 15, 6)
\end{aligned}$$

$$|\mathbf{A} \times \mathbf{B}| = \sqrt{(-22)^2 + 15^2 + 6^2} = \sqrt{745}$$

$$\sin \theta_{AB} = \frac{|\mathbf{A} \times \mathbf{B}|}{|\mathbf{A}||\mathbf{B}|} = \frac{\sqrt{745}}{\sqrt{(26)(29)}} = 0.994$$

$$\theta_{AB} = \sin^{-1} 0.994 = 83.73^\circ$$

$$\vec{A} = 3a_x + 4a_y + a_z$$

$$\vec{B} = 2a_y - 5a_z$$

$\boxed{1}$ $A \cdot B = |A|\,|B| \cos\theta_{AB}$

$$\theta_{AB} = \cos^{-1}\left(\frac{A \cdot B}{|A|\,|B|}\right)$$

$$A \cdot B = \frac{3ax}{0\,ax} + \frac{4ay}{2ay} + \frac{1az}{-5az}$$

$$[3 \times 0] + [4 \times 2] + [-5 \times 1]$$

$$0 + 8 + -5 = 3$$

$$\boxed{A \cdot B = 3}$$

$$|A| = \sqrt{3^2 + 4^2 + 1^2} = \sqrt{9 + 16 + 1}$$

$$|A| = \sqrt{26} \quad ✓$$

$$|B| = \sqrt{0^2 + 2^2 + (-5)^2} = \sqrt{0 + 4 + 25}$$

$$|B| = \sqrt{29} \quad ✓$$

$$\theta_{AB} = \cos^{-1}\left(\frac{3}{\sqrt{26}\cdot\sqrt{29}}\right) = \underline{83{,}7}$$

$\boxed{1}$

done!!

(2) $|A \times B| = |A|\,|B| \sin\theta_{AB}$

$$\theta_{AB} = \sin^{-1}\left(\frac{|A \times B|}{|A|\,|B|} \right)$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} x & y & z \\ 3 & 4 & [illegible] \\ 0 & 2 & -6 \end{vmatrix} \quad 0$$

$$\vec{A}\times\vec{B} = a_x \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix} - a_y \begin{vmatrix} 3 & 1 \\ 0 & 5 \end{vmatrix}$$

$-20 - 2$ $\quad$ $-15 - 0$

$$+ a_z \begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix}$$

$6 - 0$

$$\vec{A}\times\vec{B} = -22 a_x + 15 a_y + 6 a_z \checkmark$$

$$|A\times B| = \sqrt{(-22)^2 + (15)^2 + (6)^2}$$

$$\sqrt{484+225+36} = \sqrt{745}$$

$$\theta_{AB} = \sin^{-1}\left(\frac{\sqrt{745}}{\sqrt{26}\cdot\sqrt{29}} \right) = \underline{\underline{83{,}7}}$$