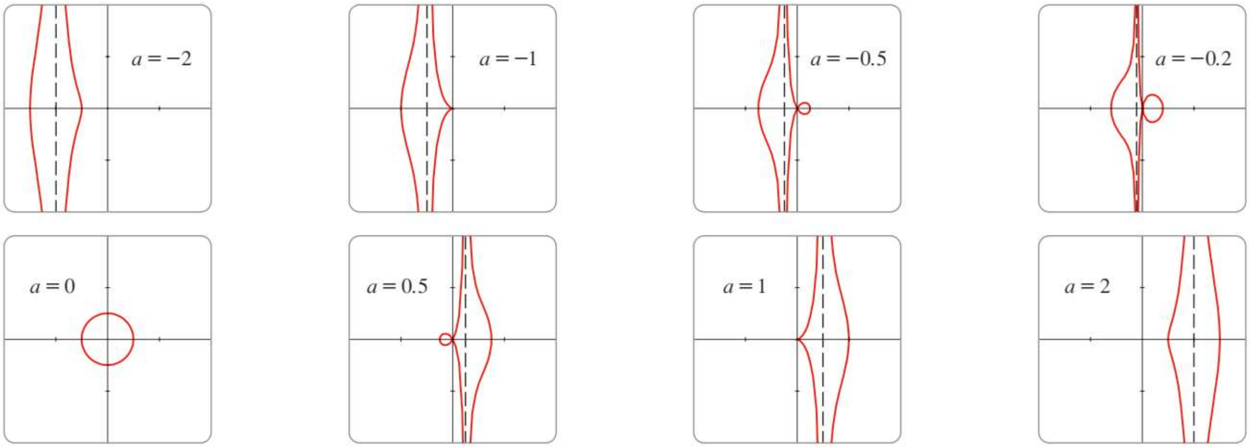


FIGURE 19
Members of the family $x = a + \cos t$, $y = a \tan t + \sin t$, all graphed in the viewing rectangle $[-4, 4]$ by $[-4, 4]$

When $a < -1$, both branches are smooth; but when a reaches -1, the right branch acquires a sharp point, called a *cusp*. For a between -1 and 0 the cusp turns into a loop, which becomes larger as a approaches 0. When $a = 0$, both branches come together and form a circle (see Example 2). For a between 0 and 1, the left branch has a loop, which shrinks to become a cusp when $a = 1$. For $a > 1$, the branches become smooth again, and as a increases further, they become less curved. Notice that the curves with a positive are reflections about the y-axis of the corresponding curves with a negative.

These curves are called **conchoids of Nicomedes** after the ancient Greek scholar Nicomedes. He called them conchoids because the shape of their outer branches resembles that of a conch shell or mussel shell. ■

10.1 Exercises

1–2 For the given parametric equations, find the points (x, y) corresponding to the parameter values $t = -2, -1, 0, 1, 2$.

1. $x = t^2 + t, \quad y = 3^{t+1}$

2. $x = \ln(t^2 + 1), \quad y = t/(t + 4)$ $\frac{t}{t+4}$

3–6 Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.

3. $x = 1 - t^2, \quad y = 2t - t^2, \quad -1 \leqslant t \leqslant 2$

4. $x = t^3 + t, \quad y = t^2 + 2, \quad -2 \leqslant t \leqslant 2$

5. $x = 2^t - t, \quad y = 2^{-t} + t, \quad -3 \leqslant t \leqslant 3$

6. $x = \cos^2 t, \quad y = 1 + \cos t, \quad 0 \leqslant t \leqslant \pi$

7–12
(a) Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.
(b) Eliminate the parameter to find a Cartesian equation of the curve.

7. $x = 2t - 1, \quad y = \frac{1}{2}t + 1$

8. $x = 3t + 2, \quad y = 2t + 3$

9. $x = t^2 - 3, \quad y = t + 2, \quad -3 \leqslant t \leqslant 3$

10. $x = \sin t, \quad y = 1 - \cos t, \quad 0 \leqslant t \leqslant 2\pi$

11. $x = \sqrt{t}, \quad y = 1 - t$

12. $x = t^2, \quad y = t^3$

13–22
(a) Eliminate the parameter to find a Cartesian equation of the curve.
(b) Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.

13. $x = 3\cos t, \quad y = 3\sin t, \quad 0 \leqslant t \leqslant \pi$

14. $x = \sin 4\theta, \quad y = \cos 4\theta, \quad 0 \leqslant \theta \leqslant \pi/2$

15. $x = \cos\theta, \quad y = \sec^2\theta, \quad 0 \leqslant \theta < \pi/2$

2)

t	x	y
-2	$\ln 5$	-1
-1	$\ln 2$	$-1/3$
0	0	0
1	$\ln 2$	$1/5$
2	$\ln 5$	$1/3$

$$\frac{-2}{-2+4} = \frac{-2}{2} = -1 \qquad \ln 1 = 0$$

$$\frac{-1}{-1+4} = \frac{-1}{3} \qquad \frac{0}{0+4} = \frac{0}{4} = 0$$

$$\frac{1}{1+5} = 1/5$$

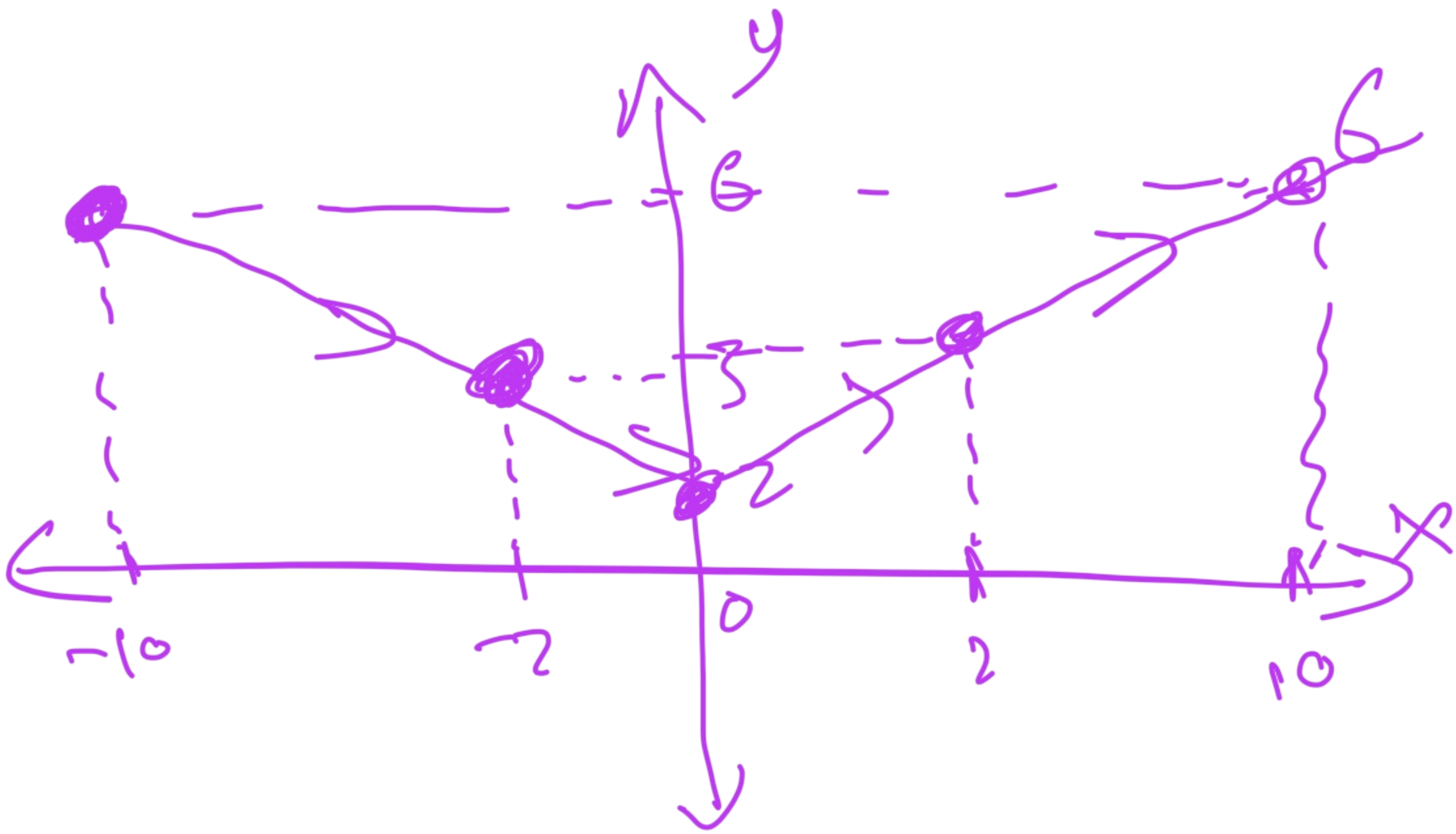
$$\frac{2}{2+4} = \frac{2}{6} = 1/3$$

(4) $x = t^3 + t$

$y = t^2 + 2$

$-2 \le t \le 2$

t	x	y
-2	-10	6
-1	-2	3
0	0	2
1	2	3
2	10	6



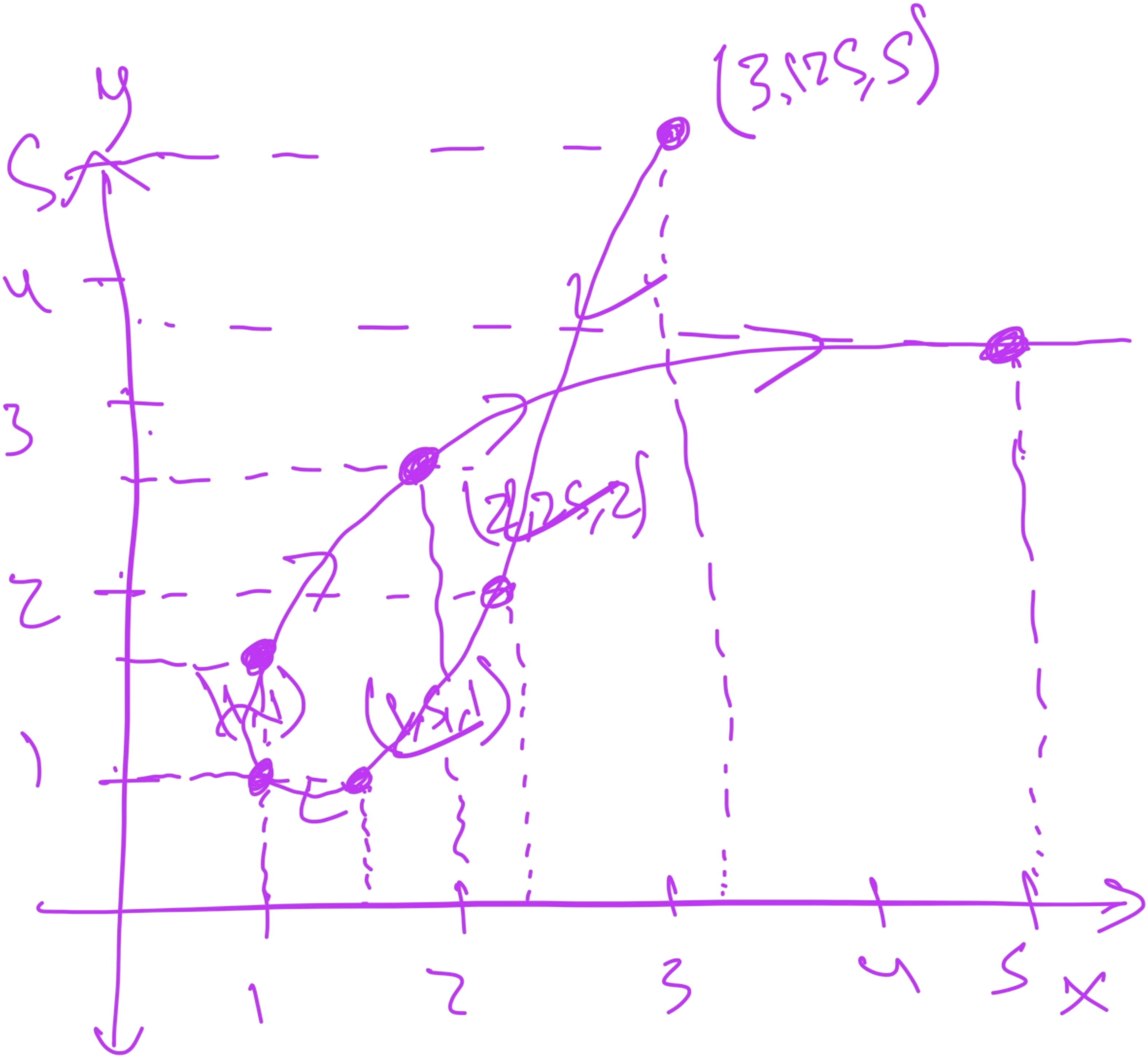
(5)

$$x = 2^t - t$$
$$y = 2^{-t} + t$$

$$-3 \le t \le 3$$

t	x	y
-3	3.125	5
-2	2.25	2
-1	1.5	1
0	1	1
1	1	1.5
2	2	2.25
3	5	3.125

$$x = 2^2 - 2 = 4 - 2 = 2$$


y
x
5
4
3
2
1
1
2
3
4
5
(3.125, 5)
(2.125, 2)
(1, 1)
(1.5, 1)

8) $x = 3t + 2$

$y = 2t + 3$

parametric eq $\rightarrow$ $x = f(t)$
$y = g(t)$

cartesian eq $\rightarrow$ $y = x$

$x = 3t + 2 \rightarrow \frac{\cancel{3}t}{\cancel{3}} = \frac{x-2}{3}$

$$\boxed{t = \frac{x-2}{3}}$$

$y = 2\left[\frac{x-2}{3}\right] + 3$

$= 2\left[\frac{x}{3} - \frac{2}{3}\right] + 3$

$$y = \frac{2}{3}x - \frac{4}{3} + \frac{3 \times 3}{1 \times 3} = 9$$

$$\boxed{y = \frac{2}{3}x + \frac{5}{3}}$$

cant

eg

line equation

10

$$x = \sin t$$

$$y = 1 - \cos t$$

$$0 \le t \le 2\pi$$

$$\boxed{\sin^2 t + \cos^2 t = 1}$$

$$\cos t = 1 - y$$

$$\boxed{x^2 + (1-y)^2 = 1}$$

$[x(y-1)]^2$

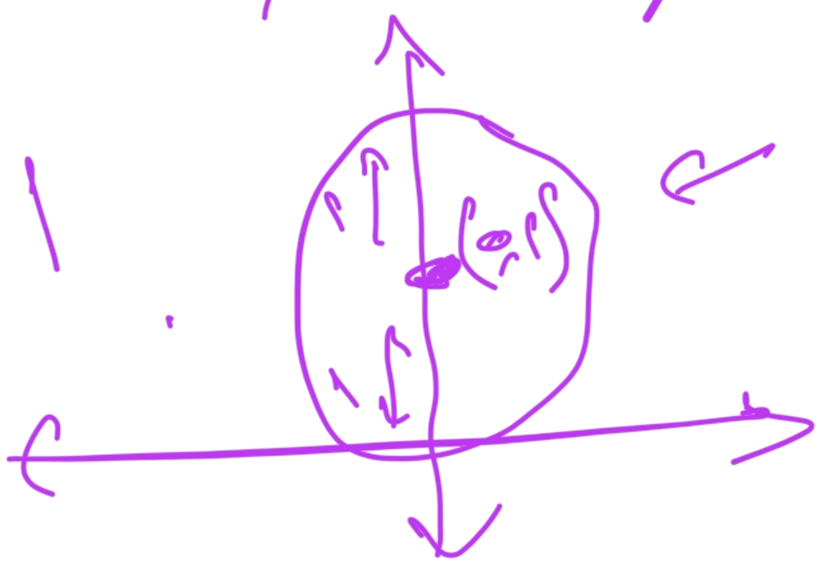
$$\boxed{x^2 + (y - \quad)^2 = \sqrt{\quad}}$$

cart eq

circle equation

center $(0, 1)$

radius $= 1$



(14) $x = \sin(4\theta)$

$y = \cos(4\theta)$ $\quad 0 \le \theta \le \pi/2$

$$\Rightarrow \boxed{\sin^2\theta + \cos^2\theta = 1}$$

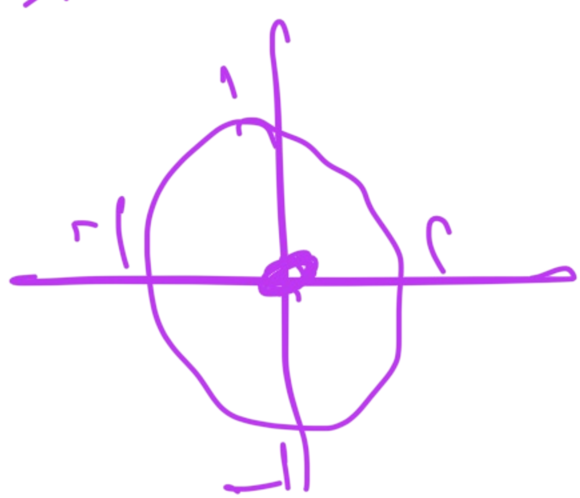
$$\sin^2(4\theta) + \cos^2(4\theta) = 1$$

$$\boxed{x^2 + y^2 = 1}$$

circle eq

center $(0, 0)$

radius $= 1$



16. $x = \csc t, \quad y = \cot t, \quad 0 < t < \pi$

17. $x = e^{-t}, \quad y = e^{t}$

18. $x = t + 2, \quad y = 1/t, \quad t > 0$

19. $x = \ln t, \quad y = \sqrt{t}, \quad t \geqslant 1$

20. $x = |t|, \quad y = \big|1 - |t|\big|$

21. $x = \sin^2 t, \quad y = \cos^2 t$

22. $x = \sinh t, \quad y = \cosh t$

23–24 The position of an object in circular motion is modeled by the given parametric equations, where t is measured in seconds. How long does it take to complete one revolution? Is the motion clockwise or counterclockwise?

23. $x = 5\cos t, \quad y = -5\sin t$

24. $x = 3\sin\left(\dfrac{\pi}{4}t\right), \quad y = 3\cos\left(\dfrac{\pi}{4}t\right)$

25–28 Describe the motion of a particle with position (x, y) as t varies in the given interval.

25. $x = 5 + 2\cos \pi t, \quad y = 3 + 2\sin \pi t, \quad 1 \leqslant t \leqslant 2$

26. $x = 2 + \sin t, \quad y = 1 + 3\cos t, \quad \pi/2 \leqslant t \leqslant 2\pi$

27. $x = 5\sin t, \quad y = 2\cos t, \quad -\pi \leqslant t \leqslant 5\pi$

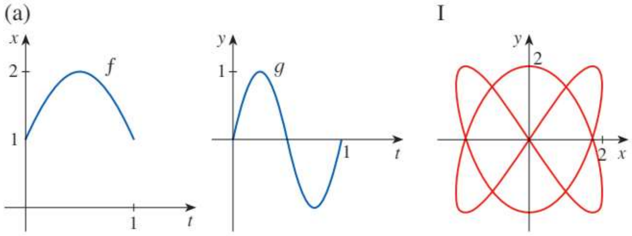
28. $x = \sin t, \quad y = \cos^2 t, \quad -2\pi \leqslant t \leqslant 2\pi$

---

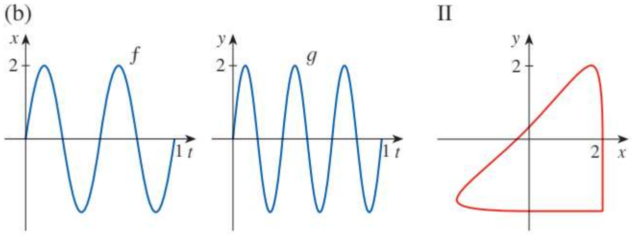
29. Suppose a curve is given by the parametric equations $x = f(t)$, $y = g(t)$, where the range of f is $[1, 4]$ and the range of g is $[2, 3]$. What can you say about the curve?

30. Match each pair of graphs of equations $x = f(t)$, $y = g(t)$ in (a)–(d) with one of the parametric curves $x = f(t)$, $y = g(t)$ labeled I–IV. Give reasons for your choices.

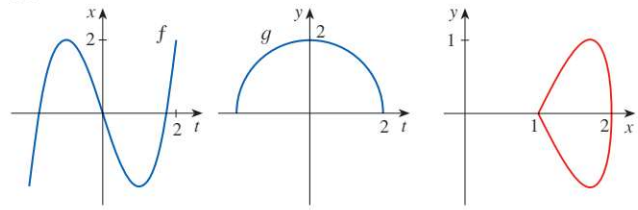
(a) I



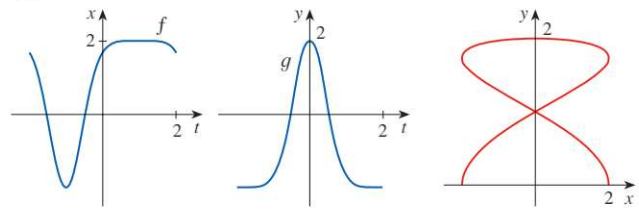
(b) II



(c) III

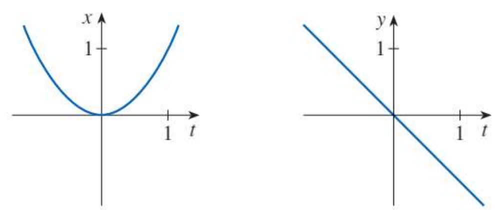


(d) IV

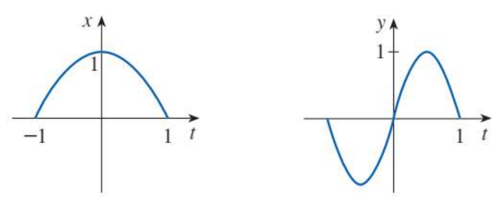


31–33 Use the graphs of $x = f(t)$ and $y = g(t)$ to sketch the parametric curve $x = f(t)$, $y = g(t)$. Indicate with arrows the direction in which the curve is traced as t increases.

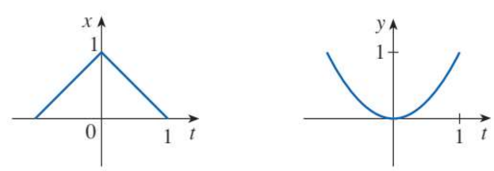
31.



32.



33.



---

34. Match the parametric equations with the graphs labeled I–VI. Give reasons for your choices.

(a) $x = t^4 - t + 1, \quad y = t^2$

(b) $x = t^2 - 2t, \quad y = \sqrt{t}$

(c) $x = t^3 - 2t, \quad y = t^2 - t$

(d) $x = \cos 5t, \quad y = \sin 2t$

(e) $x = t + \sin 4t, \quad y = t^2 + \cos 3t$

16. $x = \csc t \qquad y = \cot t$

$$0 \le t \le \pi$$

$$\Rightarrow \boxed{\frac{\cancel{\sin^2 t}}{\cancel{\sin^2 t}} + \frac{\cos^2 t}{\sin^2 t} = \frac{1}{\sin^2 t}}$$

$\therefore$

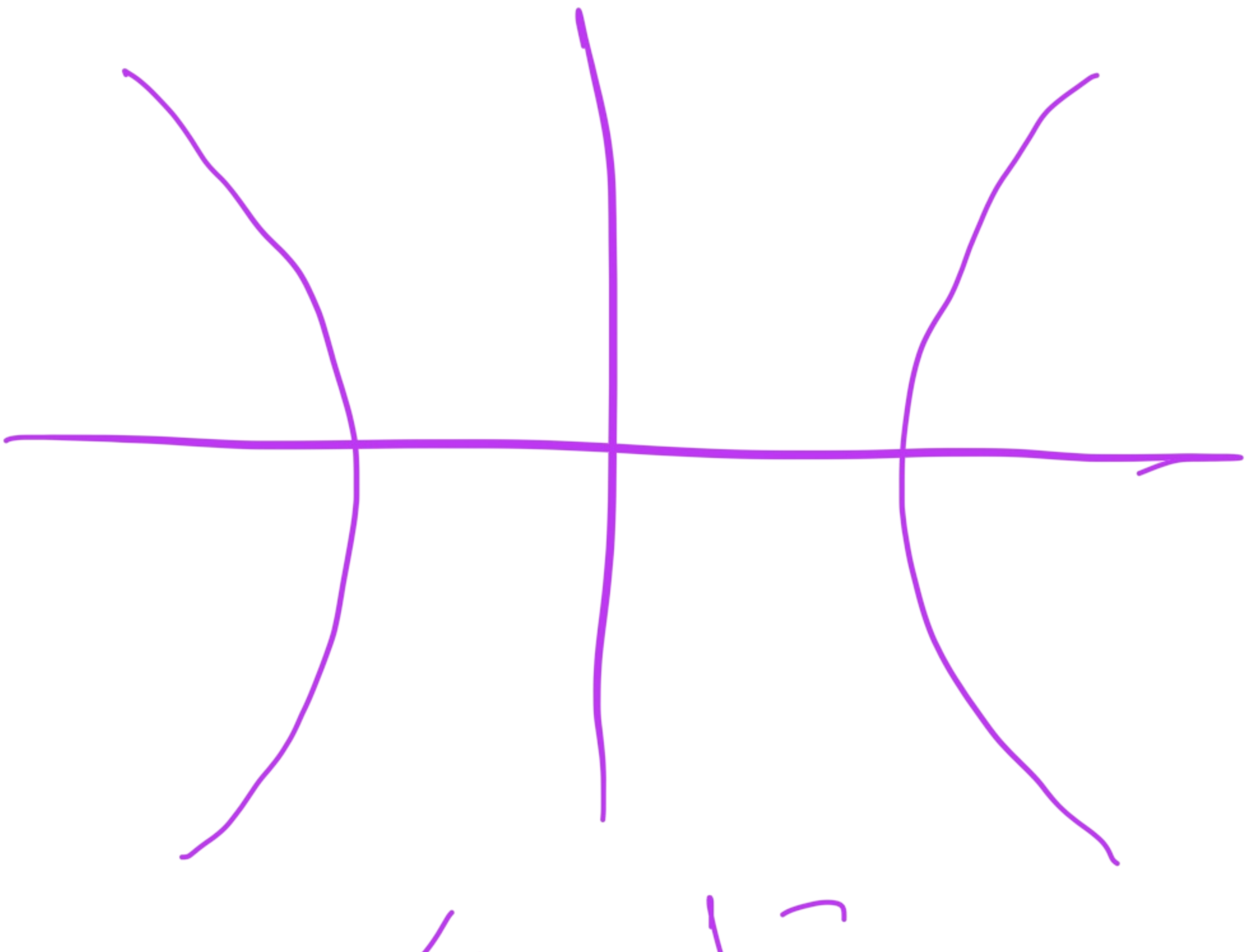
$$\boxed{1 + \cot^2 t = \csc^2 t}$$

$$\boxed{\frac{1}{\sin t} = \csc t} \qquad \boxed{\frac{1}{\cos t} = \sec t}$$

$$\boxed{\frac{\sin t}{\cos t} = \tan t} \qquad \boxed{\frac{\cos t}{\sin t} = \cot t}$$

$$\boxed{1 + y^2 = x^2}$$

↳ hyperbola



قطع زائد

(26) $x = 2 + \sin t$

$y = 1 + 3\cos t$

$$\pi/2 \le t \le 2\pi$$

$$\sin t = x - 2$$

$$\frac{\cancel{3}\cos t}{\cancel{3}} = \frac{y-1}{3}$$

$$\sin^2 t + \cos^2 t = 1$$

$$\boxed{(x-2)^2 + \left(\frac{y-1}{3}\right)^2 = 1}$$

$$\boxed{(x-2)^2 + \frac{(y-1)^2}{9} = 1}$$

Circle

$$(x-a)^2 + (y-b)^2 = r^2$$

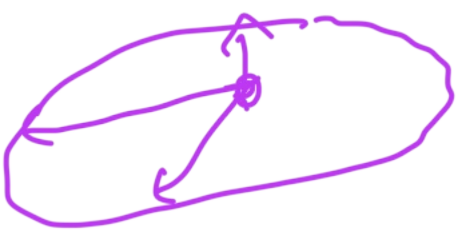
center (a, b)

radius $= r$

ellipse

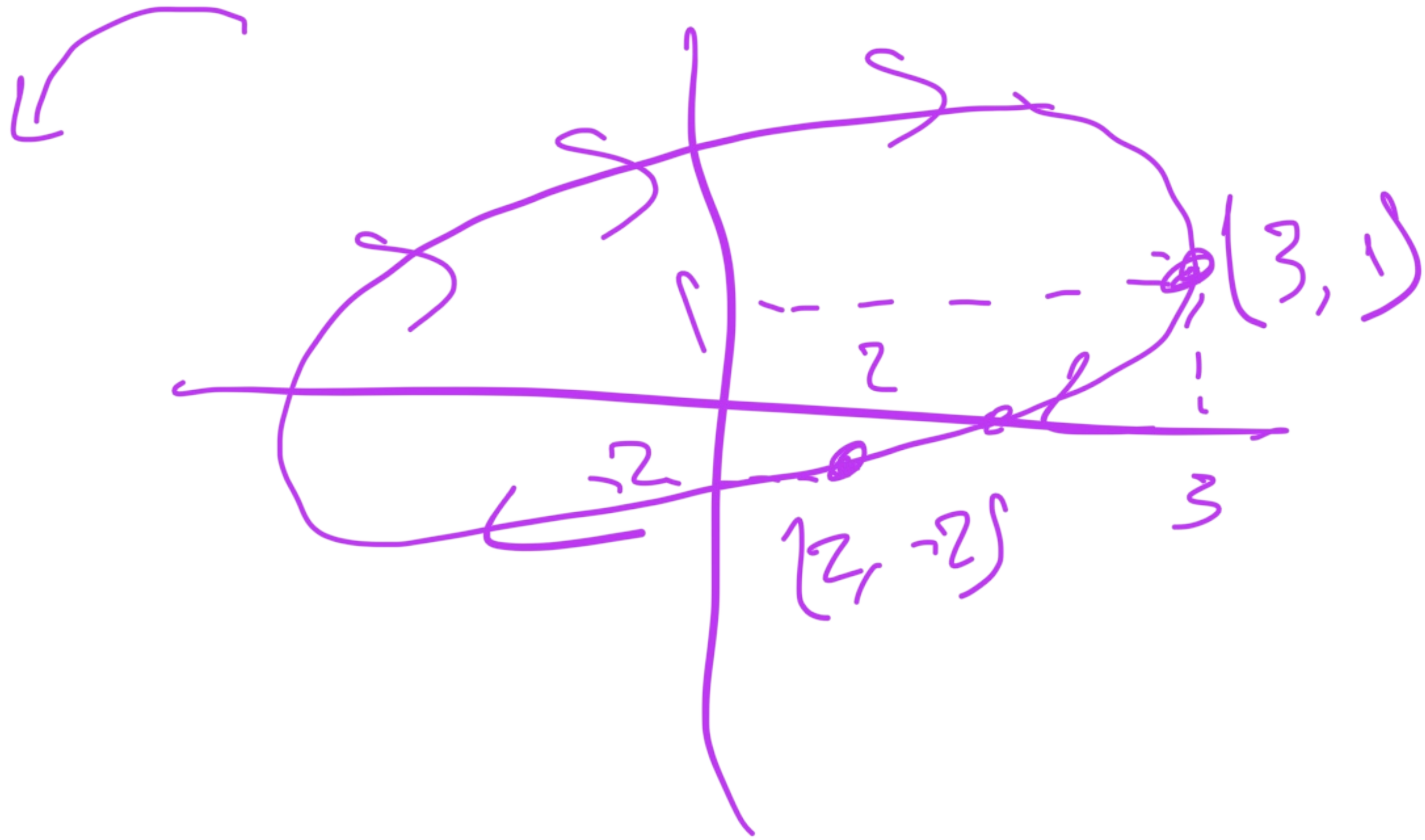
$$\frac{(x-a)^2}{m} + \frac{(y-b)^2}{k} = 1$$

[illegible]



$$(x-2)^2 + \frac{(y-1)^2}{9} = 1$$

center $(2, 1)$



$$t = \pi/2 \longrightarrow x = 2 + \overset{1}{\sin \pi/2} =$$

$$x = 3$$

$$y = 1 + 3\overset{0}{\cancel{\cos}}\, \pi/2 = 1$$

$$y = 1$$

$$t = \pi/2 \longrightarrow (3, 1)$$

$t = \pi$

$x = 2 + 5\cancel{\sin \pi} = 2$

$y = 1 + 3\underbrace{\cos \pi}_{-1} = -2$

$t = \pi \qquad (2, -2)$

27

$x = 5 \sin t$

$y = 2 \cos t$

$-\pi \le t \le 5\pi$

$\sin t = \dfrac{x}{5}$

$\cos t = y/2$

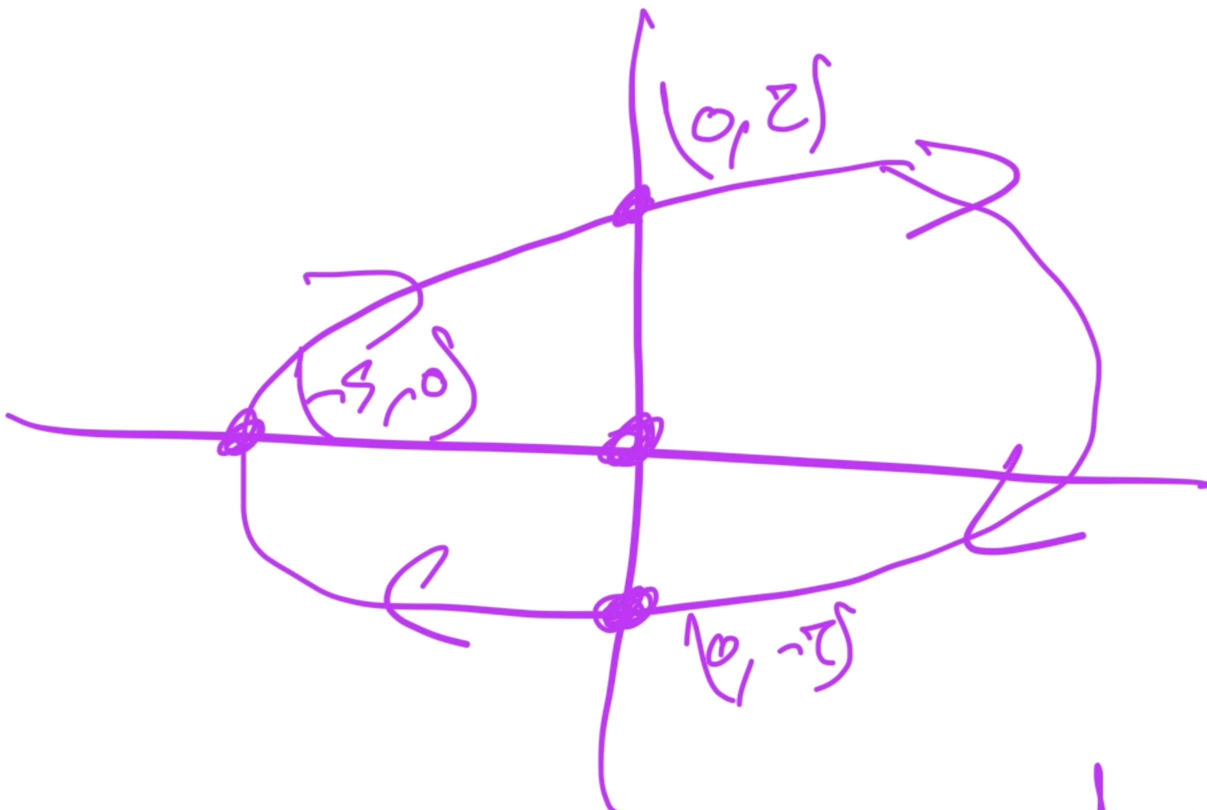
$\sin^2 + \cos^2 = 1$

$$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$$

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

ellipse

center $(0,0)$



clockwise

$t = -\pi \longrightarrow x = 0, \; y = -2$

$t = 0 \longrightarrow x = 0, \; y = 2$

$t = -\pi/2 \longrightarrow x = -5, \; y = 0$