[من إلى]

The Velocity Problem

If you watch the speedometer of a car as you drive in city traffic, you see that the speed doesn't stay the same for very long; that is, the velocity of the car is not constant. We assume from watching the speedometer that the car has a definite velocity at each moment, but how is the "instantaneous" velocity defined?

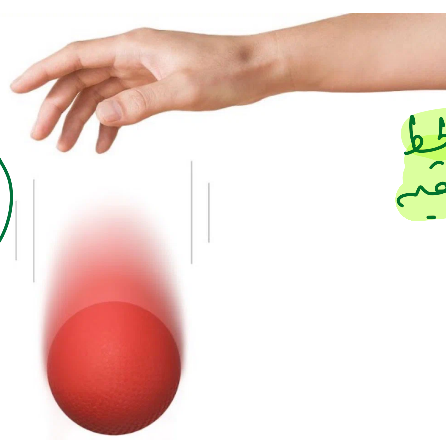
Let's consider the *velocity problem:* Find the instantaneous velocity of an object moving along a straight path at a specific time if the position of the object at any time is known. In the next example, we investigate the velocity of a falling ball. Through experiments carried out four centuries ago, Galileo discovered that the distance fallen by any freely falling body is proportional to the square of the time it has been falling. (This model for free fall neglects air resistance.) If the distance fallen after t seconds is denoted by $s(t)$ and measured in meters, then (at the earth's surface) Galileo's observation is expressed by the equation

$$s(t) = 4.9t^2$$

التغير في المسافة

$$\text{Average velocity} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

التغير في الزمن



ميل الخط المستقيم $= \frac{y_2 - y_1}{x_2 - x_1}$

معادلة ← ازاحة او position distance

t_1 t_2

الزمن

y_1 y_2

average velocity = $\dfrac{\text{change in position}}{\text{time elapsed}}$

$y_2 - y_1$

المسافة

$\dfrac{\text{التغير المكاني}}{\text{التغير الزمني}}$

$t_2 - t_1$

or $s(t_2) - s(t_1)$

EXAMPLE 3 Suppose that a ball is dropped from the upper observation deck of the CN Tower in Toronto, 450 m above the ground. Find the velocity of the ball after 5 seconds.

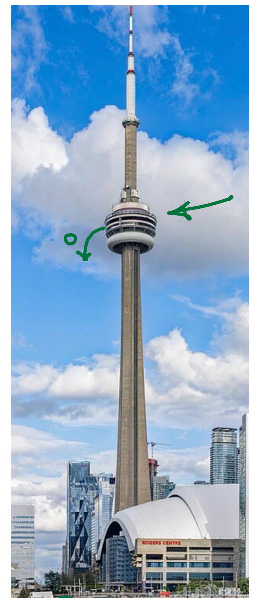
$s(t) = 4.9t^2$ ✓

$t_1 = 5\ \text{sec}$ $\qquad$ $t_2 = 5.1$

Distance $s(t)$

$s(t_1) = 4.9(5)^2$ $\qquad$ $s(t_2) = 4.9(5.1)^2$

$=$ ✓ $\qquad$ $=$ ✓



Average velocity = $\dfrac{\text{change in position}}{\text{Time elapsed}}$

$= \dfrac{s(t_2) - s(t_1)}{t_2 - t_1} = \dfrac{4.9(5.1)^2 - 4.9(5)^2}{5.1 - 5} = 49.49$ m/sec

Time interval	Average velocity (m/s)
$5 \leqslant t \leqslant 5.1$	49.49
$5 \leqslant t \leqslant 5.05$	49.245
$5 \leqslant t \leqslant 5.01$	49.049
$5 \leqslant t \leqslant 5.001$	$\boxed{49.0049}$ accurate

كلما اقتربت من 5 كلما زادت الدقة

Avg velocity $t = 5$ ← 49 m/sec

6. If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height in meters t seconds later is given by $\boxed{y = 10t - 1.86t^2.}$ معادلة ع x,y

(a) Find the average velocity over the given time intervals:

t_1 2

(i) [1, 2] (ii) [1, 1.5] (iii) [1, 1.1]

(iv) [1, 1.01] (v) [1, 1.001]

(b) Estimate the instantaneous velocity when $t = 1$. اقترب اللاشاد

$$\text{Average velocity} = \frac{\text{change in position}}{\text{Time elapsed}}.$$

$$\text{Avg } v = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

t_1 t_2

(i) [1, 2] $= \dfrac{\left(10(2) - 1.86(2)^2\right) - \left(10(1) - 1.86(1)^2\right)}{2 - 1}$

$\underset{[1,2]}{\text{Avg } v} = 4.42 \text{ m/Sec}$

t_1 t_2

(ii) [1, 1.5] $\Rightarrow \dfrac{\left(10(1.5) - 1.86(1.5)^2 - \left(10(1) - 1.86(1)^2\right)\right.}{1.5 - 1}$

[1, 1.01] (v) [1, 1.001] $= 3.5 \text{ m/Sec}$

$s(t_2)$

t_1 t_2

(iii) [1, 1.1] $\rightarrow$ $\text{Avg } v = \dfrac{\left(10(1.1) - 1.86(1.1)^2\right) - \left(10(1) - 1.86(1)^2\right)}{1.1 - 1}$

$= 6.094 \text{ m/Se}$

(iv) [1, 1.01]

Avg $v = 6.2614$ m/sec

(v) [1, 1.001]

$$\text{Avg } v = \frac{\left(10(1.001) - 1.86(1.001)^2\right) - \left(10(1) - 1.86(1)^2\right)}{1.001 - 1}$$

$$= 6.27814 \text{ m/sec}$$

Instanteneous Velocity

السرعة اللحظية

في لحظة معينة

Avg velocity

السرعة المتوسطة

المحجة

فترة معينة

[✓ ✓]

Instanteneous

اللحظية

$t = \checkmark$

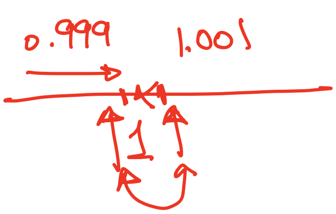
تم استنتاجها من

السرعة المتوسطة

Instanteneous velocity at $t=1$ → when time elapsed → 0

$t \to 1$

$V = 6.28$ m/sec



8. The displacement (in centimeters) of a particle moving back and forth along a straight line is given by the equation of motion $s = 2 \sin \pi t + 3 \cos \pi t$, where t is measured in seconds.

(a) Find the average velocity during each time period:

H.W (i) $[1, 2]$ → Time elapsed $t_2 - t_1 = 2 - 1 = 1$ (ii) $[1, 1.1]$

(iii) $[1, 1.01]$ (iv) $[1, 1.001]$

(b) Estimate the instantaneous velocity of the particle when $t = 1$.

$[1, 2]$

$[1, 1.1]$

$t_1 \quad t_2$

$[1, 1.01] \rightarrow$ Avg velocity $= \dfrac{s(t_2) - s(t_1)}{t_2 - t_1}$

$s = 2 \sin \pi t + 3 \cos \pi t,$

$$\text{Avg } v = \frac{\overbrace{(2 \sin \pi * 1.01 + 3 \cos \pi * 1.01)}^{s(t_1)} - (2 \sin \pi * 1 + 3 \cos \pi * 1)}{1.01 - 1}$$

$= -6.13$ cm/sec.

$t_1 \quad t_2$

$[1, 1.001] \rightarrow$ Avg $v = \dfrac{s(t_2) - s(t_1)}{t_2 - t_1}$

$$= \frac{(2 \sin \pi * 1.001 + 3 \cos \pi * 1.001) - (2 \sin \pi * 1 + 3 \cos \pi * 1)}{1.001 - 1}$$

$= -6.27$ cm/sec

The Instanteneous velocity at $t = 1$ when Time elapsed $\rightarrow 0$ $= -6.3$ cm/sec

$h \rightarrow 0$

5. The deck of a bridge is suspended 80 meters above a river. If a pebble falls off the side of the bridge, the height, in meters, of the pebble above the water surface after t seconds is given by $\boxed{y = 80 - 4.9t^2}$.

(a) Find the average velocity of the pebble for the time period beginning when $\boxed{t = 4}$ and lasting

(i) 0.1 seconds (ii) 0.05 seconds (iii) 0.01 seconds

(b) Estimate the instantaneous velocity of the pebble after 4 seconds.

$t_2 - t_1$

(i) Time elapsed $= \boxed{0.1 \text{ sec}} \rightarrow t_1 = 4$

$t_2 = \boxed{4.1}$ 0.1

$$\text{Avg velocity} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

$$= \frac{80 - 4.9(4.1)^2 - \left(80 - 4.9(4)^2\right)}{0.1}$$

$$= -39.69 \text{ ft/sec}$$

ii) Time elapsed $= 0.05$ $t_1 = 4$

$t_2 = 4.05$

$$\text{Avg velocity} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

$$= \frac{\left(80 - 4.9(4.05)^2\right) - \left(80 - 4.9(4)^2\right)}{0.05}$$

$$= -39.445 \text{ ft/sec.}$$

iii) Time elapsed = 0.01

$t_1 = 4$

$t_2 = 4.01$

$$\text{Avg velocity} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

$$= \frac{\left(80 - 4.9(4.01)^2\right) - \left(80 - 4.9(4)^2\right)}{0.01}$$

$$= -39.249 \text{ ft/sec}$$

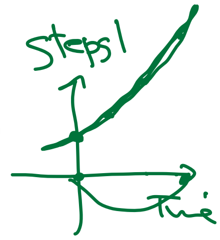
b) instanteneous velocity at $t = 4$ when Time elapsed approach 0 is -39 ft/sec

2. A student bought a smartwatch that tracks the number of steps she walks throughout the day. The table shows the number of steps recorded t minutes after 3:00 PM on the first day she wore the watch.

3:00 3:10 3:20 3:30 3:40

	t (min)	0	10	20	30	40
x	Steps	3438	4559	5622	6536	7398

y



(a) Find the slopes of the secant lines corresponding to the given intervals of t. What do these slopes represent?

(i) [0, 40] (ii) [10, 20] (iii) [20, 30]

(b) Estimate the student's walking pace, in steps per minute, at 3:20 PM by averaging the slopes of two secant lines.

a] $\text{slope} = \dfrac{\text{Change in steps}}{\text{Time ellapsed}}$

i) $[0, 40] \longrightarrow \text{slope} = \dfrac{7398 - 3438}{40 - 0} = 99$

ii) $[10, 20] \longrightarrow \text{slope} = \dfrac{5622 - 4559}{20 - 10} = 106.3$

iii) $[20, 30] \longrightarrow \text{slope} = \dfrac{6536 - 5622}{30 - 20} = 91.4$ step/

3:20

[10, 20] [20, 30]

slope

$\left(\dfrac{106.3 + 91.4}{2}\right) = 98.85$ steps/min.