Circuit II

Chapter 9

Sinusoidal Steady-State Analysis

Part 2

We can transform a sinusoid from sine form to cosine form or vice versa.

$$\sin(\omega t \pm 180^\circ) = -\sin\omega t$$
$$\cos(\omega t \pm 180^\circ) = -\cos\omega t$$
$$\sin(\omega t \pm 90^\circ) = \pm\cos\omega t$$
$$\cos(\omega t \pm 90^\circ) = \mp\sin\omega t$$

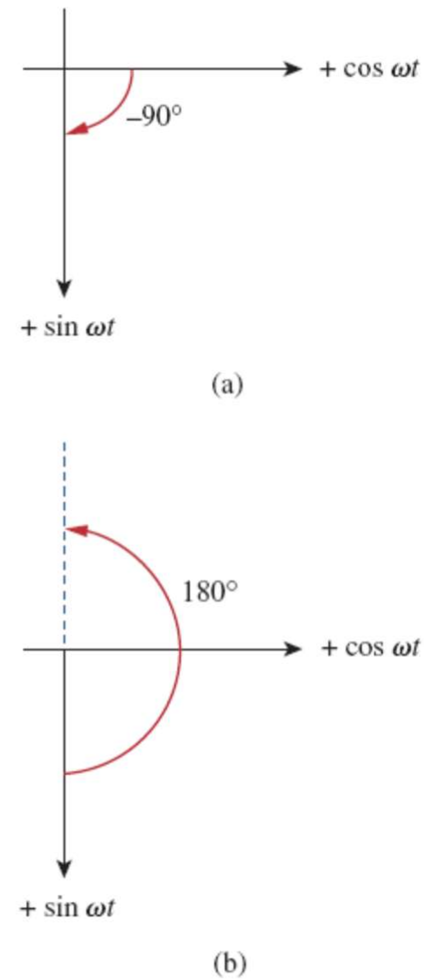


Figure 9.3
A graphical means of relating cosine and sine: (a) $\cos(\omega t - 90^\circ) = \sin\omega t$, (b) $\sin(\omega t + 180^\circ) = -\sin\omega t$.

$-\sin(\omega t) \longrightarrow \boxed{+}\sin(\omega t \pm 180)$ $\boxed{1}$

$-\cos(\omega t) \longrightarrow \boxed{+}\cos(\omega t \pm 180)$

لـ $\pm$ لا تفرق لكن الأهم انه الزاوية

نضل أقل من $\longleftarrow$ 180

Ex $-\sin(\omega t + 40)$

$40 + 180 \rightarrow 220$

$40 - 180 \rightarrow -140$

$+\sin(\omega t + 40 \pm 180)$

$\boxed{+\sin(\omega t - 140)}$

± 180

$$-\sin(\omega t) \longrightarrow \boxed{+}\cos(\omega t + 90)$$

$$+\sin(\omega t) \longrightarrow \boxed{+}\cos(\omega t - 90)$$

$$+\cos(\omega t) \longrightarrow \boxed{+}\sin(\omega t + 90)$$

$$-\cos(\omega t) \longrightarrow \boxed{+}\sin(\omega t - 90)$$

9.2 Sinusoids (4)

Example 3

Find the phase angle between $i_1 = -4\sin(377t + 25^o)$ and $i_2 = 5\cos(377t - 40^o)$, does i_1 lead or lag i_2?

Solution:

sin

Since sin(ωt+90°) = cos ωt

$$i_2 = 5\sin(377t - 40^o + 90^o) = 5\sin(377t + 50^o)$$

$$i_1 = -4\sin(377t + 25^o) = 4\sin(377t + 180^o + 25^o) = 4\sin(377t + 205^o)$$

θ_2 θ_1

therefore, i_1 leads i_2 155°.

$\theta_1 > \theta_2$

$205 - 50 = 155$

ملاحظات للتفريق بين lead و lag

1 أن يكونو كلاهما موجبين "منيرين"

2 أن يكونو كلاهما sin أو cos

3 أن يكونو كلاهما لهما نفس w الجمع t

$$i_1 = -4\sin(377t + 25) \quad ??$$

$$i_2 = +5\cos(377t - 40)$$

$$-4\sin(377t + 25)$$

$$\Rightarrow +4\cos(377t + 25 + 90)$$

$$i_1 = 4\cos(377t + 115) \leftarrow \theta_1$$

$$i_2 = 5\cos(377t - 40) \quad \theta_2$$

lead ⟶ "القائد" "متقدم" ⟵

lag ⟶ "متأخر" "الأخير"

$\theta_1 = 115$ $\qquad\qquad$ $\theta_1 > \theta_2$

$\theta_2 = -40$ $\qquad\qquad$ الأخير ⟸ القائد

c_1 lead c_2 $\qquad$ " c_1 : متقدم على c_2 "

c_2 lag c_1 $\qquad$ " c_2 متأخر عن c_1 "

i_1' lead i_2'

$$\theta_1 - \theta_2 = 115 \overset{+}{-} (-40)$$

$$115 + 40 = 155$$

$$\boxed{i_1' \text{ lead } i_2' \text{ by } 155}$$

Example 4

Calculate the phase angle between $v_1 = -10\cos(\omega t + 50^\circ)$ and $v_2 = 12\sin(\omega t - 10^\circ)$. State which sinusoid is leading.

METHOD 1 In order to compare v_1 and v_2, we must express them in the same form. If we express them in cosine form with positive amplitudes,

$$v_1 = -10\cos(\omega t + 50^\circ) = 10\cos(\omega t + 50^\circ - 180^\circ)$$

$$v_1 = 10\cos(\omega t - 130^\circ) \quad \text{or} \quad v_1 = 10\cos(\omega t + 230^\circ) \qquad \textbf{(9.2.1)}$$

$\to 230$ $\quad +180$

and

$$v_2 = 12\sin(\omega t - 10^\circ) = 12\cos(\omega t - 10^\circ - 90^\circ)$$

$$v_2 = 12\cos(\omega t - 100^\circ) \qquad \textbf{(9.2.2)}$$

It can be deduced from Eqs. (9.2.1) and (9.2.2) that the phase difference between v_1 and v_2 is 30°. We can write v_2 as

$$v_2 = 12\cos(\omega t - 130^\circ + 30^\circ) \quad \text{or} \quad v_2 = 12\cos(\omega t + 260^\circ) \qquad \textbf{(9.2.3)}$$

Comparing Eqs. (9.2.1) and (9.2.3) shows clearly that $\boxed{v_2 \text{ leads } v_1 \text{ by } 30^\circ}$.

$$V_1 = -10\cos(\omega t + 50)$$

$$\hookrightarrow +10\sin(\omega t + 50 - 90)$$

$$V_2 = +12\sin(\omega t - 10)$$

θ_1

$$V_1 = +10\sin(\omega t - 40)$$ ✓ ✓ ✓

$$V_2 = +12\sin(\omega t - 10)$$ ✓ ✓ ✓

θ_2

$\theta_1 = -40$

$\theta_2 = \underline{\underline{-10}}$

$$\boxed{\begin{array}{c}\theta_2 \\ \text{القائد}\end{array}} > \theta_1$$

$$\theta_2 - \theta_1 = -40 - (-10)$$

$+$

$$-40 + 10 = \cancel{-}\ \underline{\underline{30}}$$

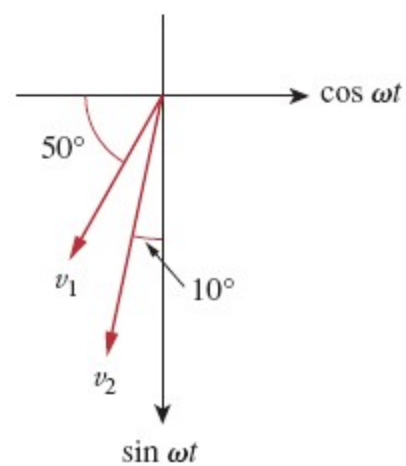
$$\boxed{V_2 \text{ lead } V_1 \text{ by } 30}$$

METHOD 2 Alternatively, we may express v_1 in sine form:

$$\begin{aligned} v_1 &= -10\cos(\omega t + 50^\circ) = 10\sin(\omega t + 50^\circ - 90^\circ) \\ &= 10\sin(\omega t - 40^\circ) = 10\sin(\omega t - 10^\circ - 30^\circ) \end{aligned}$$

But $v_2 = 12\sin(\omega t - 10^\circ)$. Comparing the two shows that v_1 lags v_2 by 30°. This is the same as saying that v_2 leads v_1 by 30°.

METHOD 3 We may regard v_1 as simply $-10\cos\omega t$ with a phase shift of $+50^\circ$. Hence, v_1 is as shown in Fig. 9.5. Similarly, v_2 is $12\sin\omega t$ with a phase shift of -10°, as shown in Fig. 9.5. It is easy to see from Fig. 9.5 that v_2 leads v_1 by 30°, that is, $90^\circ - 50^\circ - 10^\circ$.



Sinusoid-phasor transformation

Time domain representation		Phasor domain representation
$V_m \cos(\omega t + \phi)$	$\longrightarrow$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$\rightarrow \cos(\omega t - 90) \longrightarrow$	$V_m \angle \phi - 90°$
$I_m \cos(\omega t + \theta)$	$\longrightarrow$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$\rightarrow \cos(\omega t - 90) \longrightarrow$	$I_m \angle \theta - 90°$

$\boxed{\text{الأساس } \cos}$

لو عندك sin حول إل cos

9.3 Phasor (8)

Relationship between differential, integral operation in phasor listed as follow:

$$v(t) \longleftrightarrow \mathbf{V} = V\angle\phi$$

$$\frac{dv}{dt} \longleftrightarrow j\omega\mathbf{V}$$

$$\int v\,dt \longleftrightarrow \frac{\mathbf{V}}{j\omega}$$

$j = \sqrt{-1}$

[illegible] في $\boxed{\cos}$

9.3 Phasor (9)

Example 8

Use phasor approach, determine the current *i(t)* in a circuit described by the integro-differential equation.

$$4i + 8\int i\,dt - 3\frac{di}{dt} = 50\boxed{\cos(2t + 75°)}$$

ω = 2 rad/s

Answer: $i(t) = 4.642\cos(2t + 143.2^o)$ A

بكتب في $\boxed{j2}$

$2j \rightarrow$ [illegible]

$$4I + \boxed{\frac{8I}{j2}} - 3I(j2) =$$

$$50\angle 75$$

$$4\underline{\underline{I}} + -j4\underline{\underline{I}} - j6\underline{\underline{I}} = 50\angle 75$$

$$I[4 - j4 - j6] = 50\angle 75$$

$$\frac{I\,\cancel{[4 - j10]}}{\cancel{4 - j10}} = \frac{50\angle 75}{4 - j10}$$

حيث قسمت

$$\boxed{I = 4{,}642\angle 143{,}199 \quad A}$$

$$\boxed{i(t) = 4{,}642\cos(2t + 143{,}199^\circ)\,A}$$

Solution:

We transform each term in the equation from time domain to phasor domain. Keeping Eqs. (9.27) and (9.28) in mind, we obtain the phasor form of the given equation as

$$4\mathbf{I} + \frac{8\mathbf{I}}{j\omega} - 3j\omega\mathbf{I} = 50\underline{/75^\circ}$$

But $\omega = 2$, so

$$\mathbf{I}(4 - j4 - j6) = 50\underline{/75^\circ}$$

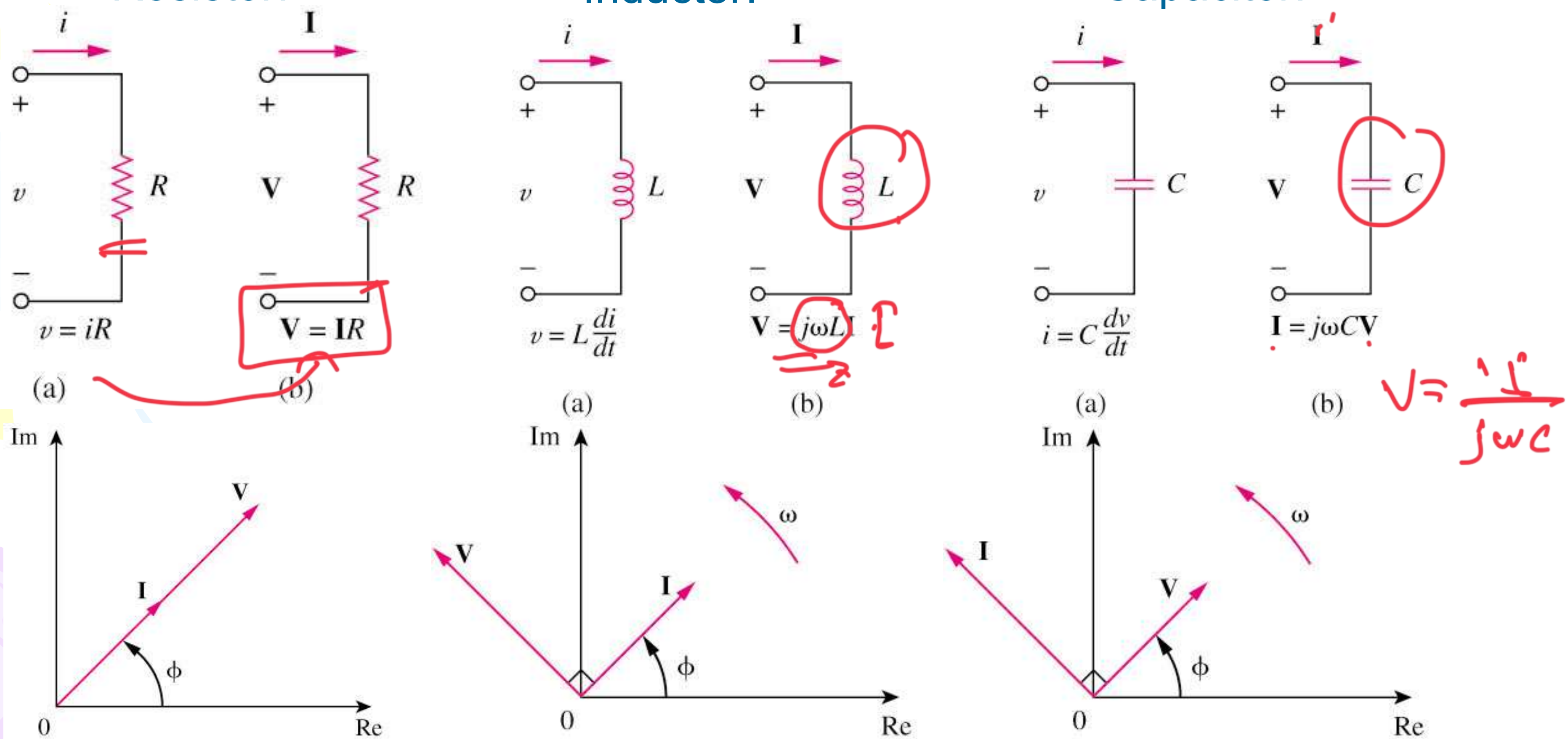
$$\mathbf{I} = \frac{50\underline{/75^\circ}}{4 - j10} = \frac{50\underline{/75^\circ}}{10.77\underline{/-68.2^\circ}} = 4.642\underline{/143.2^\circ}\text{ A}$$

Converting this to the time domain,

$$i(t) = 4.642\cos(2t + 143.2^\circ)\text{ A}$$

9.4 Phasor Relationships for Circuit Elements (1)

Resistor: Inductor: Capacitor:



حفظ مهم جزء ١ جزء ١

9.5 Impedance and Admittance (2)

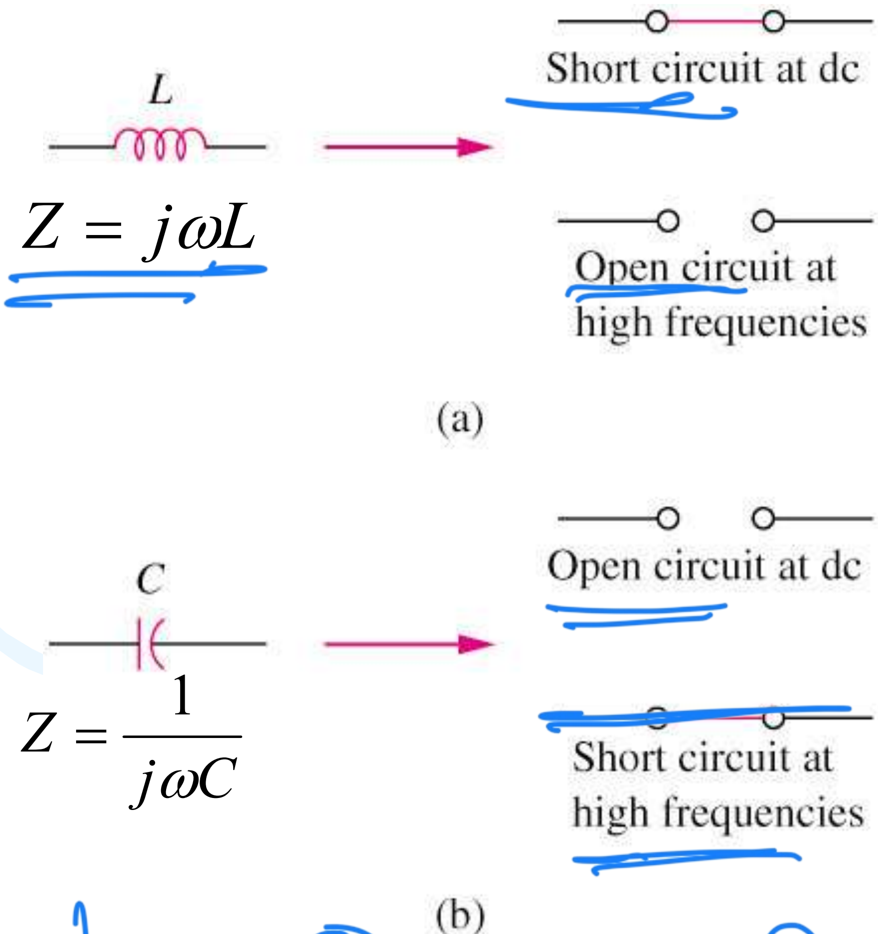
Impedances and admittances of passive elements		
Element	**Impedance**	**Admittance**
R	$Z = R$	$Y = \frac{1}{R}$
L	$Z = j\omega L$	$Y = \frac{1}{j\omega L}$
C	$Z = \frac{1}{j\omega C}$	$Y = j\omega C$

$Y = \frac{1}{Z}$

$w = 0 \Rightarrow$ DC c→

$w = \infty \Rightarrow$ AC

9.5 Impedance and Admittance (3)



$$\omega = 0; Z = 0$$

$$\omega \rightarrow \infty; \boxed{Z \rightarrow \infty}$$

$$\omega = 0; \boxed{Z \rightarrow \infty}$$

$$\omega \rightarrow \infty; \boxed{Z = 0}$$

$\frac{1}{\infty} = 0$

$R = 0 \Rightarrow$ short

$R = \infty \Rightarrow$ open

9.6 Kirchhoff's Laws in the Frequency Domain (1)

- Both KVL and KCL are hold in the phasor domain or more commonly called frequency domain.

$j = \sqrt{-1}$

- Moreover, the variables to be handled are phasors, which are complex numbers.

- All the mathematical operations involved are now in complex domain.