King Faisal University, Saudi Arabia

Department of Mechanical Engineering, College of Engineering

ME-441 CONTROL SYSTEMS

Year 1446 (2025) – 2nd Semester – SPRING 2025

Quiz – 4 Submission Deadline: **May 8, 2025** Total Marks: **10**

Name: **KFU ID:** **Section:**

Q.No.1: The transfer function of a complex control system is given below.

$$T(s) = \frac{30s^3 + 3s^2 + s}{s^6 + 8s^5 + 30s^4 + 72s^3 + 129s^2 + 160s + 100}$$

a) Observe the system's stability according to coefficient tests and mention your findings.

b) Design the Routh array for stability of this system with Routh Hurwitz stability criterion.

c) Mention the number of characteristic roots on all significant locations of s-plane.

d) Specify this system as stable, unstable, or marginally stable and mention the reason for your claim.

a) All of the same algebraic sign / none zero ⟹ No information about root location.

b) $s^6+8s^5+30s^4+72s^3+129s^2+160s+100$

s^6	1	30	129	100
s^5	8	72	160	0
s^4	21	109	100	0
s^3	$\frac{640}{21}$	$\frac{2560}{21}$	0	0
s^2	25	100	0	0
s^1	~~0~~ 50	0	0	0
s^0	100			

$$\frac{8\times 320 - 72\times 1}{8} \qquad \frac{8\times 120 - 160\times 1}{8}$$

$$\frac{21\times 72 - 109\times 8}{21} \qquad \frac{21\times 160 - 100\times 8}{21}$$

$$\frac{\frac{640}{21}\times 109 - \frac{2560\times 21}{21}}{\frac{640}{21}}$$

$$\frac{25\times \frac{2560}{21} - 100\times \frac{640}{21}}{25}$$

$$\boxed{P(s) = 25s^2 + 100}$$

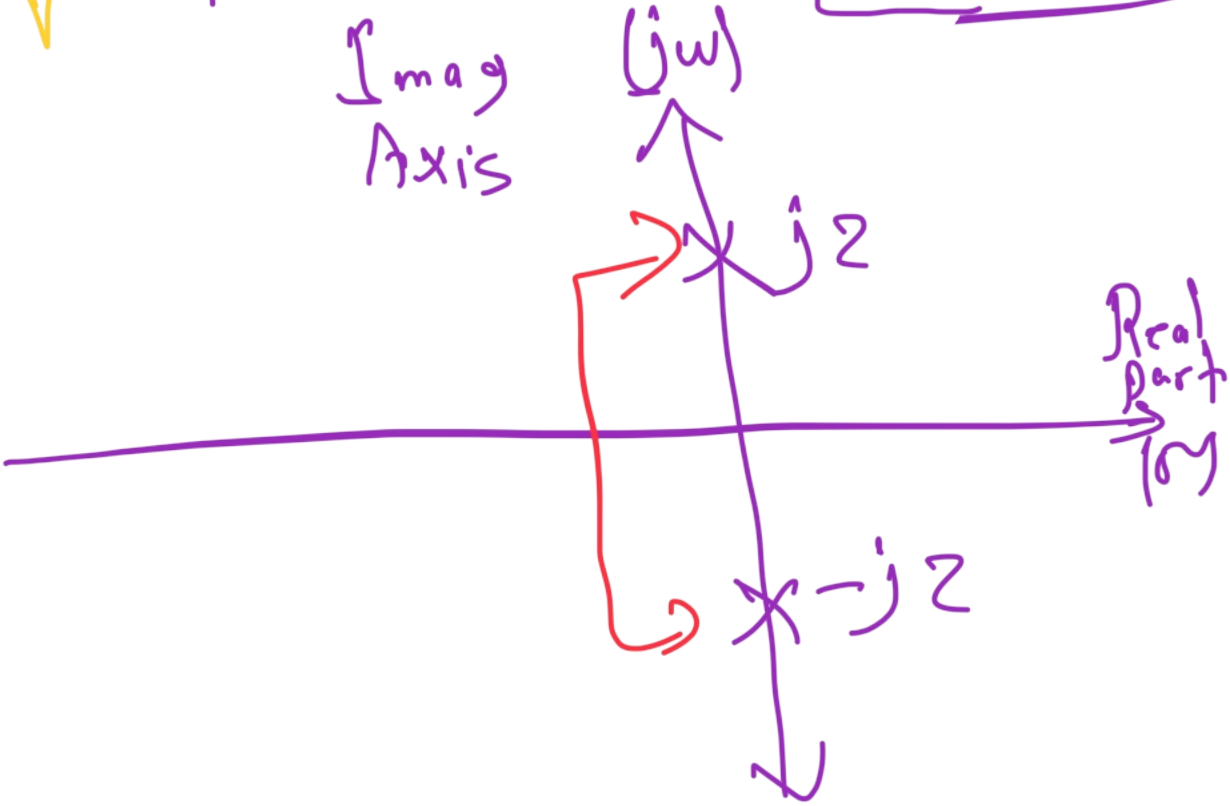
Imag Axis

$$\frac{dP(s)}{ds} = 50s$$

c) $p(s) = 25s^2 + 100 = 0$

$25s^2 = -100$

$\sqrt{s^2} = \sqrt{-4} \Longrightarrow$ $\boxed{s = \mp j2}$



d) No change in sign 1st column but we have a row zeros

$\Longrightarrow$ marginally stable

There 6 poles

- 2 poles on Imag Axis
- 4 poles on LHP

So the system is marginally stable

Problem #4: (8pts)

Transfer function of a complex control system is given below.

$$T(s) = \frac{30s^3 + 3s^2 + s}{s^5 + 2s^4 + 3s^3 + 6s^2 + 2s + 4}$$

a) Observe system's stability according to coefficient tests and mention your findings.

b) Design the Routh array for stability of this system with Routh Hurwitz stability criterion.

c) Mention the number of characteristic roots on all significant locations of s-plane.

d) Specify this system as stable, unstable, or marginally stable and mention the reason for your claim.

a) All found / No change in sign

⟹ No information about Root location;

b) $s^5 + 2s^4 + 3s^3 + 6s^2 + 2s + 4$

s^5	1	3	2
s^4	2	6	4
s^3	~~0~~ 8	~~0~~ 12	0
s^2	3	4	0
s^1	4/3	0	0
s^0	4		

$$\Rightarrow P(s) = 2s^4 + 6s^2 + 4$$

$$P'(s) = \underline{\underline{8s^3}} + \underline{\underline{12s}}$$

---

c) $P(s) = 2s^4 + 6s^2 + 4 = 0$

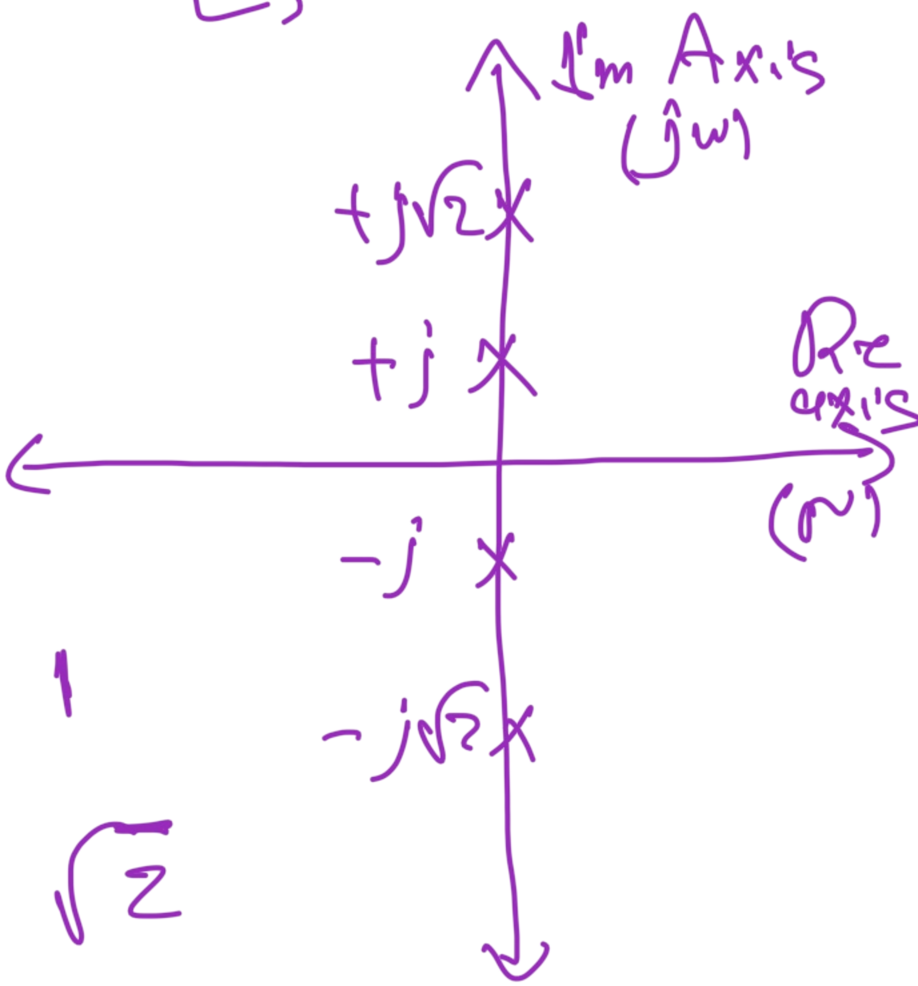
$\underline{\underline{s^2}} = x \Rightarrow 2x^2 + 6x + 4 = 0$

↳ mode → 5 → 3

$x = \sqrt{s^2} = \sqrt{-1}$

$x = \sqrt{s^2} = \sqrt{-2}$

$$\Rightarrow \begin{cases} s = \mp j1 \\ s = \mp j\sqrt{2} \end{cases}$$



d) No change in sign of 1st column

but, we have a row zeros

$\Longrightarrow$ marginally stable

There are 5 poles

↳ 4 poles on Imag Axis

↳ 1 poles on LHP