Home work (1)

2nd Semester

Section 1.2: Charge, Current, and Voltage

Problem 1.2

The units for voltage, current, and resistance are the volt (V), the ampere (A), and the ohm (Ω), respectively. Express each unit in fundamental MKS units.

Solution:

Known quantities:

MKSQ units.

Find:

Equivalent units of volt, ampere, and ohm.

Analysis:

$$\text{Voltage} = \text{Volt} = \frac{\text{Joule}}{\text{Coulomb}} \qquad V = \frac{J}{C}$$

$$\text{Current} = \text{Ampere} = \frac{\text{Coulomb}}{\text{second}} \qquad a = \frac{C}{s}$$

$$\text{Resistance} = \text{Ohm} = \frac{\text{Volt}}{\text{Ampere}} = \frac{\text{Joule} \times \text{second}}{\text{Coulomb}^2} \qquad \Omega = \frac{J \cdot s}{C^2}$$

$$\text{Conductance} = \text{Siemens or Mho} = \frac{\text{Ampere}}{\text{Volt}} = \frac{C^2}{J \cdot s}$$

Problem 1.3

A particular fully charged battery can deliver 2.7×10^6 coulombs of charge.

Solution:

Known quantities:

$Q_{Battery} = 2.7 \cdot 10^6$ C.

Find:

a) The capacity of the battery in ampere-hours.
b) The number of electrons that can be delivered.

Analysis:

a) There are 3600 seconds in one hour. Amperage is defined as 1 Coulomb per second and is directly proportional to ampere-hours.

$$2.7 \cdot 10^6 C \cdot \frac{1hr}{3600s} = 750\ AH$$

b) The charge of a single electron is $-1.602 \cdot 10^{-19}$ C. The negative sign is negligible. Simple division gives the solution:

P 1.2

Voltage: (V) $\Rightarrow \frac{\text{Joule}}{\text{coulmb}} = \frac{J}{C}$

current: (A) $= \frac{\text{coulmb}}{\text{second}} = \frac{C}{S}$

Resistance $(\Omega) = \frac{\text{Voltage}}{\text{current}} = \frac{V}{A}$

conductance $(S^1) = \frac{1}{R} = \frac{A}{V}$

$$\Omega = \frac{V}{A} = \frac{J/C}{C/S} \Rightarrow \frac{\frac{J}{C}}{\frac{C}{S}}$$

$$\Omega = \frac{J \cdot S}{C^2}$$

$$S^1 = \frac{1}{R} \Rightarrow \frac{C^2}{J \cdot S}$$

Q1.3

$Q = 2.7 \times 10^{6}$ C ←

a) 2.7×10^{6} A.h → $\frac{1h}{3600s} = 1$

1h → 60 min

1 min → 60 s

$\frac{2.7 \times 10^{6}}{3600}$ A H

$= 750$ Ah ←

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b) $Q = n\ e$ →

Q: شحنة الجسم

n: عدد electron

e: شحنة electron 1.602×10^{-19}

$n = \frac{Q}{e} = \frac{2.7 \times 10^{6}}{1.602 \times 10^{-19}} = 1.685 \times 10^{25}$ electron

$$\frac{2.7 \cdot 10^{4}\,C}{\frac{1.602 \cdot 10^{-19}\,C}{1\ electron}} = 1.685 \cdot 10^{23}\ electrons$$

Problem 1.7

An automotive battery is rated at 120 A-h. This means that under certain test conditions it can output 1 A at 12 V for 120 h (under other test conditions, the battery may have other ratings).
a. How much total energy is stored in the battery?
b. If the headlights are left on overnight (8 h), how much energy will still be stored in the battery in the morning? (Assume a 150-W total power rating for both headlights together.)

Solution:

Known quantities:

Rated discharge current of the battery; rated voltage of the battery; rated discharge time of the battery.

Find:

a) Energy stored in the battery when fully recharging
b) Energy stored in the battery after discharging

Analysis:

a) $\text{Energy} = \text{Power} \times \text{time} = (1\,A)(12\,V)(120\,hr)\left(\frac{60\,min}{hr}\right)\left(\frac{60\,sec}{min}\right)$

$$\boxed{w = 5.184 \times 10^{6}\ J}$$

b) Assume that 150 W is the combined power rating of both lights; then,

$$w_{used} = (150\,W)(8\,hrs)\left(\frac{3600\,sec}{hr}\right) = 4.32 \times 10^{6}\ J$$

$$\boxed{w_{stored} = w - w_{used} = 864 \times 10^{3}\ J}$$

Problem 1.9

Suppose the current through a wire is given by the curve shown in Figure P1.9.
a. Find the amount of charge, q, that flows through the wire between t1 = 0 and t2 = 1 s.
b. Repeat part a for t2 = 2, 3, 4, 5, 6, 7, 8, 9, and 10 s.
c. Sketch q(t) for $0 \le t \le 10$ s.

P 1.F → 120 A-h

output ⇒ 1A at 12V for 120h

a) $E = P \cdot t_{(s)} = VI \cdot t_{(s)}$ ثانية second

$E = 12 \cdot 1 \cdot 120h \cdot \frac{60 min}{1h} \cdot \frac{60 sec}{1 min}$

$E = 12 \times 120 \times 60 \times 60$

$E_{total} = 5184000\ J = 5.184 \times 10^6\ J$

b) $E_{head\ lights} = P \cdot t = 150 \times 8h \times \frac{3600\ s}{1h}$

$150 \times 8 \times 3600$

$E_{head} = 4.32 \times 10^6\ J$

$E_{remaining} = E_{total} - E_{headlight} = 5.184 - 4.32$

$= 0.864 \times 10^6\ J$

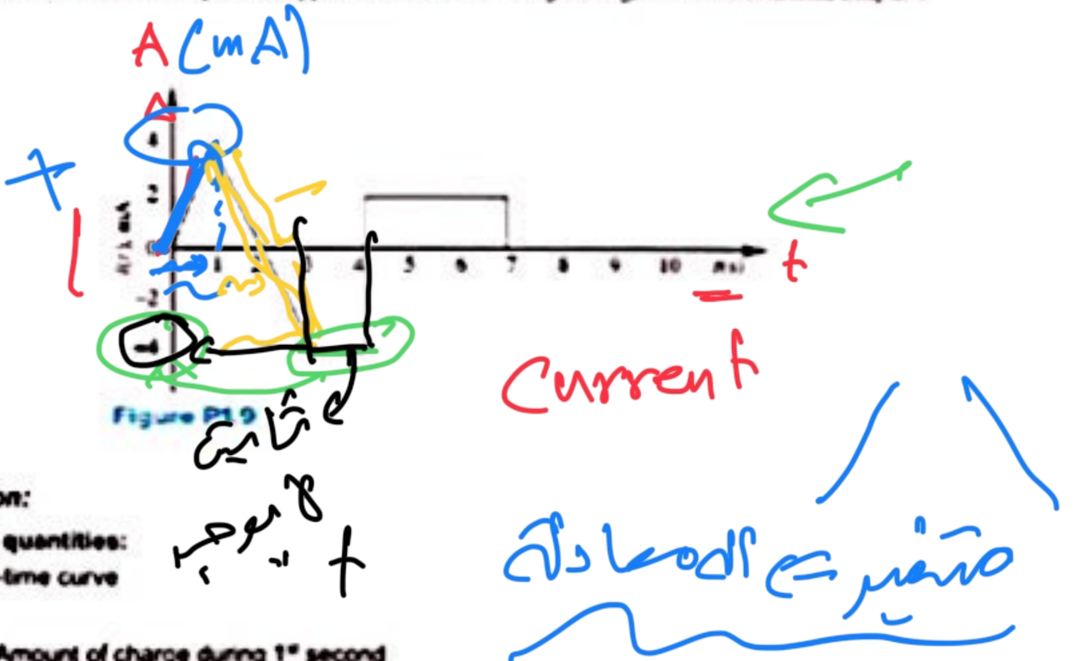


Figure P1.9

Solution:

Known quantities:

Current-time curve

Find:

a) Amount of charge during 1st second
b) Amount of charge for 2 to 10 seconds
c) Sketch charge-time curve

Analysis:

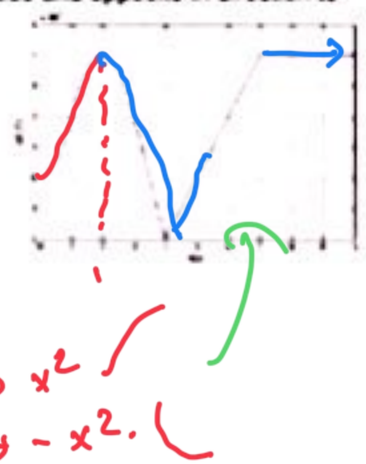
a) $i = \frac{4 \times 10^{-3} t}{1}$

$$Q_1 = \int_0^1 i\,dt = \int_0^1 4 \times 10^{-3} t\,dt = 4 \times 10^{-3} \frac{t^2}{2}\Big|_0^1 = 2 \times 10^{-3}\ \frac{\text{amp}}{\text{sec}} = 2 \times 10^{-3}\ \text{Coulombs}$$

b) The charge transferred from $t = 1$ to $t = 2$ is the same as from $t = 0$ to $t = 1$.

$Q_2 = 4 \times 10^{-3}$ Coulombs

The charge transferred from $t = 2$ to $t = 3$ is the same in magnitude and opposite in direction to that from $t = 1$ to $t = 2$. $Q_3 = 2 \times 10^{-3}$ Coulombs



$t = 4$

$Q_4 = 2 \times 10^{-3} - \int_3^4 4 \times 10^{-3} dt = 2 \times 10^{-3} - 4 \times 10^{-3} = -2 \times 10^{-3}$ Coulombs

$t = 5, 6, 7$

$Q_5 = -2 \times 10^{-3} + \int_4^5 2 \times 10^{-3} dt = 0$

$Q_6 = 0 + \int_5^6 2 \times 10^{-3} dt = 2 \times 10^{-3}$ Coulombs

$Q_7 = 2 \times 10^{-3} + \int_6^7 2 \times 10^{-3} dt = 4 \times 10^{-3}$ Coulombs

$t = 8, 9, 10\,s$

$Q = 4 \times 10^{-3}$ Coulombs

P.(1, a)

$$\int i = \int \frac{dq}{dt} \Longleftrightarrow Q(t) = \int i \, dt$$

$$i = \frac{4\times10^{-3}\,t}{1} \qquad 0 \le t \le 1$$

$$Q(t) = \int_0^1 4\times10^{-3}\, t\, dt$$

$$Q = 2\times10^{-3}\, C = 2mC$$

عمل عام

$$Q(t) = \int_0^x 4\times10^{-3}\, t\, dt$$

$$Q(t) = 4\times10^{-3} \left[\frac{t^2}{2}\right]_0^x$$

$$Q(t) = 2\times10^{-3}(x^2 - 0)$$

$$Q(t) = 2\times10^{-3}\, x^2 \; C \qquad 0 \le t \le 1$$

$1 \leq t \leq 3$

$$i(t) = \frac{-4\times10^{-3}t}{1} = -4\times10^{-3}t$$

$$Q(t) = \int i(t)\,dt = \int_1^3 -4\times10^{-3}t\,dt = -16mC$$

$$Q = -16\times10^{-3}C$$

$$Q(t) = \int_1^x -4\times10^{-3}t\,dt$$

$$= -4\times10^{-3}\left[\frac{t^2}{2}\right]_1^x$$

$$Q(t) = -2\times10^{-3}(x^2-1)\,C$$

$$3 \le t \le 4$$

$$i(t) = -4 \times 10^{-3} \, A$$

$$Q(t) = \int_3^4 -4 \times 10^{-3} \, dt$$

$$\boxed{Q(t) = -4 \, mC}$$

⋮

Section 1.4: Power and the Passive Sign Convention

Problem 1.13

Find the power delivered by the source in Figure P1.13.

Figure P1.13

Solution:
Known quantities:
Circuit shown in Figure P1.13.
Find:
Power delivered by the 3A current source.
Analysis:

Follow the counterclockwise current:

$$P = (+3A)\cdot(+10V)$$
$$P = +30W \text{ supplied}$$

Problem 1.15

Determine whether each element in Figure P1.15 is supplying or dissipating power, and how much.

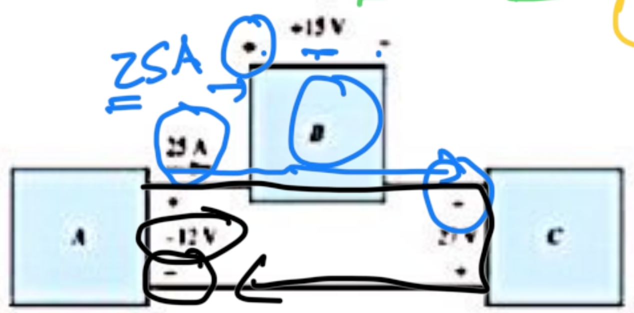


Figure P1.15

Solution:

Known quantities:

Circuit shown in Figure P1.15.

Find:

Determine power dissipated or supplied for each power source.

Analysis:

Element A:

$$P = -vi = -(-12V)(25A) = 300W \text{ (dissipating)}$$

Element B:

$$P = vi = (15V)(25A) = 375W \text{ (dissipating)}$$

Element C:

$$P = vi = (27V)(25A) = 675W \text{ (supplying)}$$

B: $P = +vi = +25 \times 15 = 375\,W$

C: $P = -vi = -25 \times 27 = -675\,W$

A: $P = -vi = +(-12) \times 25 = +300\,W$

Problem 1.16

In the circuit of Figure P1.16, find the power absorbed by the resistor R_4 and the power delivered by the current source.

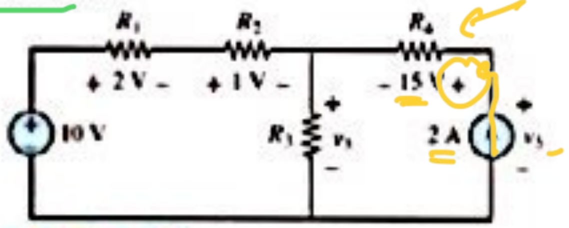


Figure P1.16

$P = +V\,i = 15 \times 2 = +30W$ ↳ absorbed

Solution:

Known quantities:

Circuit shown in Figure P1.16.

Find:

a) Power absorbed by R_4
b) Power supplied by the current source

Analysis:

a) Follow the counterclockwise current in the rightmost loop:

$$P = (2A)\cdot(15V)$$

$$P = 30W \text{ absorbed}$$

b) Use KVL at the leftmost loop to find V_3:

$$10V - 2V - 1V - V_3 = 0$$

$$V_3 = 7V$$

Use KVL at the rightmost loop to find V_5:

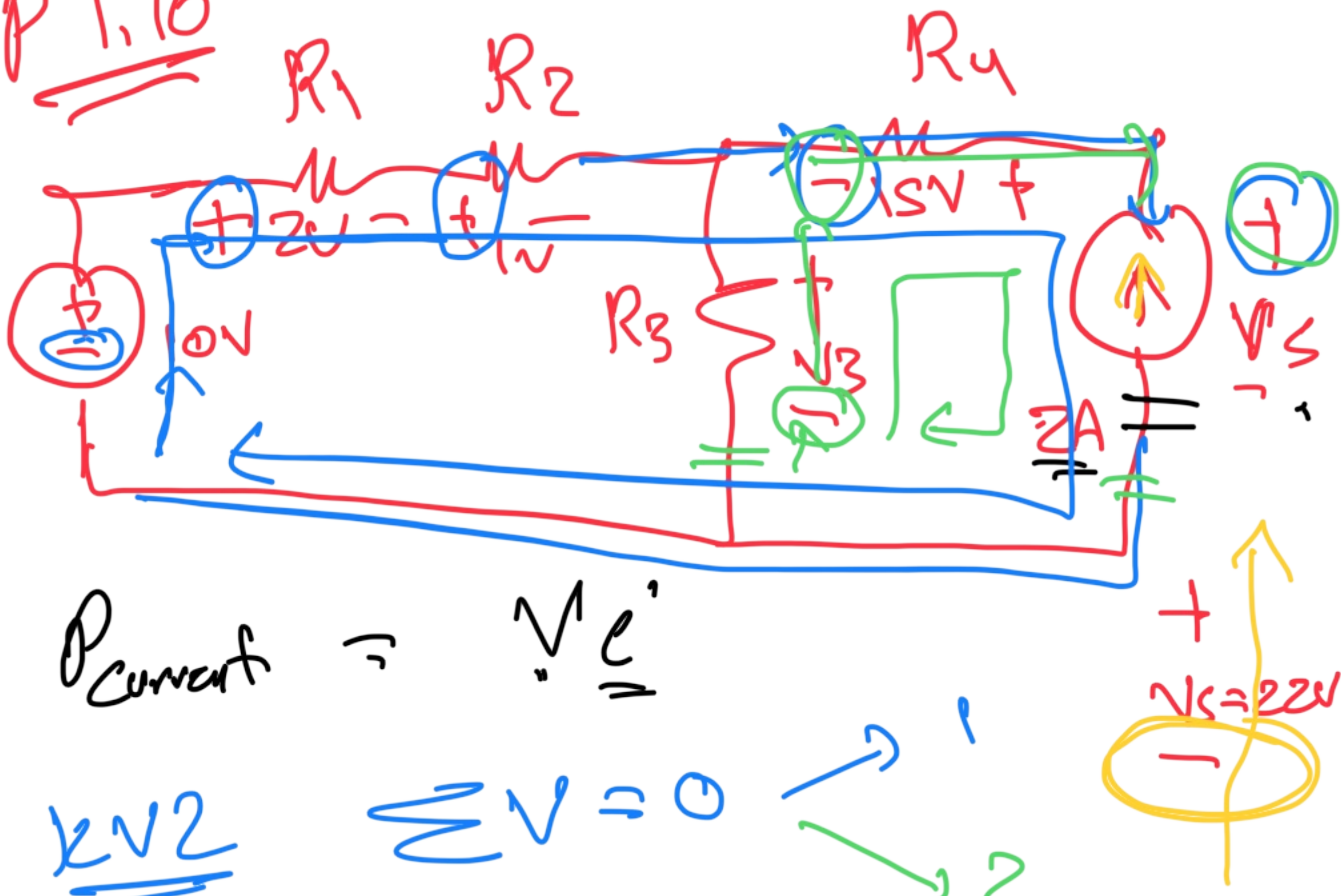
$$7V + 15V - V_5 = 0$$

$$V_5 = 22V$$

Remember that the power calculations are based upon the passive sign convention, in which current is directed from high to low potential. Positive power is power absorbed by an element. Since V_5 is positive the current is directed from low to high potential such that power is supplied by the current source.

$$P = -(+2A)\cdot(22V) = +44W \text{ supplied}$$

P 1.16



$P_{current} = V i$

KVL $\sum V = 0$ → 1, → 2

$-10 + 2 + 1 + -15 + V_s = 0$

$V_s = 10 - 2 - 1 + 15 \Rightarrow V_s = 22V$

$P = -Vi = -22 \times 2 = -44W$

$\Rightarrow +44W$ supplied

$-V_3 - 15 + V_s = 0$

Section 1.5: Kirchhoff's Laws

Problem 1.22

Use KCL to determine the unknown currents in the circuit of Figure P1.22. Assume $i_0 = 2$ A and $i_2 = -7$ A.

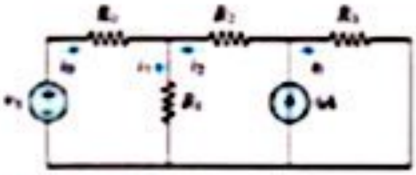


Figure P1.22

Solution:

Known quantities:

$i_0 = 2$ A, $i_2 = -7$ A

Find:

a) i_1
b) i_3

Analysis:

a) Apply KCL at the node between R_0, R_1, and R_2.

$$i_0 - i_1 + i_2 = 0$$
$$i_1 = i_0 + i_2$$
$$i_1 = -5A$$

b) Apply KCL at the node between R_2, R_3, and the current source.

$$6A + i_3 - i_2 = 0$$
$$i_3 = i_2 - 6A$$
$$i_3 = -13A$$

Problem 1.24

Use KCL to find the currents i_1, i_2, and i_3 in the circuit of Figure P1.24. Assume that $i_a = 2$ mA, $i_b = 7$ mA and $i_c = 4$ mA.

P 1.22

$i_0 = 2A$ ✓ $i_2 = -7A$ ✓

KCL ⇒ $\sum I_{in} = I_{out}$ at node

نقطة تجمع أكثر من عنصرين.

node A $6 + i_3 = i_2$

$6 + i_3 = -7 - 6$

$$\boxed{i_3 = -13A}$$

node B $i_0 + i_2 = i_1$

$2 + -7 = i_1 \Rightarrow \boxed{i_1 = -5A}$

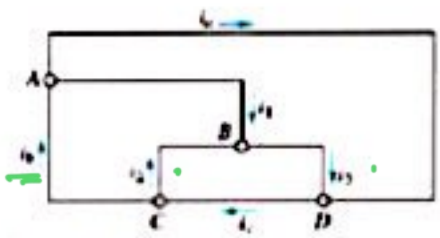


Figure P1.24

Solution:

Known quantities:

$i_a = 2$ mA, $i_b = 7$ mA, $i_c = 4$ mA

Find:

a) i_1
b) i_2
c) i_3

Analysis:

a) Use KCL at Node A.

$$i_b - i_a - i_1 = 0$$
$$i_1 = i_b - i_a$$
$$i_1 = 5mA$$

b) Use KCL at Node C.

$$i_c - i_2 - i_b = 0$$
$$i_2 = i_c - i_b$$
$$i_2 = -3mA$$

c) Use KCL at Node D.

$$i_3 + i_a - i_c = 0$$
$$i_3 = i_2 - i_a$$
$$i_3 = 2mA$$

Problem 1.25

Use KVL to find the voltages v_1, v_2, and v_3 in Figure P1.25. Assume that $v_a = 2$ V, $v_b = 4$ V, and $v_c = 5$ V.

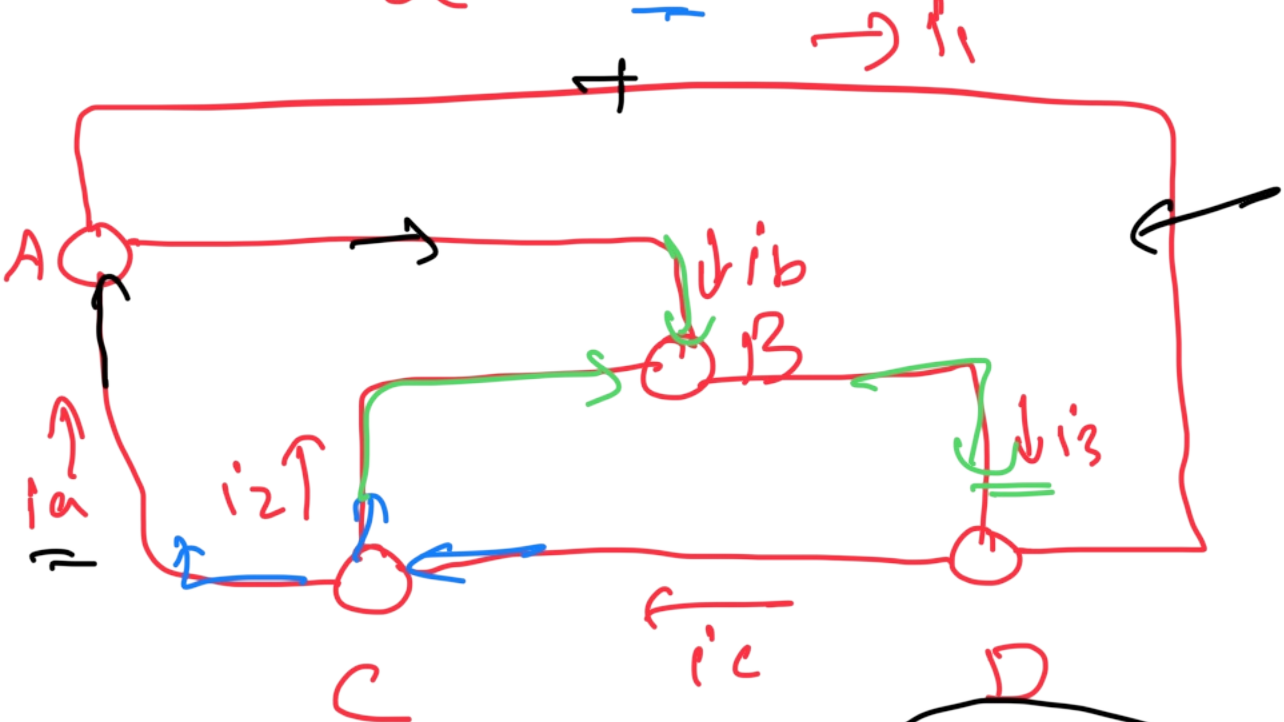
Q 1.24

i_1, i_2, i_3

$\Sigma i_{in} = \Sigma i_{out}$

$i_a = 2mA$ $i_b = 7mA$

$i_c = 4mA$



at node A $\Rightarrow$ $i_a = i_1 + i_b$

$i_1 = i_a - i_b = 2 - 7 = -5mA$

at node C $\Rightarrow$ $i_c = i_a + i_2$

$i_2 = i_c - i_a = 4 - 2 = 2mA$

at node B : $i_b + i_2 = i_3$

$7 + 2 = i_3 \Rightarrow \boxed{i_3 = 9\,mA}$

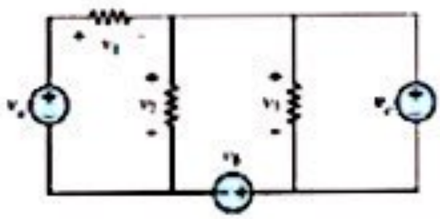


Figure P1.25

Solution:

Known quantities:

$V_a = 2$ V, $V_b = 4$ V, $V_c = 5$ V

Find:

a) V_1
b) V_2
c) V_3

Analysis:

a) Apply KVL around the right mesh.

$$V_3 - V_c = 0$$
$$V_3 = V_c$$
$$V_3 = 5V$$

b) Apply KVL around the middle mesh.

$$V_2 - V_3 - V_b = 0$$
$$V_2 = V_3 + V_b$$
$$V_2 = 9V$$

c) Apply KVL around the left mesh.

$$V_a - V_1 - V_2 = 0$$
$$V_1 = V_a - V_2$$
$$V_1 = -7V$$

Check these results by applying KVL around the outer loop. 2V = V_1 + 5V + 4V = 2V. Check!

P 1.25 KVL ⇒ $\sum V = 0$

$V_a = 2V$ $V_b = 4V$

$V_c = 5V$

KVL ⇒ $-V_3 + V_c = 0$

$\Rightarrow V_3 = V_c = 5V$

KVL ⇒ $-V_b - V_3 + V_2 = 0$

$\Rightarrow V_2 = V_b + V_3 = 4 + 5 = 9V$

$KVL \Rightarrow -V_a + V_1 + V_2 = 0$

$V_1 = V_a - V_2 = 2 - 9$

$V_1 = -7V$

$V_2 = 9V$

$V_3 = 5V$