

Electric Circuits II

Chapter 6: Inductance and Capacitance

EE 2020– Credits : 4

1

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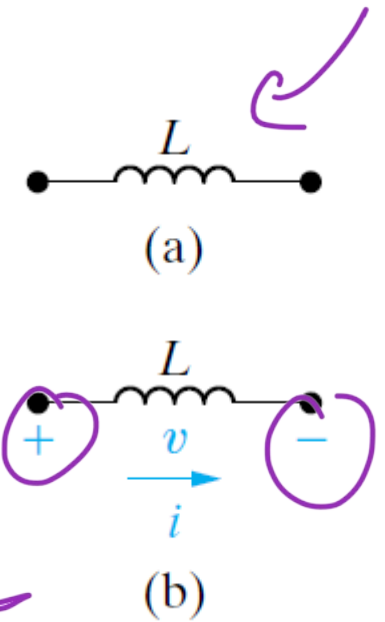
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Inductor

- Inductance is the circuit parameter used to describe an inductor.
- Inductance is symbolized by the letter L , is measured in henrys (H)
- Current reference is in the direction of the voltage drop across the inductor.
- The voltage drop across the terminals of the inductor is:



$$v = L \frac{di}{dt}$$

Notes.

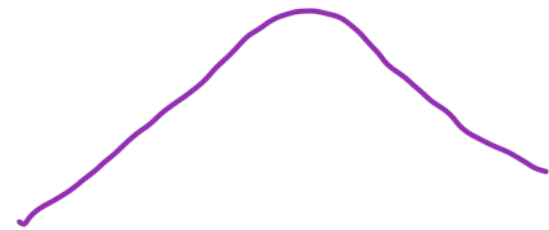
- ✓ The voltage across the terminals of an inductor is proportional to the time rate of change of the current in the inductor.
- ✓ the current is constant (DC current), the voltage across the ideal inductor is zero (Short Circuit)
- ✓ Inductor current cannot change instantaneously in an inductor; that is, the current cannot change by a finite amount in zero time (infinite voltages are not possible)
- ✓ when someone opens the switch on an inductive circuit in an actual system, WHAT will happen?! Give an example for this phenomena?

$$V_L = L \frac{di}{dt}$$

DC \Rightarrow i constant \Rightarrow L short

$$i_L(0^-) = i_L(0^+)$$

current never discontinues



Inductor

Example 6-1

The independent current source in the circuit shown in Fig. 6.2 generates zero current for $t < 0$ and a pulse $10te^{-5t}$ A, for $t > 0$.

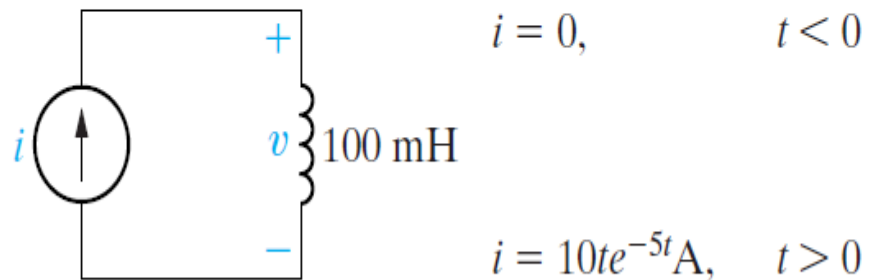


Figure 6.2 ▲ The circuit for Example 6.1.

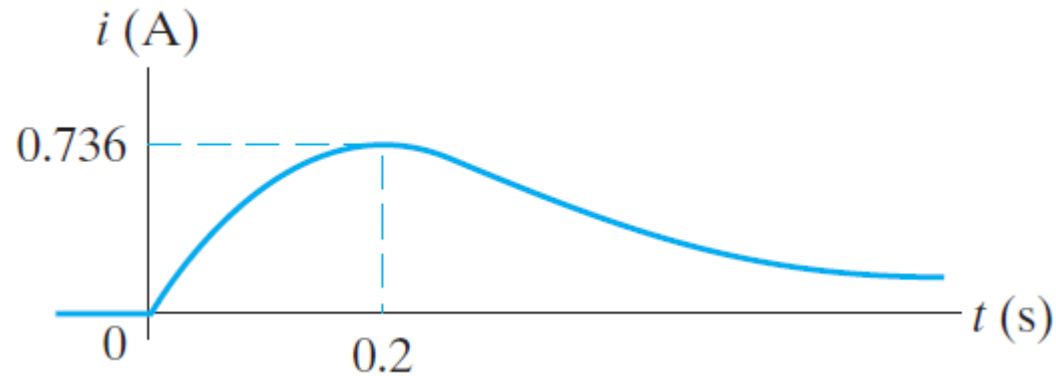
Determine the following

- Sketch the current waveform.
- At what instant of time is the current maximum?
- Express the voltage across the terminals of the 100 mH inductor as a function of time.
- Sketch the voltage waveform.
- Are the voltage and the current at a maximum at the same time?
- At what instant of time does the voltage change polarity?
- Is there ever an instantaneous change in voltage across the inductor? If so, at what time?

Inductor

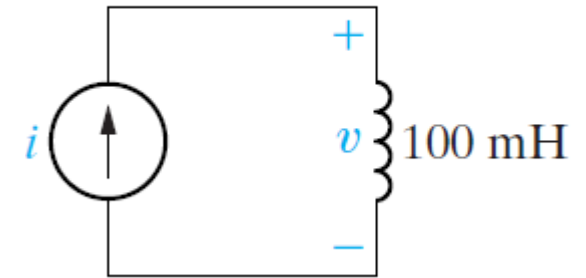
Solution

(a) Sketch the current waveform.



(b) At max $\frac{di}{dt} = 0$, then:

$$\text{b) } \frac{di}{dt} = 10(-5te^{-5t} + e^{-5t}) = 10e^{-5t}(1 - 5t) \text{ A/s; } \frac{di}{dt} = 0 \text{ when } t = \frac{1}{5} \text{ s.}$$



Inductor

Solution

c) $v = L di/dt = (0.1)10e^{-5t}(1 - 5t) = e^{-5t}(1 - 5t)$ V, $t > 0$; $v = 0, t < 0$.

d) Figure 6.4 shows the voltage waveform.

e) No; the voltage is proportional to di/dt , not i .

f) At 0.2 s, which corresponds to the moment when di/dt is passing through zero and changing sign.

g) Yes, at $t = 0$. Note that the voltage can change instantaneously across the terminals of an inductor.

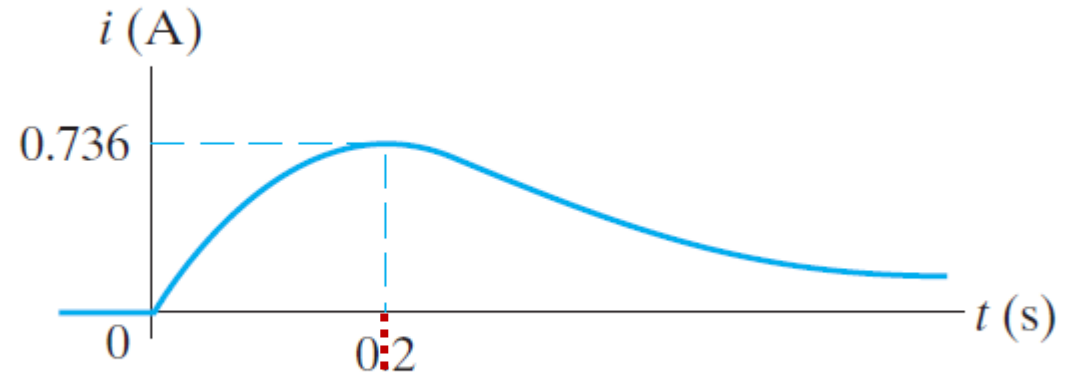


Figure 6.3 ▲ The current waveform for Example 6.1.

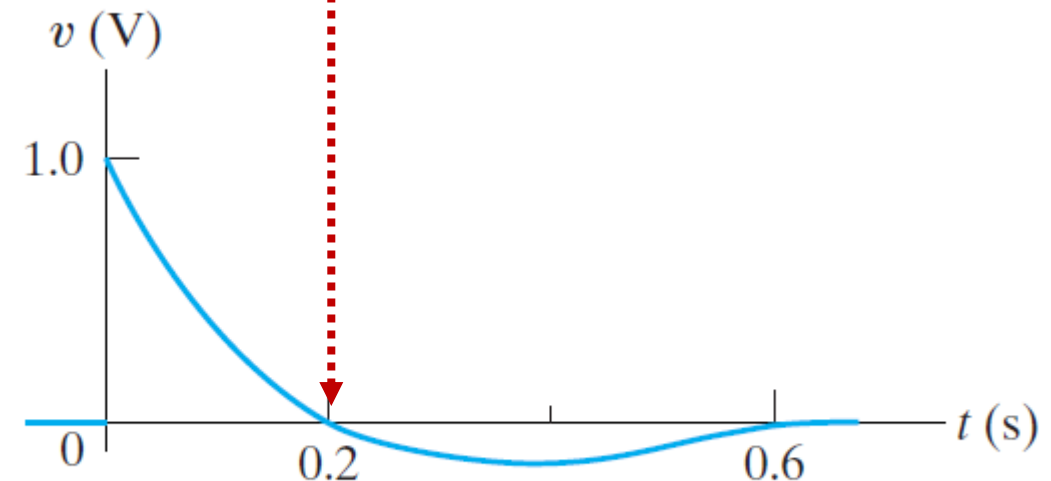


Figure 6.4 ▲ The voltage waveform for Example 6.1.

Current in an Inductor in Terms of the Voltage Across the Inductor

$$v = L \frac{di}{dt}$$



$$v dt = L di$$

Take the integrator:

$$\int_{t_0}^t di_L(t) = \frac{1}{L} \int_{t_0}^t v_L(t) dt,$$



$$i_L(t) - i_L(t_0) = \frac{1}{L} \int_{t_0}^t v_L(t) dt$$

The inductor i - v equation

$$i_L(t) = \frac{1}{L} \int_{t_0}^t v_L(t) dt + i_L(t_0)$$

t ≈ 0

Initial condition

In many practical applications, t_0 is zero

$$i_L(t) = \frac{1}{L} \int_0^t v_L(t) dt + i_L(0)$$

Inductor

Example (6-2), Determining the Current, Given the Voltage, at the Terminals of an Inductor

The voltage pulse applied to the 100 mH inductor shown in Fig. 6.5 is 0 for $t < 0$ and is given by the expression

$$\underline{v(t) = 20te^{-10t} \text{ V}}$$

for $t > 0$. Also assume $i = 0$ for $t \leq 0$.

- Sketch the voltage as a function of time.
- Find the inductor current as a function of time.
- Sketch the current as a function of time.

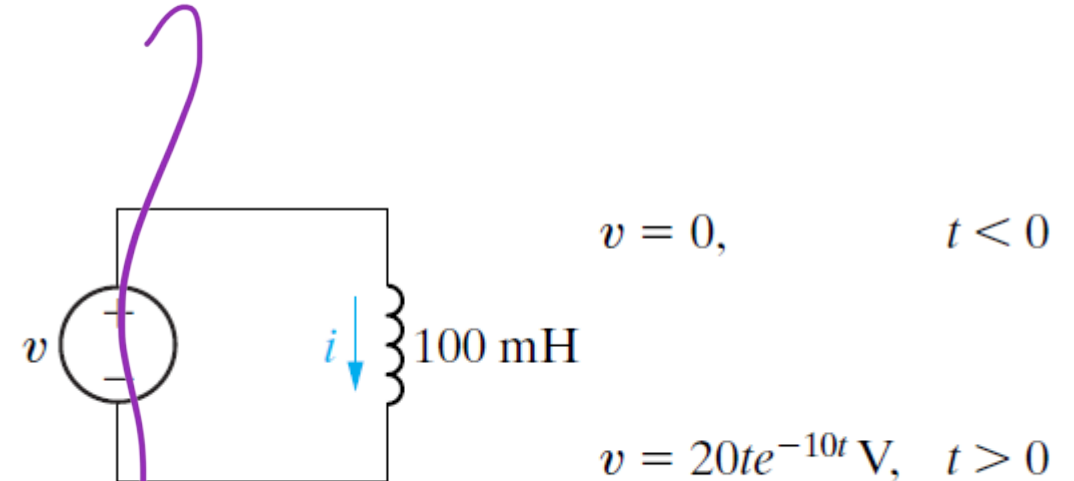


Figure 6.5 ▲ The circuit for Example 6.2.

Inductor

Solution (6-2)

a) The voltage as a function of time is shown in Fig. 6.6.

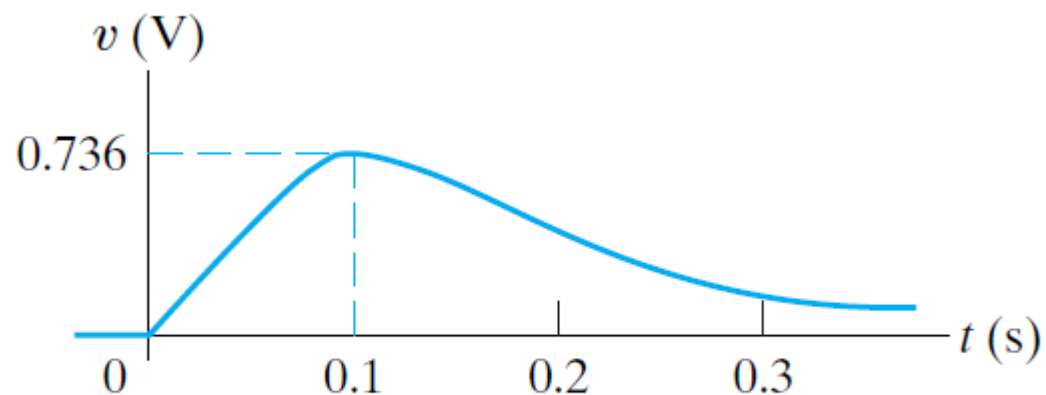


Figure 6.6 ▲ The voltage waveform for Example 6.2.

b) The current in the inductor is 0 at $t = 0$. Therefore, the current for $t > 0$ is

$$\begin{aligned} i &= \frac{1}{0.1} \int_0^t 20\tau e^{-10\tau} d\tau + 0 \quad \leftarrow \\ &= 200 \left[\frac{-e^{-10\tau}}{100} (10\tau + 1) \right] \Big|_0^t, \\ &= 2(1 - 10te^{-10t} - e^{-10t}) \text{ A}, \quad t > 0. \end{aligned}$$

Inductor

Solution (6-2)

c) Figure 6.7 shows the current as a function of time.

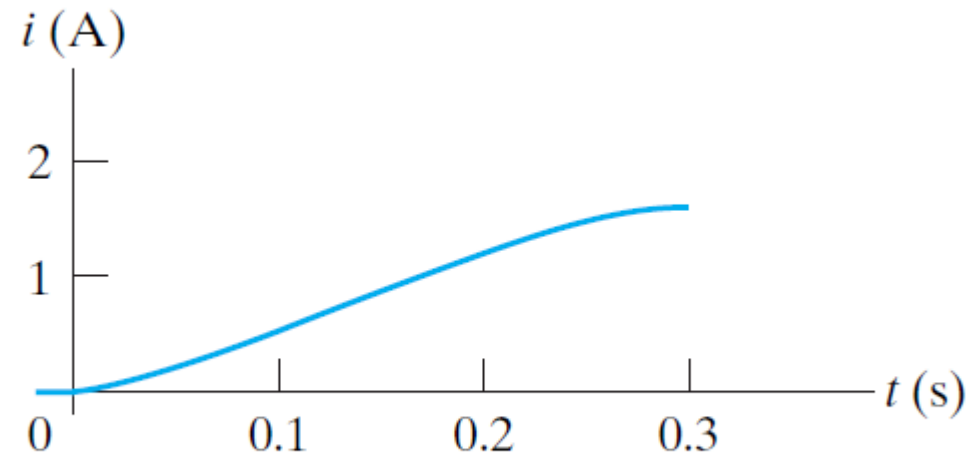


Figure 6.7 ▲ The current waveform for Example 6.2.

Power and Energy in the Inductor

The power and energy relationships for an inductor can be derived directly from the current and voltage relationships. If the current reference is in the direction of the voltage drop across the terminals of the inductor, the power is

$$p = vi \rightarrow p = Li \frac{di}{dt} \quad \text{Power in an inductor}$$

We can also express the current in terms of the voltage: $p = v \left[\frac{1}{L} \int_{t_0}^t v d\tau + i(t_0) \right]$

Power is the time rate of expending energy,

$$p = \frac{dw}{dt} = Li \frac{di}{dt} \rightarrow dw = Li di \rightarrow \int_0^w dx = L \int_0^i y dy, \quad w = \frac{1}{2} Li^2 \quad \text{Energy in an inductor}$$

$$U_L \approx L \frac{\partial c'}{\partial t} \quad ; \quad c'_L \approx \frac{1}{2} \int v + c'_L(t)$$

$$\Rightarrow \varphi = L \frac{\partial c''}{\partial t}$$

$$\Rightarrow \varphi = U_L \left[\frac{1}{L} \int v + c'_L(t) \right]$$

$$w = \int \varphi(t) dt = \frac{1}{2} L c'^2$$

Power and Energy in the Inductor

Important Notes:

- Inductors store energy (They can deliver or consume power, but energy is never lost.)
- If current is not changing, voltage across inductor is zero (a short circuit!)
- Current cannot change instantaneously (Current must change continuously)
- Voltage can change discontinuously/instantaneously

$P \neq$

constant \Rightarrow DC

DC



t_1, t_2, t_3, \dots

Power and Energy in the Inductor

Example (6-3), Determining the Current, Voltage, Power, and Energy for an Inductor

- a) For Example 6.1, plot i , v , p , and w versus time. Line up the plots vertically to allow easy assessment of each variable's behavior.
- b) In what time interval is energy being stored in the inductor?
- c) In what time interval is energy being extracted from the inductor?
- d) What is the maximum energy stored in the inductor?
- e) Evaluate the integrals

$$\int_0^{0.2} p \, dt \quad \text{and} \quad \int_{0.2}^{\infty} p \, dt,$$

Power and Energy in the Inductor

Solution

- a) The plots of i , v , p , and w follow directly from the expressions for i and v obtained in Example 6.1 and are shown in Fig. 6.8. In particular, $p = vi$, and $w = (\frac{1}{2})Li^2$.
- b) An increasing energy curve indicates that energy is being stored. Thus energy is being stored in the time interval 0 to 0.2 s. Note that this corresponds to the interval when $p > 0$.
- c) A decreasing energy curve indicates that energy is being extracted. Thus energy is being extracted in the time interval 0.2 s to ∞ . Note that this corresponds to the interval when $p < 0$.

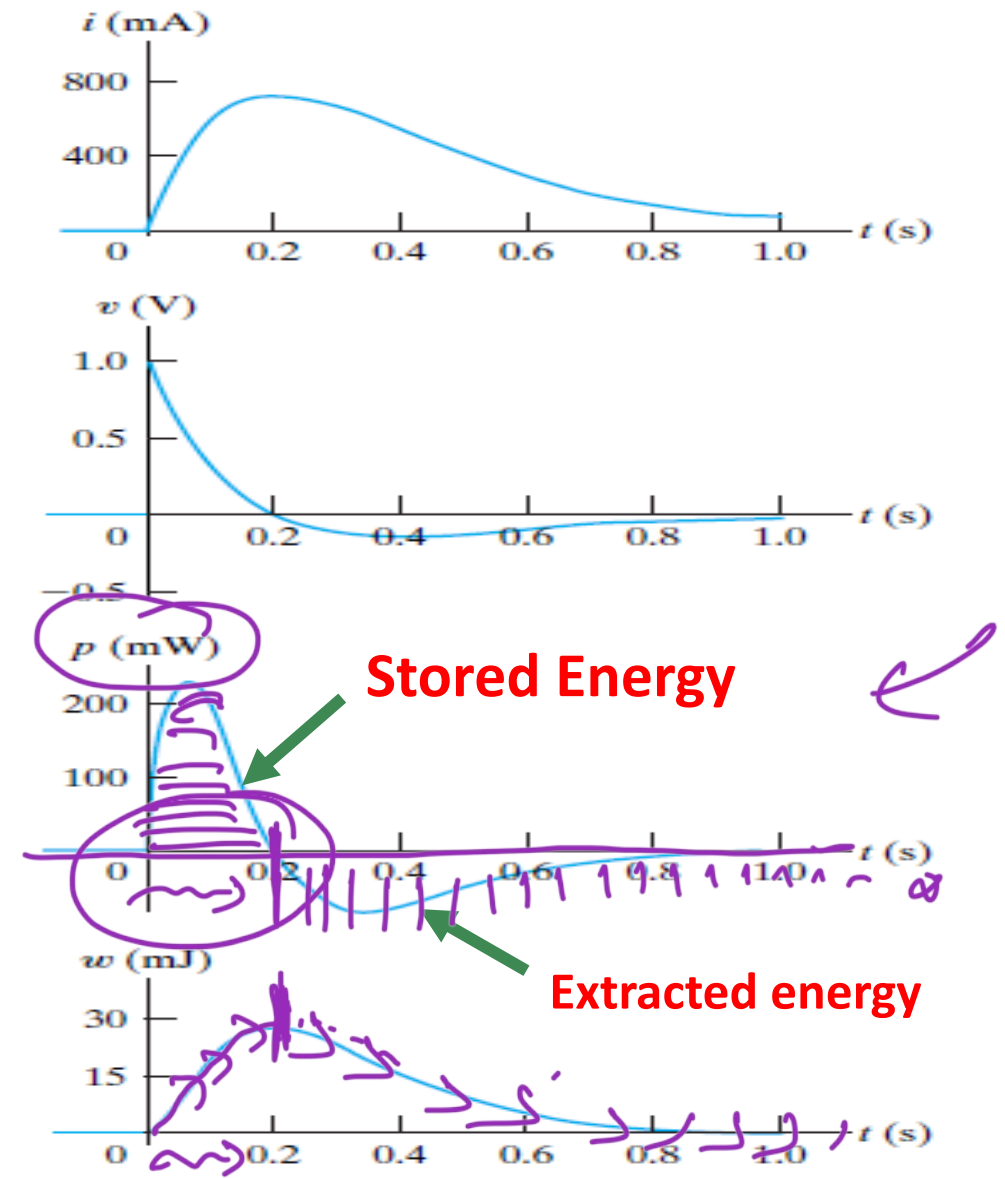


Figure 6.8 ▲ The variables i , v , p , and w versus t for Example 6.1.

Stored Energy

Extracted Energy

- ↳ P (+ ~~ت~~ اقوة)
- ↳ E (حاله التزايد)

- ↳ P - ~~ت~~ اقوة
- ↳ E (حاله التناقص)

↳

↳ *

كربس

*

C

كربس

Power and Energy in the Inductor

Solution

d) From Eq. 6.12 we see that energy is at a maximum when current is at a maximum; glancing at the graphs confirms this. From Example 6.1, maximum current = 0.736 A. Therefore, $w_{\max} = 27.07$ mJ.

e) From Example 6.1,

$$i = 10te^{-5t} \text{ A} \quad \text{and} \quad v = e^{-5t}(1 - 5t) \text{ V.}$$

Therefore,

$$p = vi = 10te^{-10t} - 50t^2e^{-10t} \text{ W.}$$

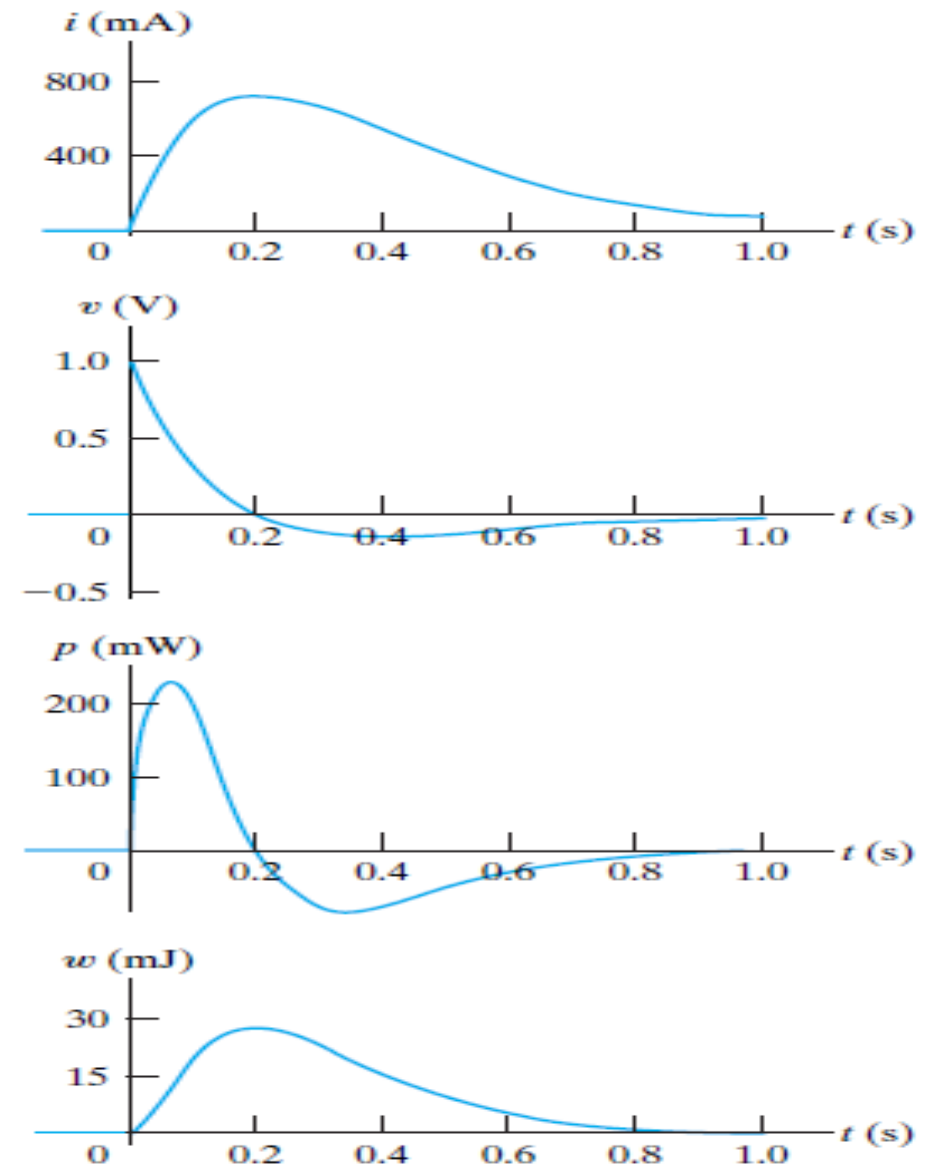


Figure 6.8 ▲ The variables i , v , p , and w versus t for Example 6.1.

Power and Energy in the Inductor

$$W \Rightarrow \int P dt$$

Solution (6-3)

Therefore,

$$p = vi = 10te^{-10t} - 50t^2e^{-10t} \text{ W.}$$

$$\int_0^{0.2} p dt = 10 \left[\frac{e^{-10t}}{100} (-10t - 1) \right]_0^{0.2}$$

$$- 50 \left\{ \frac{t^2 e^{-10t}}{-10} + \frac{2}{10} \left[\frac{e^{-10t}}{100} (-10t - 1) \right] \right\}_0^{0.2}$$

$$= 0.2e^{-2} = 27.07 \text{ mJ,}$$

Stored Energy

$$\int_{0.2}^{\infty} p dt = 10 \left[\frac{e^{-10t}}{100} (-10t - 1) \right]_{0.2}^{\infty}$$

$$- 50 \left\{ \frac{t^2 e^{-10t}}{-10} + \frac{2}{10} \left[\frac{e^{-10t}}{100} (-10t - 1) \right] \right\}_{0.2}^{\infty}$$

$$= -0.2e^{-2} = -27.07 \text{ mJ.}$$

extracted Energy

الطاقة التي تُخزن

⇒ stored Energy = Extracted Energy

$w = \int \delta = + \delta \Rightarrow$ stored

$w = \int \delta = - \delta \Rightarrow$ Extracted

Inductor (Homework) due to one week

✓ ASSESSMENT PROBLEM

Objective 1—Know and be able to use the equations for voltage, current, power, and energy in an inductor

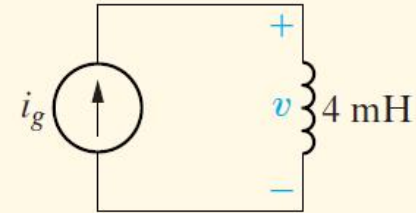
6.1 The current source in the circuit shown generates the current pulse

$$i_g(t) = 0, \quad t < 0,$$

$$i_g(t) = 8e^{-300t} - 8e^{-1200t} \text{ A}, \quad t \geq 0.$$

Find (a) $v(0)$; (b) the instant of time, greater than zero, when the voltage v passes through zero; (c) the expression for the power delivered to the inductor; (d) the instant when the power delivered to the inductor is maximum; (e) the maximum power; (f) the instant of time when the stored energy is maximum; and (g) the maximum energy stored in the inductor.

NOTE: Also try Chapter Problems 6.2 and 6.8.



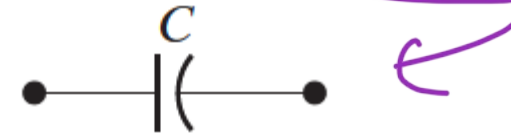
- Answer:**
- (a) 28.8 V;
 - (b) 1.54 ms;
 - (c) $-76.8e^{-600t} + 384e^{-1500t} - 307.2e^{-2400t} \text{ W}, t \geq 0$;
 - (d) 411.05 μs ;
 - (e) 32.72 W;
 - (f) 1.54 ms;
 - (g) 28.57 mJ.

Capacitor

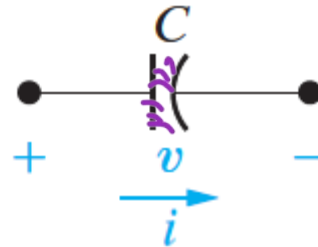


Inductor circuit

- The circuit parameter of capacitance is represented by the letter C , is measured in farads (F).
- It is symbolized graphically by two short parallel conductive plates



- Because the farad is an extremely large quantity of capacitance, practical capacitor values usually lie in the picofarad (pF) to (μ F) microfarad range.
- Assigning reference voltage and current to the capacitor, following the passive sign convention.



- The graphic symbol for a capacitor is a reminder that capacitance occurs whenever electrical conductors are separated by a dielectric, or insulating, material.
- Electric charge is not transported through the capacitor.

Capacitor

$$v \rightarrow \frac{\partial v}{\partial t}$$

- Applying a voltage to the terminals of the capacitor cannot move a charge through the dielectric, it can displace a charge within the dielectric.
- **Capacitor $i - v$ equation**



- **Notes:**

- ✓ **voltage cannot change instantaneously across the terminals of a capacitor (if the voltage changes instantaneously, the current will be infinite, a physical impossibility)**
- ✓ **If the voltage across the terminals is constant, the capacitor current is zero (Open Circuit)**

$$V_c(\infty^-) = V_c(\infty^+)$$

Capacitor

- the voltage as a function of the current

$$i_C(t)dt = Cdv$$

- Then

$$v_C(t) - v_C(t_0) = \frac{1}{C} \int_{t_0}^t i_C(t)dt$$

The capacitor voltage is:

$$v_C(t) = \frac{1}{C} \int_{t_0}^t i_C(t)dt + v_C(t_0)$$

- In many practical applications, the initial time is zero

$$\underline{v_C(t)} = \frac{1}{C} \int_0^t \underline{i_C(t)}dt + \underline{v_C(0)}$$

Capacitor

Power and energy relationships for the capacitor.

- Recall:

$$i_C(t) = C \frac{dv_C(t)}{dt}$$

$$v_C(t) = \frac{1}{C} \int_{t_0}^t i_C(t) dt + v_C(t_0)$$

- The power can be calculated as:

$$p = \frac{dw}{dt} = vi = Cv \frac{dv}{dt}$$



$$dw = Cv \frac{dv}{dt} dt$$



$$\int_0^t dw = \int_0^t C v dv$$

Capacitor energy equation

$$w(t) = \frac{1}{2} C v^2(t)$$

Series-Parallel Combinations of Inductance and Capacitance

Apply KVL.

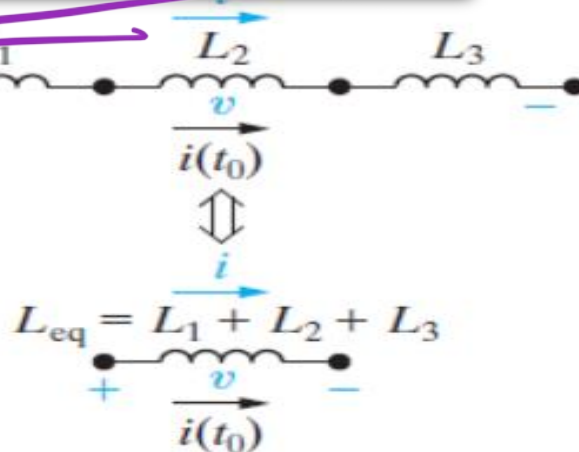
$$v_s = v_1 + v_2 + \dots + v_N$$

$$v_s = L_1 \frac{di_s}{dt} + L_2 \frac{di_s}{dt} + \dots + L_N \frac{di_s}{dt}$$

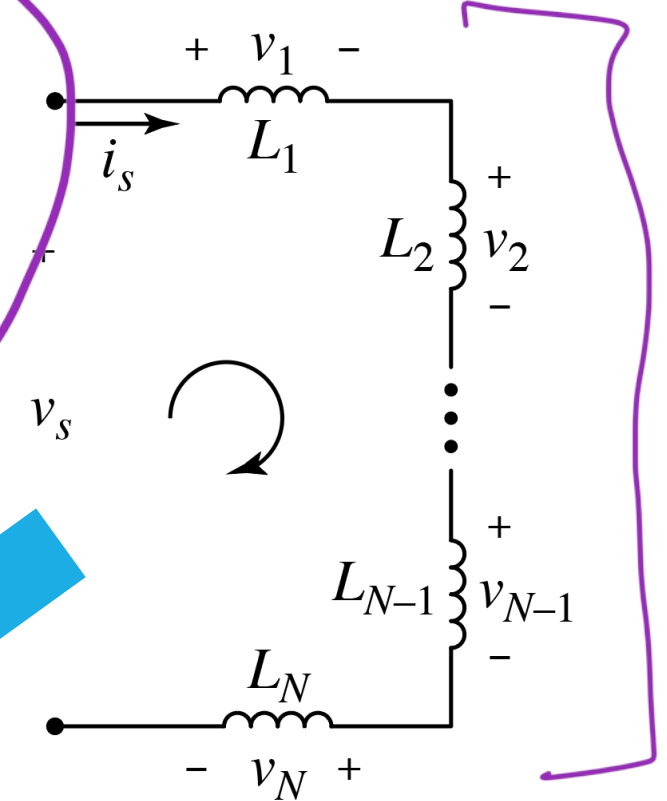
$$v_s = (L_1 + L_2 + \dots + L_N) \frac{di_s}{dt} = L_{eq} \frac{di_s}{dt}$$

$$L_{eq} = L_1 + L_2 + \dots + L_N$$

Equivalent circuit with the same initial current



Series Inductors

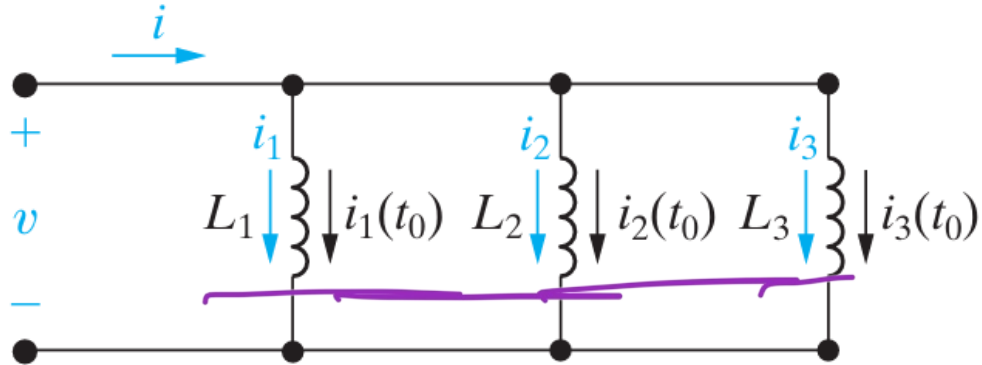


Handwritten note in purple:
Circuit diagram
do not



Series-Parallel Combinations of Inductance and Capacitance

Parallel Inductors



$$i_1(t) = \frac{1}{L_1} \int_{t_0}^t v(t) dt + i_1(t_0)$$

$$i_2(t) = \frac{1}{L_2} \int_{t_0}^t v(t) dt + i_2(t_0)$$

$$i_3(t) = \frac{1}{L_3} \int_{t_0}^t v(t) dt + i_3(t_0)$$

Apply KCL:

$$i = i_1 + i_2 + i_3$$

$$i(t) = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \right) \int_{t_0}^t v(t) dt + (i_1(t_0) + i_2(t_0) + i_3(t_0))$$

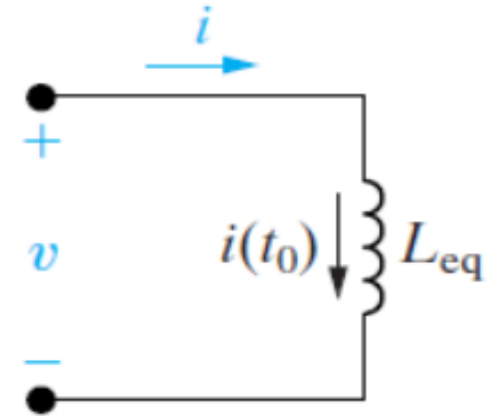
$$i(t) = \left(\frac{1}{L_{eq}} \right) \int_{t_0}^t v(t) dt + (i(t_0))$$

and



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

$$i(t_0) = i_1(t_0) + i_2(t_0) + i_3(t_0)$$



Series-Parallel Combinations of Inductance and Capacitance

Capacitors in Series

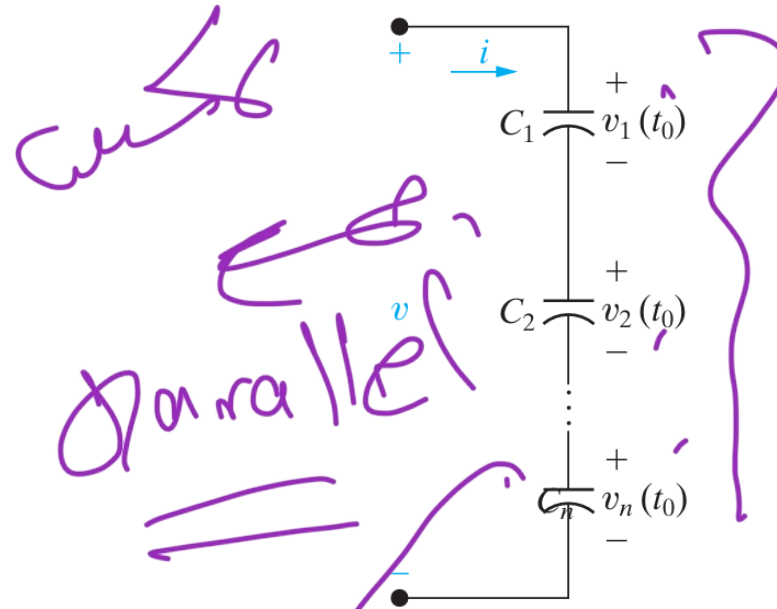
$$v = v_1 + v_2 + \dots + v_N$$

$$v_1(t) = \frac{1}{C_1} \int_{t_0}^t i(t) dt + v_1(t_0)$$

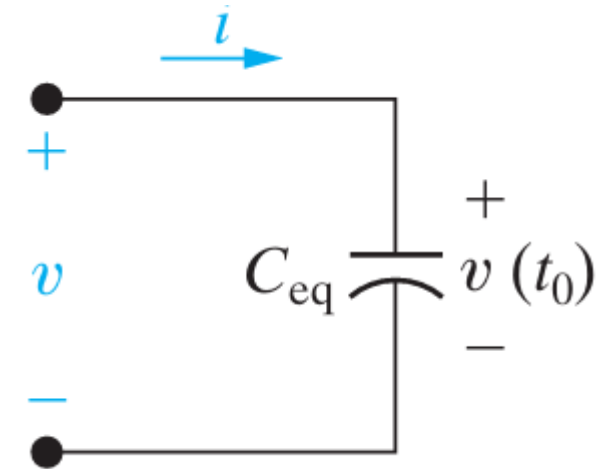
$$v_2(t) = \frac{1}{C_2} \int_{t_0}^t i(t) dt + v_2(t_0)$$

$$v(t) = \left(\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N} \right) \int_{t_0}^t i(t) dt + (v_1(t_0) + \dots + v_N(t_0))$$

$$v(t) = \frac{1}{C_{eq}} \int_{t_0}^t i(t) dt + v(t_0)$$



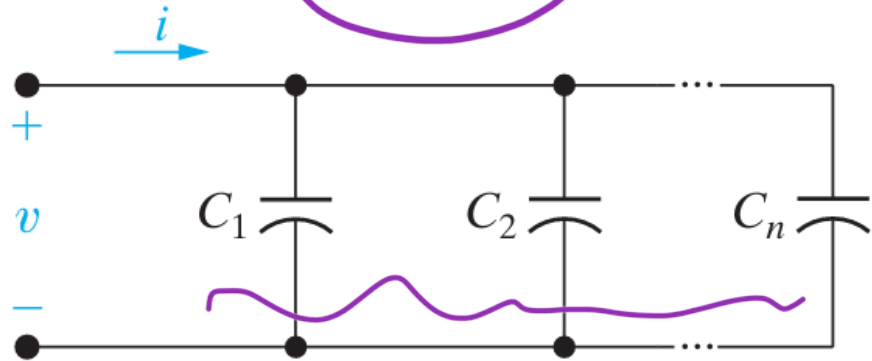
Equivalent circuit



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}$$

Series-Parallel Combinations of Inductance and Capacitance

Capacitors in Parallel

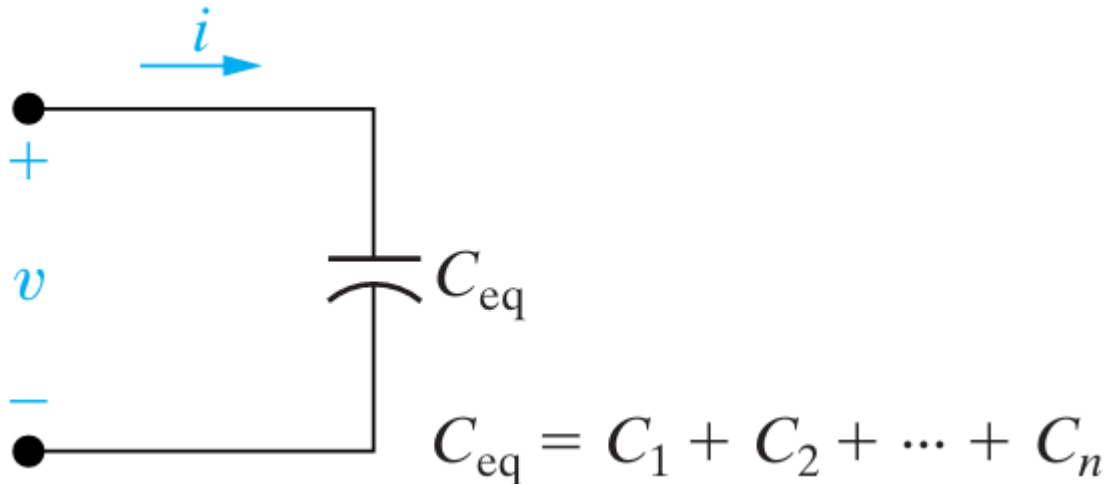


$$i = i_1 + i_2 + \dots + i_N$$

$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + \dots + C_N \frac{dv}{dt}$$

$$i = (C_1 + C_2 + \dots + C_N) \frac{dv}{dt} = C_{eq} \frac{dv}{dt}$$

Equivalent circuit

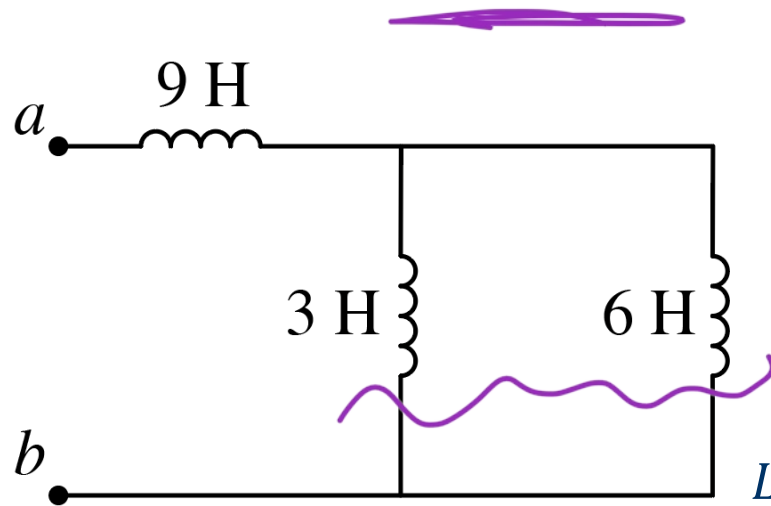


$$C_{eq} = C_1 + C_2 + \dots + C_N$$

Series Z_{eq}

Series-Parallel Combinations of Inductance and Capacitance

What is equivalent inductance between nodes *a* and *b*?



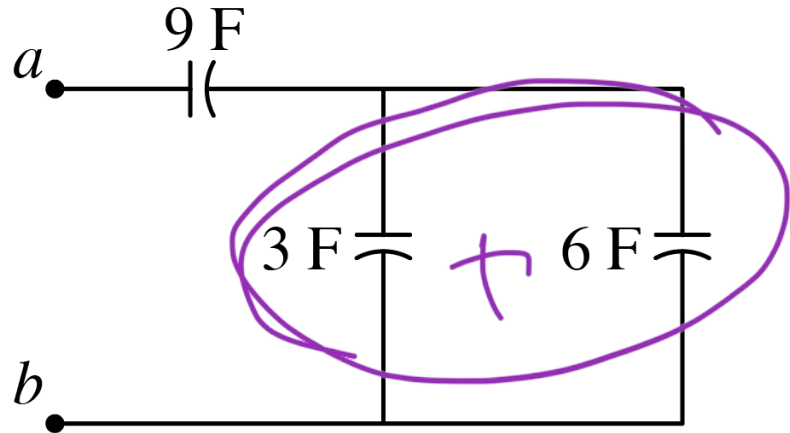
$$L_{eq} = ?$$

$$L_{eq} = 9 + \underline{(3 \parallel 6)} = 9 + \underline{2} = \underline{11} \text{ H}$$



Series-Parallel Combinations of Inductance and Capacitance

What is equivalent capacitance between nodes *a* and *b*?

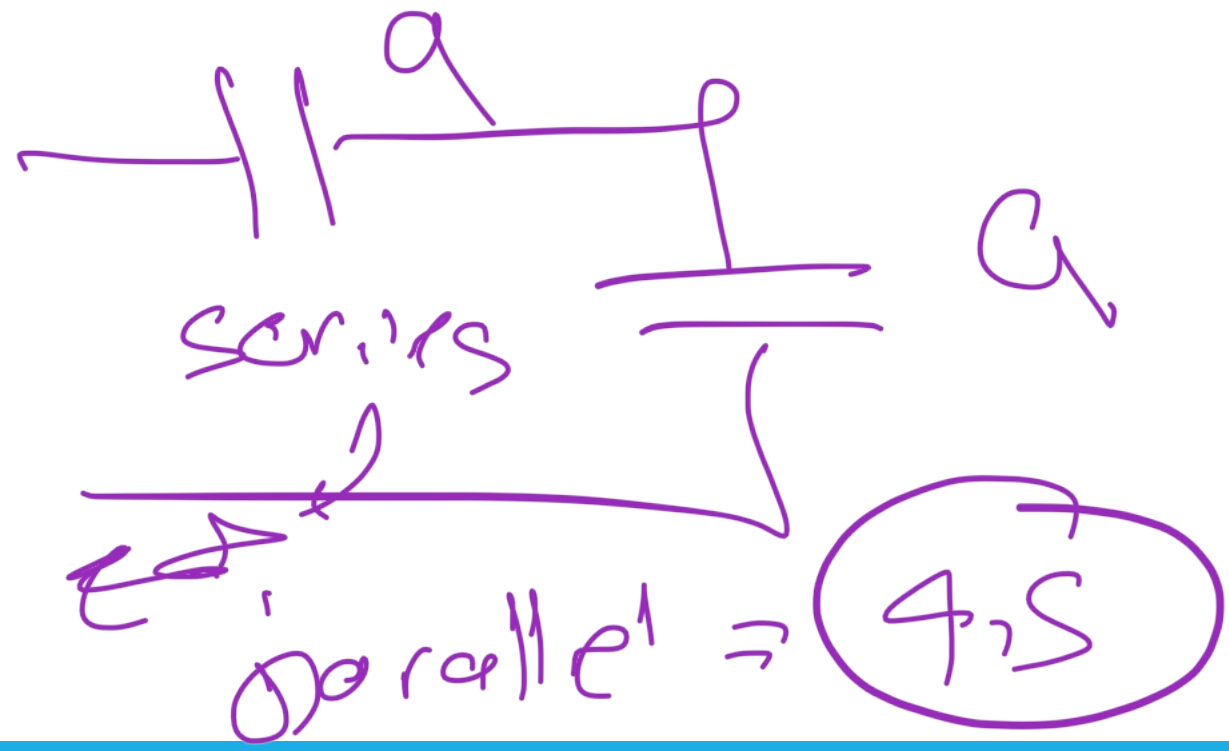


parallel → series
 $C_{eq} = ?$

$$C_{eq} = 9 + (3 \parallel 6) = 9 + 9 = 4.5 \text{ F}$$

↓

$$\frac{1}{\frac{1}{9} + \frac{1}{9}} = \frac{81}{18}$$



Tutorial (Capacitor)

Example 6-4: Determining Current, Voltage, Power, and Energy for a Capacitor

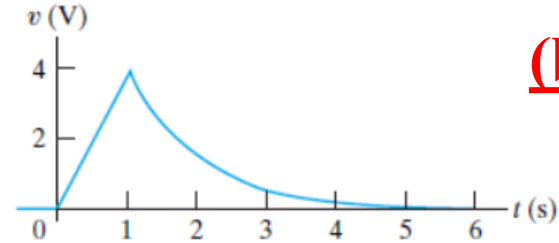
The voltage pulse described by the following equations is impressed across the terminals of a $0.5 \mu\text{F}$ capacitor:

$$v(t) = \begin{cases} 0, & t \leq 0 \text{ s;} \\ 4t \text{ V}, & 0 \text{ s} \leq t \leq 1 \text{ s;} \\ 4e^{-(t-1)} \text{ V}, & t \geq 1 \text{ s.} \end{cases}$$

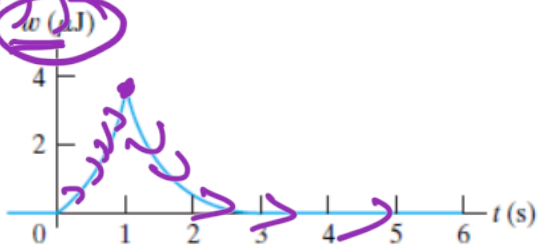
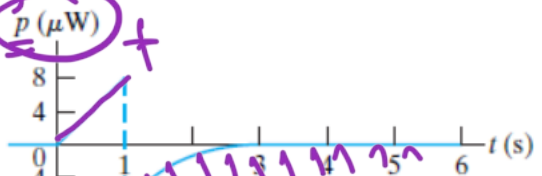
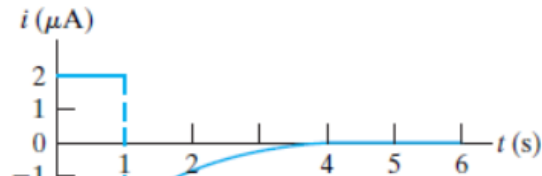
- Derive the expressions for the capacitor current, power, and energy.
- Sketch the voltage, current, power, and energy as functions of time. Line up the plots vertically.
- Specify the interval of time when energy is being stored in the capacitor.
- Specify the interval of time when energy is being delivered by the capacitor.
- Evaluate the integrals

$$\int_0^1 p \, dt \quad \text{and} \quad \int_1^{\infty} p \, dt$$

and comment on their significance.



(b)



Stored Energy
0 → 1

Extracted Energy
1 → ∞

Tutorial (Capacitor)



power bank

Example 6-5:

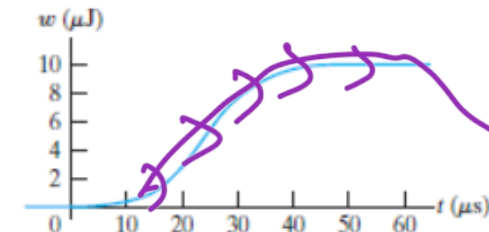
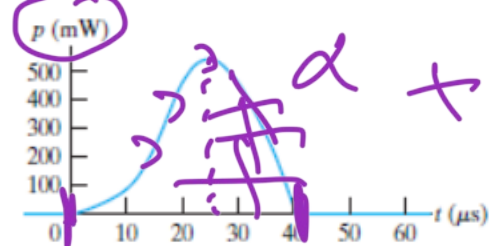
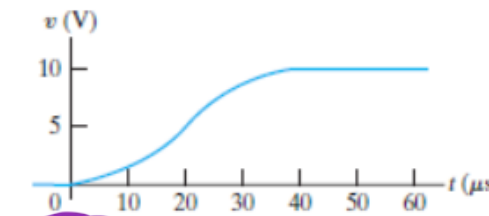
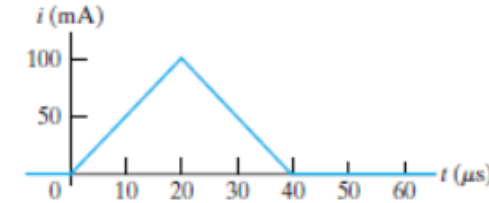
Finding v , p , and w Induced by a Triangular Current Pulse for a Capacitor

An uncharged $0.2 \mu\text{F}$ capacitor is driven by a triangular current pulse. The current pulse is described by

$$i(t) = \begin{cases} 0, & t \leq 0; \\ 5000t \text{ A}, & 0 \leq t \leq 20 \mu\text{s}; \\ 0.2 - 5000t \text{ A}, & 20 \leq t \leq 40 \mu\text{s}; \\ 0, & t \geq 40 \mu\text{s}. \end{cases}$$

- Derive the expressions for the capacitor voltage, power, and energy for each of the four time intervals needed to describe the current.
- Plot i , v , p , and w versus t . Align the plots as specified in the previous examples.
- Why does a voltage remain on the capacitor after the current returns to zero?

shared data



(b)

Homework :Due Date: One Week

Chapter (6) in textbook, solve the following problems and you will deliver it hard copy.

Problem : 6.1

problem : 6.3

problem : 6.8

problem : 6.22

problem : 6.24

problem : 6.27

problem : 6.32

problem : 6.34

problem: 6.35