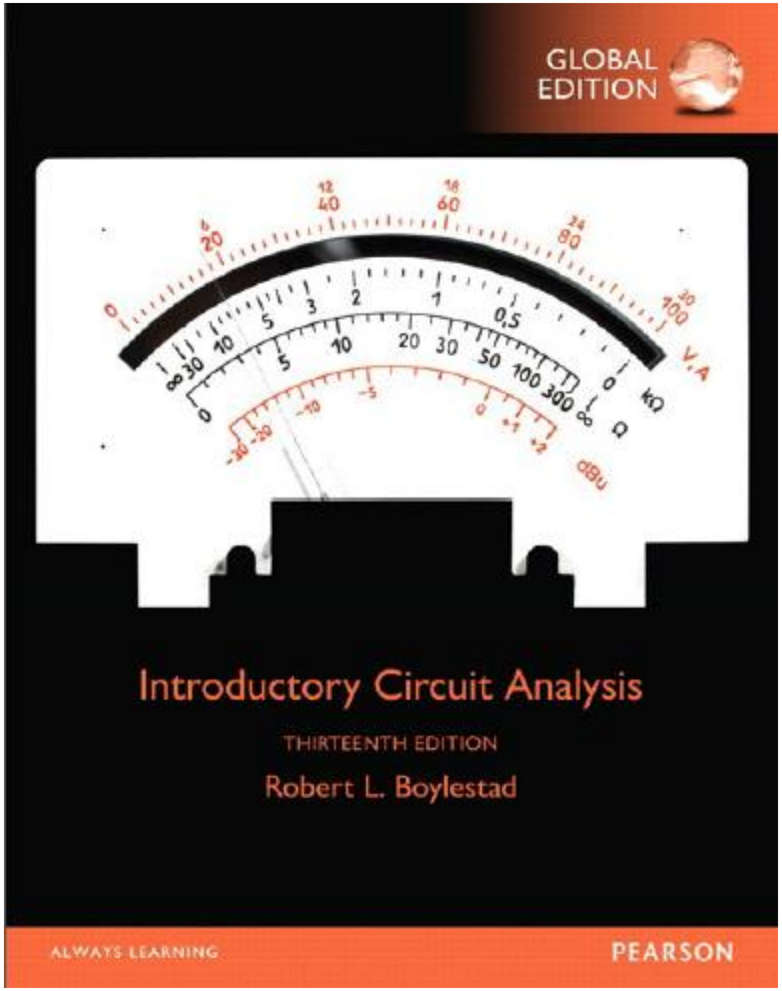
EEE 101: Electrical Circuits I

WEEK 3

Chapter 6

Parallel Circuits

Textbook



Textbook:

Robert L. Boylestad “ Introductory Circuit Analysis “ Twelfth Edition Pearson

[ISBN 13:-978-1-292-09895-1]

Chapter 6: DC Parallel Circuits

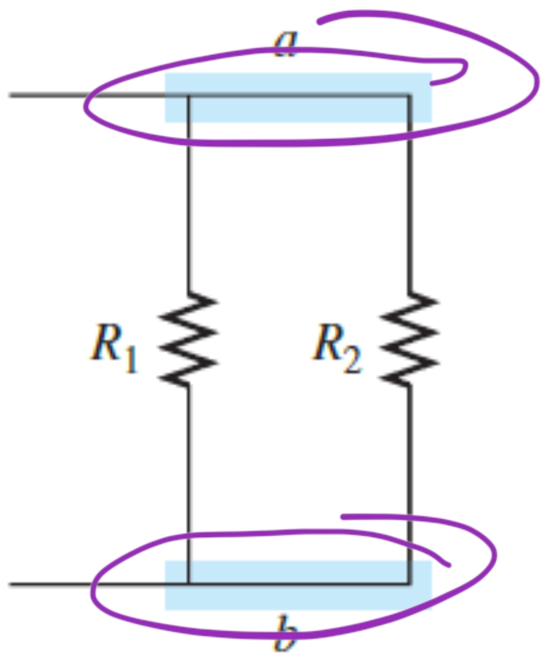
Topics to be covered in this chapter

[illegible]

6.2 Parallel Resistors

The term parallel is used so often to describe a physical arrangement between two elements that most individuals are aware of its general characteristics.
In general,
two elements, branches, or circuits are in parallel if they have two points in common.

For instance, in Fig. 6.1(a), the two resistors are in parallel because they are connected at points *a* and *b*.



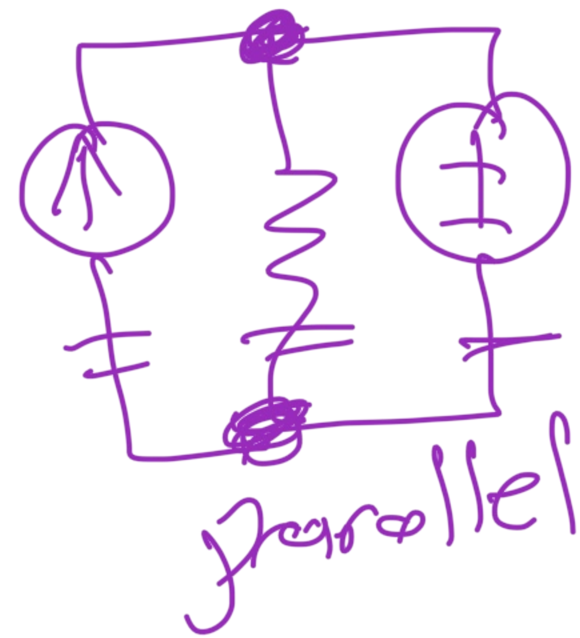


Fig. 6.1 (a): Parallel resistors

Both ends were *not* connected as shown, the resistors would not be in parallel.
In Fig. 6.1(b), resistors R_1 and R_2 are in parallel because they again have points *a* and *b* in common. R_1 is not in parallel with R_3 because they are connected at only one point (*b*). Further, R_1 and R_3 are not in series because a third connection appears at point *b*.

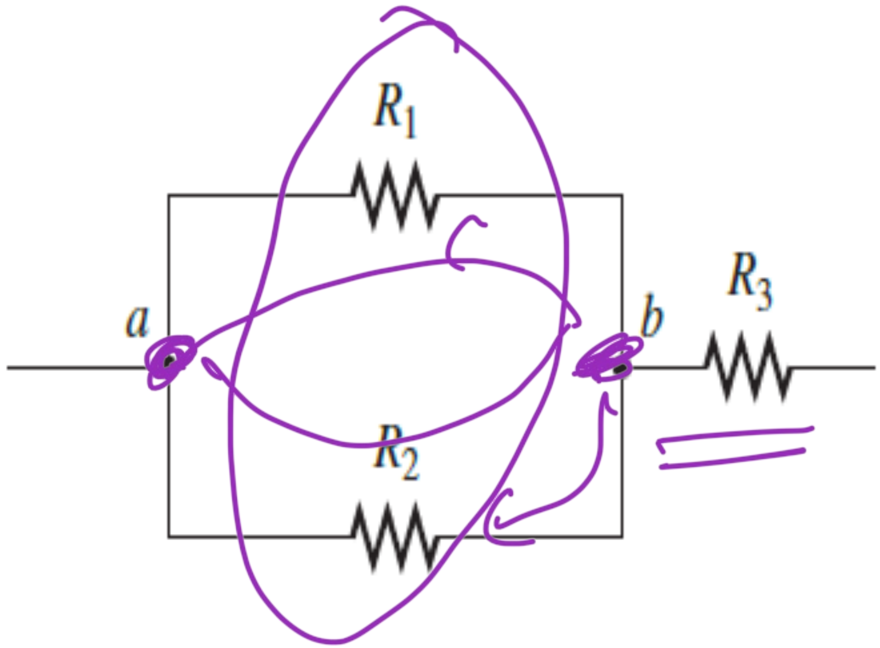
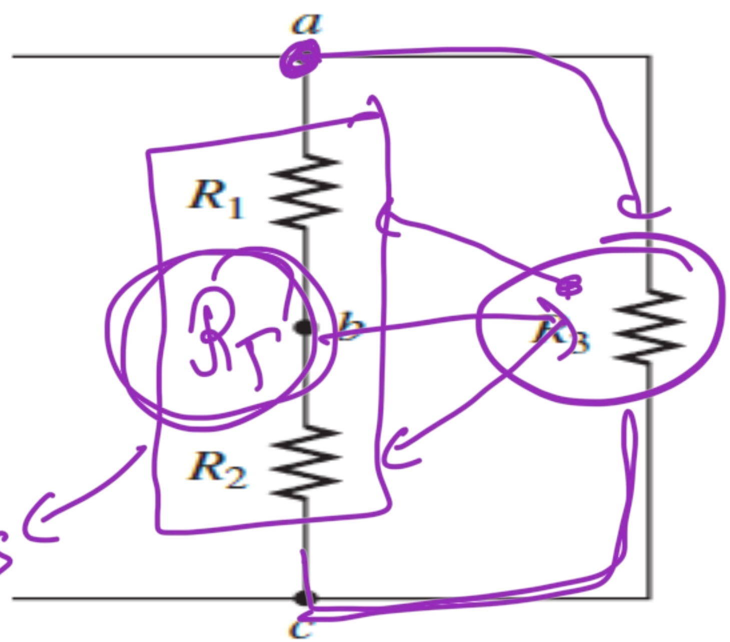


Fig. 6.1 (b): R1 and R2 are in parallel

The same can be said for resistors R_2 and R_3. In Fig. 6.1(c), resistors R_1 and R_2 are in series because they have only one point in common that is not connected elsewhere in the network. Resistors R_1 and R_3 are not in parallel because they have only point *a* in common. In addition, they are not in series because of the third connection to point *a*. The same can be said for resistors R_2 and R_3. In a broader context, it can be said that the series combination of resistors R_1 and R_2 is in parallel with resistor R_3

R_3 not parallel to R_1 to R_2



series

R_T parallel R_3

Fig. 6.1 (c) : R3 is in parallel with R3 is in parallel with the series combination of R1 and R2. the series combination of R1 and R2.

On schematics, the parallel combination can appear in a number of ways, as shown in Fig. 6.2. In each case, the three resistors are in parallel. They all have points *a* and *b* in common.

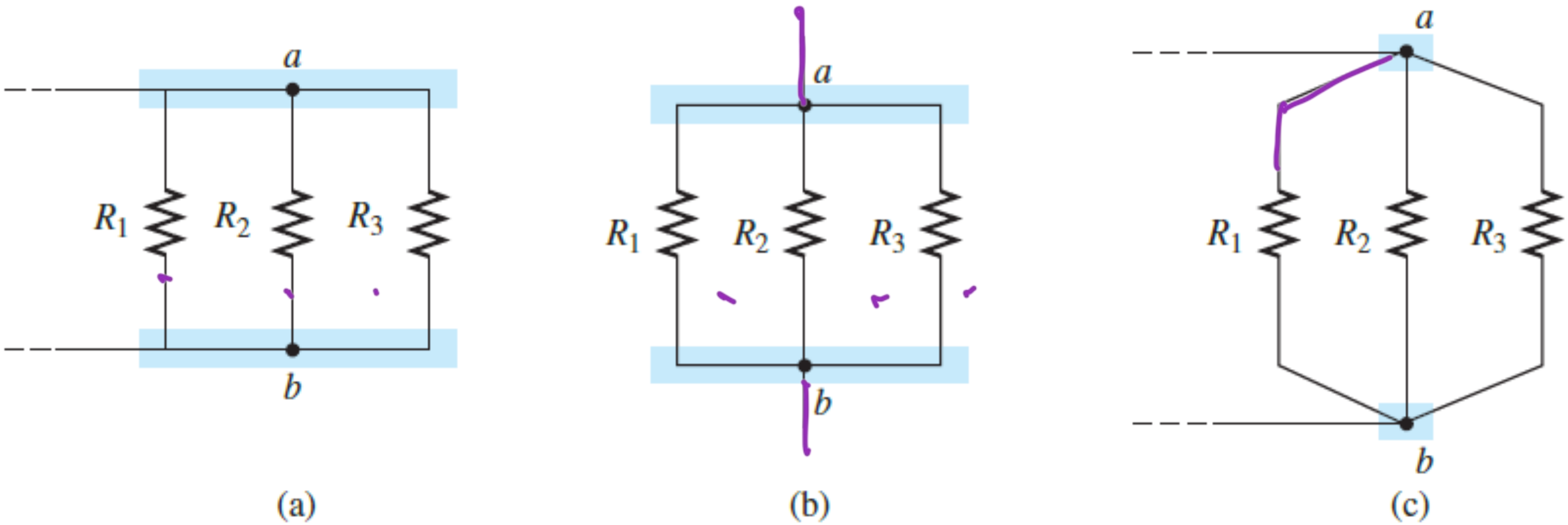


FIG. 6.2
Schematic representations of three parallel resistors.

لازم تقلبه

For resistors in parallel as shown in Fig. 6.3, the total resistance is determined from the following equation:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N} \tag{6.1}$$

OR

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}} \tag{6.3}$$

أفضل

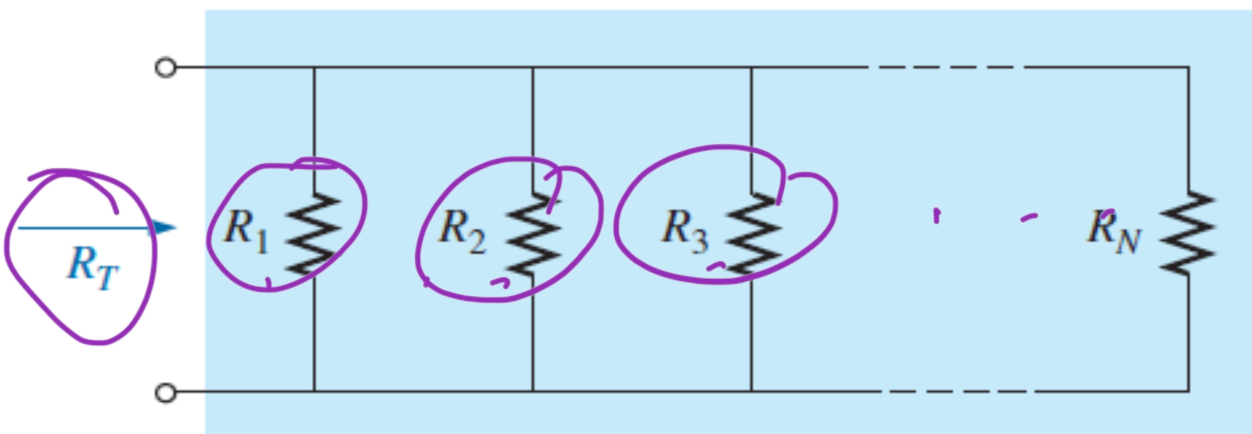


FIG. 6.3
Parallel combination of resistors.

Example 6.1 : Find the total resistance of the configuration in Fig. 6.6.

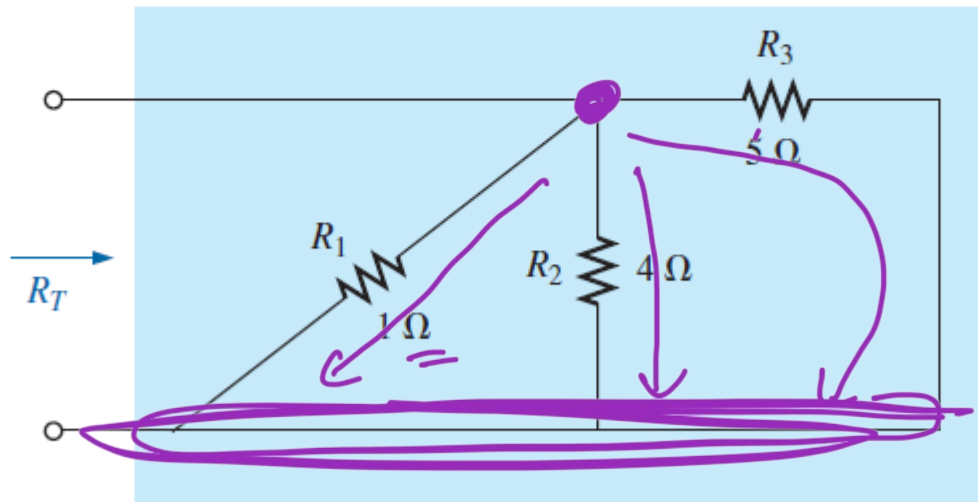


FIG. 6.6

Solution:

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{1\ \Omega} + \frac{1}{4\ \Omega} + \frac{1}{5\ \Omega}} = 0.689\ \Omega$$

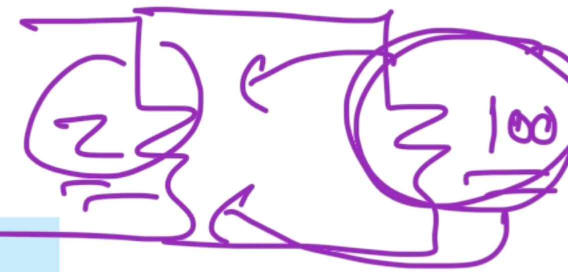
$$R_T = \frac{1}{\frac{1}{1} + \frac{1}{4} + \frac{1}{5}}$$

$$\frac{20}{29}\ \Omega$$

$$= 0{,}689\ \Omega$$

Example 6.2 : What is the effect of adding another resistor of 100 in parallel with the parallel resistors as shown in Fig. 6.8?

$$R = \frac{1}{\frac{1}{3} + \frac{1}{6}} = 2\,\Omega$$



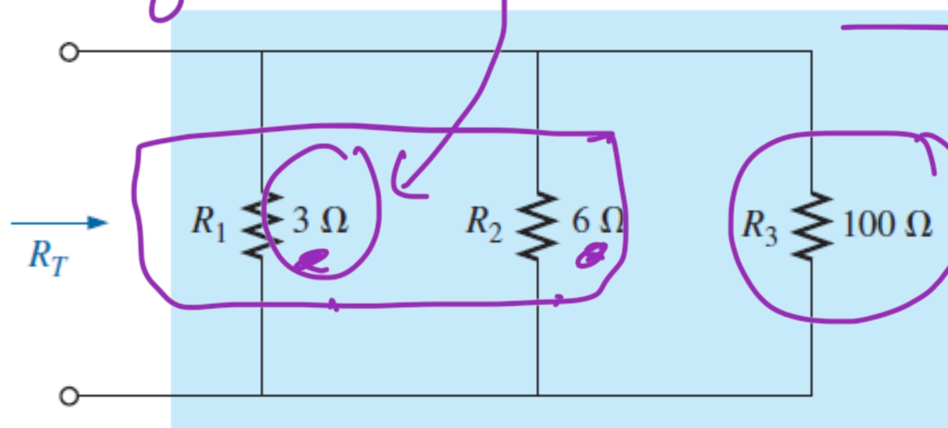


FIG. 6.8

Adding a parallel 100 Ω resistor to the network in Fig. 6.4.

Solution:

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{3\,\Omega} + \frac{1}{6\,\Omega} + \frac{1}{100\,\Omega}} = \boxed{1.96\,\Omega} = \frac{100}{51}$$

Series : $R_T = R_1 + R_2 + R_3 + \dots$

ل R_T تكون أكبر من أكبر مقاومة

$R_T = 1+2+3 = 6\Omega$

Parallel : $R_T = \dfrac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots}$

ل R_T تكون أصغر من أصغر مقاومة

Example 6.3 : Find the total resistance for the configuration in Fig. 6.10.

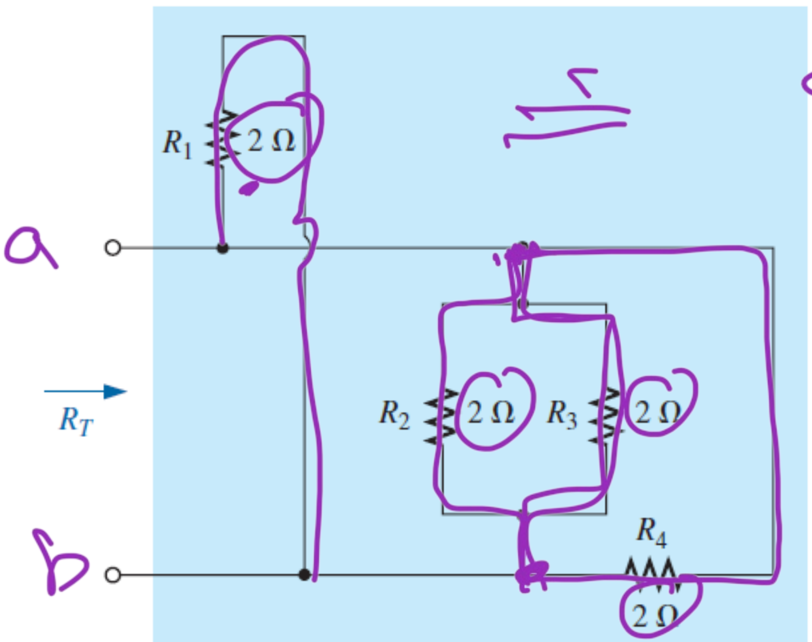


FIG. 6.10
Parallel configuration for Example 6.6.

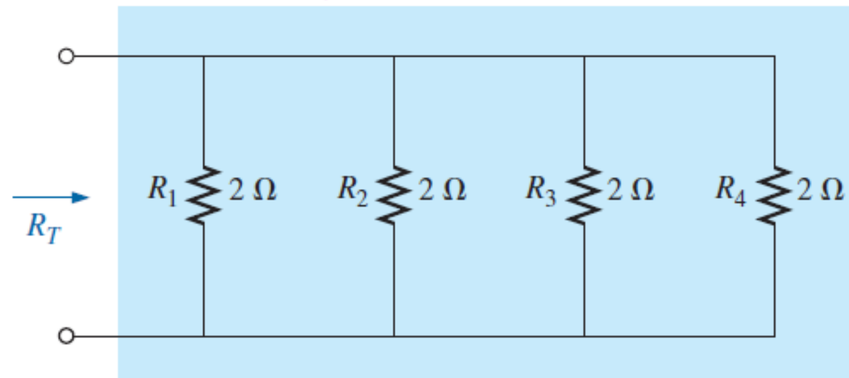


FIG. 6.11
Network in Fig. 6.10 redrawn.

Solution: ***The total resistance of N parallel resistors of equal value is the resistance of one resistor divided by the number (N) of parallel resistors.***

$$\boxed{R_T = \frac{R}{N}}$$

$$R_T = \frac{R}{N} = \frac{2\ \Omega}{4} = \mathbf{0.5\ \Omega}$$

$$\boxed{R_T = \frac{R}{N}} = \frac{2}{4} = 0{,}5\ \Omega$$

في حال كانت
كل المقاومات متساوية

Special Case: Two Parallel Resistors

The total resistance of two parallel resistors is simply the product of their values divided by their sum.

$$\boxed{R_T = \frac{R_1 R_2}{R_1 + R_2}} \qquad (6.5)$$

ضربهم

جمعهم

Example 6.4 : Find the total resistance for the configuration in Fig. 6.4.

Solution:

$$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \frac{18}{9}\ \Omega = \mathbf{2\ \Omega}$$

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{1}{3\ \Omega} + \frac{1}{6\ \Omega}} = \mathbf{2\ \Omega}$$

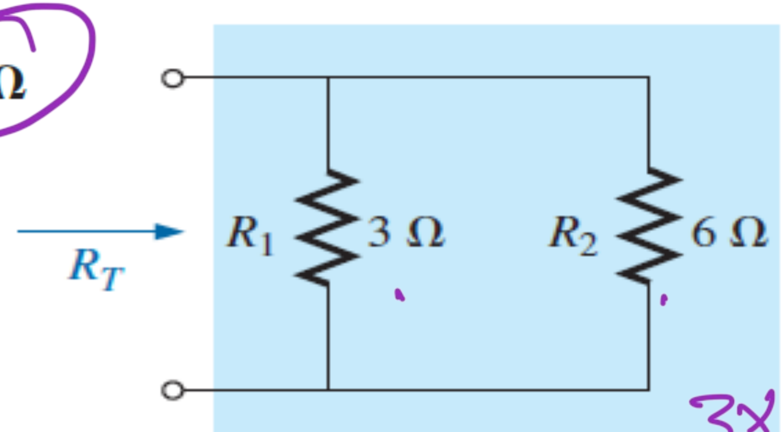


FIG. 6.4

$$\frac{3 \times 6}{3+6} = \frac{18}{9} = 2\Omega$$

Example 6.5 : Determine the value of R_2 in Fig. 6.15 to establish a total resistance of 9kΩ.

Solution:

$$\boxed{R_T = \frac{R_1 R_2}{R_1 + R_2}}$$

$$R_T(R_1 + R_2) = R_1 R_2$$

$$R_T R_1 + R_T R_2 = R_1 R_2$$

$$R_T R_1 = R_1 R_2 - R_T R_2$$

$$R_T R_1 = (R_1 - R_T) R_2$$

and

$$R_2 = \frac{R_T R_1}{R_1 - R_T}$$

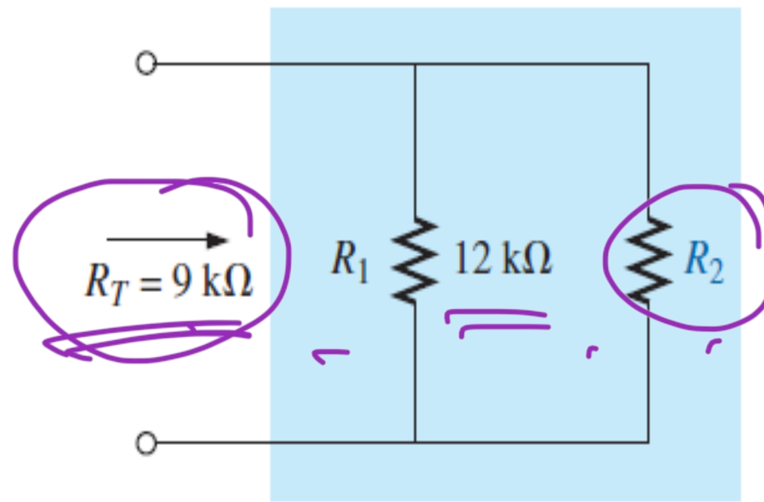


FIG. 6.15

Substituting values gives

$$R_2 = \frac{(9\ \text{k}\Omega)(12\ \text{k}\Omega)}{12\ \text{k}\Omega - 9\ \text{k}\Omega} = \frac{108}{3}\text{k}\Omega = \mathbf{36\ k\Omega}$$

سؤال على الأول حاسم

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$\boxed{4 = \frac{1}{\frac{1}{12} + \frac{1}{x}}}$$

$$\boxed{x = R_2 = 36\ k\Omega}$$

Example 6.6 : Determine the value of R_1 , R_2 , and R_3 in Fig. 6.16 to establish a total resistance of 16kΩ. Consider $R_2 = 2R_1$ and $R_3 = 2R_2$

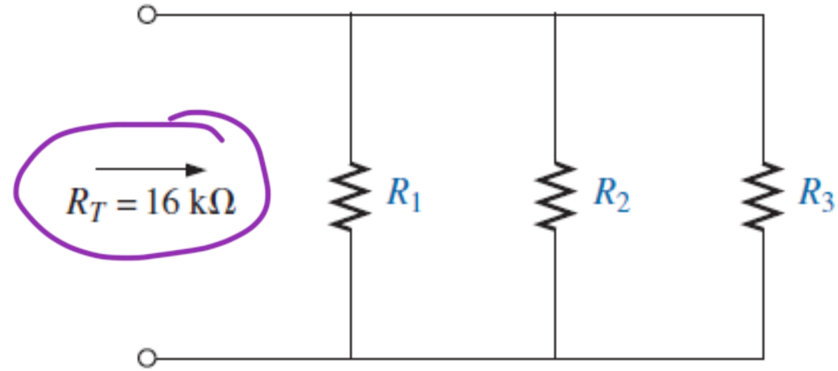


FIG. 6.16

Solution:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

However, $R_2 = 2R_1$ and $R_3 = 2R_2 = 2(2R_1) = 4R_1$

so that
$$\frac{1}{16\ \text{k}\Omega} = \frac{1}{R_1} + \frac{1}{2R_1} + \frac{1}{4R_1}$$

and
$$\frac{1}{16\ \text{k}\Omega} = \frac{1}{R_1} + \frac{1}{2}\left(\frac{1}{R_1}\right) + \frac{1}{4}\left(\frac{1}{R_1}\right)$$

or
$$\frac{1}{16\ \text{k}\Omega} = 1.75\left(\frac{1}{R_1}\right)$$

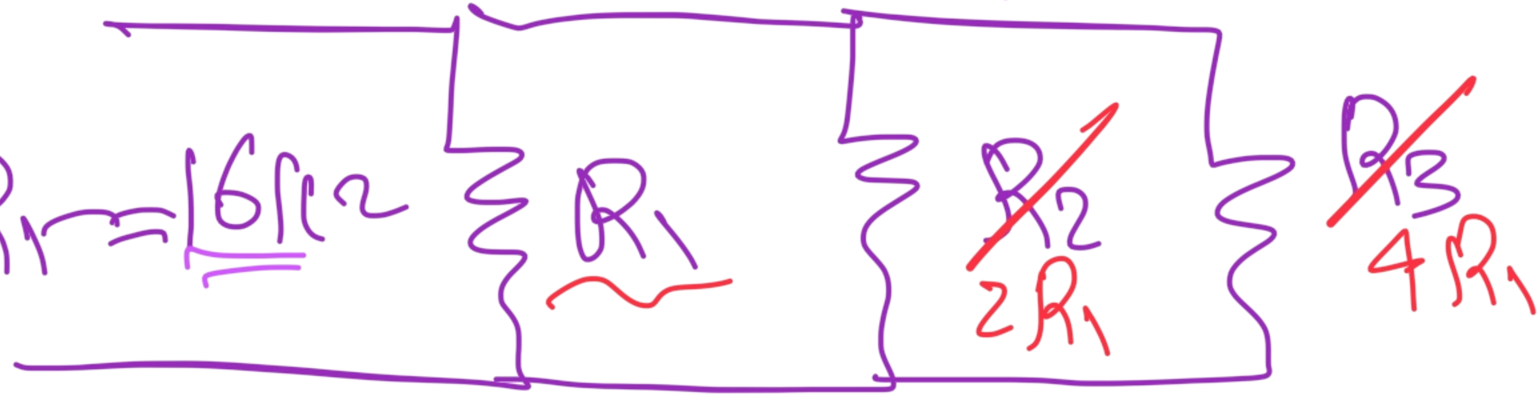
resulting in $R_1 = 1.75(16\ \text{k}\Omega) = \mathbf{28\ k\Omega}$

so that $R_2 = 2R_1 = 2(28\ \text{k}\Omega) = \mathbf{56\ k\Omega}$

and $R_3 = 2R_2 = 2(56\ \text{k}\Omega) = \mathbf{112\ k\Omega}$

$R_1 = x$

$R_1 = 16\Omega$



$$R_2 = \boxed{2R_1}$$

$$R_3 = 2R_2 = 2(2R_1) = 4R_1$$

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$16 = \frac{1}{\frac{1}{x} + \frac{1}{2x} + \frac{1}{4x}}$$

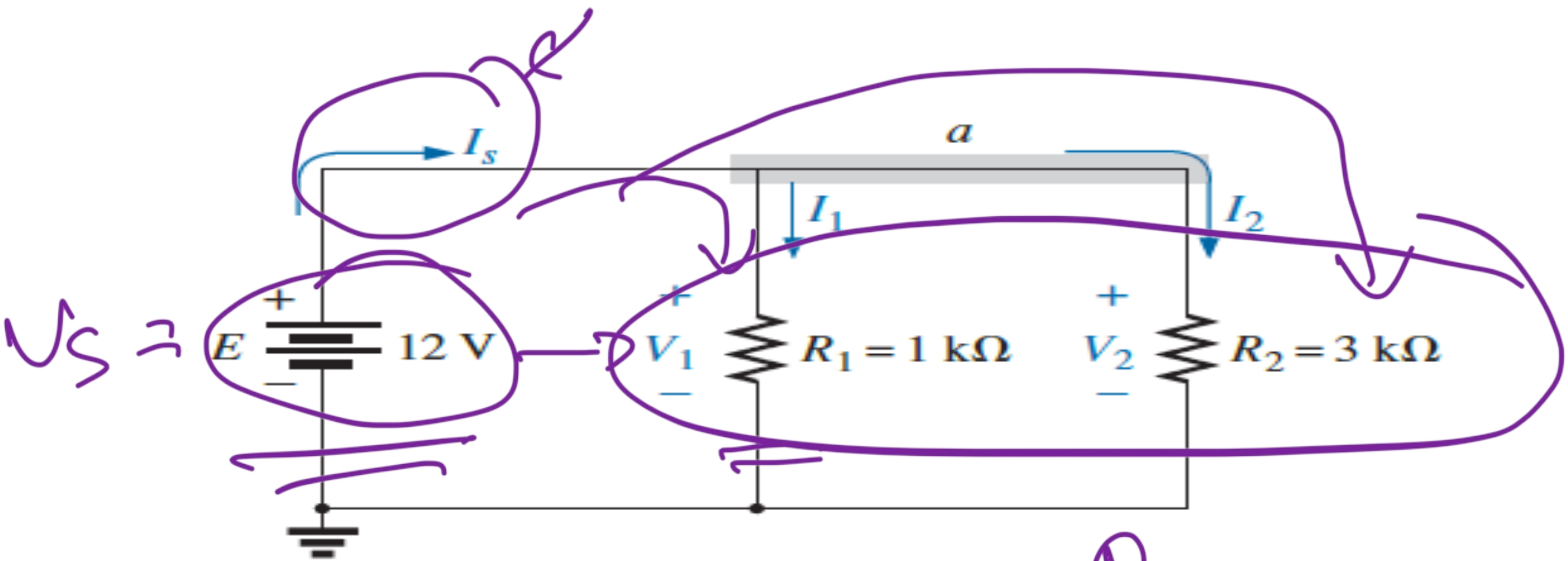
$$\Rightarrow x = R_1 = 28\,k\Omega$$

$$R_2 = 2 \times 28 = 56\,k\Omega$$

$$R_3 = 2 \times 56 = 112\,k\Omega$$

6.3 Parallel Circuits

A parallel circuit can now be established by connecting a supply across a set of parallel resistors as shown in Fig. 6.18. The positive terminal of the supply is directly connected to the top of each resistor, while the negative terminal is connected to the bottom of each resistor. Therefore, it should be quite clear that the applied voltage is the same across each resistor.



R_T

FIG. 6.18
Parallel network.

$$I_s = \frac{E}{R_T}$$

Parallel

$I_s = I_1 + I_2$ التيار بتوزع

$V_s = V_1 = V_2$ الجهد بيبقى ثابت

Properties of DC Parallel Circuits

a. The voltage is always the same across parallel elements.

$$\boxed{V_1 = V_2 = E} \qquad (6.6)$$

b. The source current *Is* can then be determined using Ohm's law:

$$\boxed{I_s = \frac{E}{R_T}} \qquad (6.7)$$

Since the voltage is the same across parallel elements, the current through each resistor can also be determined using Ohm's law. That is,

$$\boxed{I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} \quad \text{and} \quad I_2 = \frac{V_2}{R_2} = \frac{E}{R_2}} \qquad (6.8)$$

c. For single-source parallel networks, the source current (I_s) is always equal to the sum of the individual branch currents.

$$\boxed{I_s = I_1 + I_2} \qquad (6.9)$$

Example 6.7 : For the parallel network in Fig. 6.22:

a. Find the total resistance.

b. Calculate the source current. I_s

c. Determine the current through each parallel branch.

d. Show that Eq. (6.9) is satisfied.

Parallel

Solution:

$E = V_1 = V_2$

a. Using Eq. (6.5) gives

$$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(9\ \Omega)(18\ \Omega)}{9\ \Omega + 18\ \Omega} = \frac{162}{27}\ \Omega = \mathbf{6\ \Omega}$$

b. Applying Ohm's law gives

$$I_s = \frac{E}{R_T} = \frac{27\ \text{V}}{6\ \Omega} = \mathbf{4.5\ A}$$

c. Applying Ohm's law gives

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{27\ \text{V}}{9\ \Omega} = \mathbf{3\ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{27\ \text{V}}{18\ \Omega} = \mathbf{1.5\ A}$$

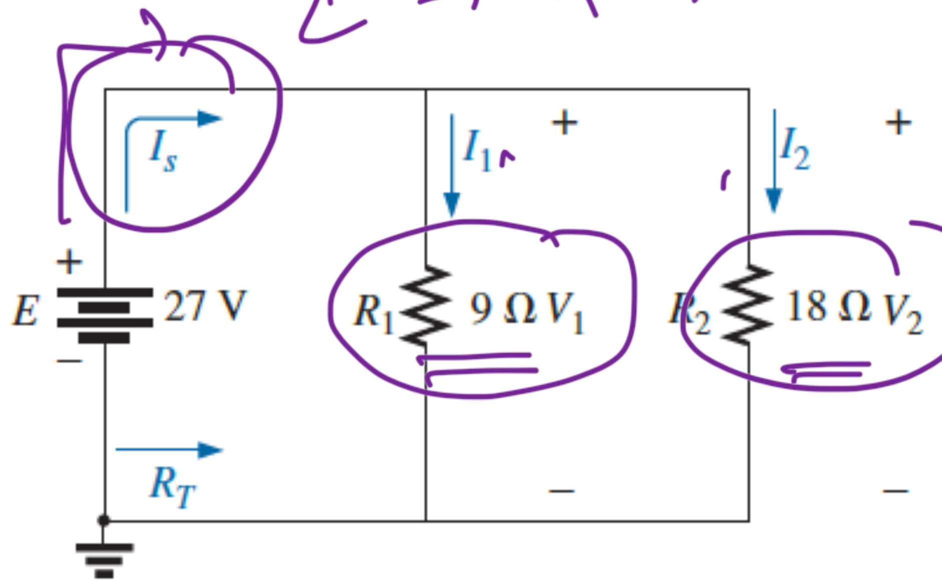


FIG. 6.22

Parallel network for Example 6.12.

d. Substituting values from parts (b) and (c) gives

$$I_s = \mathbf{4.5\ A} = I_1 + I_2 = 3\ \text{A} + 1.5\ \text{A} = \mathbf{4.5\ A} \quad \text{(checks)}$$

$$\boxed{I_s = I_1 + I_2}$$

a) $R_T = \dfrac{1}{\frac{1}{9}+\frac{1}{18}} = 6\ \Omega$

b) $I_S = \dfrac{E}{R_T} = \dfrac{27}{6} = \underline{\underline{4{,}5}}\ A$

c) $I_1 = \dfrac{E}{R_1} = \dfrac{27}{9} = \underline{\underline{3}}\ A$

$I_2 = \dfrac{E}{R_2} = \dfrac{27}{18} = \underline{\underline{1{,}5}}\ A$

Example 6.7: **For the parallel network in Fig. 6.23.**

a. Find the total resistance.

b. Calculate the source current.

c. Determine the current through each branch.

$\rightarrow \frac{E}{R_T} = \frac{24}{\bigcirc} = 2{,}529\ A$

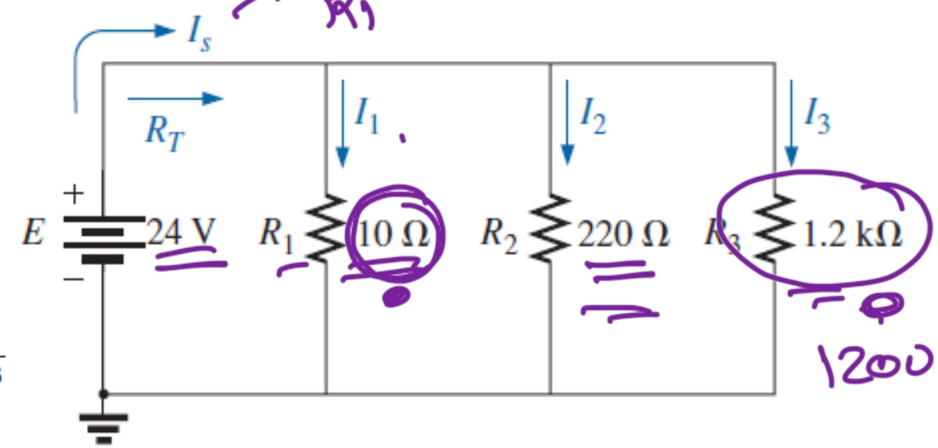


FIG. 6.23

Solution:

a. Applying Eq. (6.3) gives

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{10\ \Omega} + \frac{1}{220\ \Omega} + \frac{1}{1.2\ k\Omega}}$$

$$= \frac{1}{100 \times 10^{-3} + 4.545 \times 10^{-3} + 0.833 \times 10^{-3}} = \frac{1}{105.38 \times 10^{-3}}$$

$$R_T = \mathbf{9.49\ \Omega}$$

$$R_T = \frac{1}{\frac{1}{10} + \frac{1}{220} + \frac{1}{1200}} = 9{,}49\ \Omega = \frac{13200}{1391}$$

b. Using Ohm's law gives

$$I_s = \frac{E}{R_T} = \frac{24\ V}{9.49\ \Omega} = \mathbf{2.53\ A}$$

c. Applying Ohm's law gives

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{24\ V}{10\ \Omega} = \boxed{\mathbf{2.4\ A}}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{24\ V}{220\ \Omega} = \mathbf{0.11\ A}$$

$$I_3 = \frac{V_3}{R_3} = \frac{E}{R_3} = \frac{24\ V}{1.2\ k\Omega} = \mathbf{0.02\ A}$$

في حالة series " جهد "

أكبر مقاومة رح يكون عليها أكبر جهد

في حالة parallel = " تيار "

أصغر مقاومة رح يكون عليها أكبر تيار,

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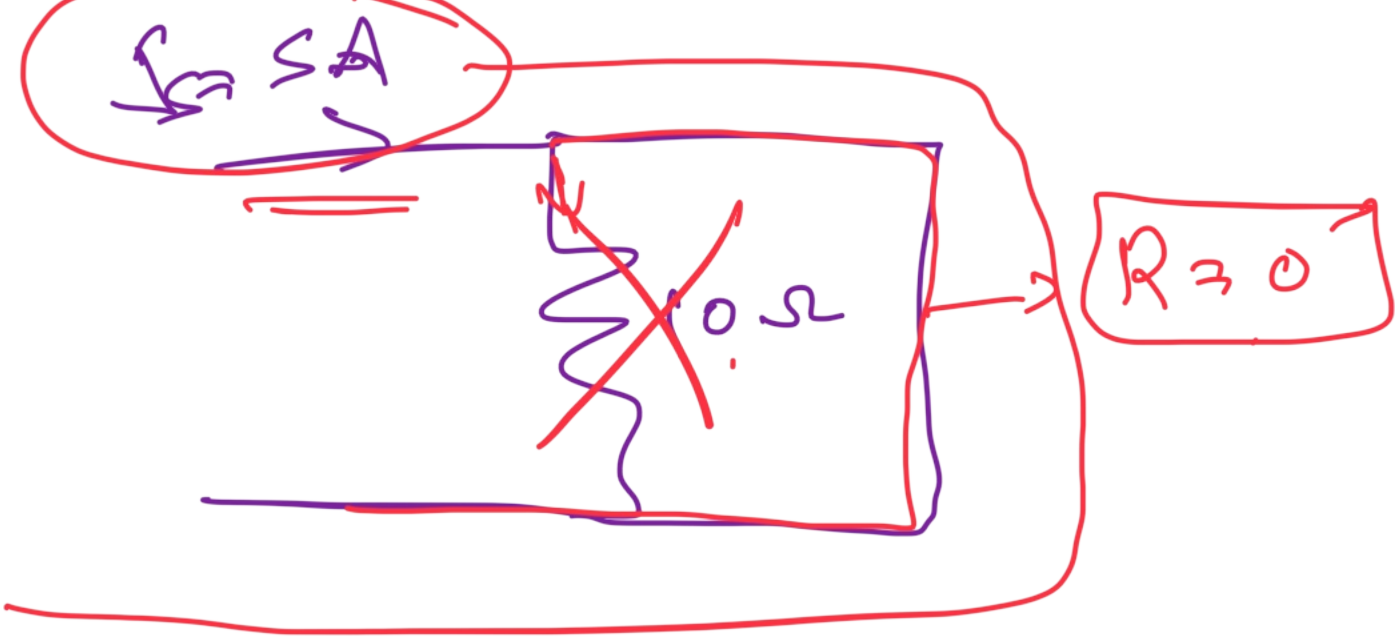
* التيار دائما يختار المقاومة الأصغر ليمر من خلالها.


5A
0 Ω
R = 0

6.4 Power Distribution in a Parallel Circuit

For any network composed of resistive elements, the power applied by the source will equal that dissipated by the resistive elements.

For the parallel circuit in Fig. 6.28:

$$\boxed{P_E = P_{R_1} + P_{R_2} + P_{R_3}} \tag{6.10}$$

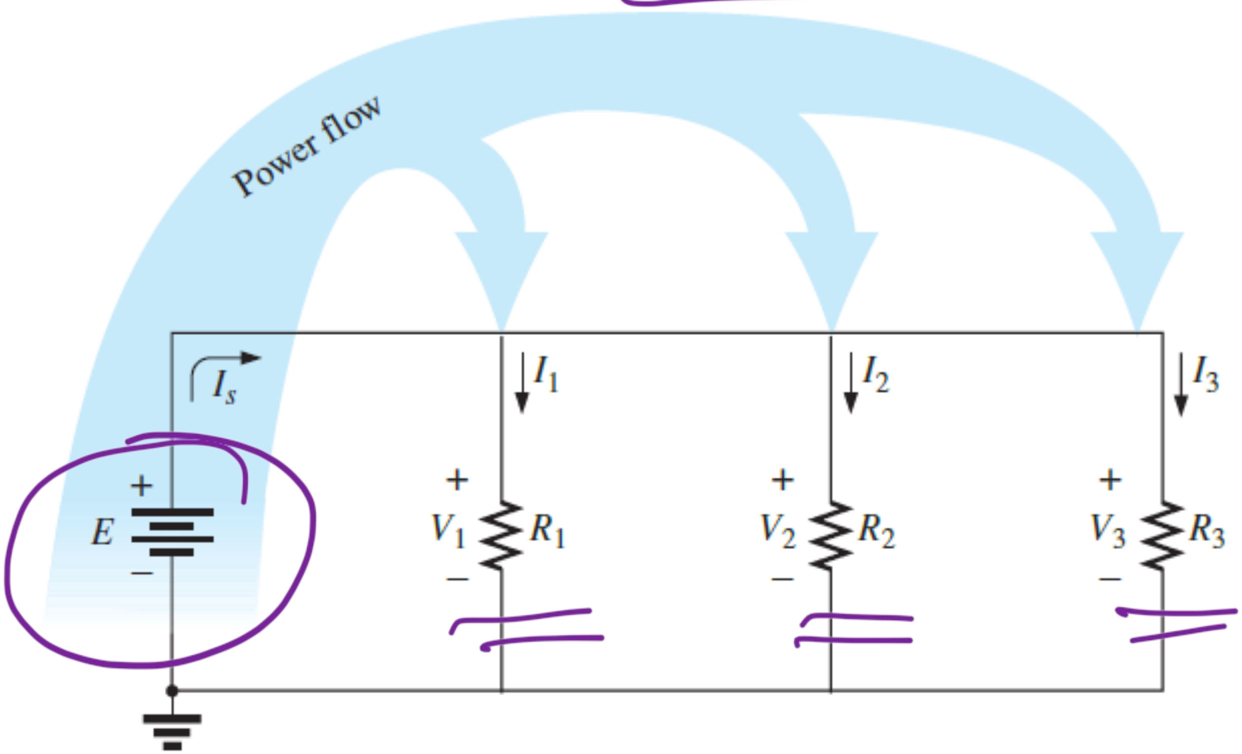


FIG. 6.28
Power flow in a dc parallel network.

The power delivered by the supply can be determined using

$$\boxed{P_E = EI_s} \quad \text{(watts, W)} \qquad \textbf{(6.11)}$$

The power dissipated by the resistive elements can be determined by any of the following forms (shown for resistor R_1 only):

$$\boxed{P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1}} \quad \text{(watts, W)} \qquad \textbf{(6.12)}$$

$E = V_1 = V_2$

In the equation $P = V^2/R$, the voltage across each resistor in a parallel circuit will be the same. The only factor that changes is the resistance in the denominator of the equation.

The result is that in a parallel resistive network, the larger the resistor, the less is the power absorbed.

Example 6.8 : For the parallel network in Fig. 6.29 (all standard values):

a. Determine the total resistance R_T.

b. Find the source current and the current through each resistor.

c. Calculate the power delivered by the source.

d. Determine the power absorbed by each parallel resistor.

e. Verify Eq. (6.10).

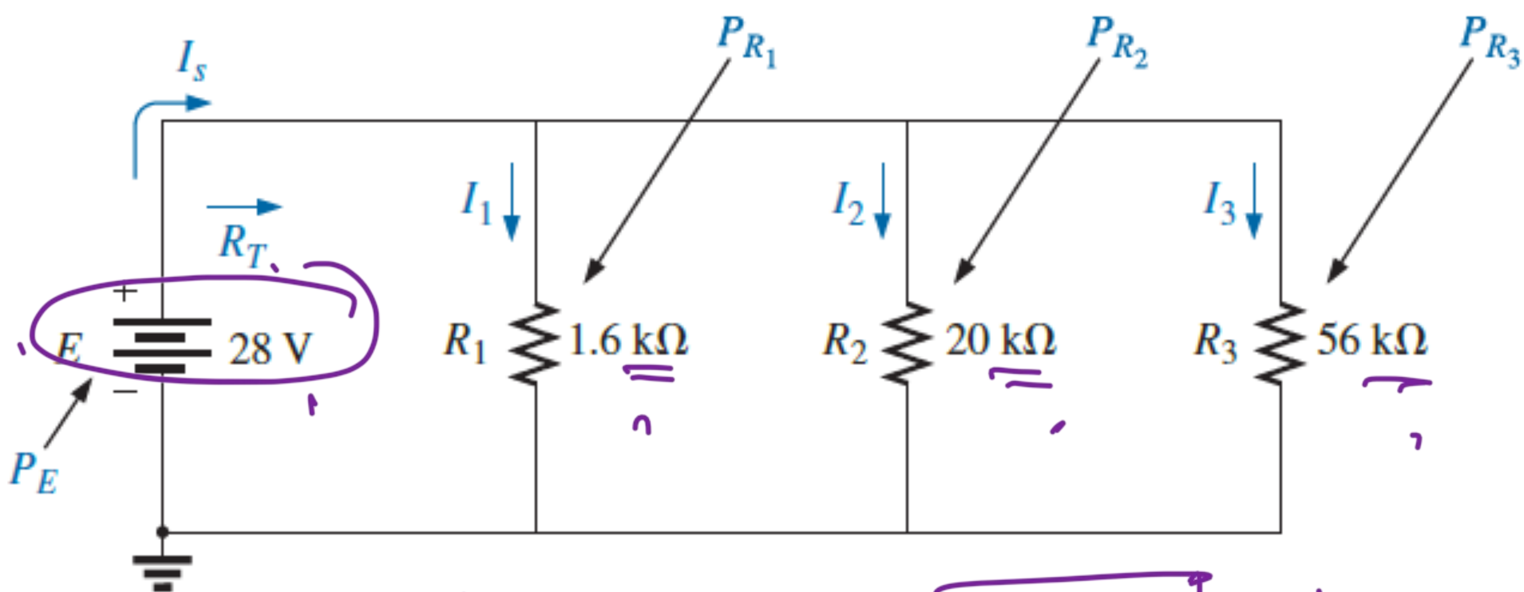


FIG. 6.29

a) $R_T = \dfrac{1}{\dfrac{1}{1,6} + \dfrac{1}{20} + \dfrac{1}{56}} = \boxed{\dfrac{140}{97}} = 1,44\ k\Omega$

b) $I_s = \dfrac{E}{R_T} = 19{,}4 \text{ mA}$

c) $P = IV = 19{,}4 \times 28 = 543{,}2 \text{ mW}$

mA

d) $P_{R_1} = E^2/R_1 = 28^2/1600 = 490 \text{ mW}$

$P_{R_2} = E^2/R_2 = 28^2/20000 = 39{,}2 \text{ mW}$

$P_{R_3} = E^2/R_3 = 28^2/56000 = 14 \text{ mW}$

Solution:

a. $$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{1.6\text{ k}\Omega} + \frac{1}{20\text{ k}\Omega} + \frac{1}{56\text{ k}\Omega}}$$

$$= \frac{1}{625 \times 10^{-6} + 50 \times 10^{-6} + 17.867 \times 10^{-6}} = \frac{1}{692.867 \times 10^{-6}}$$

and $R_T = \mathbf{1.44\text{ k}\Omega}$

d.

b. $$I_s = \frac{E}{R_T} = \frac{28\text{ V}}{1.44\text{ k}\Omega} = \mathbf{19.44\text{ mA}}$$

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{28\text{ V}}{1.6\text{ k}\Omega} = \mathbf{17.5\text{ mA}}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{28\text{ V}}{20\text{ k}\Omega} = \mathbf{1.4\text{ mA}}$$

$$I_3 = \frac{V_3}{R_3} = \frac{E}{R_3} = \frac{28\text{ V}}{56\text{ k}\Omega} = \mathbf{0.5\text{ mA}}$$

e. $$P_E = P_{R_1} + P_{R_2} + P_{R_3}$$

$$\mathbf{543.2\text{ mW}} = 490\text{ mW} + 39.2\text{ mW} + 14\text{ mW} = \mathbf{543.2\text{ mW}}$$

c. $$P_E = EI_s = (28\text{ V})(19.4\text{ mA}) = \mathbf{543.2\text{ mW}}$$

c. $P_E = EI_s = (28\text{ V})(19.4\text{ mA}) = \mathbf{543.2\text{ mW}}$

d. $P_1 = V_1I_1 = EI_1 = (28\text{ V})(17.5\text{ mA}) = \mathbf{490\text{ mW}}$

$P_2 = I_2^2R_2 = (1.4\text{ mA})^2(20\text{ k}\Omega) = \mathbf{39.2\text{ mW}}$

$$P_3 = \frac{V_3^2}{R_3} = \frac{E^2}{R_3} = \frac{(28\text{ V})^2}{56\text{ k}\Omega} = \mathbf{14\text{ mW}}$$

A review of the results clearly substantiates the fact that the larger the resistor, the less is the power absorbed.

e. $P_E = P_{R_1} + P_{R_2} + P_{R_3}$

$\mathbf{543.2\text{ mW}} = 490\text{ mW} + 39.2\text{ mW} + 14\text{ mW} = \mathbf{543.2\text{ mW}}$ (checks)

6.5 Kirchhoff's Current Law

- **Kirchhoff's current law (KCL):** *The algebraic sum of the currents entering and leaving a junction (or region) of a network is zero.*

$$\boxed{\Sigma I_{i} = \Sigma I_{o}} \quad \text{in} \quad \text{out} \qquad (6.13)$$

- I_i representing the current entering, or "in," and I_o representing the current leaving, or "out."
  - In Fig. 6.30, for example, the shaded area can enclose an entire system or a complex network, or it can simply provide a connection point (junction) for the displayed currents. In each case, the current entering must equal that leaving, as required by Eq. (6.13):

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_4 = I_2 + I_3$$
$$4\text{ A} + 8\text{ A} = 2\text{ A} + 10\text{ A}$$
$$\mathbf{12\text{ A}} = \mathbf{12\text{ A}}\ \text{(checks)}$$

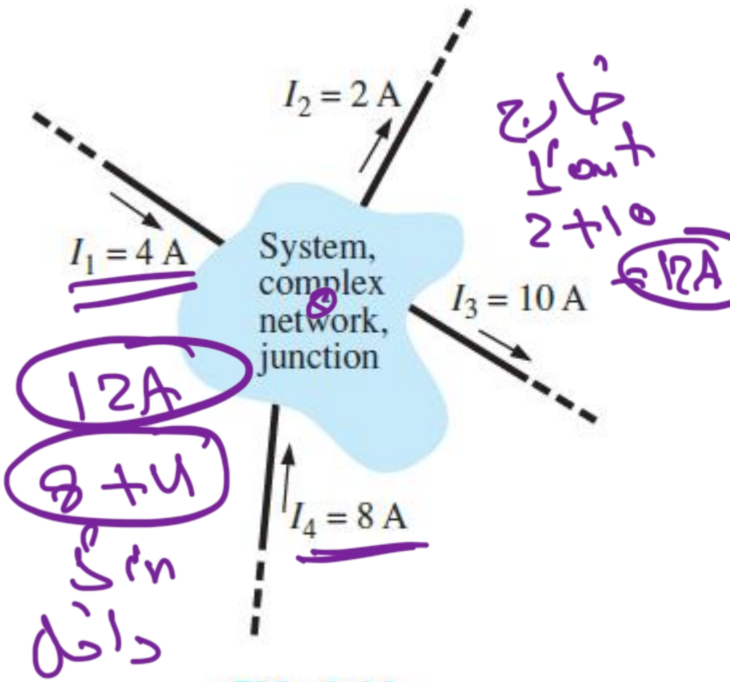


FIG. 6.30
Introducing Kirchhoff's current law.

مجموع التيار الداخل عند أي نقطة
= مجموع التيار الخارج من نفس النقطة

Example 6.7 : **Determine currents I_3 and I_4 in Fig. 6.32 using Kirchhoff's current law.**

$\Sigma I_{in} = \Sigma I_{out}$

Solution:

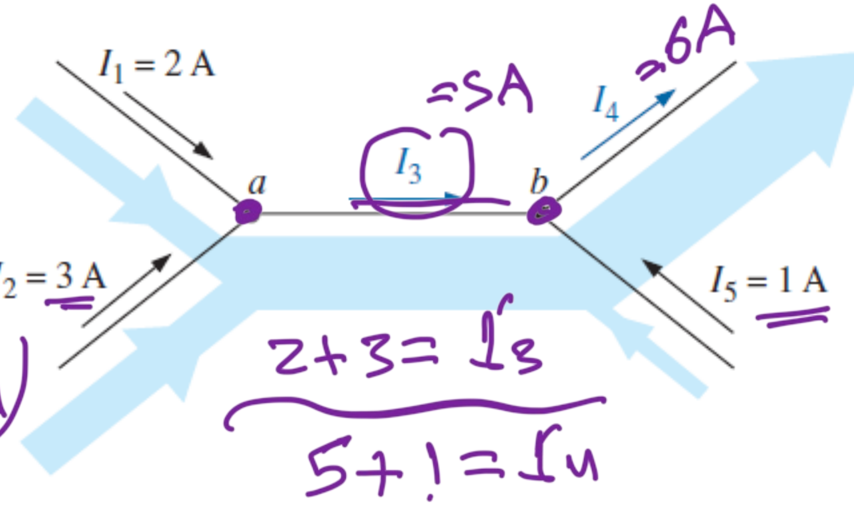


FIG. 6.32

At node *a*

$$\Sigma I_i = \Sigma I_o$$

$$I_1 + I_2 = I_3$$

$$2\text{ A} + 3\text{ A} = I_3 = \mathbf{5\ A}$$

At node *b*, using the result just obtained,

$$\Sigma I_i = \Sigma I_o$$

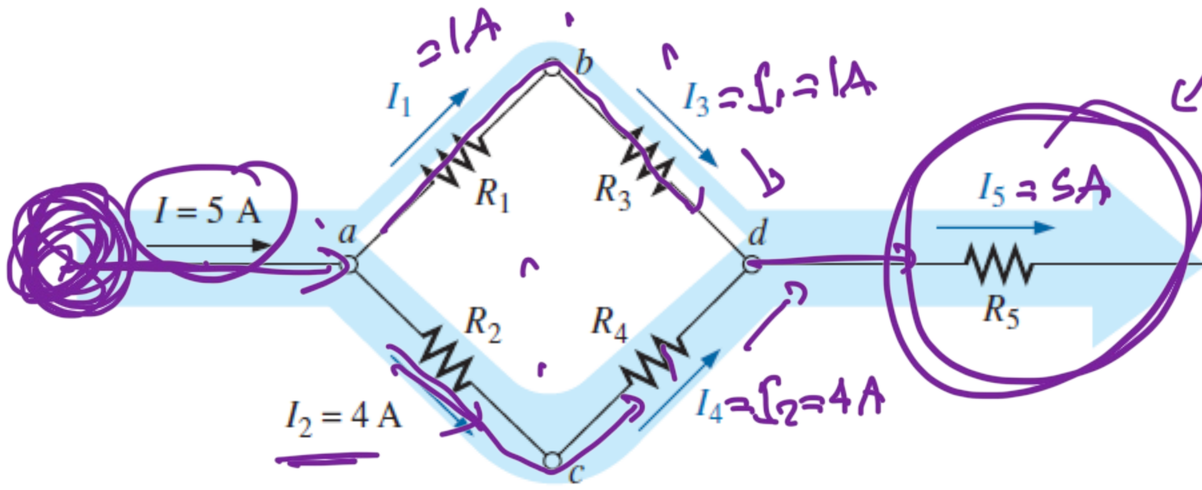
$$I_3 + I_5 = I_4$$

$$5\text{ A} + 1\text{ A} = I_4 = \mathbf{6\ A}$$

Note that in Fig. 6.32, the width of the blue-shaded regions matches the magnitude of the current in that region.

Example 6.8 : **Determine currents I_1, I_3, I_4, and I_5 for the network in Fig. 6.33.**

Solution:



$$\Sigma I_i = \Sigma I_o$$
$$I_5 = 5 \text{ A}.$$

At node a

$$\Sigma I_i = \Sigma I_o$$
$$I = I_1 + I_2$$
$$5 \text{ A} = I_1 + 4 \text{ A}$$

and
$$I_1 = 5 \text{ A} - 4 \text{ A} = \mathbf{1 \text{ A}}$$

At node b

$$\Sigma I_i = \Sigma I_o$$
$$I_1 = I_3$$
$$I_3 = I_1 = \mathbf{1 \text{ A}}$$

At node c

$$\Sigma I_i = \Sigma I_o$$
$$I_2 = I_4$$

and
$$I_4 = I_2 = \mathbf{4 \text{ A}}$$

At node d

$$\Sigma I_i = \Sigma I_o$$
$$I_3 + I_4 = I_5$$
$$1 \text{ A} + 4 \text{ A} = I_5 = \mathbf{5 \text{ A}}$$

node
النقطة التي تربط ا عناصر ببعض

Example 6.9 : For the parallel dc network in Fig. 6.35:

a. Determine the source current *Is*.

b. Find the source voltage *E.*

c. Determine *R*3.

d. Calculate *RT*.

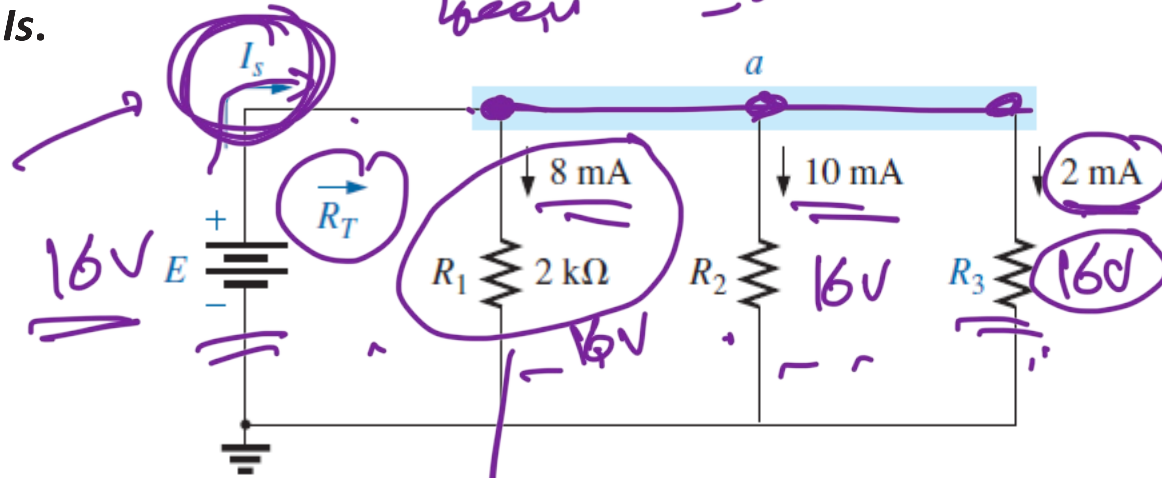


FIG. 6.35

Solution:

a. $\Sigma I_i = \Sigma I_o$

$$I_s = I_1 + I_2 + I_3$$

$$I_s = 8\text{ mA} + 10\text{ mA} + 2\text{ mA} = \mathbf{20\text{ mA}}$$

b. Apply Ohm's law gives

$$E = V_1 = I_1R_1 = (8\text{ mA})(2\text{ k}\Omega) = \mathbf{16\text{ V}}$$

c. Apply Ohm's law in a different form

$$R_3 = \frac{V_3}{I_3} = \frac{E}{I_3} = \frac{16\text{ V}}{2\text{ mA}} = \boxed{\mathbf{8\text{ k}\Omega}}$$

d. Apply Ohm's law again gives

$$\boxed{R_T = \frac{E}{I_s}} = \frac{16\text{ V}}{20\text{ mA}} = \boxed{\mathbf{0.8\text{ k}\Omega}}$$

$\Sigma I_{in} = \Sigma I_{out}$

$I_s = 8+10+2$

$= 20\text{mA}$

$V = IR = 8\times 2 = 16\text{V}$

$R_3 = \frac{V}{I} = \frac{16}{2} = 8\text{k}\Omega$

$R_2 = \frac{V}{I} = \frac{16}{10} = 1{,}6\text{k}\Omega$

6.6 Current Divider Rule

- For two parallel elements of equal value, the current will divide equally.
- For parallel elements with different values, the smaller the resistance, the greater is the share of input current.
- For parallel elements of different values, the current will split with a ratio equal to the inverse of their resistance values.

$$\boxed{\frac{I_1}{I_2} = \frac{R_2}{R_1}} \tag{6.14}$$

The current through any branch of a parallel resistive network is equal to the total resistance of the parallel network divided by the resistance of the resistor of interest and multiplied by the total current entering the parallel configuration.

$$\boxed{I_x = \frac{R_T}{R_x} I_T} \tag{6.15}$$

parallel $\Leftarrow$ current divider

- **For two parallel resistors, the current through one is equal to the resistance of the other times the total entering current divided by the sum of the two resistances.**

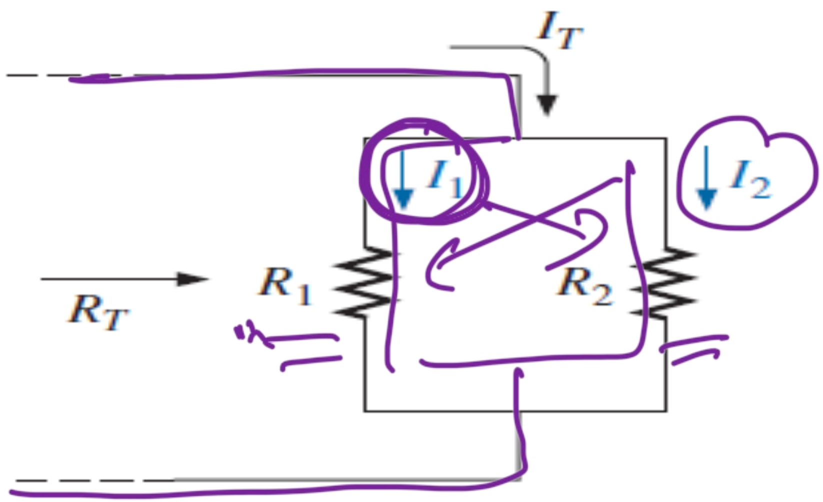


FIG. 6.42

Deriving the current divider rule for the special case of only two parallel resistors.

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I_T \quad \text{(6.16a)}$$

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I_T \quad \text{(6.16b)}$$

Example 6.10 :

a. Determine the current I_1 for the network of Fig. 6.39 using the ratio rule.

b. Determine the current I_3 for the network of Fig. 6.39 using the ratio rule.

c. Determine the current I_s using Kirchhoff's current law.

$$\frac{I_1}{I_2} = \frac{R_2}{R_1} \quad I_2$$

$$I_1 = \frac{R_2}{R_1} I_2$$

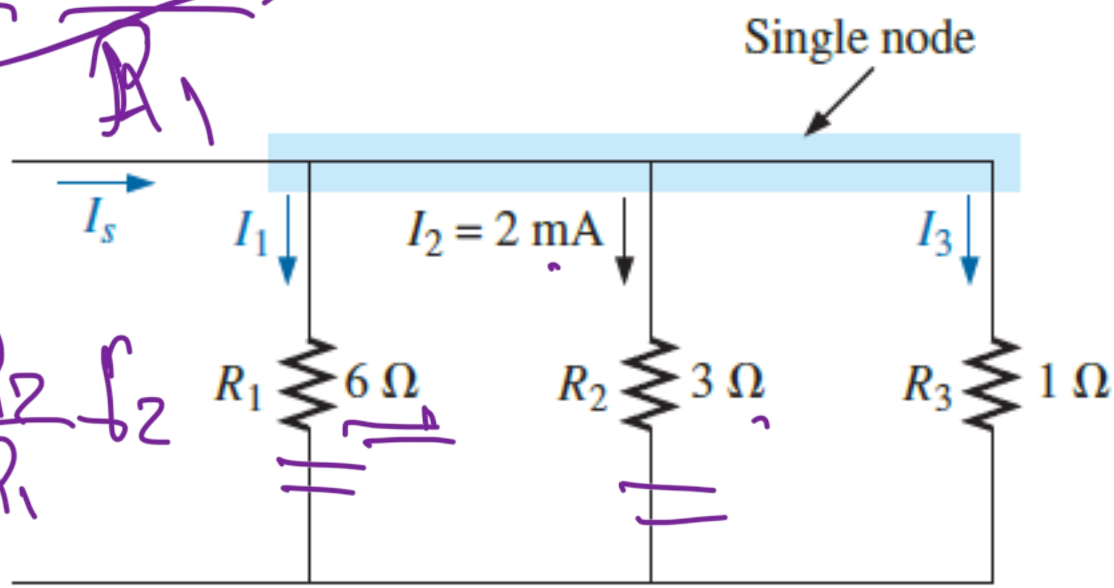


FIG. 6.39

Solution:

a. **Applying the ratio rule:**

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

$$\frac{I_1}{2\text{ mA}} = \frac{3\ \Omega}{6\ \Omega}$$

$$I_1 = \frac{1}{2}(2\text{ mA}) = \boxed{\mathbf{1\ mA}}$$

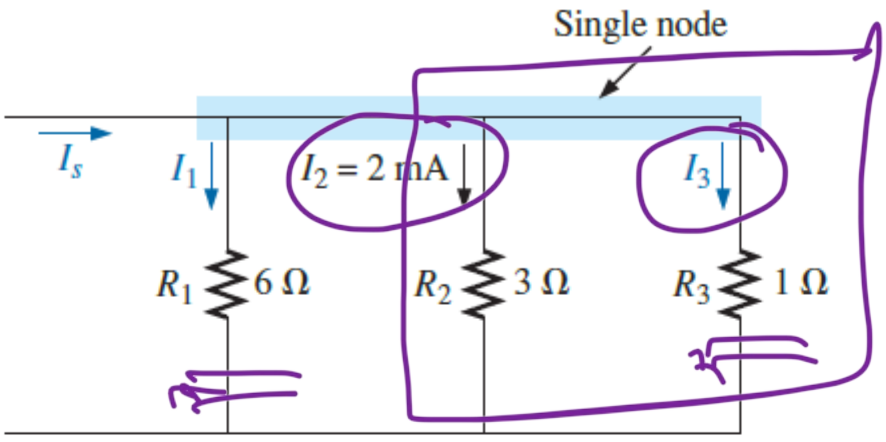


FIG. 6.39

b. **Applying the ratio rule:**

$$\frac{I_2}{I_3} = \frac{R_3}{R_2}$$

$$\frac{2\text{ mA}}{I_3} = \frac{1\ \Omega}{3\ \Omega}$$

$$I_3 = 3(2\text{ mA}) = \mathbf{6\ mA}$$

c. **Applying Kirchhoff's current law:**

$$\Sigma I_i = \Sigma I_o$$

$$I_s = I_1 + I_2 + I_3$$

$$= 1\text{ mA} + 2\text{ mA} + 6\text{ mA}$$

$$= \mathbf{9\ mA}$$

$$\frac{I_2}{I_3} = \frac{R_3}{R_2} \Rightarrow \frac{2\times 3}{I_3} = \frac{1}{3} \Rightarrow I_3$$

$$\boxed{I_3 = 6\,mA}$$

$$\frac{I_3}{I_1} = \frac{R_1}{R_3} \Rightarrow I_1 \Rightarrow I_3 = \frac{6}{1} \times 1$$

$$\boxed{6\,mA}$$

Example 6.11 : **For the parallel network in Fig. 6.41, determine current I_1 using Eq. (6.15).**

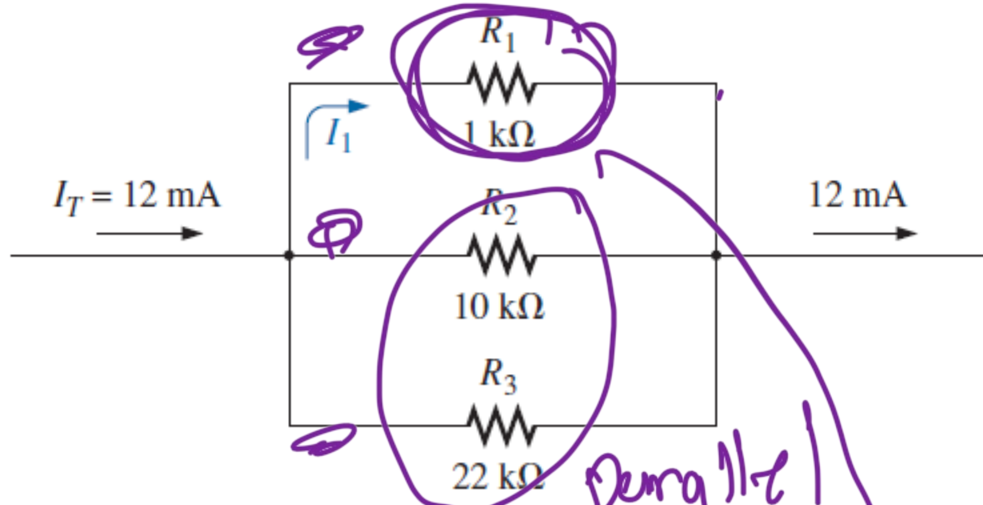


FIG. 6.41

parallel

6,875k Ω

Solution:

$$R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$$

$$= \frac{1}{\dfrac{1}{1\ \text{k}\Omega} + \dfrac{1}{10\ \text{k}\Omega} + \dfrac{1}{22\ \text{k}\Omega}}$$

$$= \frac{1}{1 \times 10^{-3} + 100 \times 10^{-6} + 45.46 \times 10^{-6}}$$

$$= \frac{1}{1.145 \times 10^{-3}} = \mathbf{873.01\ \Omega}$$

$$I_1 = \frac{R_T}{R_1} I_T$$

$$= \frac{(873.01\ \Omega)}{1\ \text{k}\Omega}(12\ \text{mA}) = (0.873)(12\ \text{mA}) = \mathbf{10.48\ mA}$$

With the smallest parallel resistor receives the majority of the current.

$$I_1 = \frac{6,875}{1+6,875} \times 12 = \underline{10,476\,mA}$$

$$I_1 = \frac{R_1}{R_1} \times I_T$$

مجموع
مقلوب

"Parallel"

$$\frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots}$$

[illegible]

Example 6.11 : **Determine current I_2 for the network in Fig. 6.43 using the current divider rule.**

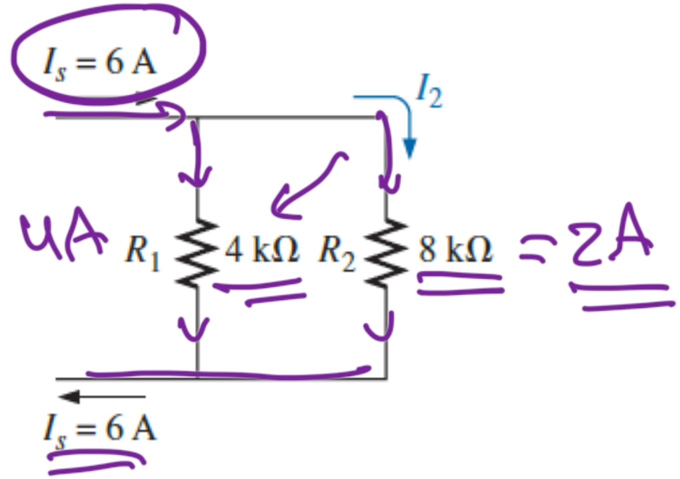


FIG. 6.43

Solution:

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right)I_T$$

$$= \left(\frac{4\text{ k}\Omega}{4\text{ k}\Omega + 8\text{ k}\Omega}\right)6\text{ A} = (0.333)(6\text{ A}) = \mathbf{2\text{ A}}$$

Eq. (6.15) gives

$$I_2 = \frac{R_T}{R_2}I_T$$

$$R_T = 4\text{ k}\Omega \| 8\text{ k}\Omega = \frac{(4\text{ k}\Omega)(8\text{ k}\Omega)}{4\text{ k}\Omega + 8\text{ k}\Omega} = 2.667\text{ k}\Omega$$

$$I_2 = \left(\frac{2.667\text{ k}\Omega}{8\text{ k}\Omega}\right)6\text{ A} = (0.333)(6\text{ A}) = \mathbf{2\text{ A}}$$

$$I_2 = \frac{4}{4+8} \times 6$$

$$= \boxed{2A}$$

6.7 Voltage Sources in parallel

- Voltage sources can be placed in parallel only if they have the same voltage.

The primary reason for placing two or more batteries or supplies in parallel is to increase the current rating above that of a single supply. For example, in Fig. 6.46, two ideal batteries of 12 V have been placed in parallel. The total source current using Kirchhoff's current law is now the sum of the rated currents of each supply. The resulting power available will be twice that of a single supply if the rated supply current of each is the same.

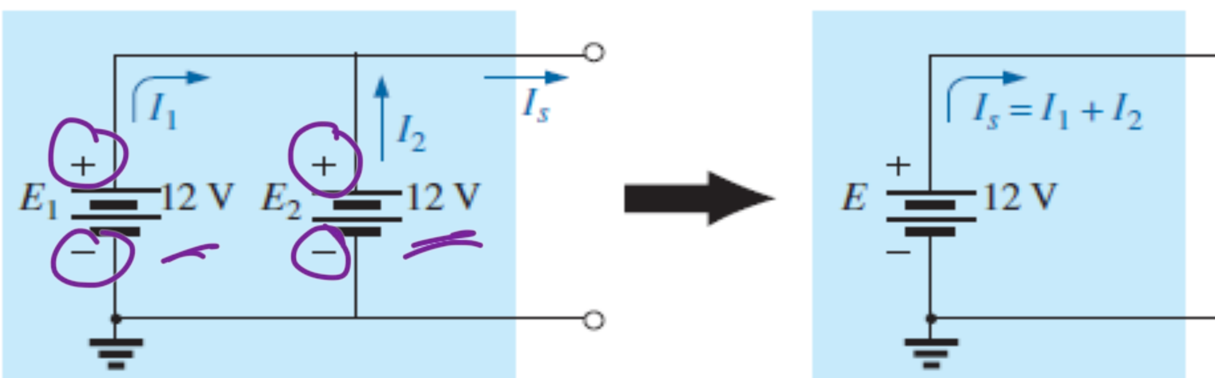
That is,

① أنه يكون نفس القيمة

② أنه يكونوا

with $I_1 = I_2 = I$

then $P_T = E(I_1 + I_2) = E(I + I) = E(2I) = 2(EI) = 2P_{\text{(one supply)}}$



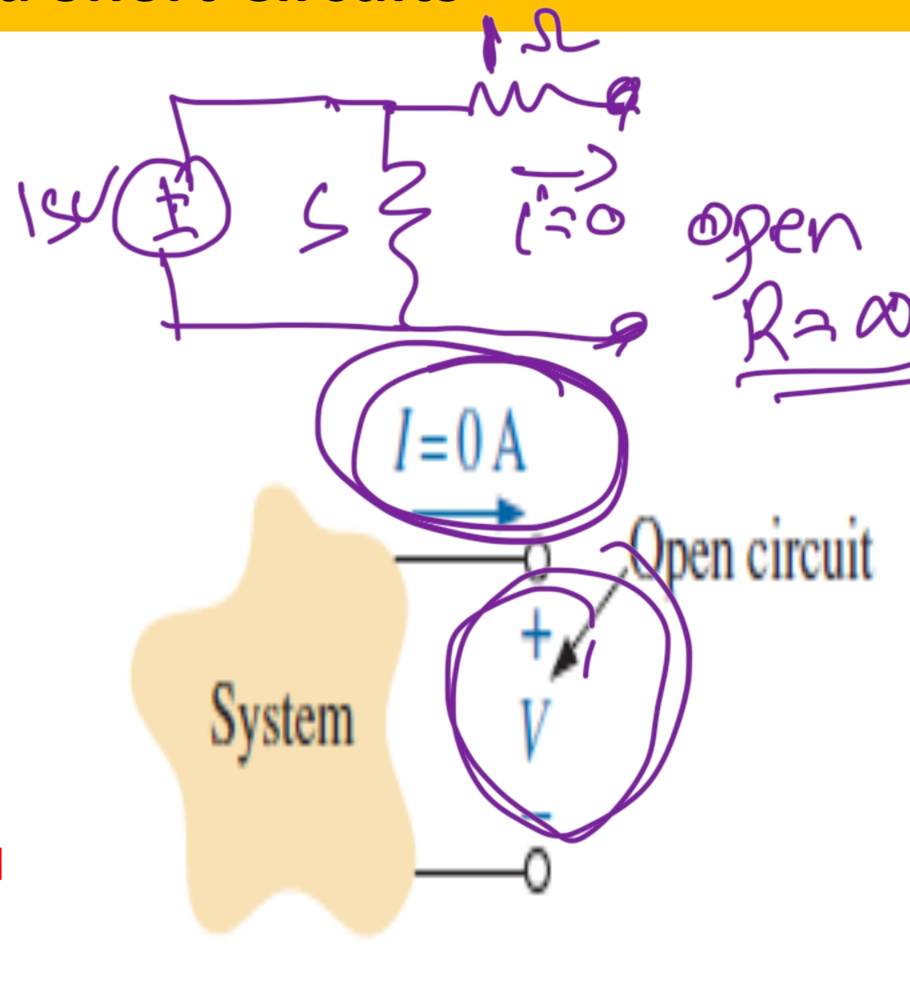
موصلين بنفس القطبية

FIG. 6.46

" سيارة كاشحن سيارة بطارية ثانية "

Open Circuit

- **An open circuit can have a potential difference (voltage) across its terminals, but the current is always zero.**
- **An open circuit is two isolated terminals not connected by an element of any kind, as shown in Fig. 6.48(a). Since a path for conduction does not exist, the current associated with an open circuit must always be zero. The voltage across the open circuit, however, can be any value, as determined by the system it is connected to .**

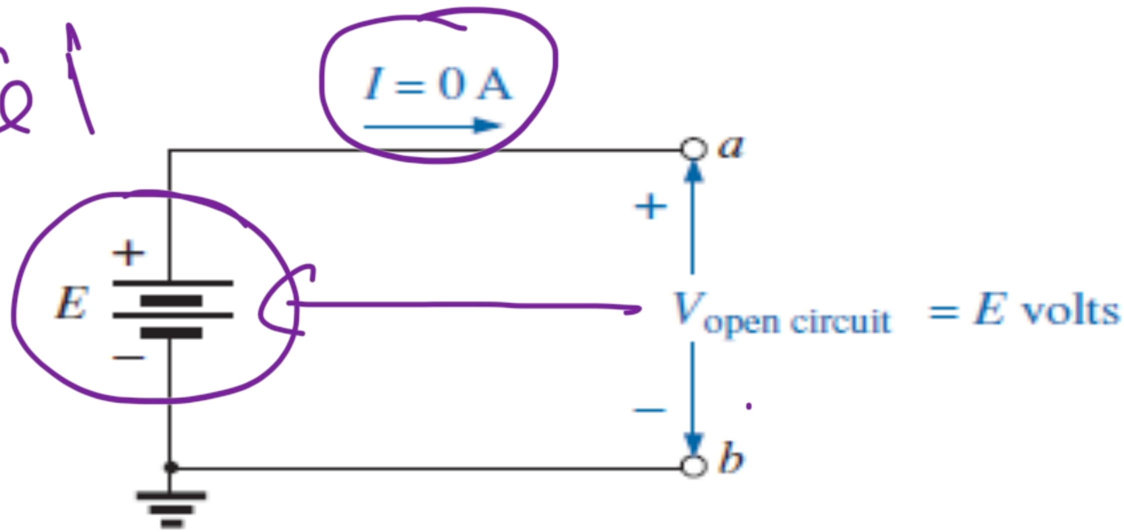


(a)

Open Circuit

- In Fig. 6.48(b), an open circuit exists between terminals *a* and *b*.
- The voltage across the open-circuit terminals is the supply voltage, but the current is zero due to the absence of a complete circuit.

أقرب بطارية عليه

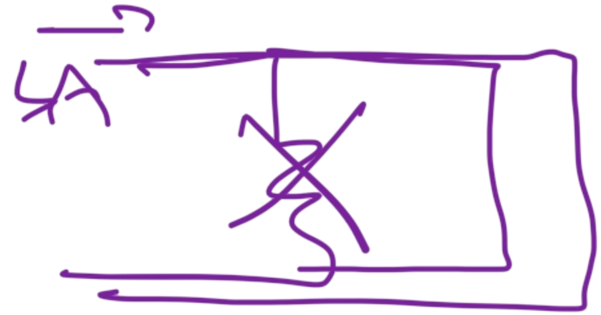


(b)

FIG. 6.48

Short Circuit

R=0



- A short circuit can carry a current of a level determined by the external circuit, but the potential difference (voltage) across its terminals is always zero volts.

- A short circuit is a very low resistance, direct connection between two terminals of a network, as shown in Fig. 6.50.
- The current through the short circuit can be any value, as determined by the system it is connected to, but the voltage across the short circuit is always zero volts because the resistance of the short circuit is assumed to be essentially zero ohms and $V = IR = I(0) = 0$ V.

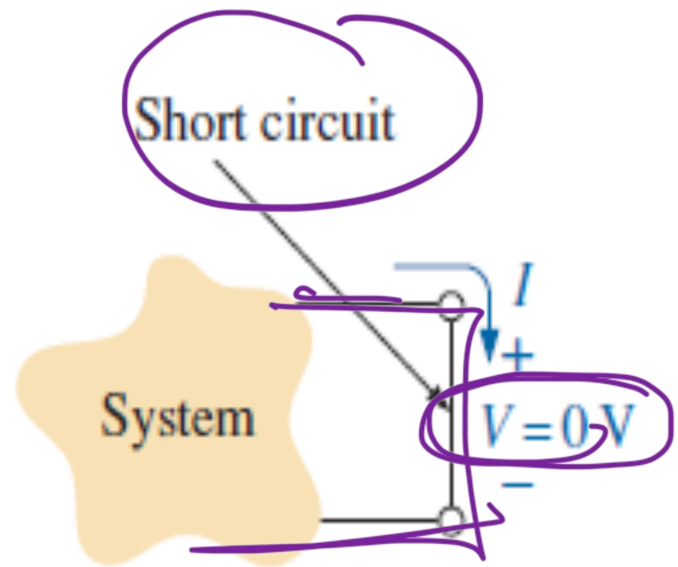


FIG. 6.50
Defining a short circuit.

Home work

Problem 1 : For the parallel network in Fig. 6.82:

a. Find the total resistance.

b. What is the voltage across each branch?

c. Determine the source current and the current through each branch.

d. Verify that the source current equals the sum of the branch currents.

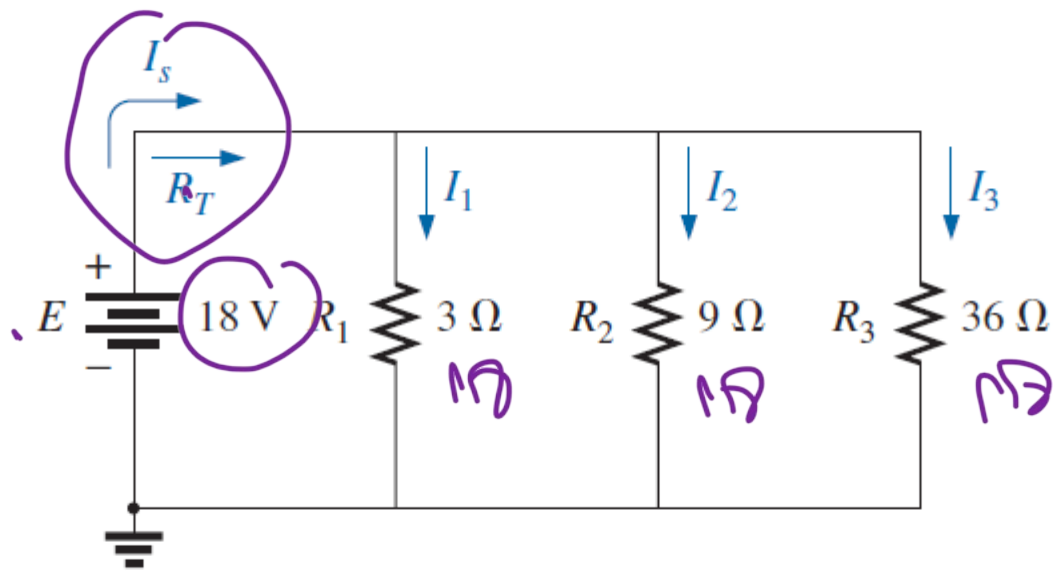


FIG. 6.82

Problem 2: **Given the information provided in Fig. 6.86, find the unknown quantities: *E*, *R*$_1$, and *I*$_3$.**

10,8

36V = 36

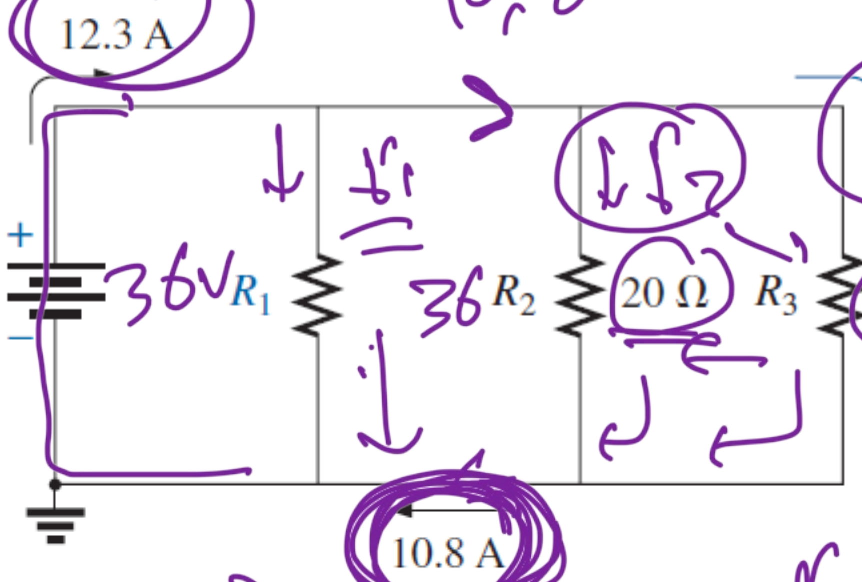


FIG. 6.86

$I_2 + I_3 = 10{,}8$

$I_1 + 10{,}8 = 12{,}3$

$I_1 = 12{,}3 - 10{,}8 = \boxed{1{,}5\,A}$

current divider

$$I_2 = \frac{4}{4+20} \times 10{,}8 = 1{,}8\,A$$

$$I_3 = \frac{20}{4+20} \times 10{,}8 = 9\,A$$

$$E = V_1 = V_2 = V_3 = 9 \times 4 = 36\,V$$

$$R_1 = \frac{V}{I} = \frac{36}{1{,}5} = \boxed{24\,\Omega}$$

Problem 3: Eight holiday lights are connected in parallel as shown in Fig. 6.90.

a. If the set is connected to a 120 V source, what is the current through each bulb if each bulb has an internal resistance of 1.8 k?
b. Determine the total resistance of the network.
c. Find the current drain from the supply.
d. What is the power delivered to each bulb?
e. Using the results of part (d), what is the power delivered by the source?
f. If one bulb burns out (that is, the filament opens up), what is the effect on the remaining bulbs? What is the effect on the source current? Why?

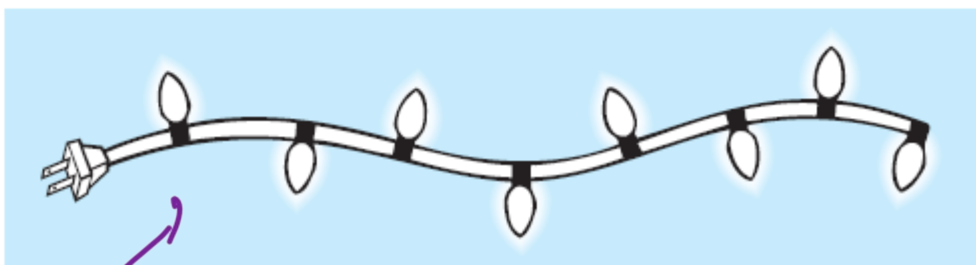


FIG. 6.90

$$I_s = I_1 + I_2 + I_3 + \cdots$$

a) $I_s = \frac{E}{R_T} = \frac{120}{225} = 0,533A$

$\frac{8}{15}A$

b) $R_T = \frac{R}{N} = \frac{1,8}{8} = \boxed{225\,\Omega}$

c

$I = \frac{I_s}{8} = \frac{\frac{\cancel{8}}{15}}{\cancel{8}} = \frac{1}{15} = \boxed{66,67\,mA}$

Problem 4 : **Using the information provided in Fig. 6.97, find the branch resistors R_1 and R_3, the total resistance R_T, and the voltage source E.**

Parallel

$$R_T = \frac{1}{\frac{1}{3}+\frac{1}{6}+\frac{1}{4}}$$

$\boxed{1{,}33\,k\Omega}$

$\boxed{\frac{4}{3}\,k\Omega}$

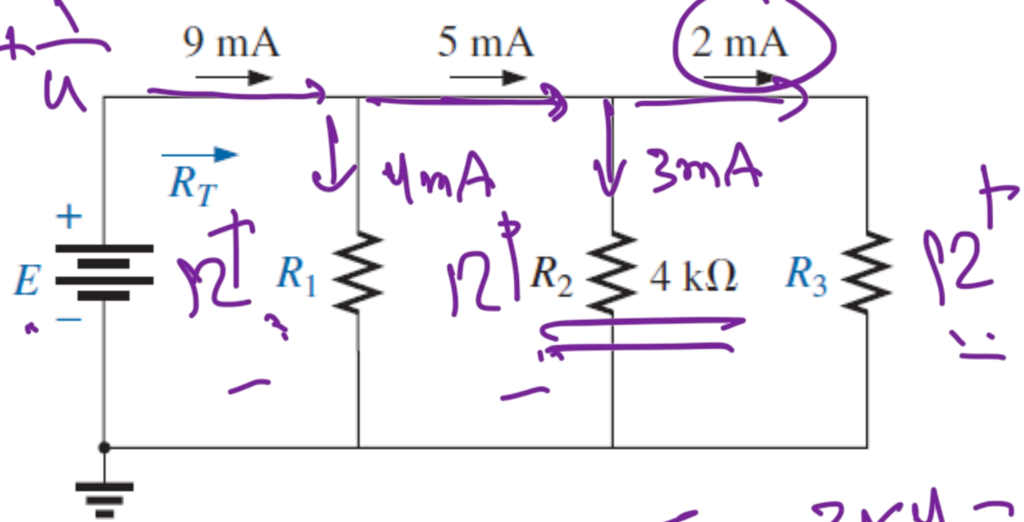


FIG. 6.97

$$E = 3 \times 4 = 12V$$

$$R_1 = \frac{E}{I} = \frac{12}{4} = 3k\Omega$$

$$R_3 = \frac{E}{I} = \frac{12}{2} = 6k\Omega$$

Problem 5 : **Determine the currents for the configurations in Fig. 6.100.**

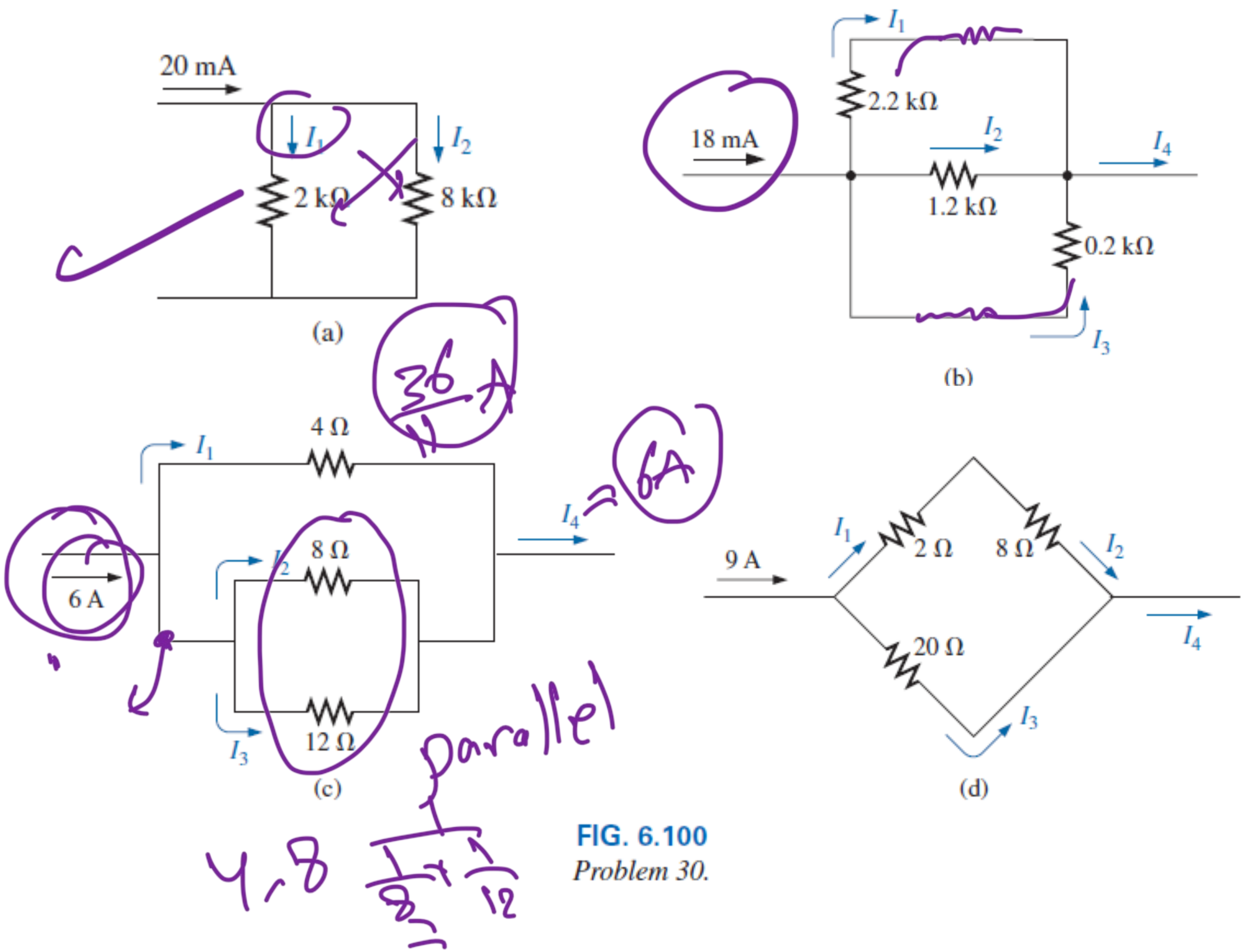
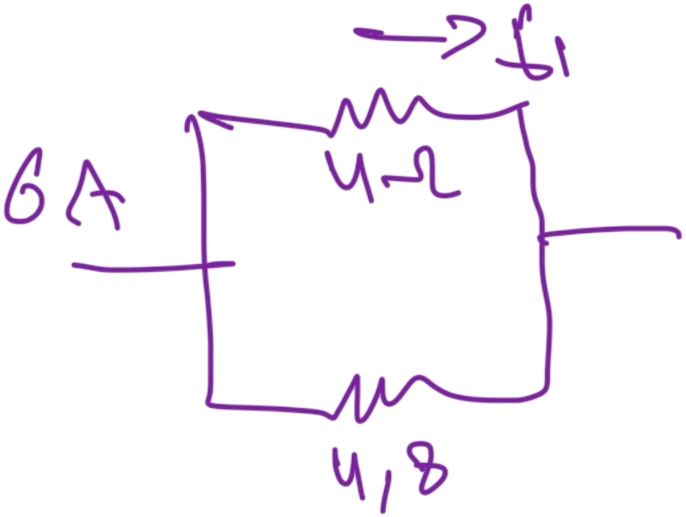
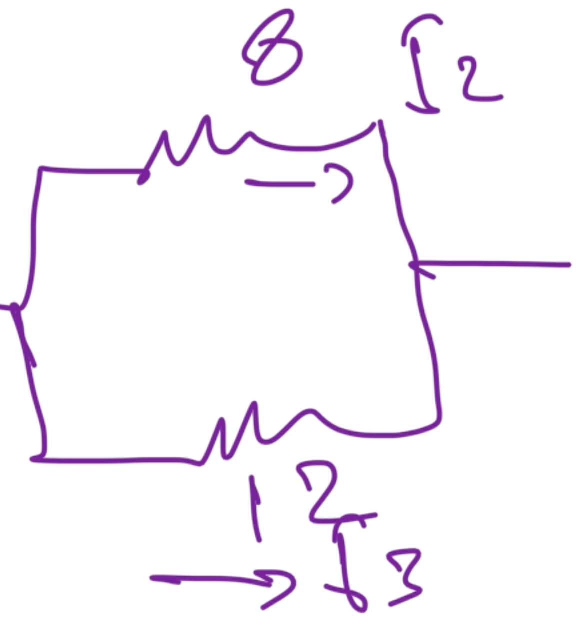


FIG. 6.100
Problem 30.



$$I_1 = \frac{4,8}{4+4,8} \times 6 = \frac{36}{11} A$$

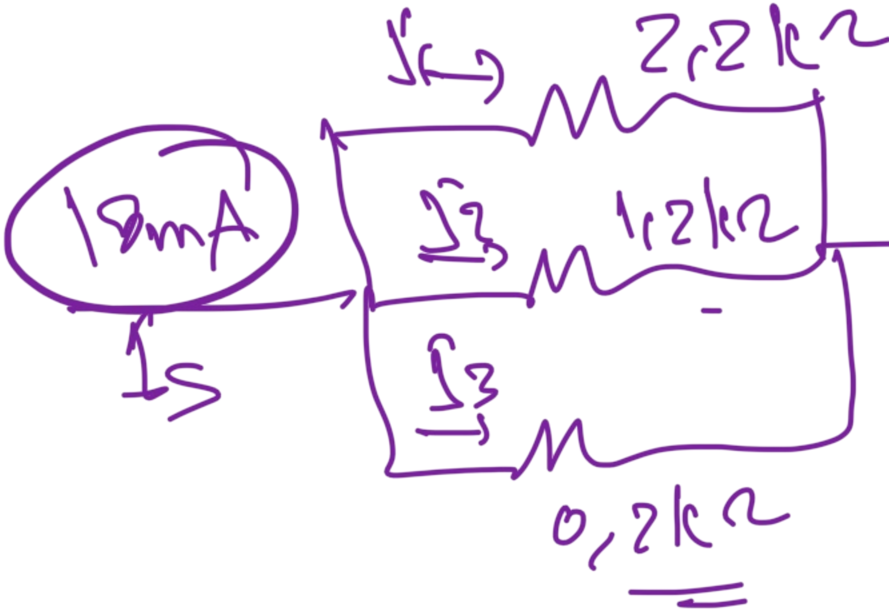
$$6 - \frac{36}{11} = \frac{30}{11} A$$



$$I_2 = \frac{12}{12+8} \times \frac{30}{11}$$

$$I_2 = \frac{18}{11} A$$

$$I_3 = \frac{8}{12+8} \times \frac{30}{11} = \boxed{\frac{12}{11} A}$$



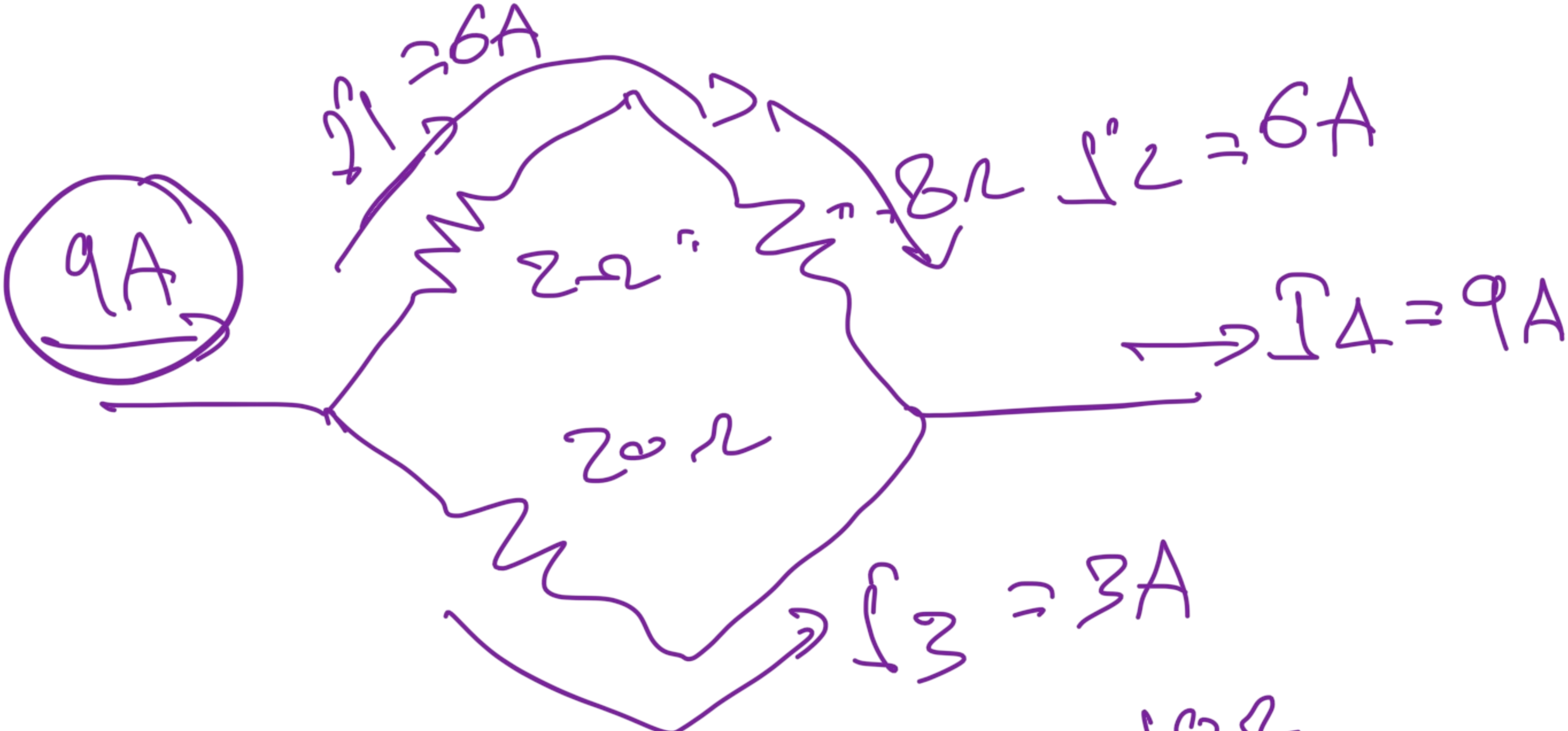
$I_4 = 18mA$

$$R_T = \frac{1}{\frac{1}{0{,}2} + \frac{1}{1{,}2} + \frac{1}{2{,}2}}$$

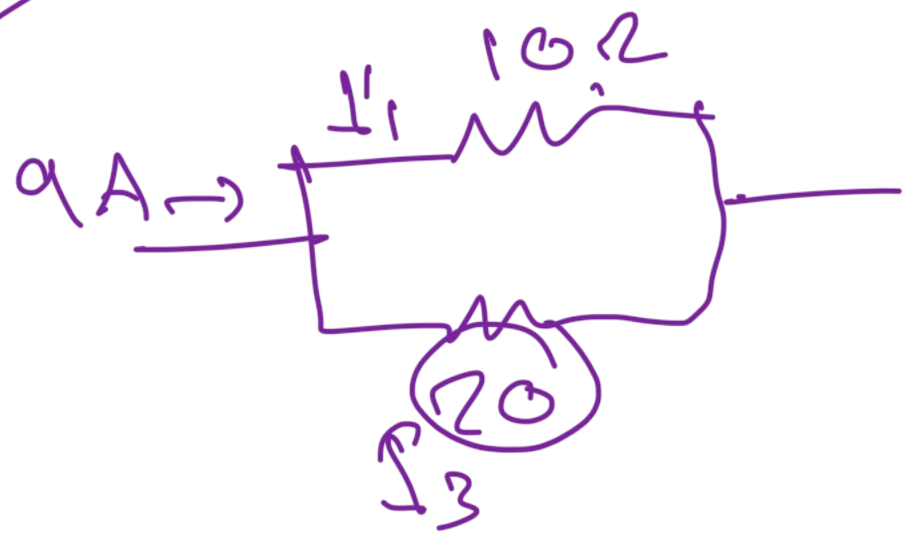
$$I_1 = \frac{R_T}{R_1} * I_s = 1{,}3mA \qquad R_T = \frac{66}{415} k\Omega$$

$$I_2 = \frac{R_T}{R_2} * I_s = 2{,}39\ mA$$

$$I_3 = \frac{R_T}{R_3} * I_s = 14{,}31\ mA$$



$$I_1 = I_2$$



$$I_1 = \frac{20}{20+10} \times 9 = 6\,A$$

$$I_3 = \frac{10}{20+10} \times 9 = 3\,A$$