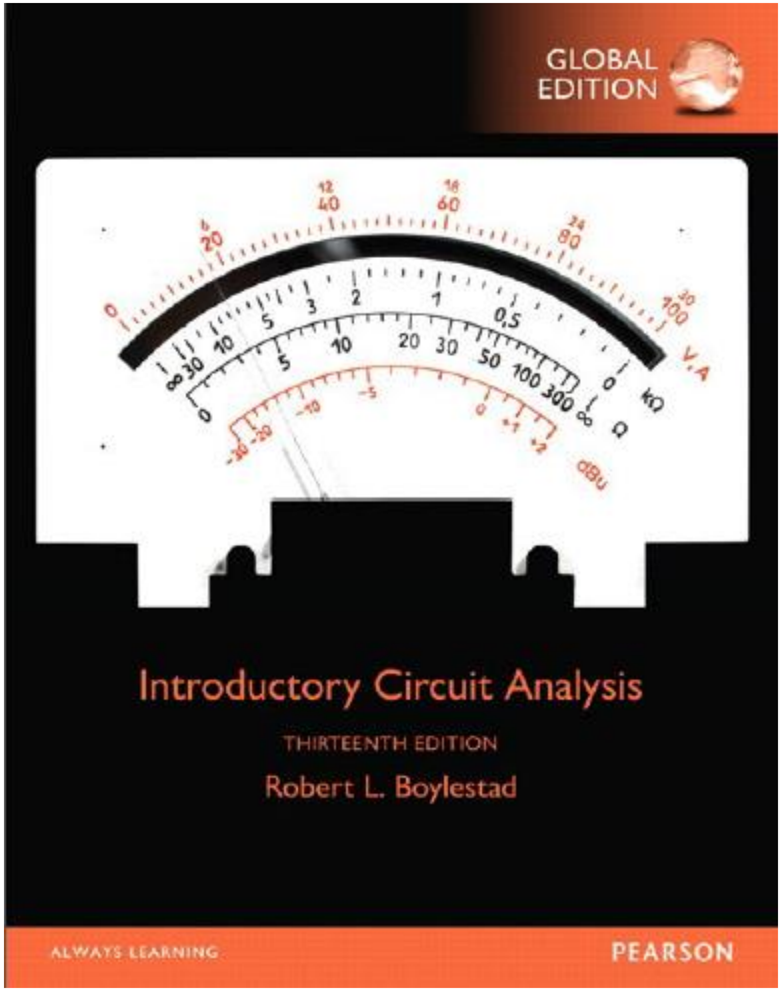
EEE 101: Electrical Circuits I

WEEK 2

Chapter 5

Series Circuits

Textbook



Textbook:

Robert L. Boylestad “ Introductory Circuit Analysis “ Twelfth Edition Pearson

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Chapter 5 : DC Series Circuits

Topics to be covered in this chapter

5.2 Series Resistors

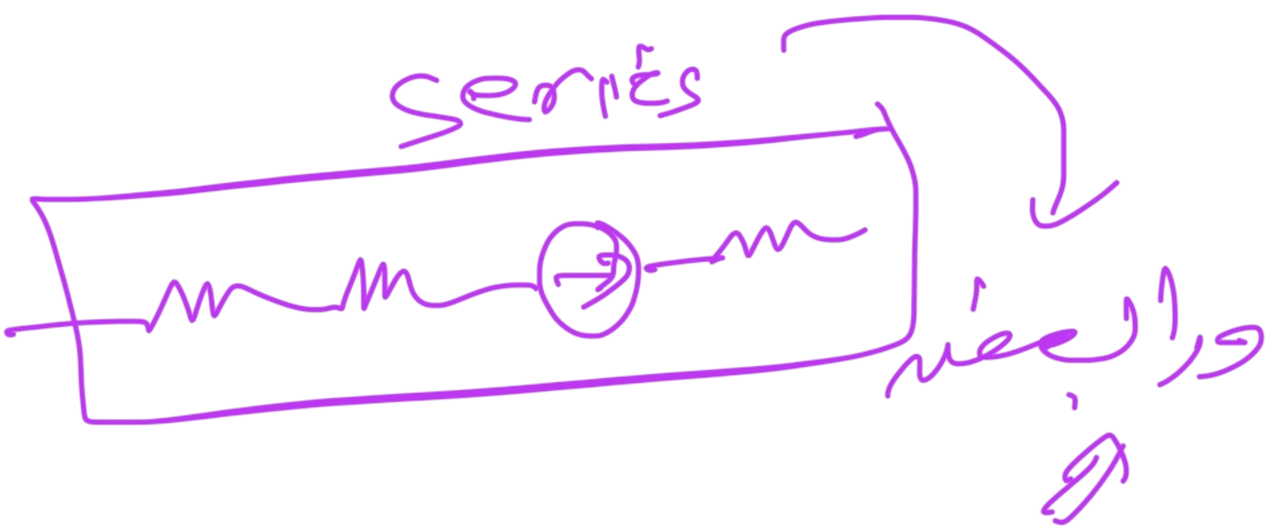
5.3 Series Circuits

5.4 Power Distribution in a Circuit Circuits

5.5 Voltage sources in series

5.6 Kirchhoff's Voltage Law

5.7 Voltage Division in a series circuit



5.2 Series Resistors

a. In Fig. 5.4, one terminal of resistor *R*2 is connected to resistor *R*1 on one side, and the remaining terminal is connected to resistor *R*3 on the other side, resulting in one, and only one, (one point) connection between adjoining resistors. When connected in this manner, the resistors have established a series connection. If three elements were connected to the same point, as shown in Fig. 5.5, there would not be a series connection between resistors *R*1 and *R*2.

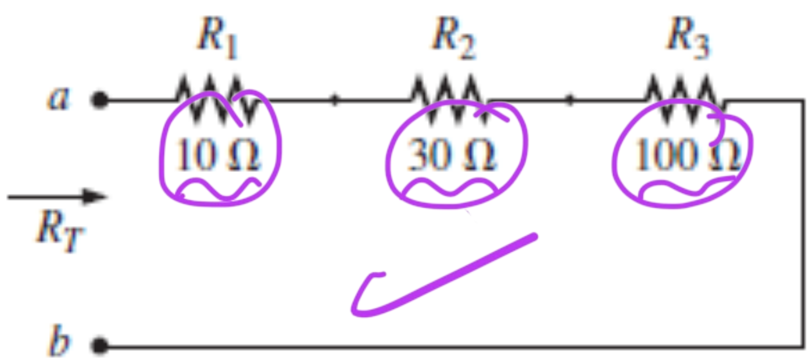


FIG. 5.4
Series connection of resistors.

Rth ✓
Req ✓
= 140
RT ✓
Rin ✓

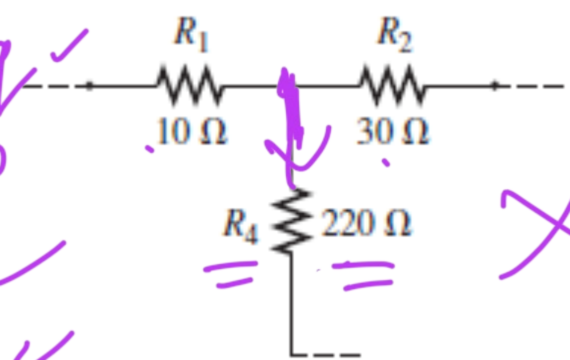


FIG. 5.5
Configuration in which none of the resistors are in series.

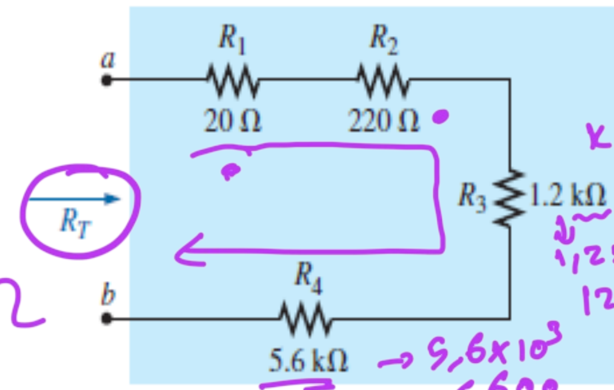
b. The total resistance of a series configuration is the sum of the resistance levels.

$$R_T = R_1 + R_2 + R_3 + R_4 + \ldots\ldots + R_N \qquad \text{Ohm } (\Omega)$$

20 + 220 + 1200 + 5600

= 7040 Ω

Example 5.1 : Determine the total resistance of the series connection in Fig 5.6.



k = 10^3

1,2 x 10^3

1200 Ω

→ 5,6 x 10^3

5600

FIG. 5.6

Series connection of resistors for Example 5.1.

Solution:

7,04 kΩ

$R_T = R_1 + R_2 + R_3 + R_4$

$R_T = 20\ \Omega + 220\ \Omega + 1.2\ k\Omega + 5.6\ k\Omega$

$\boxed{R_T = 7.04\ k\Omega}$

shift Eng = 7,04 x 10^3 Ω

Example 5.2 : Determine the total resistance of the series connection in Fig 5.7

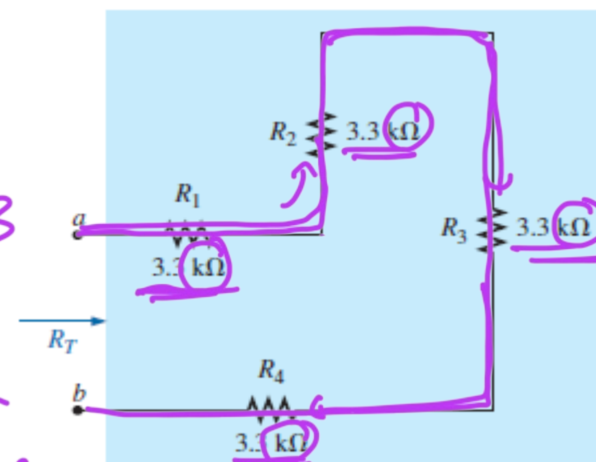


FIG. 5.7

Series connection of four resistors of the same value (Example 5.2).

Solution:

3,3 + 3,3 + 3,3 + 3,3

13,2 kΩ

OR

4 x 3,3 = 13,2 kΩ

$R_T = NR$

$R_T = (4)(3.3k\Omega)$

$R_T = 13.2\ k\Omega$

Series Circuits

a. Two elements are in series if

1. They have only one terminal in common (i.e., one lead of one is connected to only one lead of the other).

2. The common point between the two elements is not connected to another current-carrying element.

series

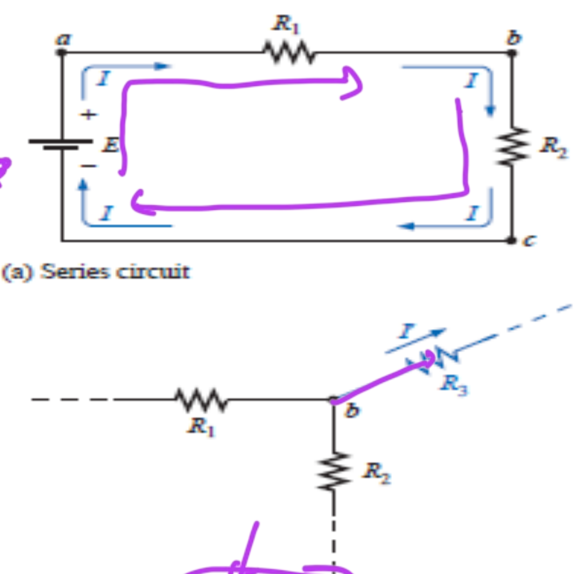
b. The total resistance of a set of resistances in series is the sum of the resistance levels.

$$R_T = R_1 + R_2 + R_3 + R_4 + \ldots\ldots + R_N \quad \text{Ohm } (\Omega)$$

A circuit consists of any number of elements joined at terminal point providing at least one closed path which charge can flow.

Fig. 5.4(a) has three elements joined at three terminals a, b &c to provide a closed path for the current *I*.

Fig 5.4 (b) the resistance *R1* and *R2* are not longer in series due to violation of rule number 2 of the above definition of series elements.



(a) Series circuit

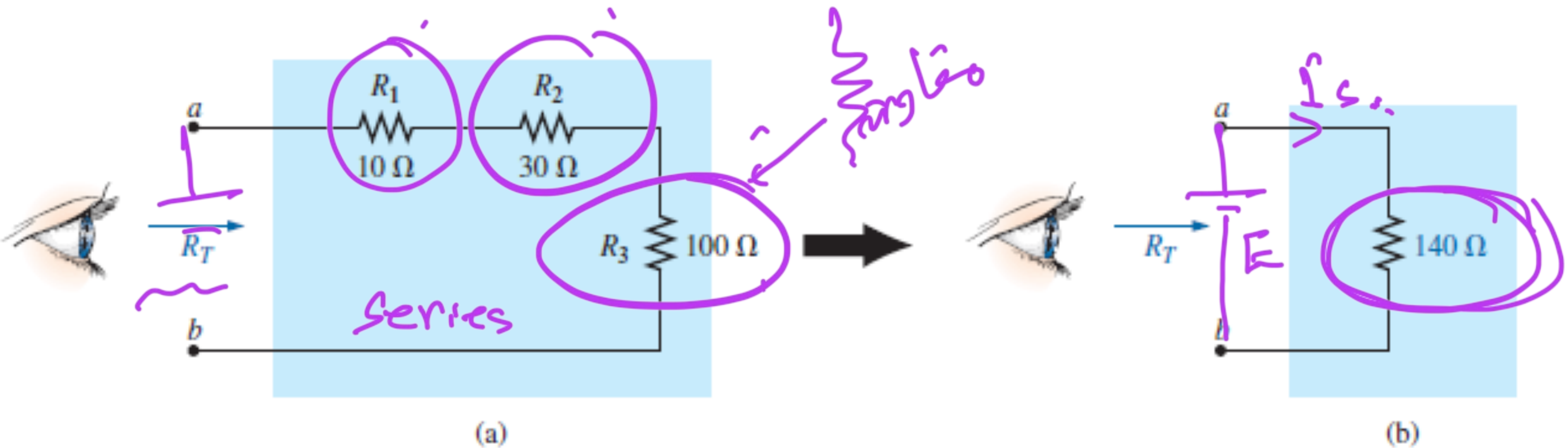
(b) R_1 and R_2 are not in series

FIG. 5.4

(a) Series circuit; (b) situation in which R_1 and R_2 are not in series.

$\Rightarrow$ Same current $I_s = I_1 = I_2 = \ldots$

It is important to realize that when a dc supply is connected, it does not "see" the individual connection of elements but simply the total resistance "seen" at the connection terminals, as shown in Fig. 5.13(a). In other words, it reduces the entire configuration to one such as in Fig. 5.13(b) to which Ohm's law can easily be applied.



100+30+10=140

FIG. 5.13

Resistance "seen" at the terminals of a series circuit.

قانون ohm

$$\boxed{I_s = \frac{E}{R_T}} \qquad (5.3)$$

جهد

مقاومة

$$V = I * R$$

$$\frac{V}{R} = \frac{I * \cancel{R}}{\cancel{R}}$$

Properties of DC Series Circuits

a. The direction of conventional current in a series dc circuit is such that it leaves the positive terminal of the supply and returns to the negative terminal, as shown in Fig. 5.12.

b. In any configuration, if two elements are in series, the current must be the same. However, if the current is the same for two adjoining elements, the elements may or may not be in series.

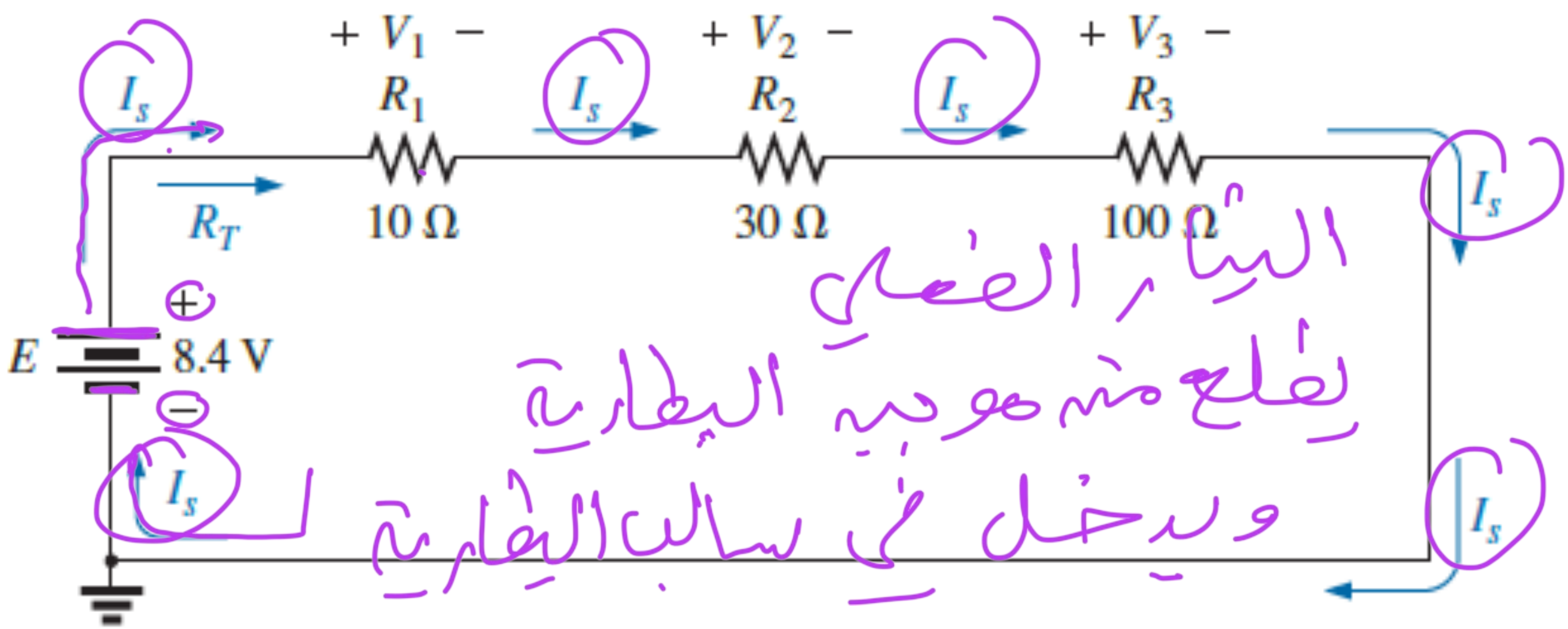


FIG. 5.12

Schematic representation for a dc series circuit.

c. The polarity of the voltage across a resistor is determined by the direction of the current in Fig 5.14

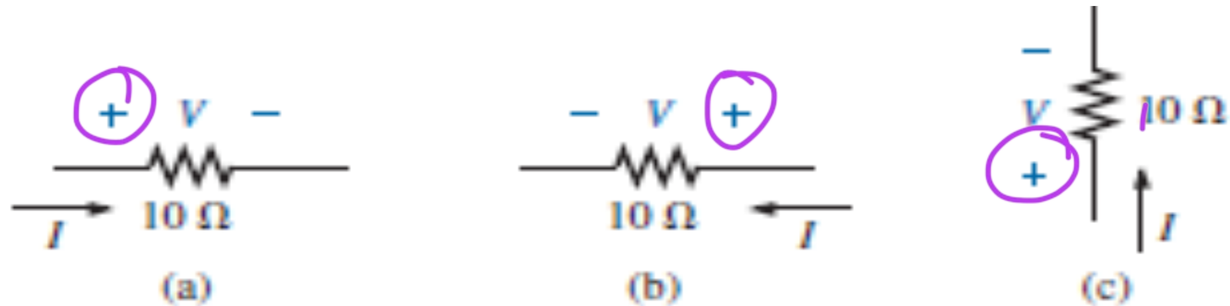
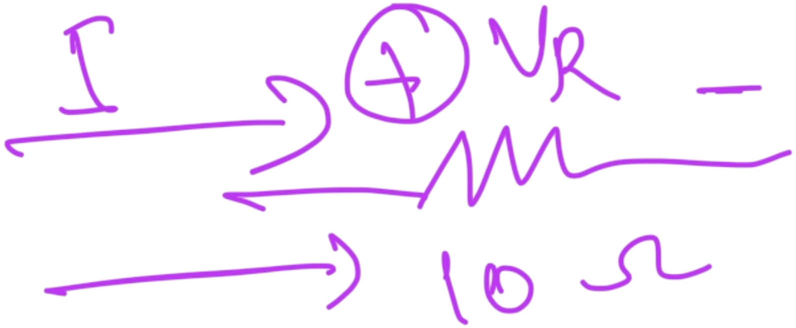


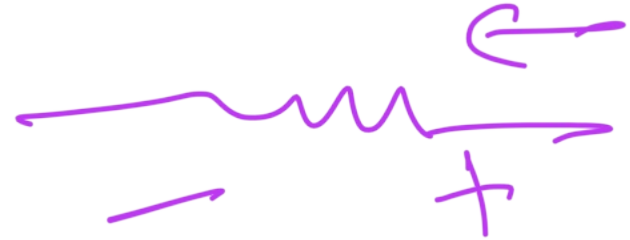
FIG. 5.14

Inserting the polarities across a resistor as determined by the direction of the current.

d. The magnitude of the voltage drop across each resistor can then be found by applying Ohm's law using only the resistance of each resistor. That is,

$$\boxed{\begin{aligned} V_1 &= I_1R_1 \\ V_2 &= I_2R_2 \\ V_3 &= I_3R_3 \end{aligned}} \qquad (5.4)$$


I
+ VR −
10 Ω


− +


− +


+ −

Example 5.3 : For the series circuit in Fig 5.15:

series

a. Find the total resistance R_T.
b. Calculate the resulting source current I_s.
c. Determine the voltage across each resistor.

Solution:

a. $R_T = R_1 + R_2 + R_3$
$\quad = 2\ \Omega + 1\ \Omega + 5\ \Omega$
$R_T = \mathbf{8\ \Omega}$

b. $I_s = \dfrac{E}{R_T} = \dfrac{20\text{ V}}{8\ \Omega} = \mathbf{2.5\text{ A}}$

c. $V_1 = I_1R_1 = I_sR_1 = (2.5\text{ A})(2\ \Omega) = \mathbf{5\text{ V}}$
$V_2 = I_2R_2 = I_sR_2 = (2.5\text{ A})(1\ \Omega) = \mathbf{2.5\text{ V}}$
$V_3 = I_3R_3 = I_sR_3 = (2.5\text{ A})(5\ \Omega) = \mathbf{12.5\text{ V}}$

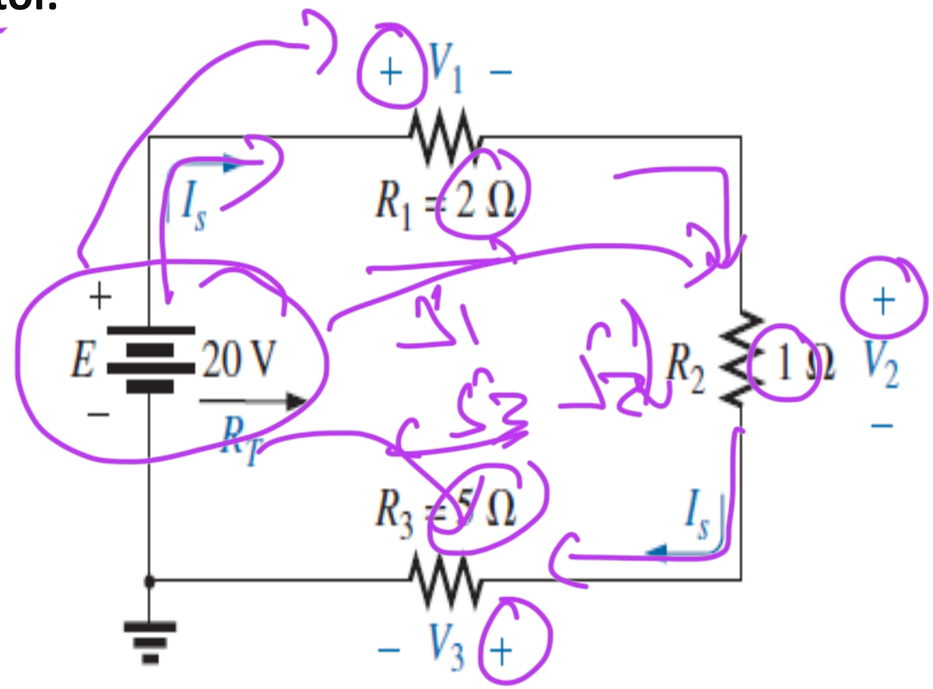


FIG. 5.15

a) $R_T = 2 + 1 + 5 = 8\ \Omega$

b) $I_S = \dfrac{E}{R_T} = \dfrac{20}{8} = \boxed{2{,}5\ A}$

c) $I_S = I_1 = I_2 = I_3$

$V_{R_1} = I R_1 = 2{,}5 \times 2 = 5\ V$

$V_{R_2} = I R_2 = 2{,}5 \times 1 = 2{,}5\ V$

$V_{R_3} = I R_3 = 2{,}5 \times 5 = 12{,}5\ V$

$\underline{\underline{20\ V}}$

Series circuit

$\Rightarrow I_s = I_1 = I_2 = I_3 \cdots$

التيار يكون متساوي

$\Rightarrow V_s = V_1 + V_2 + V_3 + \cdots$

الجهد يتوزع على كل المقاومات .

$\Rightarrow R_T = R_1 + R_2 + R_3 + \cdots$

Example 5.4 : For the series circuit in Fig 5.18: Given *R_T* and *I_3* , calculate *R_1* and *E* .

Solution:

Since we are given the total resistance, it seems natural to first write the equation for the total resistance and then insert what we know.

$$R_T = R_1 + R_2 + R_3$$

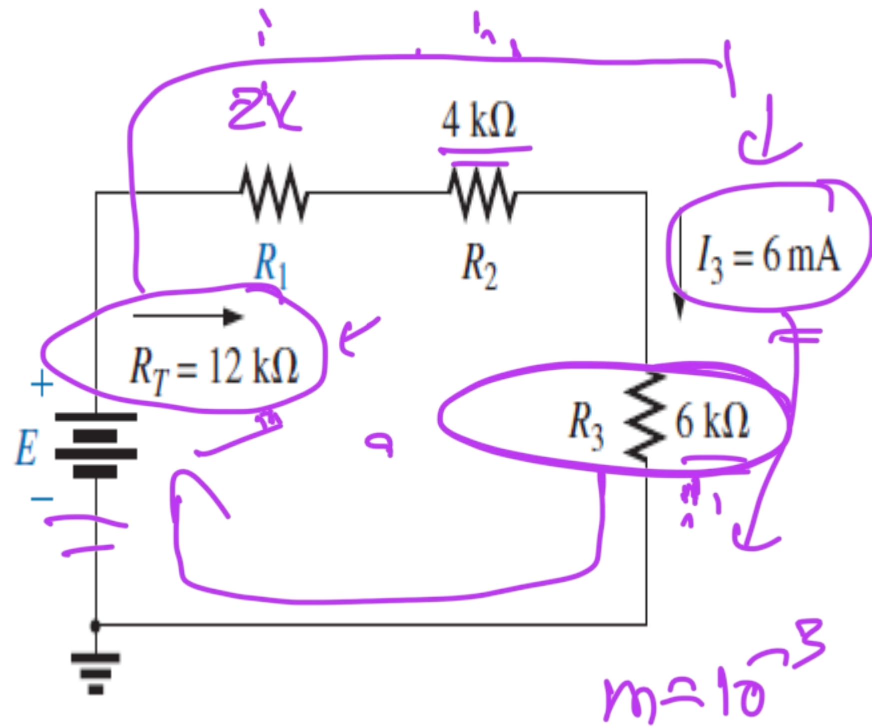


FIG. 5.18

We find that there is only one resistance unknown, and it can be determined with some simple mathematical manipulations. That is,

$$R_T = R_1 + R_2 + R_3$$
$$12\ k\Omega = R_1 + 4\ k\Omega\ \ + + 6\ k\Omega$$
$$R_1 = 12\ k\ \Omega - 10\ k\Omega$$
$$R_1 = 2\ k\Omega$$

The dc voltage can be determined directly from Ohm's law.

$$E = I_S \, . \, R_T = I_3\, R_T = (6mA)(12\ k\Omega\ ) = 72\ V$$

$$R_T = R_1 + R_2 + R_3$$

$$12 = R_1 + \underbrace{4 + 6}_{=10}$$

$$-10$$

$$2 = R_1 \Rightarrow \boxed{R_1 = 2\,k\Omega}$$

$$E = I_s * R_T$$

$$E = 6 \times \cancel{10^{-3}} * 12 \times \cancel{10^{3}} = \boxed{72V}$$

5.4 Power Distribution in a series Circuit

In any electrical system, the power applied will equal the power dissipated or absorbed. For any series circuit, such as that in Fig. 5.21,

the power applied by the dc supply must equal that dissipated by the resistive elements.

In equation form,

يشمل عام
سواء كان
series
أو لا

$$\boxed{P_E = P_{R_1} + P_{R_2} + P_{R_3}} \qquad (5.5)$$

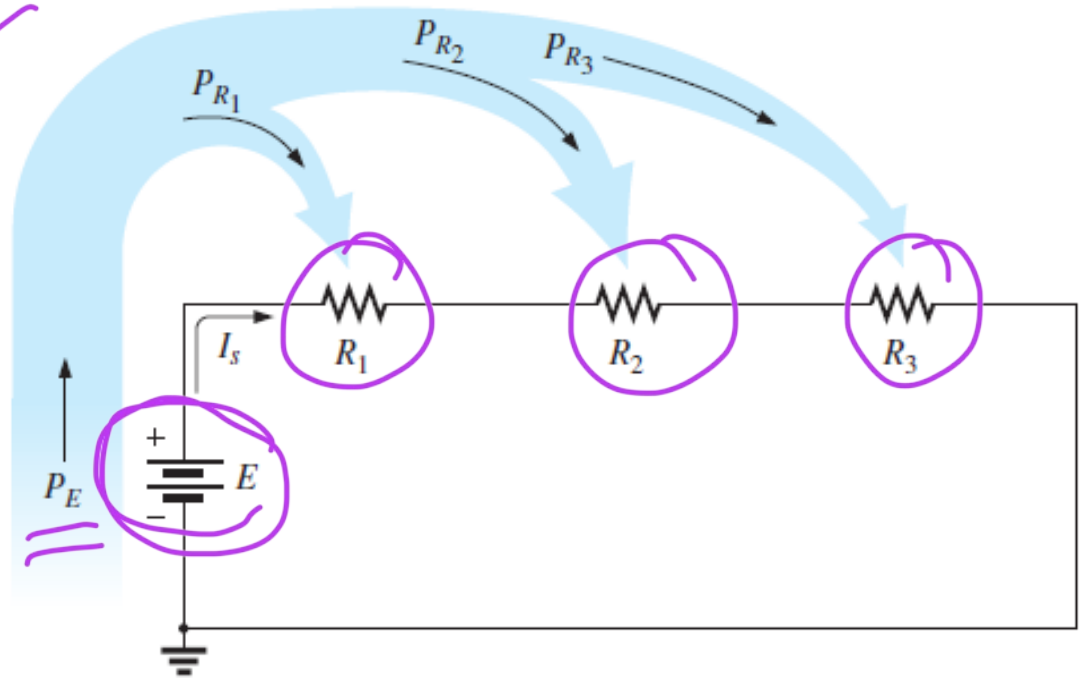


FIG. 5.21
Power distribution in a series circuit.

The power delivered by the supply can be determined using

VI

$$\boxed{P_E = EI_s} \quad \text{(watts, W)} \tag{5.6}$$

الناتج من مصدر تغذية تيار او جهد

The power dissipated by the resistive elements can be determined by any of the following forms (shown for resistor *R*1 only):

$V = IR$

$P = VI$

$I^2 R$

$P = \frac{V^2}{R}$

$$\boxed{P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1}} \quad \text{(watts, W)} \tag{5.7}$$

للمقاومة

Since the current is the same through series elements, you will find in the following examples that in a series configuration, **maximum power** is delivered to the largest resistor.

في حالة series المقاومة رح توخذ الأكبر power

Example 5.5 : For the series circuit in Fig 5.22:

رح تنتجة

a. Determine the total resistance R_T.
b. Calculate the current I_s.
c. Determine the voltage across each resistor.
d. Find the power supplied by the battery.

رح تستهلكة

e. Determine the power dissipated by each resistor.
f. Comment on whether the total power supplied equals to the total power dissipated.

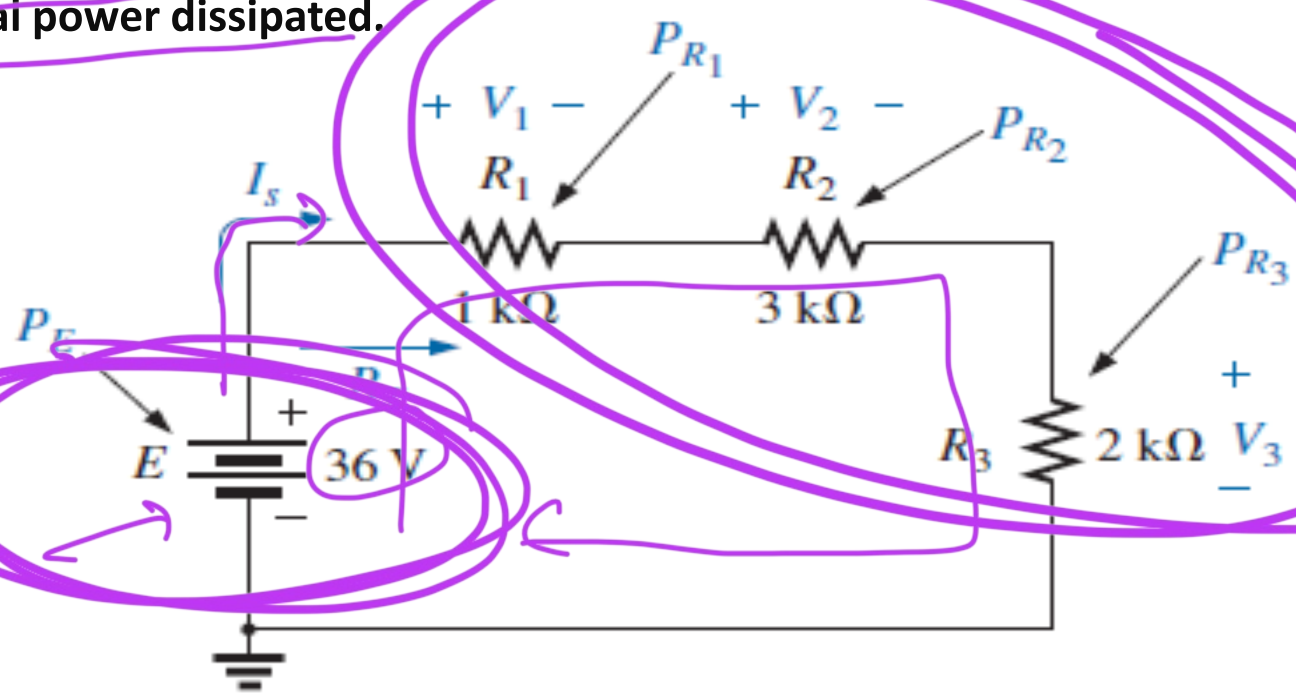


FIG. 5.22

a) $R_T = 1 + 3 + 2 = 6\,k\Omega$

b) $I_s = \dfrac{E}{R_T} = \dfrac{36}{6 \times 10^3} = 6\,mA$

c) $V_{R_1} = 6\cancel{m} \times 1\cancel{k} = \underline{6V}$

$V_{R_2} = 6\cancel{m} \times 3\cancel{k} = 18V$

$V_{R_3} = 6\cancel{m} \times 2\cancel{k} = \underline{12V}$

d) $P = IV = 6 \times 10^{-3} \times 36 = \boxed{216\,mW}$

c) $P_{R_1} = VI = 6 \times 6 \times 10^{-3} = 36\ mW$

$+$

$P_{R_2} = I^2 R_2 = (6 \times 10^{-3})^2 \times 3 \times 10^3 = 108\ mW$

$+$

$P_{R_3} = \frac{V^2}{R_3} = \frac{12^2}{2 \times 10^3} = 72\ mW$

$$\boxed{216\ mW}$$

Solution:

a. $R_T = R_1 + R_2 + R_3$
$= 1\ \text{k}\Omega + 3\ \text{k}\Omega + 2\ \text{k}\Omega$
$R_T = \mathbf{6\ k\Omega}$

b. $I_s = \dfrac{E}{R_T} = \dfrac{36\ \text{V}}{6\ \text{k}\Omega} = \mathbf{6\ mA}$

c. $V_1 = I_1R_1 = I_sR_1 = (6\ \text{mA})(1\ \text{k}\Omega) = \mathbf{6\ V}$
$V_2 = I_2R_2 = I_sR_2 = (6\ \text{mA})(3\ \text{k}\Omega) = \mathbf{18\ V}$
$V_3 = I_3R_3 = I_sR_3 = (6\ \text{mA})(2\ \text{k}\Omega) = \mathbf{12\ V}$

d. $P_E = EI_s = (36\ \text{V})(6\ \text{mA}) = \mathbf{216\ mW}$

e. $P_1 = V_1I_1 = (6\ \text{V})(6\ \text{mA}) = \mathbf{36\ mW}$
$P_2 = I_2^2R_2 = (6\ \text{mA})^2(3\ \text{k}\Omega) = \mathbf{108\ mW}$
$P_3 = \dfrac{V_3^2}{R_3} = \dfrac{(12\ \text{V})^2}{2\ \text{k}\Omega} = \mathbf{72\ mW}$

f. $P_E = P_{R_1} + P_{R_2} + P_{R_3}$
$216\ \text{mW} = 36\ \text{mW} + 108\ \text{mW} + 72\ \text{mW} = \mathbf{216\ mW}$ (checks)

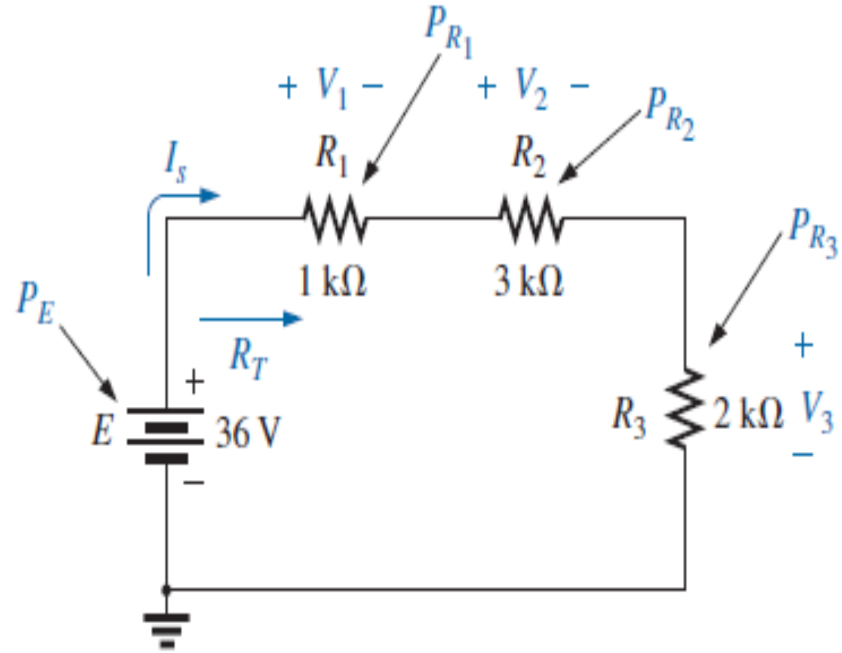
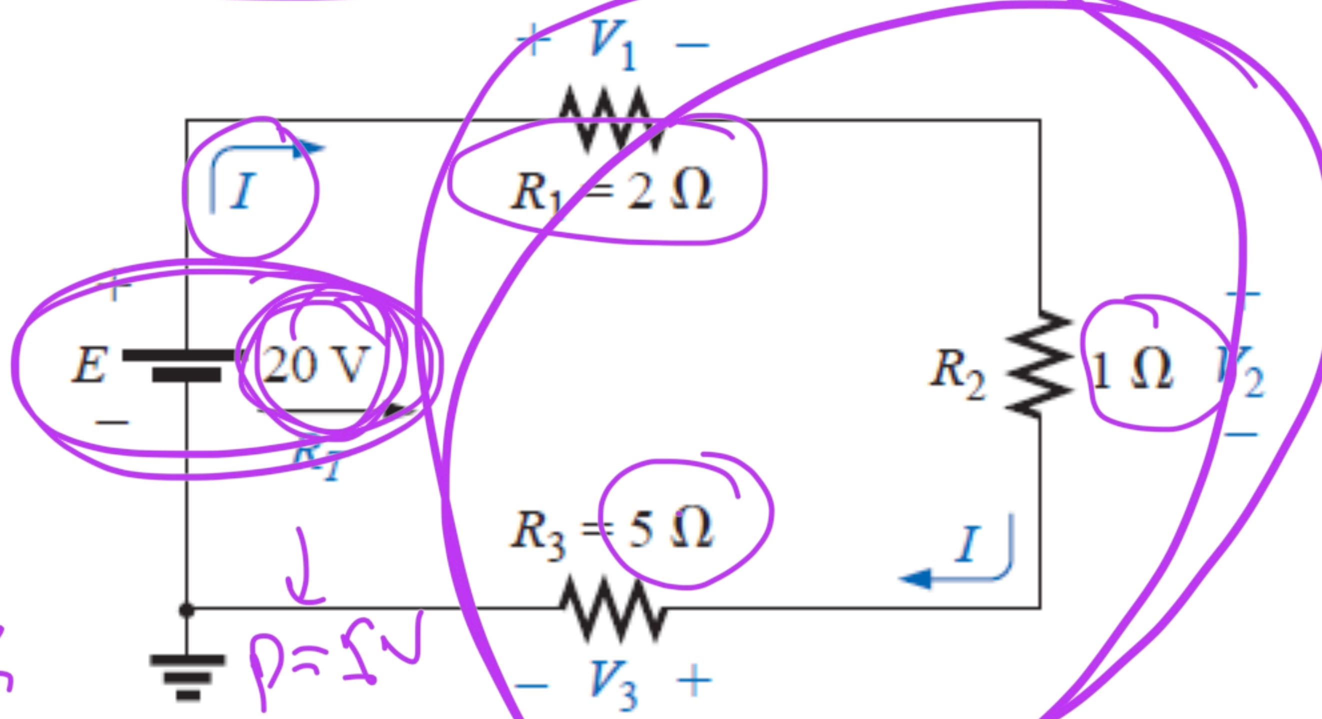


FIG. 5.22

Example 5.6 : For the series circuit in Fig 5.23:

a. Determine the total resistance R_T.
b. Calculate the current I_s.
c. Determine the voltage across each resistor.
d. Find the power supplied by the battery.
e. Determine the power dissipated by each resistor.
f. Comment on whether the total power supplied equals to the total power dissipated.

$V = IR$



$I = \frac{E}{R_T}$

$P = IV$

FIG 5.23

Solution:

a. $R_T = R_1 + R_2 + R_3 = 2\ \Omega + 1\ \Omega + 5\ \Omega$ $= 8\ \Omega$

b. $I_s = \dfrac{E}{R_T} = \dfrac{20\ \text{V}}{8\ \Omega} = \mathbf{2.5\ A}$

c. $V_1 = IR_1 = (2.5\ \text{A})(2\ \Omega) = \mathbf{5\ V}$
$V_2 = IR_2 = (2.5\ \text{A})(1\ \Omega) = \mathbf{2.5\ V}$
$V_3 = IR_3 = (2.5\ \text{A})(5\ \Omega) = \mathbf{12.5\ V}$

d. $P_1 = V_1I_1 = (5\ \text{V})(2.5\ \text{A}) = \mathbf{12.5\ W}$
$P_2 = I_2^2R_2 = (2.5\ \text{A})^2(1\ \Omega) = \mathbf{6.25\ W}$
$P_3 = V_3^2/R_3 = (12.5\ \text{V})^2/5\ \Omega = \mathbf{31.25\ W}$

e. $P_{\text{del}} = EI = (20\ \text{V})(2.5\ \text{A}) = \boxed{\mathbf{50\ W}}$
$P_{\text{del}} = P_1 + P_2 + P_3$
$50\ \text{W} = 12.5\ \text{W} + 6.25\ \text{W} + 31.25\ \text{W}$
$50\ \text{W} = 50\ \text{W}$ (checks)

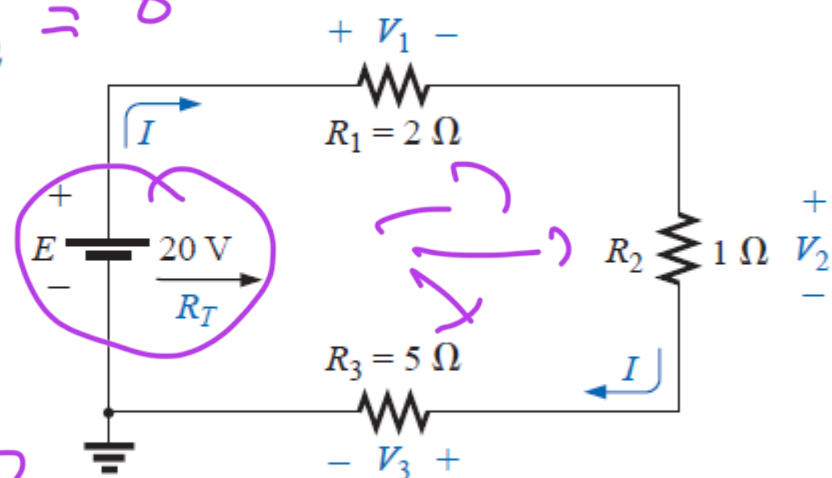


FIG 5.23

5.5 Voltage Sources in series

Voltage sources can be connected in series, as shown in Fig. 5.23, to increase or decrease the total voltage applied to a system. The net voltage is determined <u>by summing the sources with the same polarity and subtracting the total of the sources with the opposite polarity</u>. The net polarity is the polarity of the larger sum

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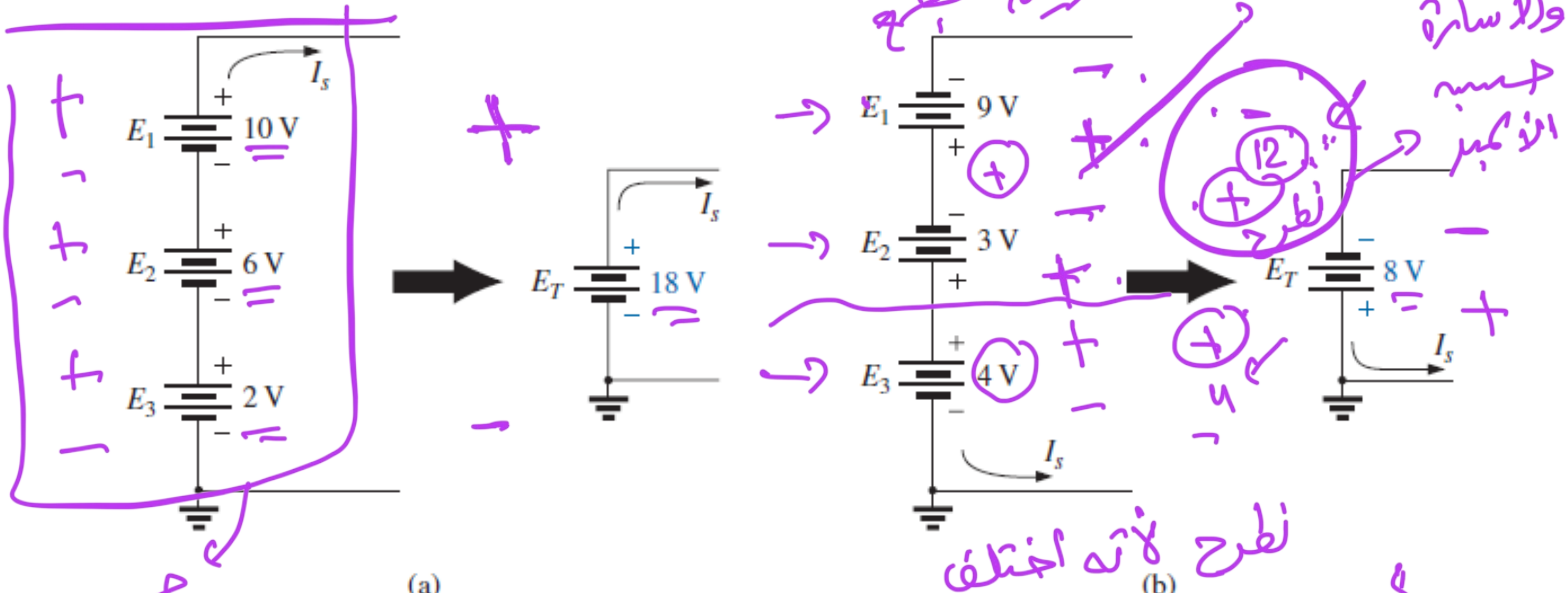
والإشارة حسب الأكبر



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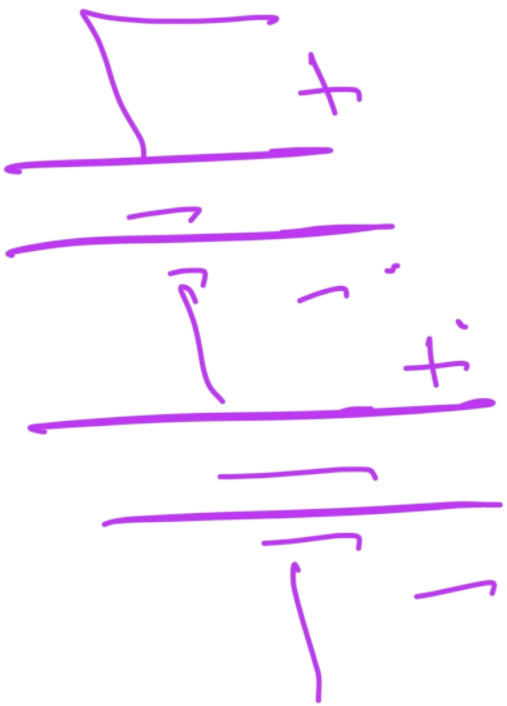
نطرح لأنه اختلف الترتيب

FIG. 5.23

Reducing series dc voltage sources to a single source.

حالة توصيل البطاريات :-

(1) series يعني ورا بعض



In Fig. 5.23(a), for example, the sources are all “pressuring” current to follow a clockwise path, so the net voltage is

$$E_T = E_1 + E_2 + E_3 = 10\ V + 6\ V + 2\ V = 18\ V$$

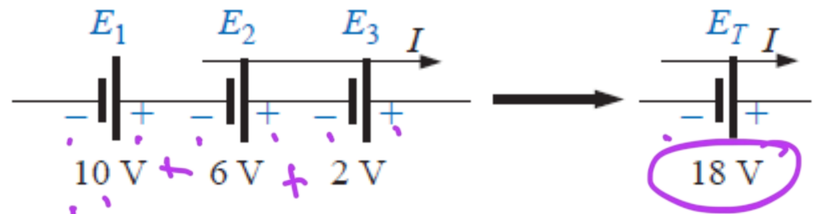
as shown in the figure. In Fig. 5.23(b), however, the 4 V source is “pressuring” current in the clockwise direction while the other two are trying to establish current in the counter-clockwise direction. In this case, the applied voltage for a counter-clockwise direction is greater than that for the clockwise direction.

The result is the counter-clockwise direction for the current as shown in Fig. 5.23(b). The net effect can be determined by finding the difference in applied voltage between those supplies “pressuring” current in one direction and the total in the other direction.
In this case,

$$E_T = E_1 + E_2 - E_1 = 9\ V + 3\ V - 4\ V = 8\ V$$

Example 5.7 : Reduce the voltage sources into a single source Fig 5.24 in the followings :

Solution:



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نجمع

Fig 5.24 (a)

Solution:

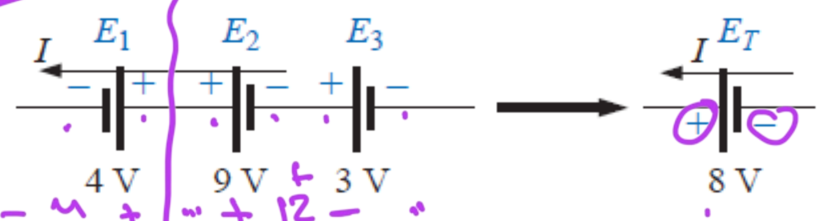

Fig 5.24 (b)

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اختلف الترتيب

Solution:

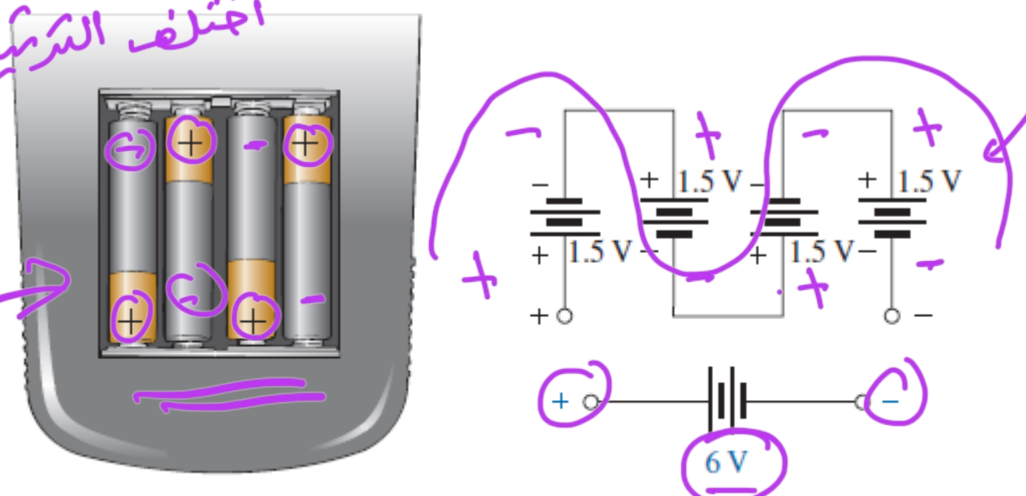


Fig 5.24 (c)

5.6 Kirchhoff’s Voltage Law

The law, called Kirchhoff’s voltage law (KVL), was developed by Gustav Kirchhoff (Fig. 5.25) in the mid-1800s. It is a cornerstone of the entire field and, in fact, will never be outdated or replaced.

The application of the law requires that we define a closed path of investigation, permitting us to start at one point in the network, travel through the network, and find our way back to the original starting point.

FIG. 5.25
Gustav Robert Kirchhoff.
Library of Congress Prints and Photographs Division

German (Königsberg, Berlin)
(1824–87),
Physicist
Professor of Physics, University of Heidelberg

Although a contributor to a number of areas in the physics domain, he is best known for his work in the electrical area with his definition of the relationships between the currents and voltages of a network in 1847. Did extensive research with German chemist Robert Bunsen (developed the *Bunsen burner*), resulting in the discovery of the important elements of *cesium and rubidium.*

The path does not have to be circular, square, or any other defined shape; it must simply provide a way to leave a point and get back to it without leaving the network. In Fig. 5.26, if we leave point *a* and follow the current, we will end up at point *b.* Continuing, we can pass through points *c* and *d* and eventually return through the voltage source to point *a,* our starting point.

The path ***abcda*** is therefore a closed path, or closed loop.

The law specifies that مجموع الجهود داخل أي مسار مغلق = 0

the algebraic sum of the potential rises and drops around a closed path (or closed loop) is zero.

KVL
k
voltage
law

In symbolic form it can be written as

$$\boxed{\Sigma V = 0}$$

$$\boxed{\Sigma_C V = 0} \quad \text{(Kirchhoff's voltage law in symbolic form)} \quad (5.8)$$

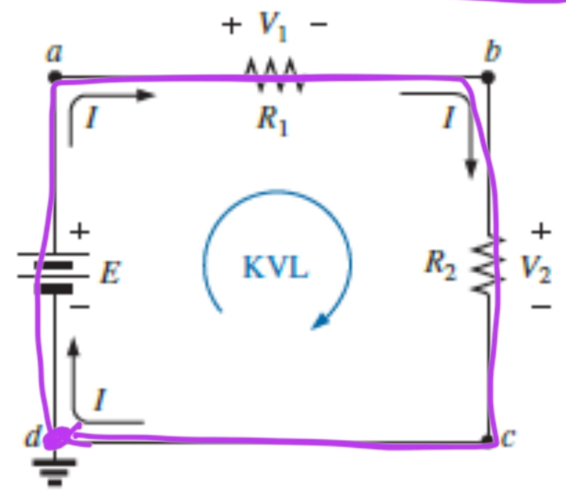
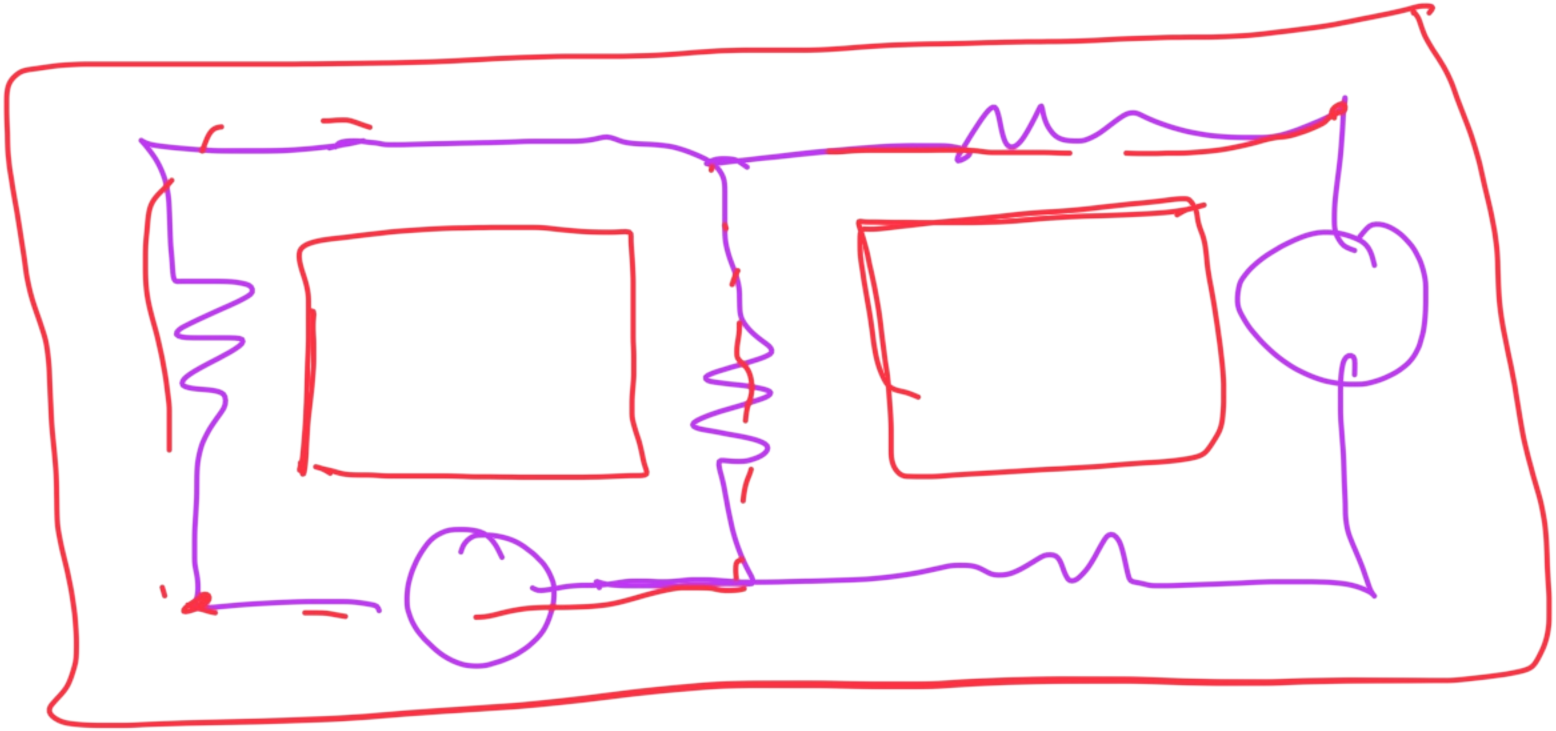


FIG. 5.26
Applying Kirchhoff's voltage law to a series dc circuit.



يوجد هنا 3 مسارات مغلقة

Kirchhoff's Voltage Law

The term *algebraic* simply means paying attention to the signs that result in the equations as we add and subtract terms.

The first question that often arises is, Which way should I go around the closed path? Should I always follow the direction of the current? To simplify matters, this text will always try to move in a clockwise direction.

By selecting a direction, you eliminate the need to think about which way would be more appropriate. Any direction will work as long as you get back to the starting point.

Another question is, How do I apply a sign to the various voltages as I proceed in a clockwise direction?

For a particular voltage, we will assign a positive sign when proceeding from the negative to positive potential , a positive experience such as moving from a negative checking balance to a positive one. The opposite change in potential level results in a negative sign.

$$\boxed{V_s = V_1 + V_2 + V_3}$$ series

In Fig. 5.26, as we proceed from point *d* to point *a* across the voltage source, we move from a negative potential (the negative sign) to a positive potential (the positive sign), so a positive sign is given to the source voltage *E*. As we proceed from point *a* to point *b*, we encounter a positive sign followed by a negative sign, so a drop in potential has occurred, and a negative sign is applied. Continuing from *b* to *c*, we encounter another drop in potential, so another negative sign is applied. We then arrive back at the starting point *d*, and the resulting sum is set equal to zero as defined by Eq. (5.8).

Writing out the sequence with the voltages and the signs results in the following:

$$+E - V_1 - V_2 = 0$$

which can be rewritten as: $E = V_1 + V_2$

The result is particularly interesting because it tells us that

The applied voltage of a series dc circuit will equal the sum of the voltage drops of the circuit.

Kirchhoff's voltage law can also be written in the following form:

$$\boxed{\Sigma_{\circlearrowright} V_{\text{rises}} = \Sigma_{\circlearrowright} V_{\text{drops}}} \qquad (5.9)$$

To demonstrate that the direction that you take around the loop has no effect on the results, let's take the counterclockwise path and compare results. The resulting sequence appears as

$$-E + V_2 + V_1 = 0$$

yielding the same result of $E = V_1 + V_2$

Example 5.7 : Use Kirchhoff's voltage law to determine the unknown voltage for the circuit in Fig. 5.27.

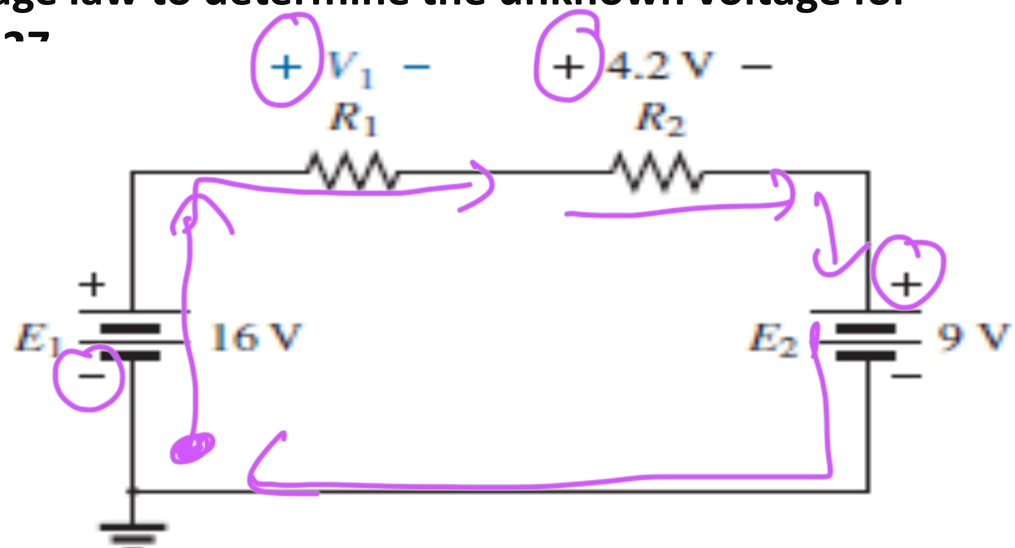


FIG. 5.27
Series circuit to be examined in Example 5.8.

Solution:

$+E_1 - V_1 - V_2 - E_2 = 0$
and $V_1 = E_1 - V_2 - E_2$
$= 16\text{ V} - 4.2\text{ V} - 9\text{ V}$
so $V_1 =$ **2.8 V**

$$\Sigma V = 0$$

$$-16 + V_1 + 4{,}2 + 9 = 0$$

$$V_1 = 16 - 9 - 4{,}2 = \boxed{2{,}8\,V}$$

Example 5.8 : **Using Kirchhoff's voltage law, determine the unknown voltage for the circuit in Fig. 5.30.**

Solution:

$$+60\text{ V} - 40\text{ V} - V_x + 30\text{ V} = 0$$
$$V_x = 60\text{ V} + 30\text{ V} - 40\text{ V} = 90\text{ V} - 40\text{ V}$$
$$V_x = \mathbf{50\ V}$$

$$-60 + 40 + V_x - 30 = 0$$

$$V_x = 60 - 40 + 30 \qquad 60 - 10 = \boxed{50V}$$

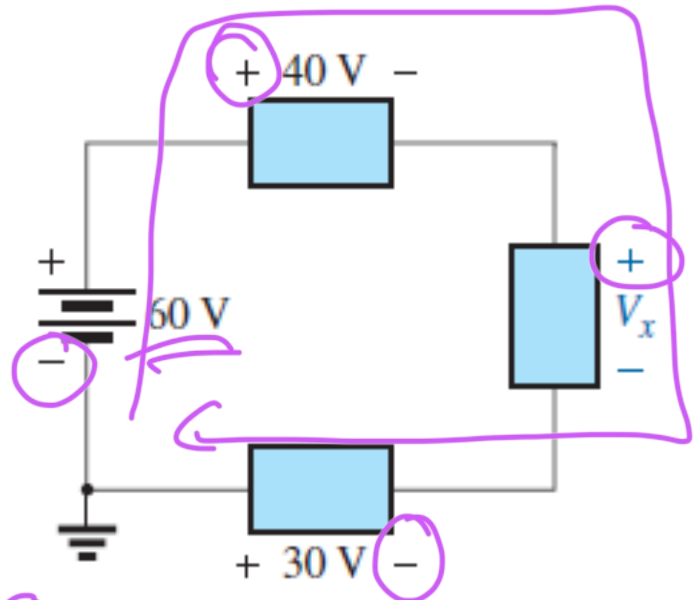


FIG. 5.30
Series configuration to be examined in Example 5.11.

Example 5.9 : **Determine the voltage *Vx* for the circuit in Fig. 5.31. Note that the polarity of *Vx* was not provided.**

$V_x = 18V$

Solution:

$$-6\text{ V} - 14\text{ V} - V_x + 2\text{ V} = 0$$
$$V_x = -20\text{ V} + 2\text{ V}$$
$$V_x = \mathbf{-18\text{ V}}$$

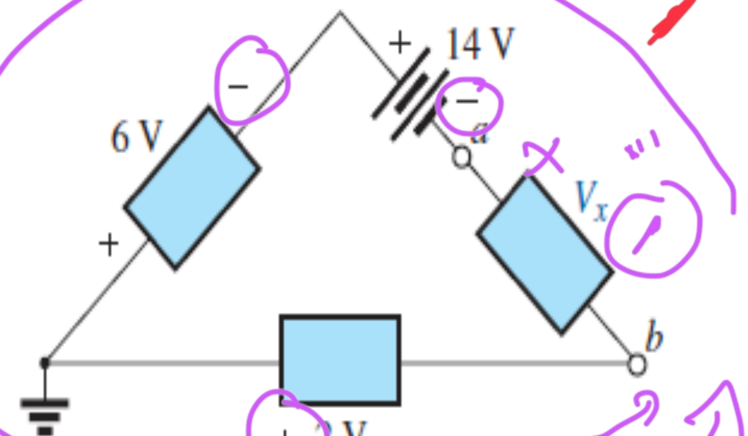


FIG. 5.31
Applying Kirchhoff's voltage law to a circuit in which the polarities have not been provided for one of the voltages (Example 5.12).

$-V_x + -14 + -6 + 2 = 0$

$V_x = -14 - 6 + 2 = -18V$

Example 5.10 : **Determine the unknown voltage V_x for the circuit in Fig. 5.28.**

Solution:

$+E - V_1 - V_x = 0$
and

$V_x = E - V_1 = 32\text{ V} - 12\text{ V} = 20\text{ V}$

For the clockwise path, including resistor R_3, the following results:

$+V_x - V_2 - V_3 = 0$
and

$V_x = V_2 + V_3 = 6\text{ V} + 14\text{ V} = 20\text{ V}$

providing exactly the same solution.

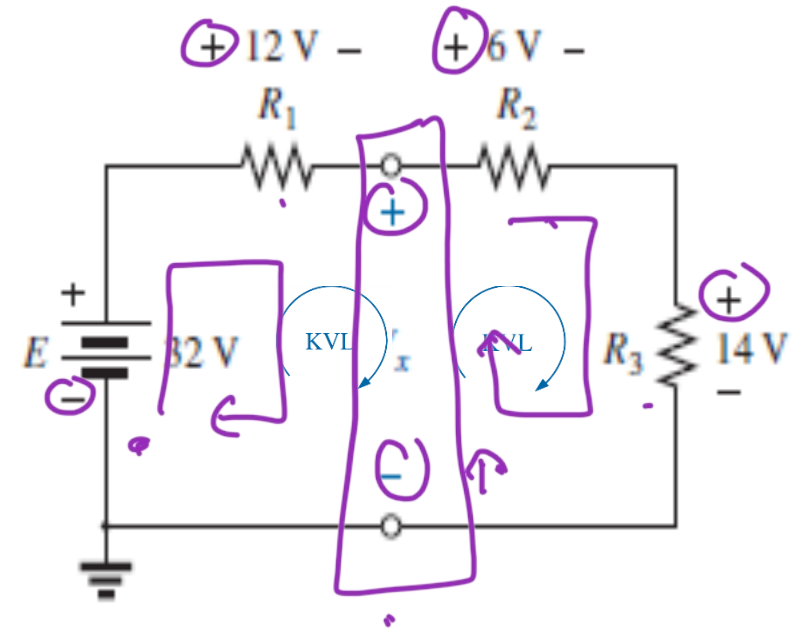


FIG. 5.28
Series dc circuit to be analyzed in Example 5.9.

$-32 + 12 + V_x = 0$

$V_x = 32 - 12 = 20\text{ V}$

$-V_x + 6 + 14 = 0 \Rightarrow V_x = 20\text{ V}$

Example 5.11 : Using Kirchhoff's voltage law, determine voltages V_1 and V_2 for the network in Fig. 5.29.

$-25+V_1-15=0$

$V_1=15+25=40V$

Solution:

For path 1, starting at point *a* in a clockwise direction,

$+25\text{ V} - V_1 + 15\text{ V} = 0$

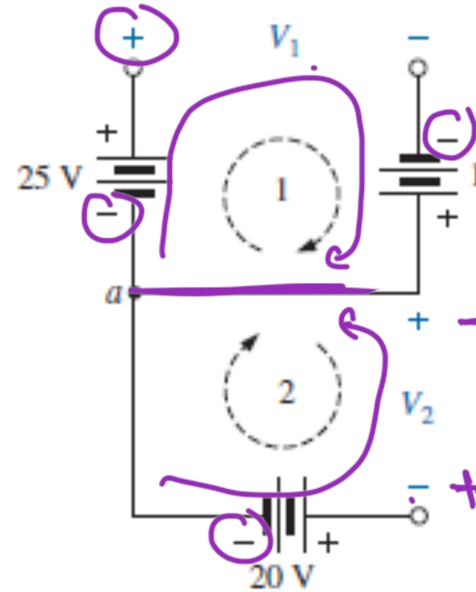
and $V_1 = \mathbf{40\ V}$

For path 2, starting at point *a* in a clockwise direction,

$-V_2 - 20\text{ V} = 0$

and $V_2 = \mathbf{20\ V}$

The minus sign in the solution simply indicates that the actual polarities are different from those assumed.



$+-20-V_2=0$

$V_2=-20V$

FIG. 5.29
Combination of voltage sources to be examined in Example 5.10.

Example 5.12 : For the series circuit in Fig. 5.32

a. Determine V_2 using Kirchhoff's voltage law.

b. Determine current I_2.

c. Find R_1 and R_3.

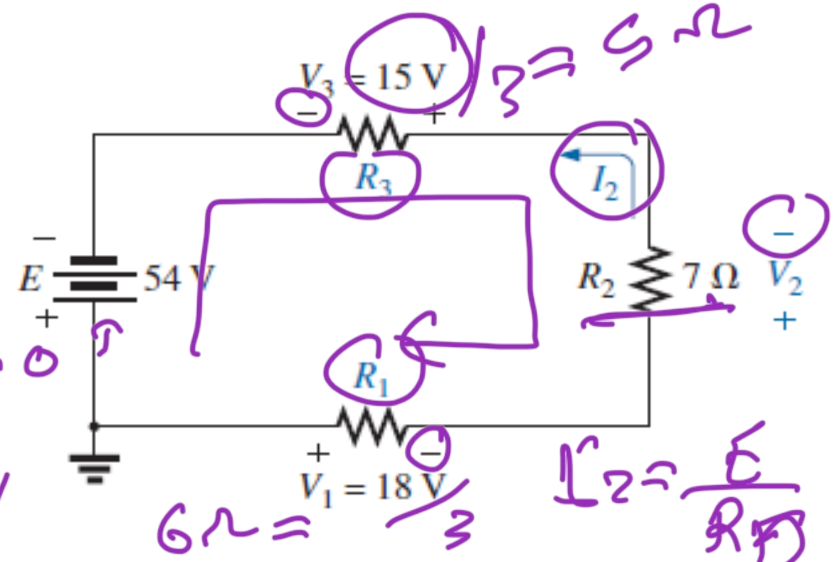


FIG. 5.32

Series configuration to be examined in Example 5.13.

Solution:

$54 + -15 + -V_2 - 18 = 0$

$V_2 = 54 - 15 - 18 = 21V$

$I_2 = \frac{E}{R_2}$

$\frac{21}{7} = 3A$

$V = IR \rightarrow R = \frac{V}{I}$

a. Applying Kirchhoff's voltage law in the clockwise direction starting at the negative terminal of the supply results in

$$-E + V_3 + V_2 + V_1 = 0$$

and $$E = V_1 + V_2 + V_3 \text{ (as expected)}$$

so that $$V_2 = E - V_1 - V_3 = 54\text{ V} - 18\text{ V} - 15\text{ V}$$

and $$V_2 = \mathbf{21\ V}$$

b. $$I_2 = \frac{V_2}{R_2} = \frac{21\text{ V}}{7\ \Omega}$$

$$I_2 = \mathbf{3A}$$

c. $$R_1 = \frac{V_1}{I_1} = \frac{18\text{ V}}{3\text{ A}} = \mathbf{6\ \Omega}$$

with $$R_3 = \frac{V_3}{I_3} = \frac{15\text{ V}}{3\text{ A}} = \mathbf{5\ \Omega}$$

5.6 Voltage Division in a series Circuit

The voltage across series resistive elements will divide as the magnitude of the resistance levels.

In other words, in a series resistive circuit, the larger the resistance, the more of the applied voltage it will capture.

① $I_s = \frac{E}{R_T}$

In addition, the ratio of the voltages across series resistors will be the same as the ratio of their resistance levels.

② $V_{R_1} = I_s * R_1$

$V_S = V_1 + V_2 + V_3$

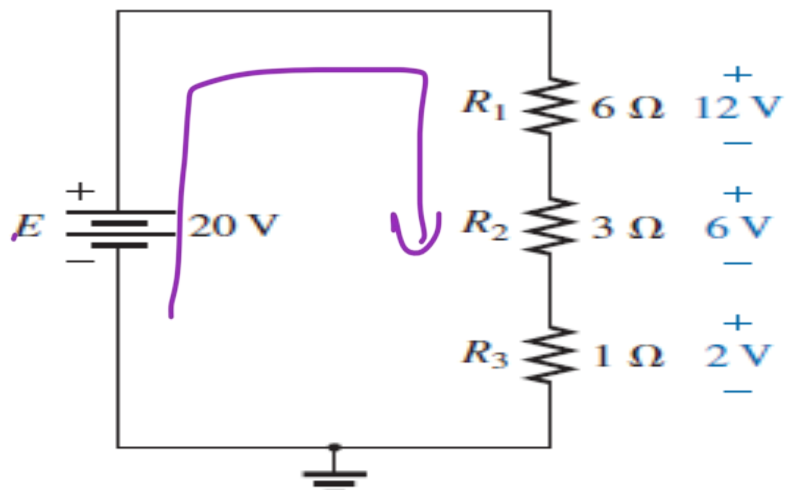


FIG. 5.33
Revealing how the voltage will divide across series resistive elements.

Voltage Divider Rule (VDR)

series

- **The voltage divider rule (VDR) permits the determination of the voltage across a series resistor without first having to determine the current of the circuit.**

- **The voltage across a resistor in a series circuit is equal to the value of that resistor times the total applied voltage divided by the total resistance of the series configuration.**
- **Formula :**

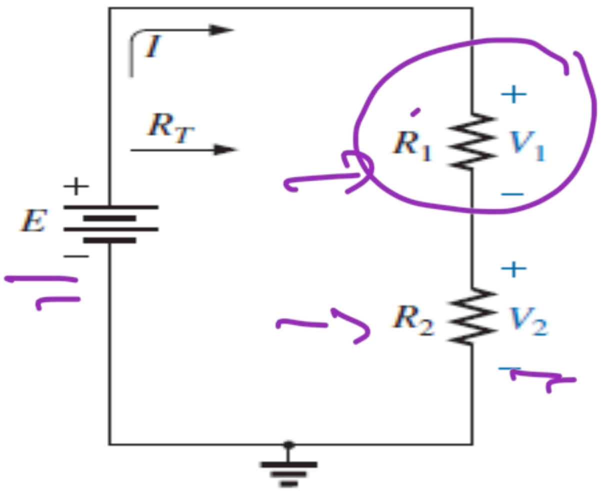


FIG. 5.36
Developing the voltage divider rule.

$$\boxed{V_x = R_x \frac{E}{R_T}} \qquad \text{(voltage divider rule)} \qquad \textbf{(5.10)}$$

$$V_1 = I_1R_1 = \left(\frac{E}{R_T}\right)R_1 = R_1\frac{E}{R_T}$$

$$V_2 = I_2R_2 = \left(\frac{E}{R_T}\right)R_2 = R_2\frac{E}{R_T}$$

$$\boxed{V_{R_1} = \frac{R_1}{R_1+R_2} * E}$$

$$\boxed{V_{R_2} = \frac{R_2}{R_1+R_2} * E}$$

Example 5.13 : **Using the voltage divider rule, determine voltages *V*1 and *V*3 for the series circuit in Fig. 5.38.**

$$V_1 = \frac{2}{2+5+8} * 45 = 6V$$

Solution:

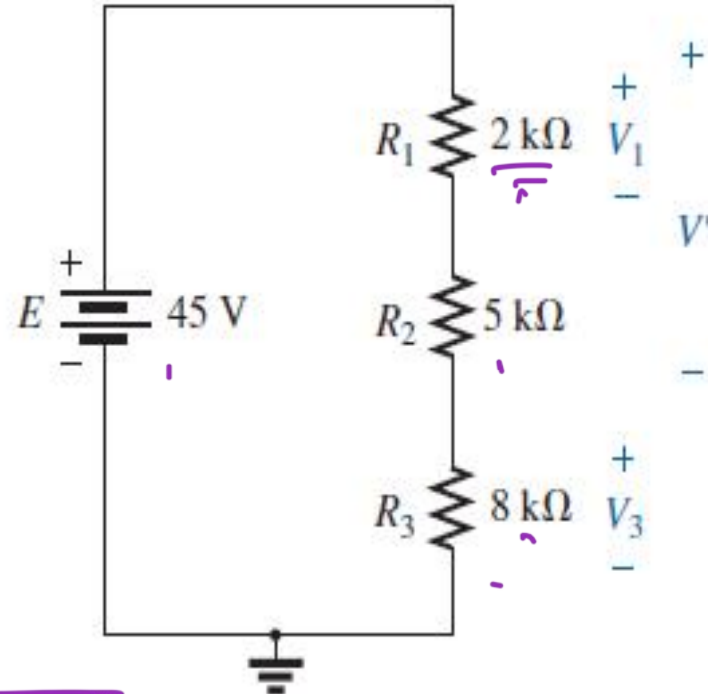


FIG. 5.38
Series circuit to be investigated in Examples 5.16 and 5.17.

$$R_T = R_1 + R_2 + R_3$$
$$= 2\ k\Omega + 5\ k\Omega + 8\ k\Omega$$
$$R_T = 15 k\Omega$$

$$V_1 = R_1\frac{E}{R_T} = 2\ k\Omega\left(\frac{45\ V}{15 k\Omega}\right) = \mathbf{6\ V}$$

$$V_3 = R_3\frac{E}{R_T} = 8\ k\Omega\left(\frac{45\ V}{15\ \Omega}\right) = \mathbf{24\ V}$$

$$V_3 = \frac{8}{2+5+8} * 45 = \boxed{24V}$$

<u>Example 5.14</u> : Using the voltage divider rule, determine voltages $V1$ and $V2$ for the series circuit in Fig. 5.62.

Solution:

Redrawing the network with the standard battery symbol results in the network in Fig. 5.63.

Applying the voltage divider rule,

$$V_1 = \frac{R_1 E}{R_1 + R_2} = \frac{(4\,\Omega)(24\text{ V})}{4\,\Omega + 2\,\Omega} = \mathbf{16\text{ V}}$$

$$V_2 = \frac{R_2 E}{R_1 + R_2} = \frac{(2\,\Omega)(24\text{ V})}{4\,\Omega + 2\,\Omega} = \mathbf{8\text{ V}}$$

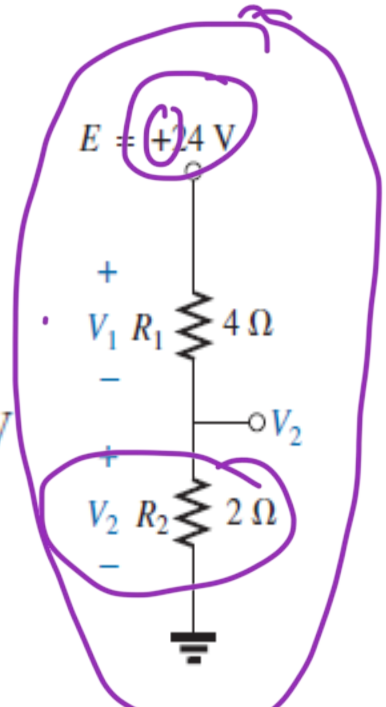


FIG. 5.62
Example 5.26.

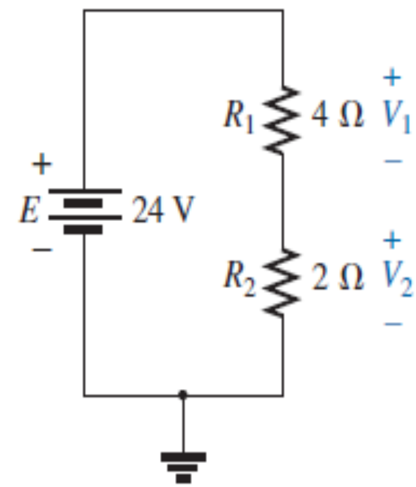
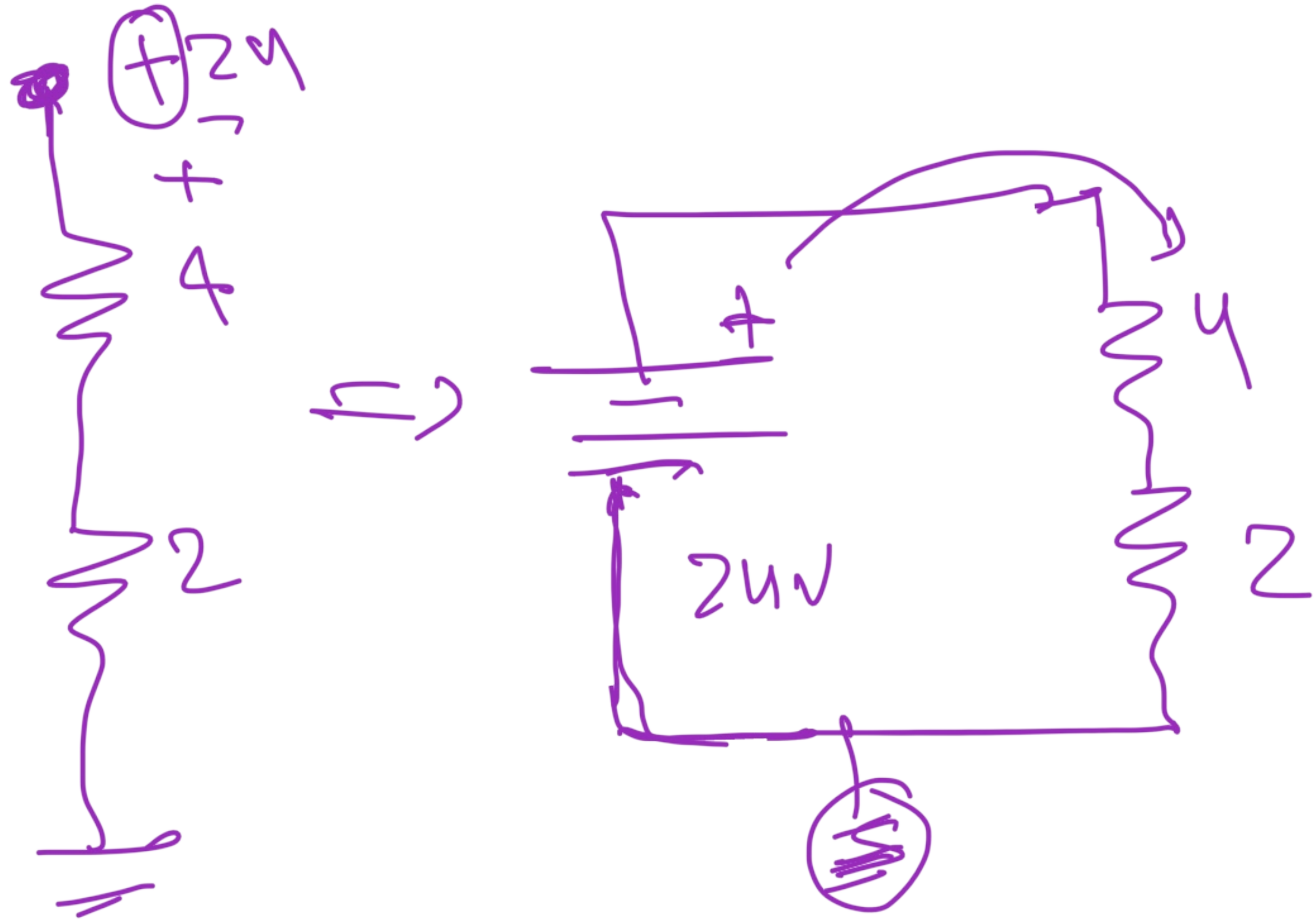


FIG. 5.63
Circuit of Fig. 5.62 redrawn.



+ 24
4
2
24V
4
2

Home work

Problem 1 : For the series configuration in Fig. 5.92, constructed using standard value resistors:

a. Without making a single calculation, which resistive element will have the most voltage across it? Which will have the least?

b. Which resistor will have the most impact on the total resistance and the resulting current? Find the total resistance and the current .

$V = IR$

الأقل

الجهد الأعلى

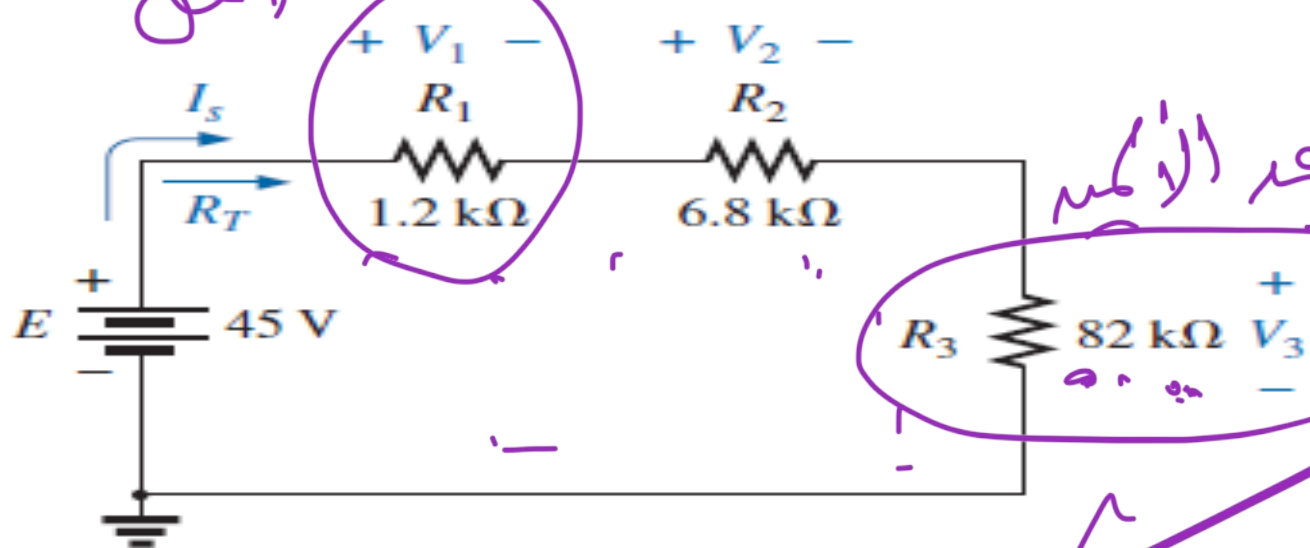


FIG. 5.92
Problem 8.

Problem 2: For the circuit of Fig. 5.80:

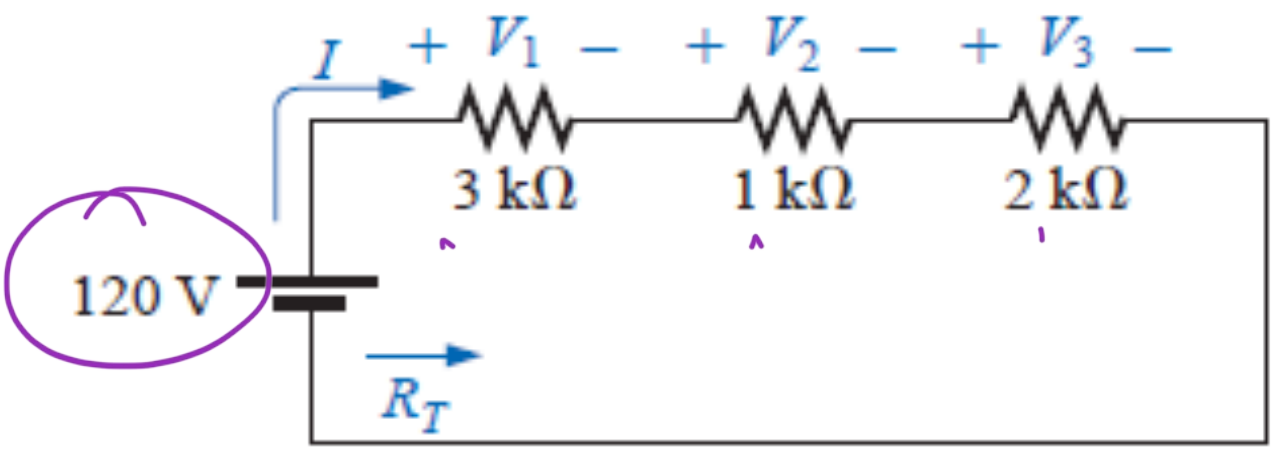
a. Find the total resistance, current, and unknown voltage drops.

b. Verify Kirchhoff's voltage law around the closed loop.

c. Find the power dissipated by each resistor, and note whether the power delivered is equal to the power dissipated.

d. If the resistors are available with wattage ratings of 1/2, 1, and 2 W, what minimum wattage rating can be used for each resistor in this circuit?.

60



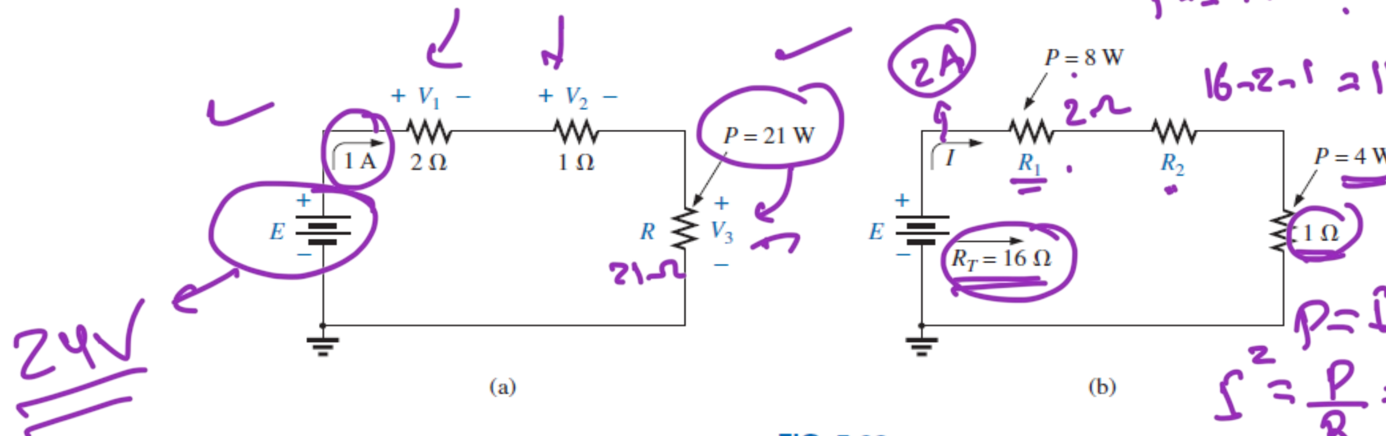
$$V_1 = \frac{3}{3+1+2} \times 120$$

FIG. 5.80

Problem 10.

Problem 3: Find the unknown quantities for the circuits in Fig. 5.98 using the information provided.

$P = I^2 R \Rightarrow 8 = 4 \times R_1 \Rightarrow R_1 = 2$



24V

$P = I^2 R$

$I^2 = \frac{P}{R} = \frac{4}{1}$

$\sqrt{I^2} = \sqrt{4}$

$I = 2$

FIG. 5.98
Problem 14.

Problem 4: Combine the series voltage sources in Fig. 5.101 into a single voltage source between points *a* and *b.*

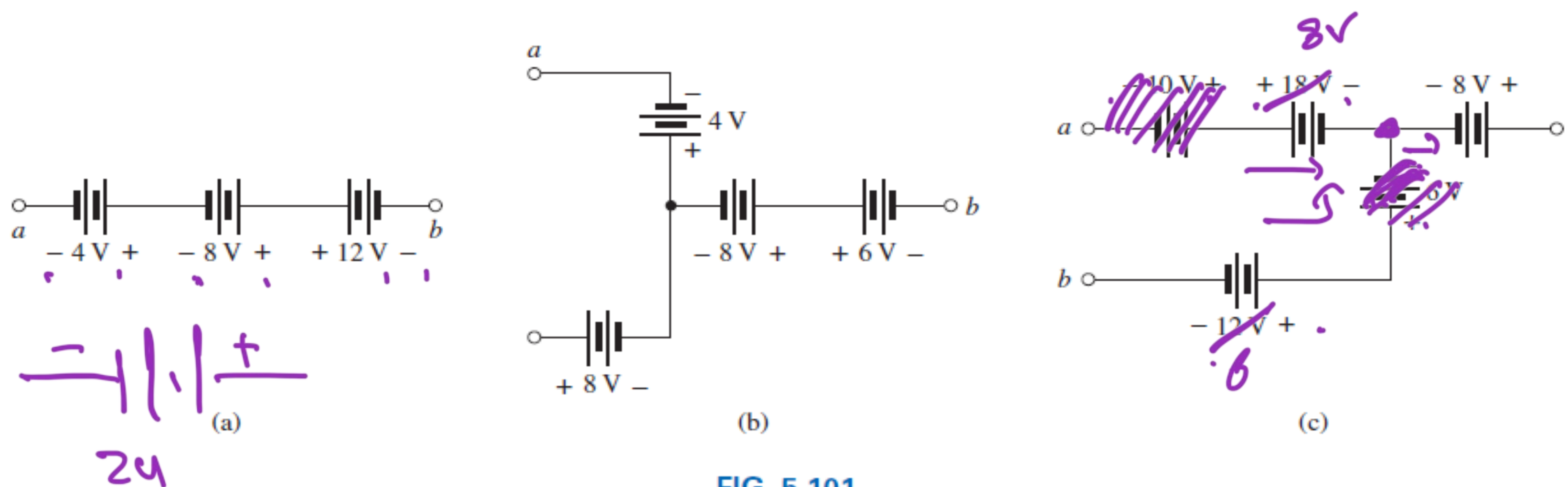
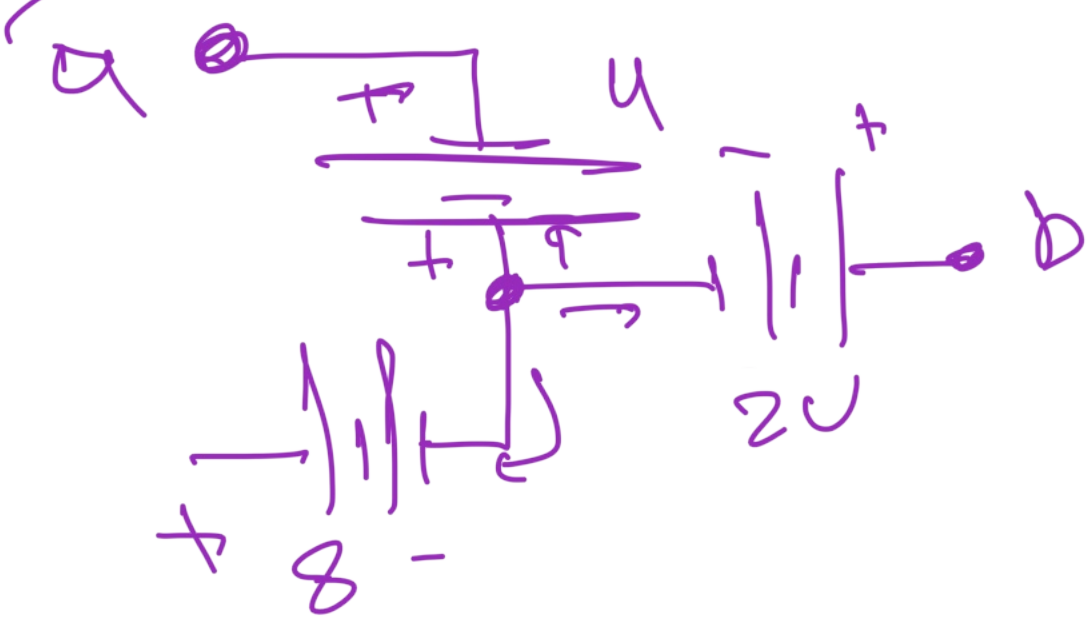


FIG. 5.101
Problem 17.

$$P = I^2 R \rightarrow R = \frac{P}{I^2} = \frac{21}{1^2} = 21\,\Omega$$

$$R_T = 2 + 1 + 21 = \underline{\underline{24\,\Omega}}$$

$$E = I_s R_T = 1 \times 24 = 24\,V$$



Problem 5: Find the unknown voltage source and resistor for the networks in Fig. 5.103. First combine the series voltage sources into a single source. Indicate the direction of the resulting current.

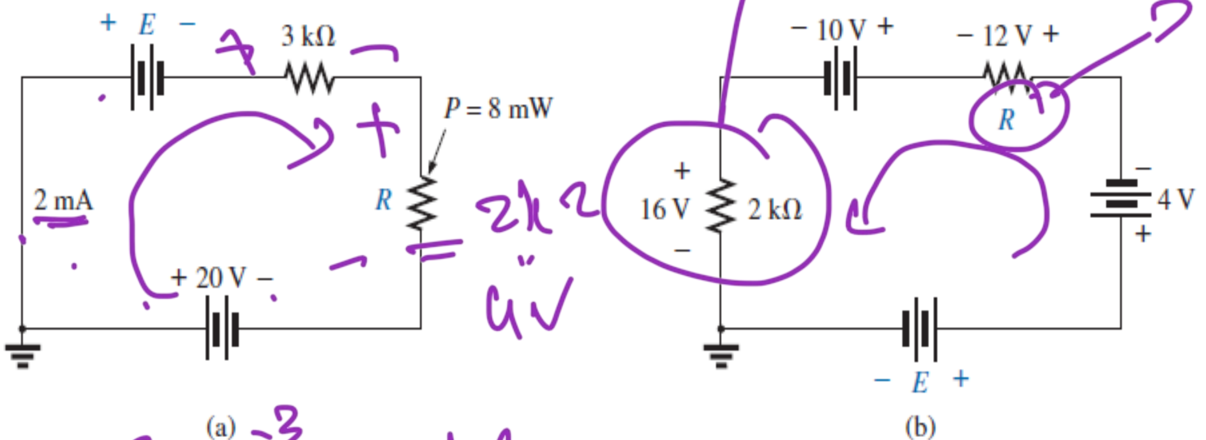
$P = I^2 R$

$V = IR$

$6V$

$I = \frac{16}{2} = 8mA$

$R = \frac{V}{I} = \frac{12}{8m} = 1.5k\Omega$



$2k\Omega$

$4V$

FIG. 5.103
Problem 19.

$R = \frac{P}{I^2} = \frac{8 \times 10^{-3}}{(2 \times 10^{-3})^2} = 2k\Omega$

$E + 6 + 4 - 20 = 0$

$E = 10V$

$-E + 4 + 12 + 10 + 16 = 0$

$\boxed{E = 42V}$

Problem 6: Using Kirchhoff's voltage law, find the unknown voltages for the circuits in Fig. 5.104.

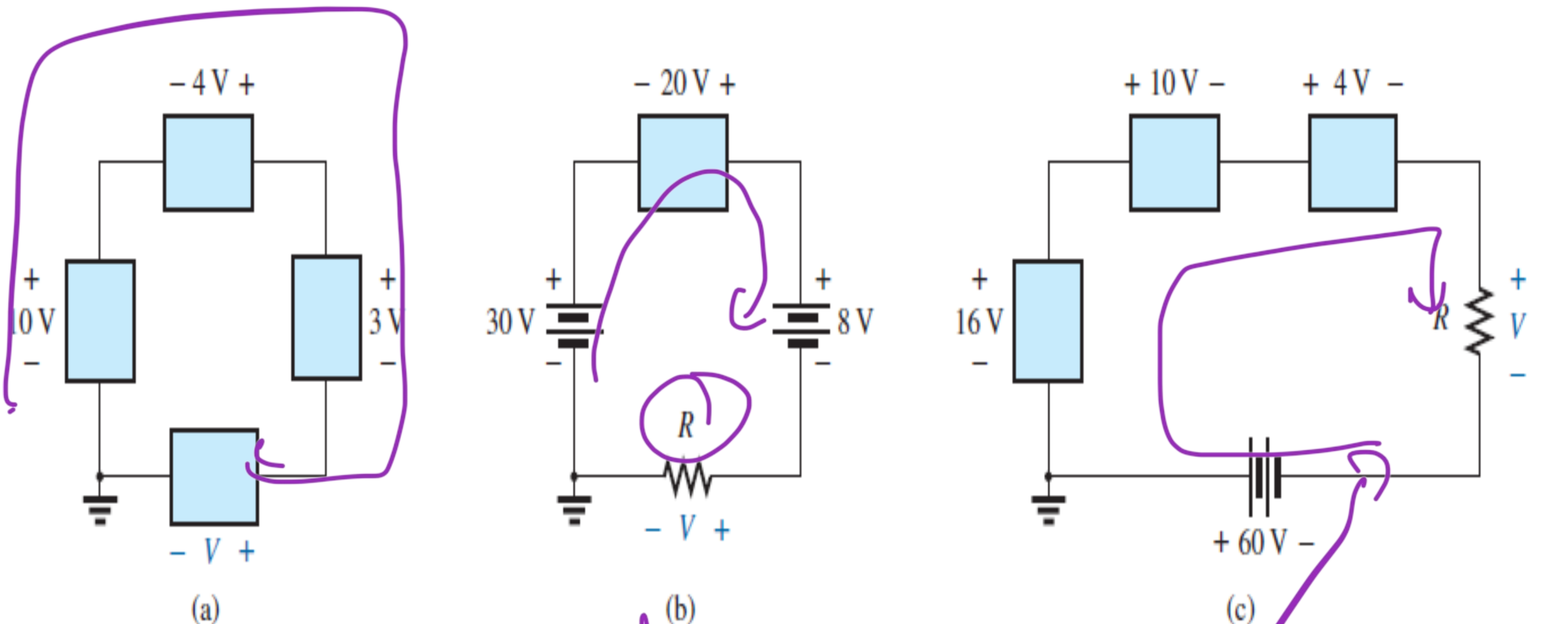


FIG. 5.104
Problem 20.

$-10 + -4 + 3 + V = 0$

$V = 10 + 4 - 3 = 11V$

$-60 - 16 + 10 + 4 + V = 0$

$V = 60 + 16 - 10 - 4$

$\boxed{V = 62V}$

$$\underbrace{-30 - 20 + 8}\overbrace{+ V = 0}$$

$$V = 30 + 20 - 8 = 42V$$

Problem 7: Using the voltage divider rule or Kirchhoff's voltage law, determine the unknown voltages for the configurations in Fig. 5.111.

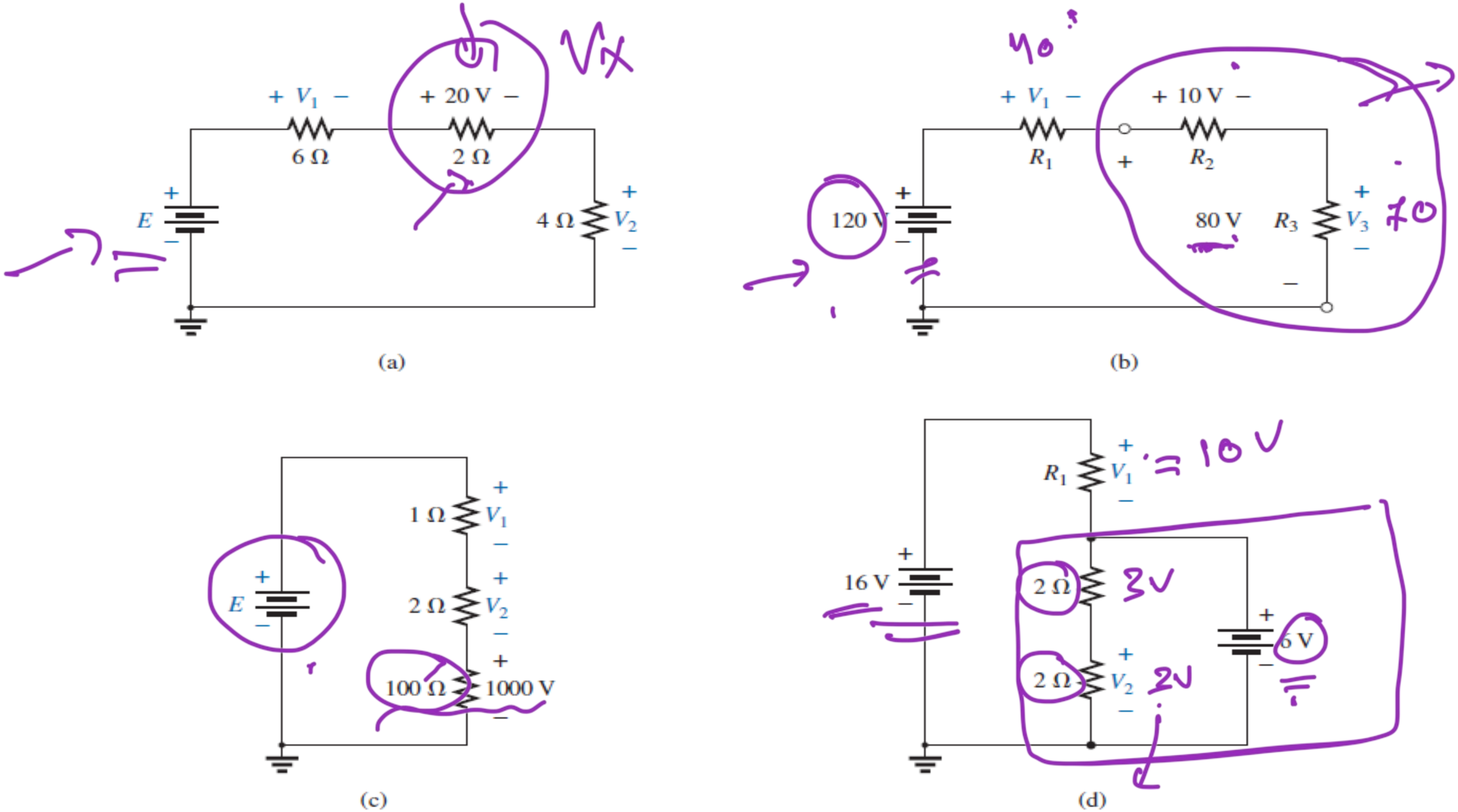


FIG. 5.111
Problem 27.

$$\underline{V_x} = \frac{2}{2+4+6} \times E$$

$$\frac{12}{2} \times 20 = \cancel{12}\times\frac{\cancel{2}}{\cancel{12}}E \rightarrow E = \frac{\cancel{20}^{10}\times 12}{2_1} = 120V$$

$$1000 = \frac{100}{\cancel{100+2+1}\,103} \times E \Rightarrow E = \frac{103}{100} \times 1000V$$

$$= 1030V$$