EE 210 ELECTRIC CIRCUITS II

Lecture 1

Mutual Inductance and Transformers

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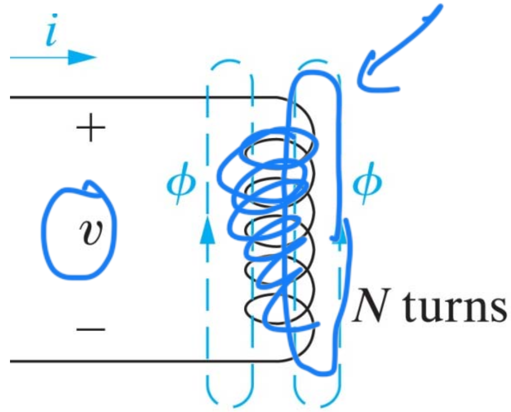
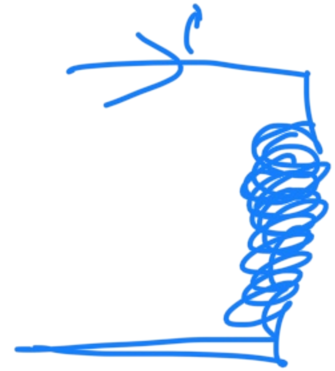
Magnetically Coupled Circuits

- The circuits studied so far can be considered as **conductively coupled**, since loops affect each other by current conduction.
- When two loops with or without contact affect each other through magnetic fields, they are said to be **magnetically coupled**.
- The **transformer** is a device designed based on the concept of magnetic coupling.
- In preparation for the study of transformers, we will first make a brief recap of **self inductance** and then discuss the concept of **mutual inductance**.

Faraday's Law





$$\underbrace{\lambda}_{\text{Flux linkage}} = \underbrace{\mathcal{N}}_{\#\text{ of turns}} \times \underbrace{\phi}_{\text{Flux [Weber]}}$$

- Consider a coil of $\mathcal{N}$ turns, through which a current i is flowing.
- **Faraday's Law:** The voltage induced in the coil is given by the rate-of-change of the flux linkage:

$$v(t) = \frac{d\lambda(t)}{dt} = \boxed{\mathcal{N}\frac{d\phi(t)}{dt}}$$

Self Inductance

L

- The magnitude of the flux is given by:

$\phi = \mathcal{P} \times \mathcal{N} \times i$; $\mathcal{P}$: permeance of the field occupied by the flux

- The **permeance** is flux-dependent for magnetic materials (like iron, nickel, cobalt), whereas it is **constant** for **nonmagnetic** materials.

AC

i'(t)

- When the core material of the coil is nonmagnetic:

$$v(t) = \mathcal{N}\frac{d\,\phi(t)}{dt} = \underbrace{\mathcal{N}^2\mathcal{P}}_{\mathcal{L}} \times \frac{di(t)}{dt}$$

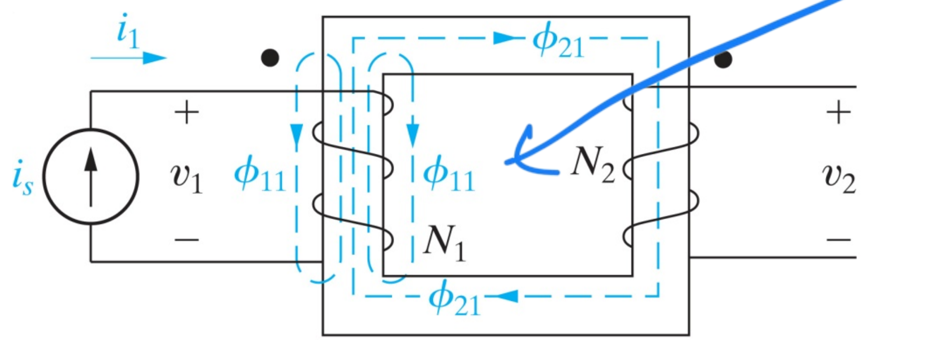
N NP

- The proportionality constant is the **self-inductance**:

$$\mathcal{L} = \mathcal{N}^2\mathcal{P}$$

$$\boxed{V_L(t) = L\,\frac{\partial i}{\partial t}}$$

Magnetically-Coupled Coils

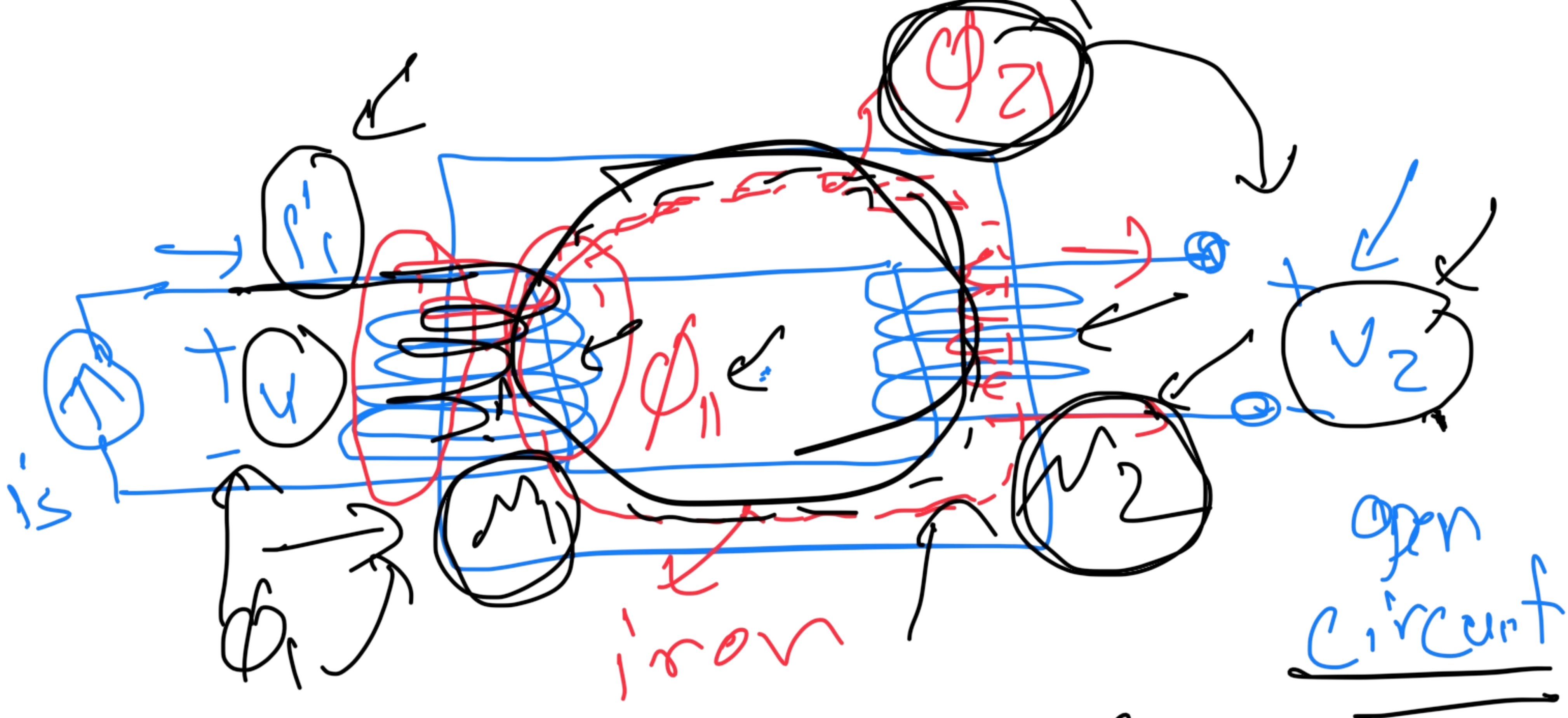


- Now consider two neighboring coils wound on a nonmagnetic core, with a current in the first one:

$$\underbrace{\phi_1}_{\text{total flux}} = \underbrace{\mathcal{P}_{11}\mathcal{N}_1 \times i_1}_{\phi_{11}\text{: flux linking coil-1}} + \underbrace{\mathcal{P}_{21}\mathcal{N}_1 \times i_1}_{\phi_{21}\text{: flux linking coil-2}} = \underbrace{\left(\mathcal{P}_{11} + \mathcal{P}_{21}\right)}_{\mathcal{P}_1}\mathcal{N}_1 \times i_1$$

- Using Faraday's law, we find:

$$v_1(t) = \mathcal{N}_1 \frac{d\,\phi_1(t)}{dt} = \underbrace{\mathcal{N}_1^2\mathcal{P}_1}_{\mathcal{L}_1} \frac{di_1(t)}{dt}; \quad v_2(t) = \mathcal{N}_2 \frac{d\,\phi_{21}(t)}{dt} = \underbrace{\mathcal{N}_2\mathcal{N}_1\mathcal{P}_{21}}_{\mathcal{M}_{21}} \frac{di_1(t)}{dt}$$



$$\phi_1 = \phi_{11} + \phi_{21}$$

total flux

$$\phi_1 = \overset{\phi_{11}}{P_{11} N_1 i_1'} + \overset{\phi_{21}}{P_{21} N_1 i_1'}$$

$$N_1 i_1' (\underbrace{P_{11} + P_{21}}_{P}) = P_1 N_1 i_1'$$

$$v_1(t) = N_1 \frac{d\phi_1}{dt}$$

$$v_1(t) = \underbrace{N_1 N_1 P_1}_{L_1} \frac{di_1'}{dt}$$

$$v_2(t) = N_2 \frac{\partial \phi_{21}}{\partial t}$$

$$v_2(t) = N_2 N_1 P_{21} \frac{\partial i_1}{\partial t}$$

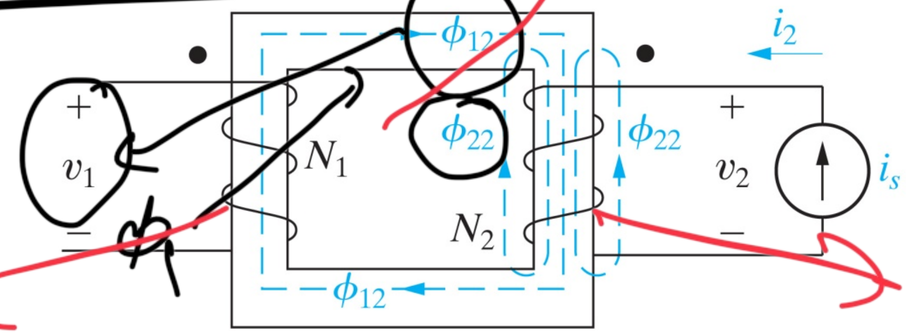
M_{21}

Mutual inductance

$$\boxed{v_2(t) = M_{21}\, \partial i_1 / \partial t}$$

Mutual Inductance

open



- When the current is fed to the second coil:

$$\mathcal{v}_2(t) = \mathcal{N}_2 \frac{d\,\phi_2(t)}{dt} = \underbrace{\mathcal{N}_2^2 \mathcal{P}_2}_{\mathcal{L}_2} \frac{di_2(t)}{dt}; \quad \mathcal{v}_1(t) = \mathcal{N}_1 \frac{d\,\phi_1(t)}{dt} = \underbrace{\mathcal{N}_1 \mathcal{N}_2 \mathcal{P}_{12}}_{\mathcal{M}_{12}} \frac{di_2(t)}{dt}$$

- For nonmagnetic core materials, we have:

$$\mathcal{P}_{12} = \mathcal{P}_{21} \Rightarrow \mathcal{M}_{12} = \mathcal{M}_{21} = \mathcal{M}$$

- $\mathcal{L}_1$ and $\mathcal{L}_2$ are the self inductances, whereas $\mathcal{M}$ is the **mutual inductance** between the coils.

The Coefficient of Coupling

$-\frac{1}{2}$

- Recall the inductance expressions:

$$\mathcal{L}_1 = \mathcal{N}_1^2\mathcal{P}_1 \text{ and } \mathcal{L}_2 = \mathcal{N}_2^2\mathcal{P}_2 \Rightarrow \mathcal{L}_1\mathcal{L}_2 = \mathcal{N}_1^2\mathcal{N}_2^2\mathcal{P}_1\mathcal{P}_2$$

- For nonmagnetic core materials, we have:

$$\mathcal{P}_{21} = \mathcal{P}_{12} \Rightarrow \mathcal{L}_1\mathcal{L}_2 = \underbrace{\left(\mathcal{N}_1\mathcal{N}_2\mathcal{P}_{12}\right)^2}_{\mathcal{M}^2}\underbrace{\left(1+\frac{\mathcal{P}_{11}}{\mathcal{P}_{12}}\right)\left(1+\frac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right)}_{1/k^2}$$

- Coupling is quantified by the **coefficient of coupling:**

$$k = \left(\left(1+\frac{\mathcal{P}_{11}}{\mathcal{P}_{12}}\right)\left(1+\frac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right)\right)^{-1/2} \in [0,1]; \;\; \mathcal{M} = k\sqrt{\mathcal{L}_1\mathcal{L}_2}$$

$k \in (0.5,1)$: tightly coupled; $k \in (0,0.5)$: loosely coupled

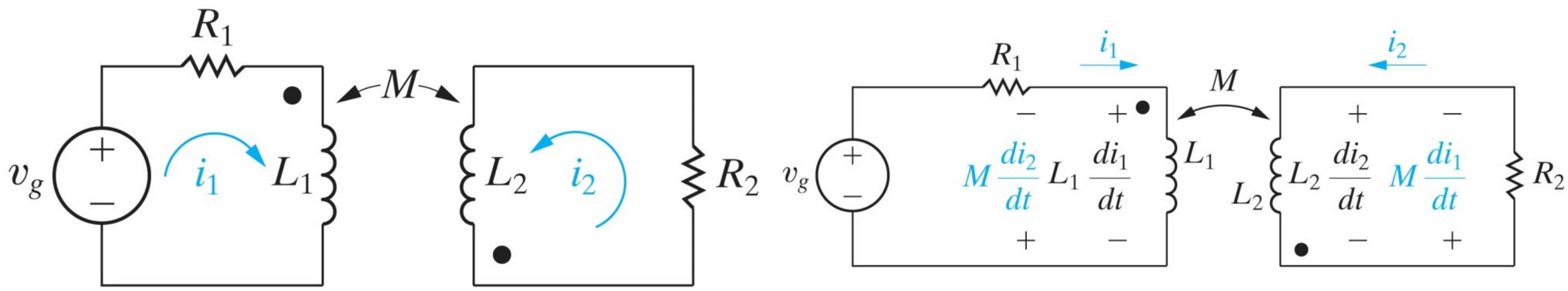
$$\boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$$

[1] $0 \le K \le 1$

[2] لا يساوي 0 $\; 0 \le K \le 0.5 \rightarrow$ loosely coupled

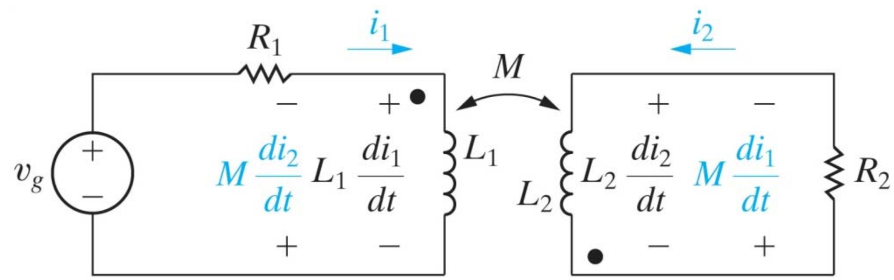
كويس $\leftarrow$ $0.5 \le K \le 1 \rightarrow$ tightly coupled ✓

Voltage Polarity and the Dot Convention



- The polarity of self-induced voltage is identified from the direction of the current.
- The polarity of the mutually-induced voltage is identified based on the dot convention.
- **Dot convention:** When the current enters (leaves) the dotted terminal of a coil, the polarity of the voltage it induces in the other coil is positive (negative) at its dotted terminal.

Application of the Dot Convention



- The easiest way to analyze circuits containing mutual inductance is to use mesh currents.
- KVL is then applied with the addition of the mutually induced voltage with appropriate polarity:

$$v_g = R_1 i_1 + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$0 = R_2 i_2 + L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

$V_L = L\frac{di}{dt}$

إذا دخل أحدهما من Dot والآخر لا يكون سالب

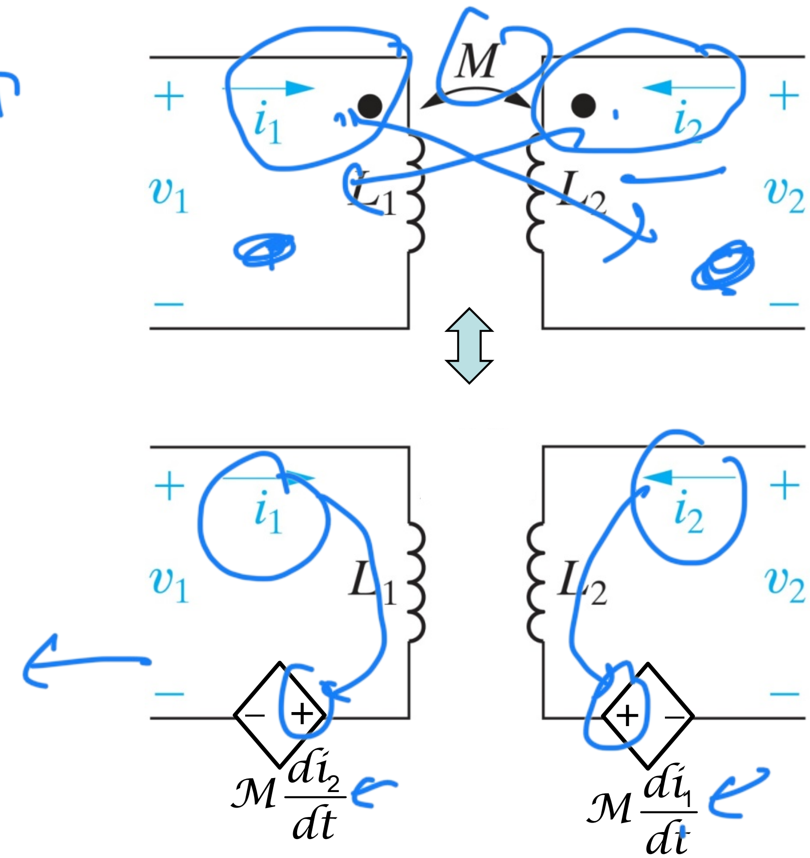
$$V_g = R_1 i_1 + L_1 \frac{\partial i_1}{\partial t} - M \frac{\partial i_2}{\partial t}$$

$$0 = R_2 i_2 + L_2 \frac{\partial i_2}{\partial t} - M \frac{\partial i_1}{\partial t}$$

Equivalent Circuit for Mutual Inductance

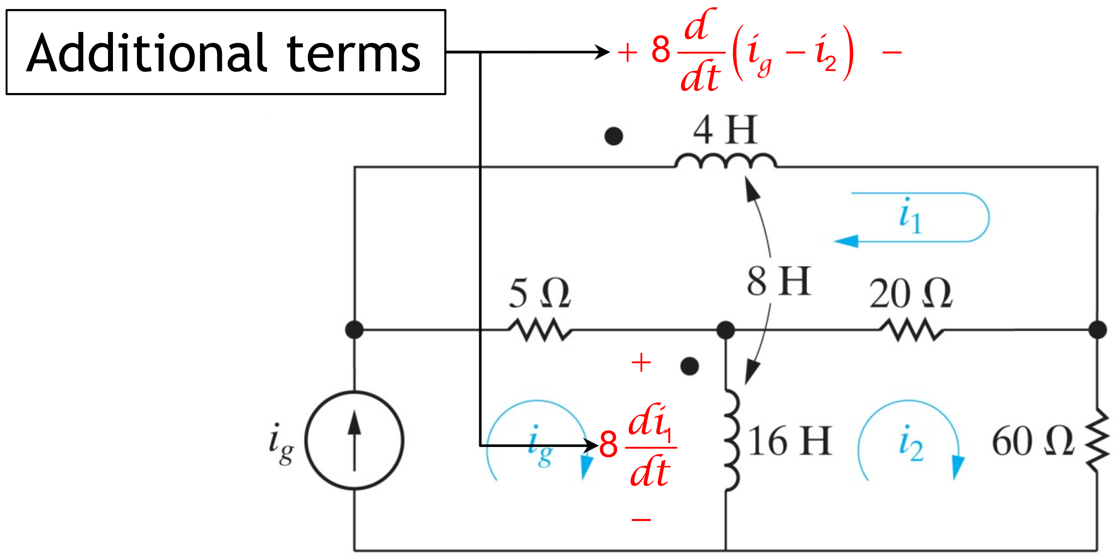
Dot

Dot +M



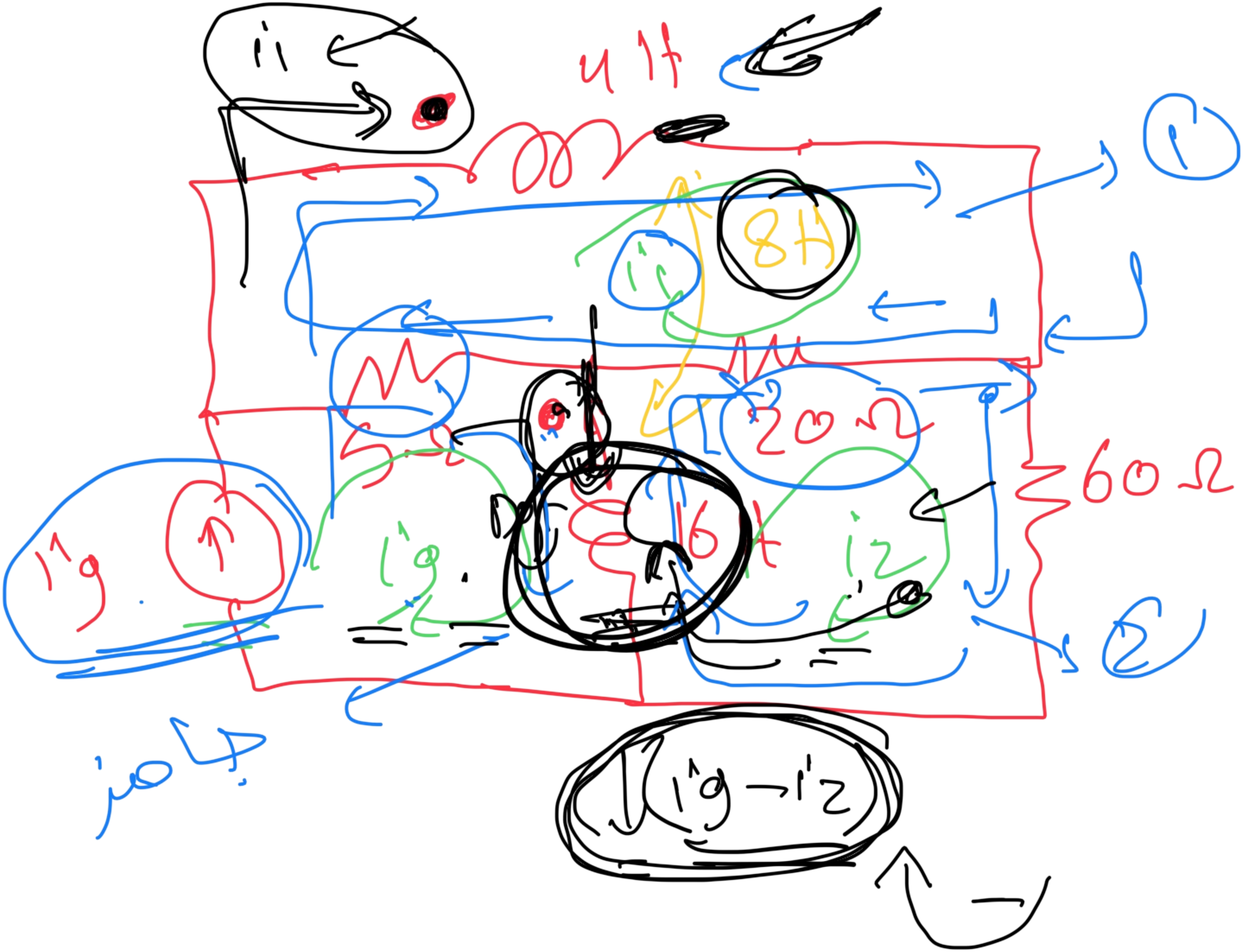
- The dependent source voltages are determined by the derivatives of the currents. The polarities of the sources are identified from the dot convention.

Example: Mesh Current Equations



$$i_1 \text{ mesh} \quad : \quad 4\frac{di_1}{dt} + 8\frac{d}{dt}\left(i_g - i_2\right) + 20\left(i_1 - i_2\right) + 5\left(i_1 - i_g\right) = 0$$

$$i_2 \text{ mesh} \quad : \quad 20\left(i_2 - i_1\right) + 60 i_2 + 16\frac{d}{dt}\left(i_2 - i_g\right) - 8\frac{di_1}{dt} = 0$$


8H
20Ω
60Ω

mesh #1

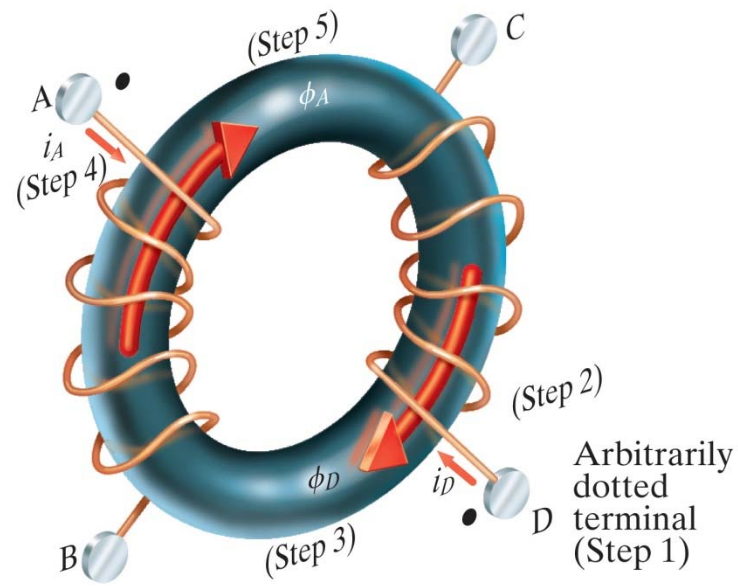
$$20(i_1 - i_2) + 5(i_1 - i_g) + 4\frac{di_1}{dt} + 8\frac{d}{dt}(i_g - i_2) = 0$$

mesh #2

$$60 i_2 + 16\frac{d}{dt}(i_2 - i_g) + 20(i_2 - i_1) - 8\frac{d}{dt}i_1 = 0$$

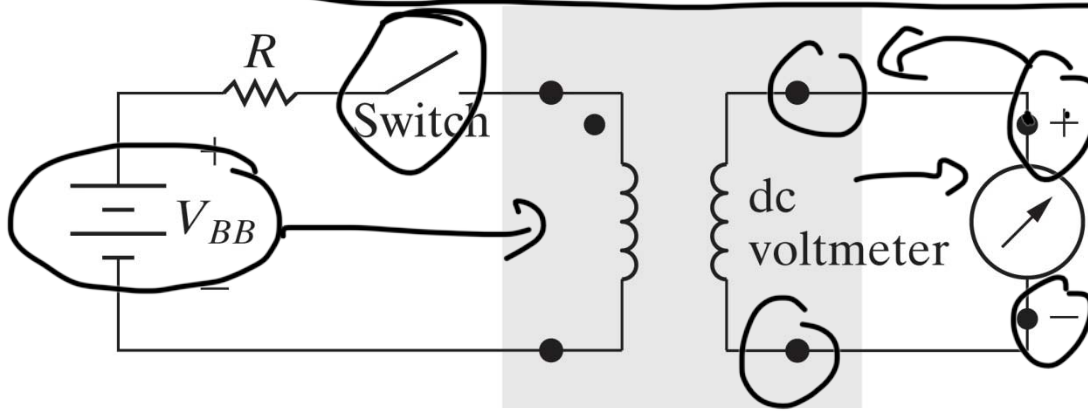
Procedure for Determining Dot Markings

- Arbitrarily select and mark a terminal, say D, with a dot.
- Assign a current i_D to it.
- Determine the direction of the induced magnetic flux, ϕ_D, based on the right-hand rule.
- Arbitrarily pick a terminal of the second coil, say A, and apply the same steps again.
- If the directions ϕ_A and ϕ_D are the same, then place a dot on A.
- If the directions are opposite, place a dot on the other terminal.



Experimental Setup for Dot Marking

AC



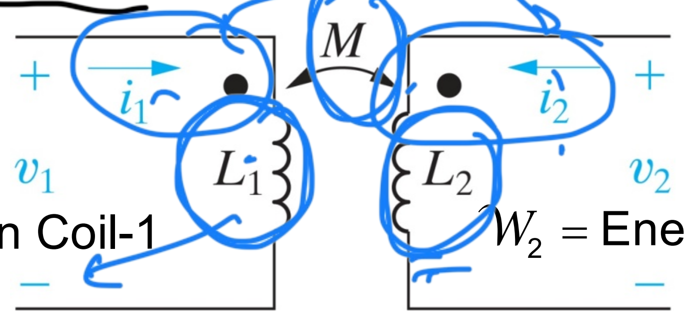
+

−

- Put a dot on the terminal to which the resistor is connected.
- Observe the momentary deflection of the DC voltmeter when the switch is closed.
- If it is upscale/downscale, put a dot on the terminal connected to the positive/negative terminal of the voltmeter.

$W_L = \frac{1}{2} L i^2$

Energy Calculations



$\mathcal{W}_1$ = Energy Stored in Coil-1

$\mathcal{W}_2$ = Energy Stored in Coil-2

- Assume zero initial energy.
- Increase i_1 from zero to I_1:

$$\mathcal{W}_1^1 = \int_{i_1=0}^{i_1=I_1} i_1 \cdot \mathcal{L}_1 \frac{di_1}{dt} dt = \int_0^{I_1} \mathcal{L}_1 i_1 \, di_1 = \frac{1}{2} \mathcal{L}_1 I_1^2; \quad \mathcal{W}_2^1 = 0$$

- Keep $i_1 = I_1$ constant; increase i_2 from zero to I_2:

$$\mathcal{W}_1^2 = \int_{i_2=0}^{i_2=I_2} I_1 \cdot \mathcal{M}_{12} \frac{di_2}{dt} dt = I_1 I_2 \mathcal{M}_{12}; \quad \mathcal{W}_2^2 = \int_{i_2=0}^{i_2=I_2} i_2 \cdot \mathcal{L}_2 \frac{di_2}{dt} dt = \frac{1}{2} \mathcal{L}_2 I_2^2$$

- Total energy stored in the coils:

$$\mathcal{W} = \mathcal{W}_1^1 + \mathcal{W}_1^2 + \mathcal{W}_2^1 + \mathcal{W}_2^2 = \frac{1}{2} \mathcal{L}_1 I_1^2 + \frac{1}{2} \mathcal{L}_2 I_2^2 + \mathcal{M}_{12} I_1 I_2$$

$M = M_{12} = M_{21}$

Total Energy Stored in the Coils

- Total energy stored in the coils at time t (with the dot marking not specified) :

$$\mathcal{W}(t) = \frac{1}{2}\mathcal{L}_1 i_1^2(t) + \frac{1}{2}\mathcal{L}_2 i_2^2(t) \pm \mathcal{M}_{12} i_1(t) i_2(t)$$

- When the order of the procedure is reversed:

self

Mutual

$$\mathcal{W}(t) = \frac{1}{2}\mathcal{L}_1 i_1^2(t) + \frac{1}{2}\mathcal{L}_2 i_2^2(t) \pm \mathcal{M}_{21} i_1(t) i_2(t)$$

- For linear coupling media $\mathcal{M}_{12} = \mathcal{M}_{21} = \mathcal{M}$, which means that the total energies stored are the same.
- Determining the sign: If both currents are entering or leaving the dotted terminals, then the sign is positive; otherwise the sign is negative.

اذا كان علا الشارتين
زي بعض داخلين على dot
أو خارجين منه

⟶

لو أحدهما داخل
dot والآخر منه

⟶ undot

Positivity of the Total Energy

- Consider finding the i_2 that minimizes $\mathcal{W}$:

$$\mathcal{W}(i_2) = \frac{1}{2}\mathcal{L}_1 i_1^2 + \frac{1}{2}\mathcal{L}_2 i_2^2 \pm \mathcal{M} i_1 i_2$$

$$\Rightarrow \frac{d\mathcal{W}(i_2)}{di_2} = \mathcal{L}_2 i_2 \pm \mathcal{M} i_1 \text{ and } \frac{d^2\mathcal{W}(i_2)}{di_2^2} = \mathcal{L}_2 > 0$$

- Equate the first derivative to zero find the minimum:

$$\mathcal{L}_2 i_2^\circ \pm \mathcal{M} i_1 = 0 \Rightarrow i_2^\circ = \mp \frac{\mathcal{M}}{\mathcal{L}_2} i_1 = \mp k \sqrt{\frac{\mathcal{L}_1}{\mathcal{L}_2}} i_1$$

- Since the energy is ≥ its minimum value, we have:

$$\mathcal{W}(i_2) \geq \mathcal{W}(i_2^\circ) = \frac{1}{2}\mathcal{L}_1 i_1^2 + \left(\frac{1}{2}\mathcal{L}_2 i_2^\circ \pm \mathcal{M} i_1\right) i_2^\circ = \frac{1}{2}\left(1 - k^2\right)\mathcal{L}_1 i_1^2 \geq 0$$

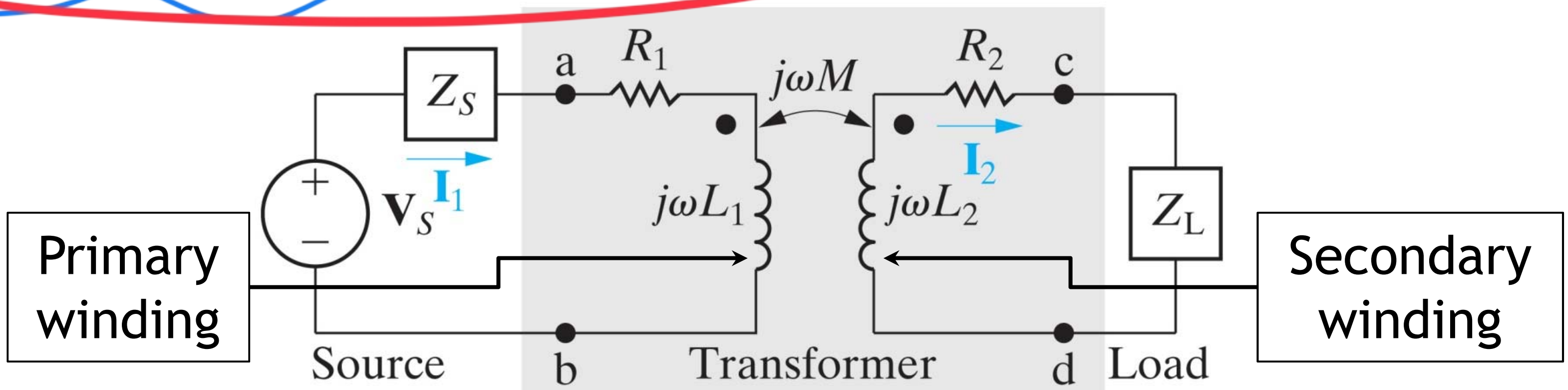
DC → → short

Transformers

- Transformer is a device based on magnetic coupling.
- In communication circuits, transformers are used to match impedances and eliminate DC signals.
- In power circuits, transformers are used to establish AC voltage levels that facilitate the transmission, distribution and consumption of electrical power.
- We will first analyze the steady-state behavior of the **linear transformer**, which is common in communication systems.
- We will then study the **ideal transformer**, which models the ferromagnetic transformer used in power systems.

Transformers

The Linear Transformer



R_1	Resistance of the primary winding
R_2	Resistance of the secondary winding
L_1	Self-inductance of the primary winding
L_2	Self-inductance of the secondary winding
M	Mutual inductance
V_S	Source voltage
Z_S	Source impedance
Z_L	Load impedance
I_1	Primary current
I_2	Secondary current

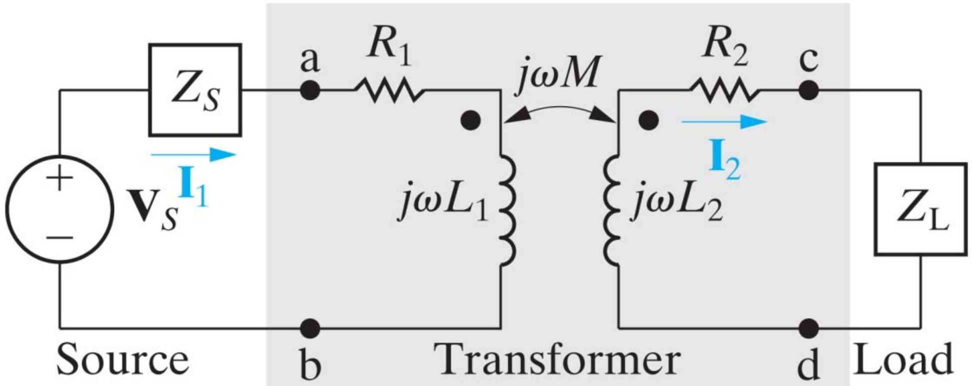
Primary winding

"Voltage source"

Secondary winding

"load"

Analysis of a Linear Transformer



- Phasor analysis is adapted easily: $dx/dt \leftrightarrow j\omega \times X$

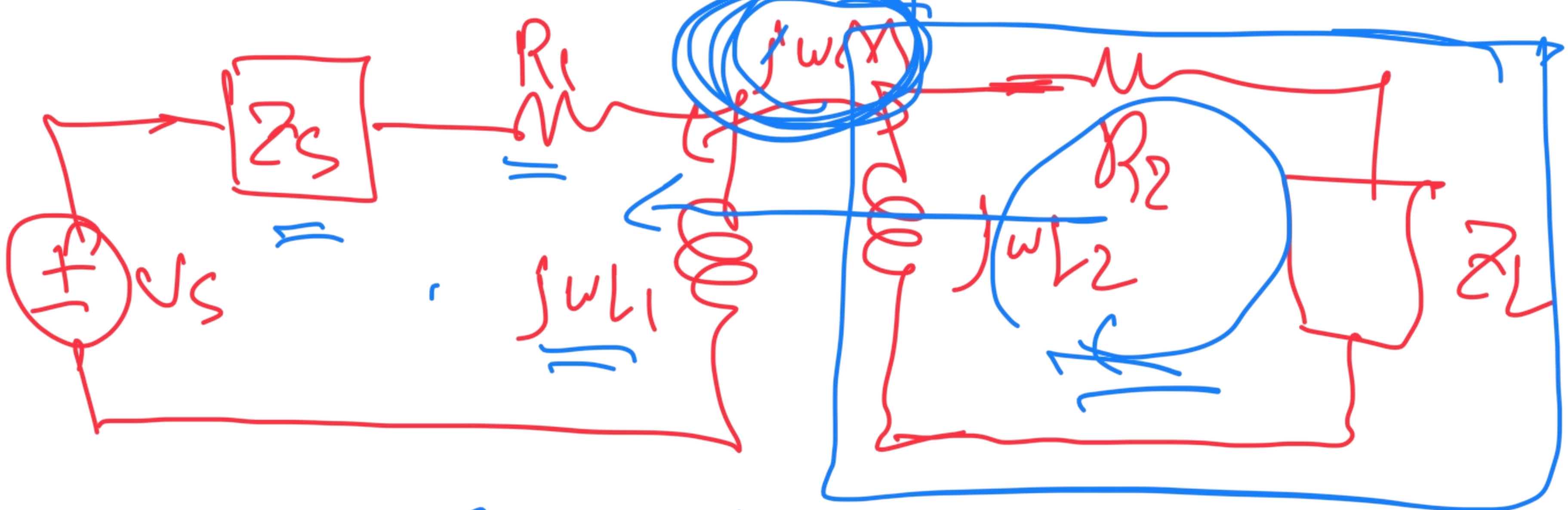
$$\underbrace{\left(Z_S + R_1 + j\omega L_1\right)}_{Z_{11}} I_1 - j\omega M I_2 = V_S$$

$Z_{11} \longleftarrow$ Self impedance of the primary

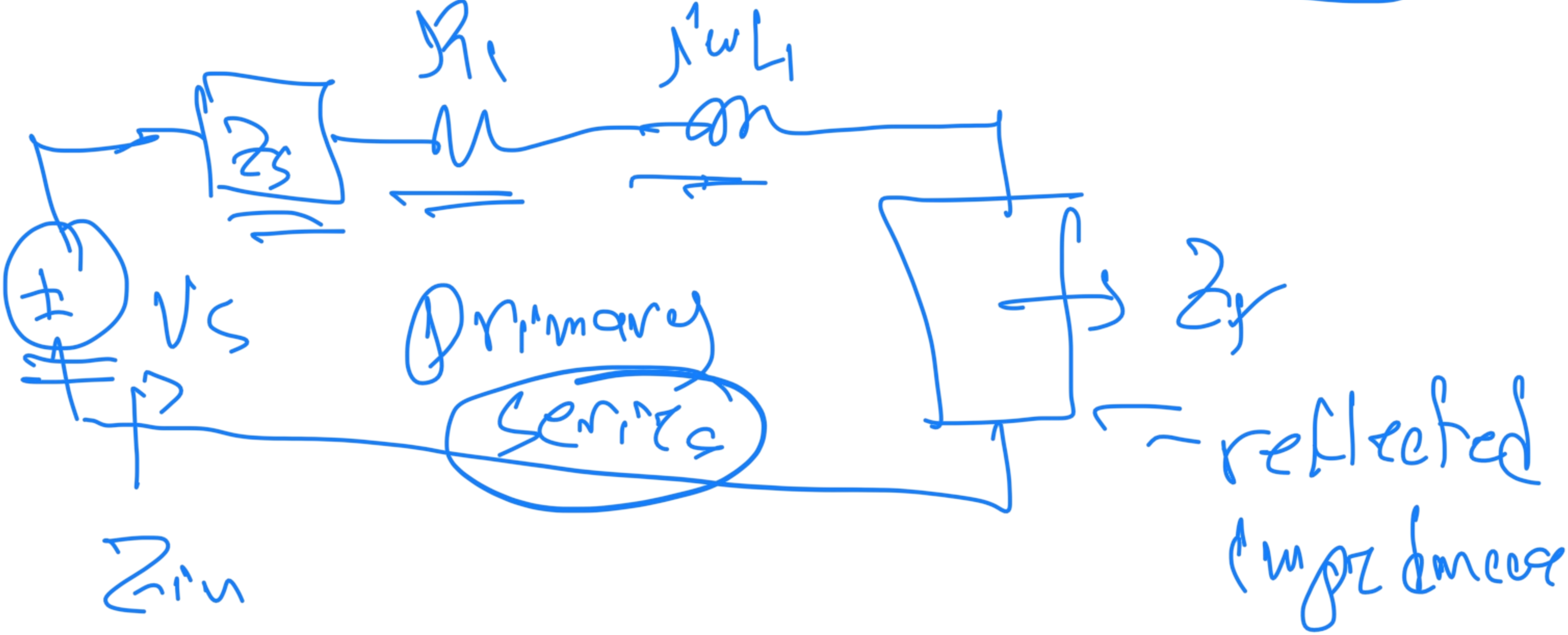
$$\underbrace{\left(Z_L + R_2 + j\omega L_2\right)}_{Z_{22}} I_2 - j\omega M I_1 = 0$$

$Z_{22} \longleftarrow$ Self impedance of the secondary

$$\Rightarrow I_2 = \frac{j\omega M}{Z_{22}} I_1 \quad \text{and} \quad I_1 = \frac{Z_{22}}{Z_{11}Z_{22} + \omega^2 M^2} V_S$$



Zs
R1
jωM
R2
jωL1
jωL2
ZL
Vs

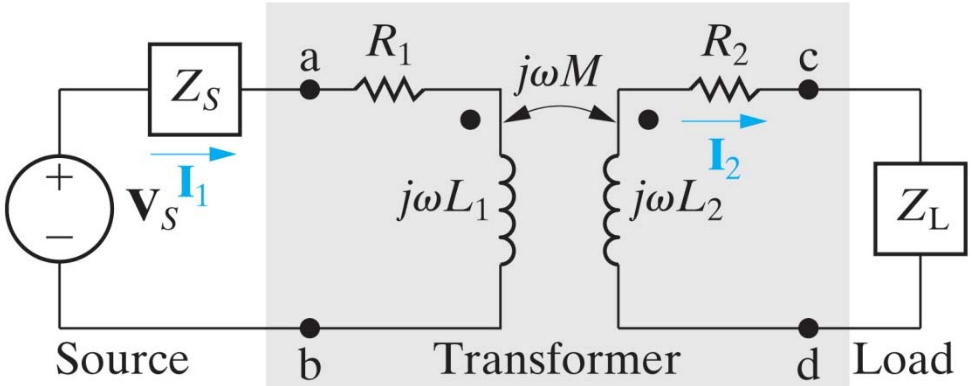


R1
jωL1
Zs
Vs
Primary
series
Zr
reflected
impedance
Zin

$$Z_r = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}$$

$$Z_{in} = Z_s + R_1 + j\omega L_1 + Z_r$$

The Impedance Seen by the Source



- The impedance seen from the nodes a and b is:

$$Z_{ab} = \frac{V_s}{I_1} - Z_s = Z_{11} + \frac{\omega^2 M^2}{Z_{22}} - Z_s = R_1 + j\omega L_1 + \boxed{\frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}}$$

Z_r: Reflected impedance

- A key role of the transformer is thus revealed: Changing the impedance seen by the source:

$$Z_L \to Z_{ab} = R_1 + j\omega L_1 + Z_r(Z_L)$$

The Reflected Impedance

- Impedance of the second coil plus load is transmitted to the primary side via the mutual inductance:

$$Z_r = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}$$

- When the load impedance is $Z_L = R_L + jX_L$, we have:

$$Z_r = \boxed{\frac{\omega^2 M^2}{(R_2 + R_L)^2 + (\omega L_2 + X_L)^2}} \left[(R_2 + R_L) - j(\omega L_2 + X_L) \right]$$

Scaling Factor

- The impedance is thus conjugated and then scaled during the reflection.

$$Z_r = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}$$

$Z_L = R_L + jX_L$

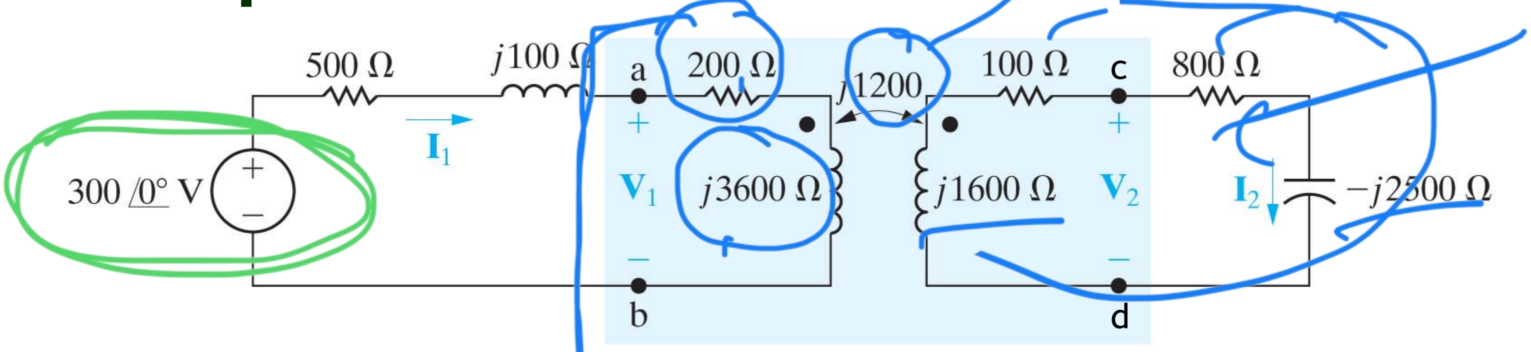
$$\text{scaling factor} = \frac{\omega^2 M^2}{(R_2 + R_L)^2 + (\omega L_2 + X_2)^2}$$

ρ

$$\boxed{Z_r = \rho \left[(R_2 + R_L) - j(\omega L_2 + X_L) \right]}$$

$\omega M = 1200$

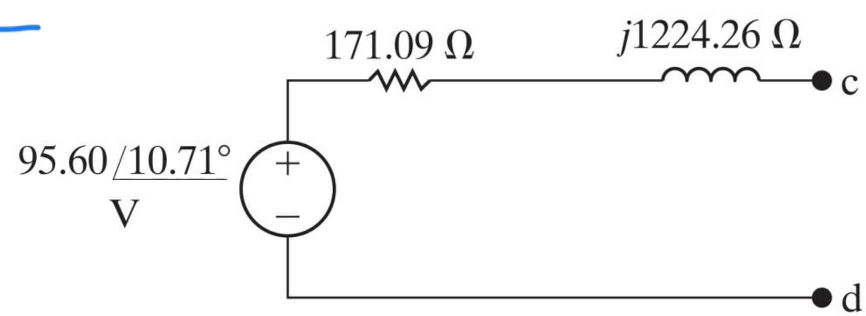
Example: Linear Transformer Circuit



- Scaling factor: $= \dfrac{1200^2}{\left|800+100+1600j-2500j\right|^2} = \dfrac{1200^2}{\left|900\text{-}900j\right|^2} = \dfrac{1200^2}{900^2+900^2} = \dfrac{8}{9}$
- Reflected impedance: $= \dfrac{8}{9}\left(900+900j\right) = 800+800j\ \Omega$
- Impedance seen from a - b: $= 200+3600j+800+800j = 1000+4400j\ \Omega$
- Thevenin equivalent seen from c – d:

$$\mathcal{Z}_{\text{th}} = 100+1600j+\frac{1200^2}{700^2+3700^2}\left(700-3700j\right)$$
$$= 171.09+1224.26j\ \Omega$$

$$\mathcal{V}_{\text{th}} = 1200j \times \frac{300}{500+200+\left(100+3600\right)j}$$
$$= 95.6\angle 10.71^{\circ}\ \text{V}$$



Scaling factor

$$Z_r = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}$$

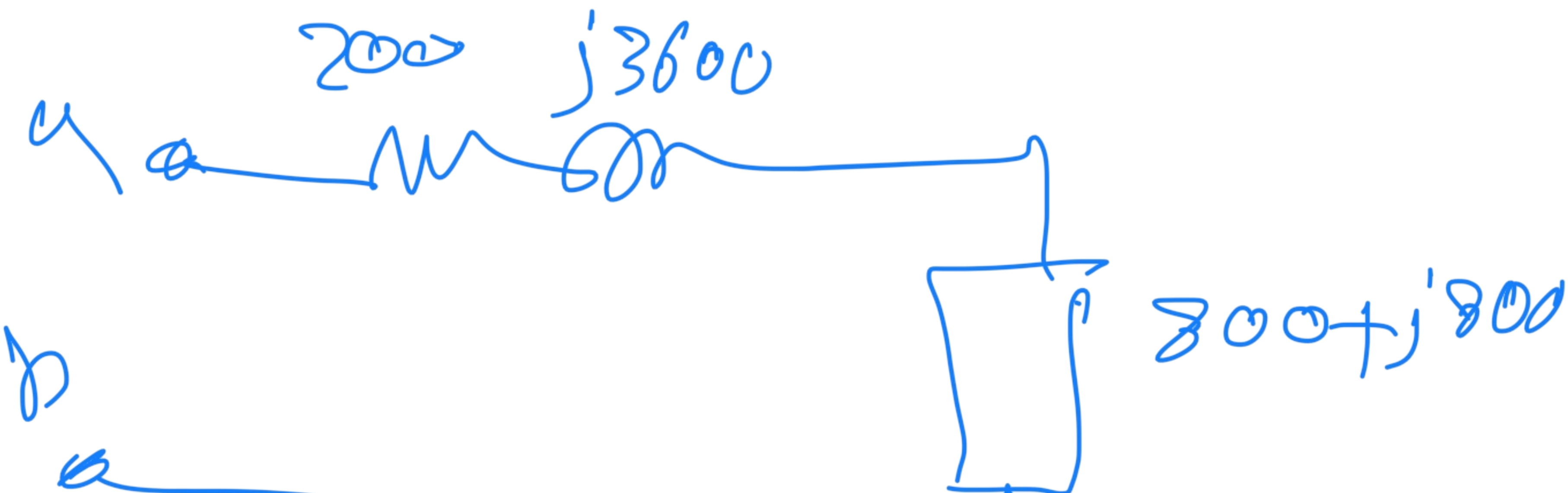
[illegible]

$$Z_r = \frac{1200^2}{\underbrace{100 + 800}_{\text{Real part}} \underbrace{- j2500 + j1600}_{\text{imag part}}} \rightarrow -j900$$

$$f = \frac{1200^2}{900^2 + 900^2} = \frac{8}{9}$$

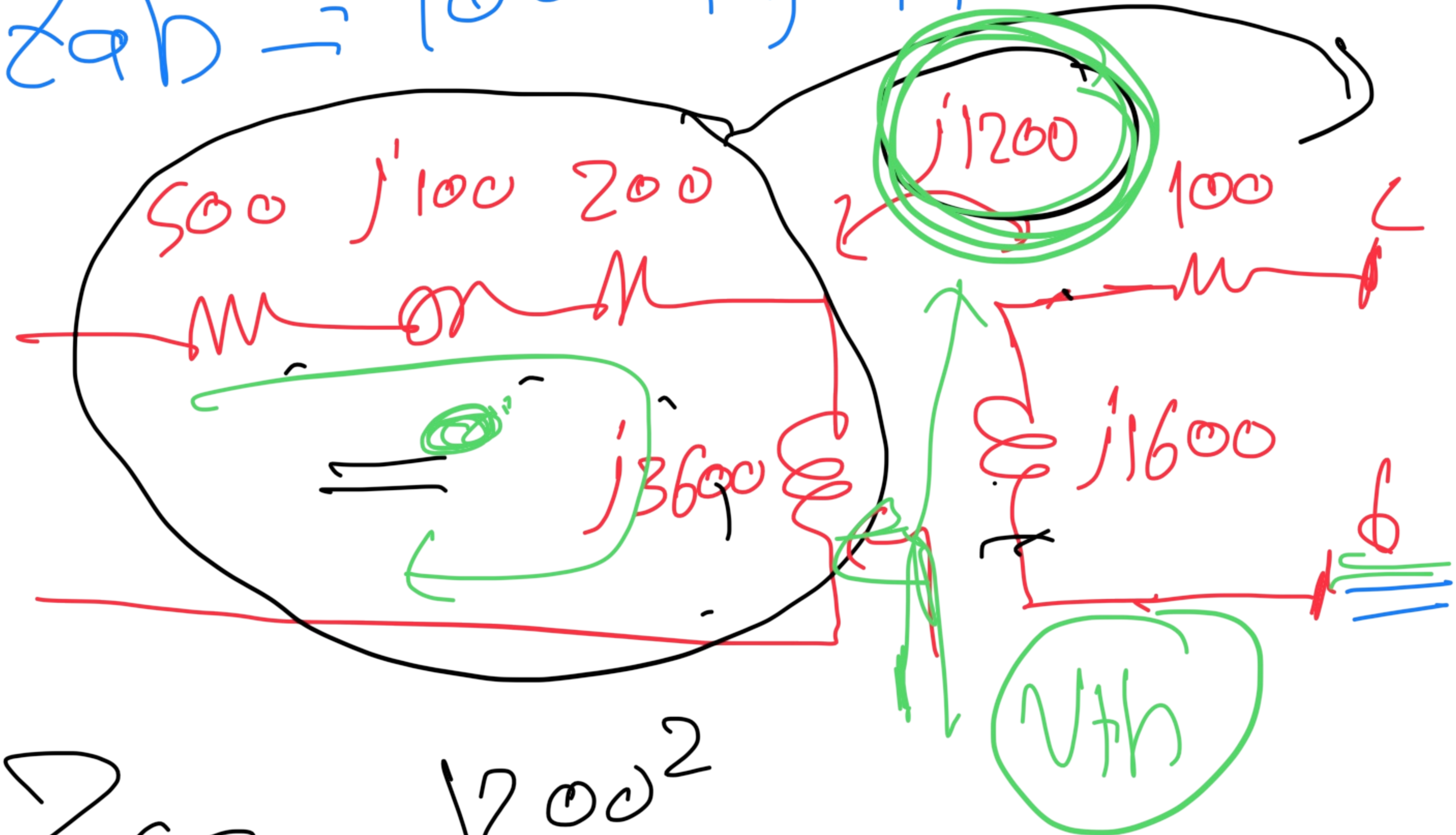
$$Z_r = \frac{1200^2}{100+800-j2500+j1600}$$

$$Z_r = 800+j800$$

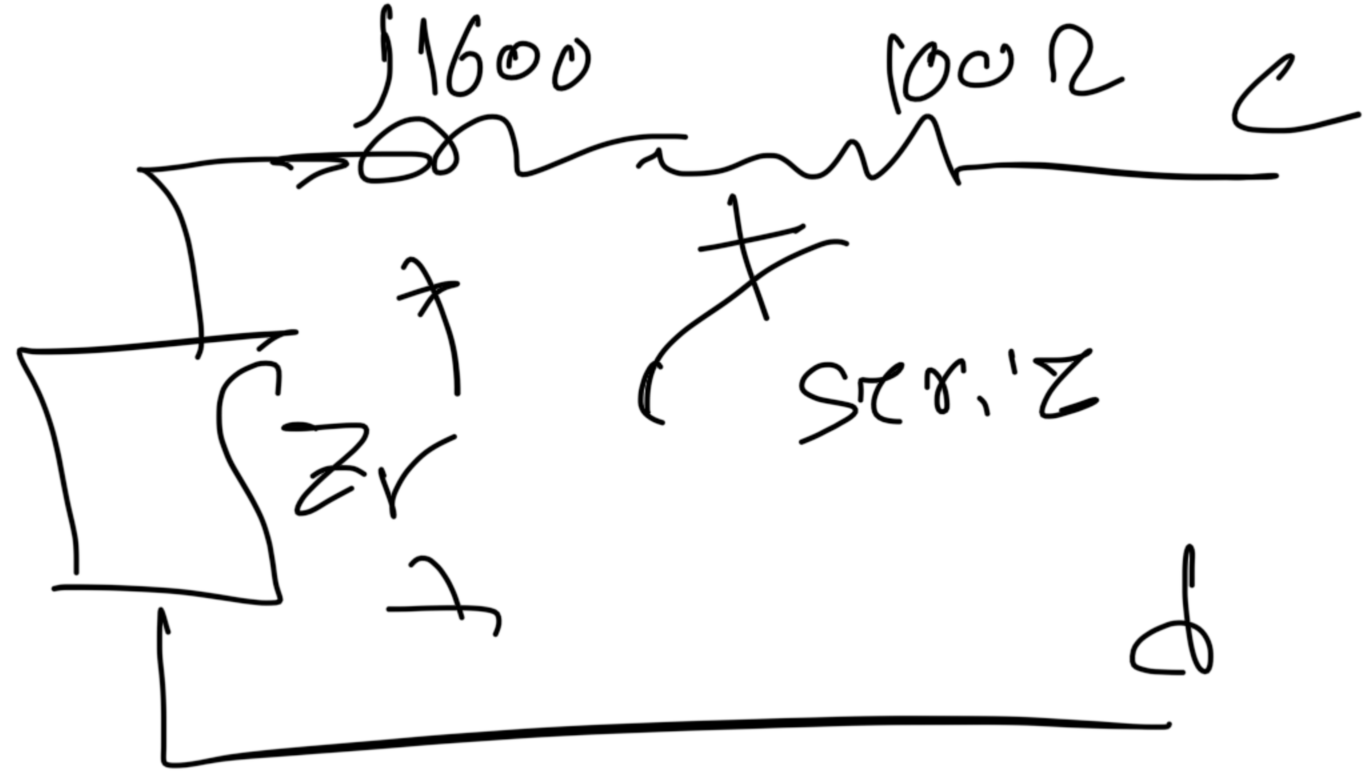


$$200+j3600+800+j800$$

$$Z_{ab} = 1000 + j4400\ \Omega$$



$$Z_r = \frac{j1200^2}{500 + j100 + 200 + j3600}$$

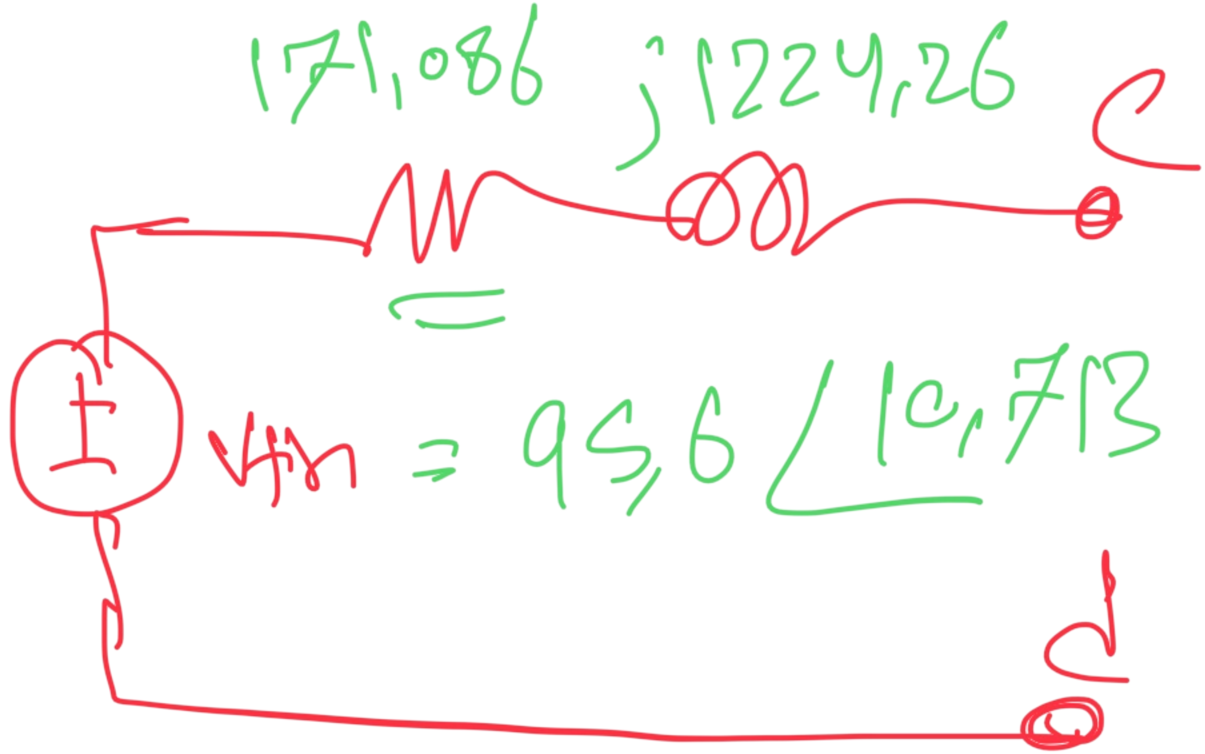


$$Z_r = 71{,}086 - j375{,}7\ \Omega$$

$$Z_{cd} = 171{,}086 + j1724{,}26\ \Omega$$

$R \qquad + \quad X_L \to L$

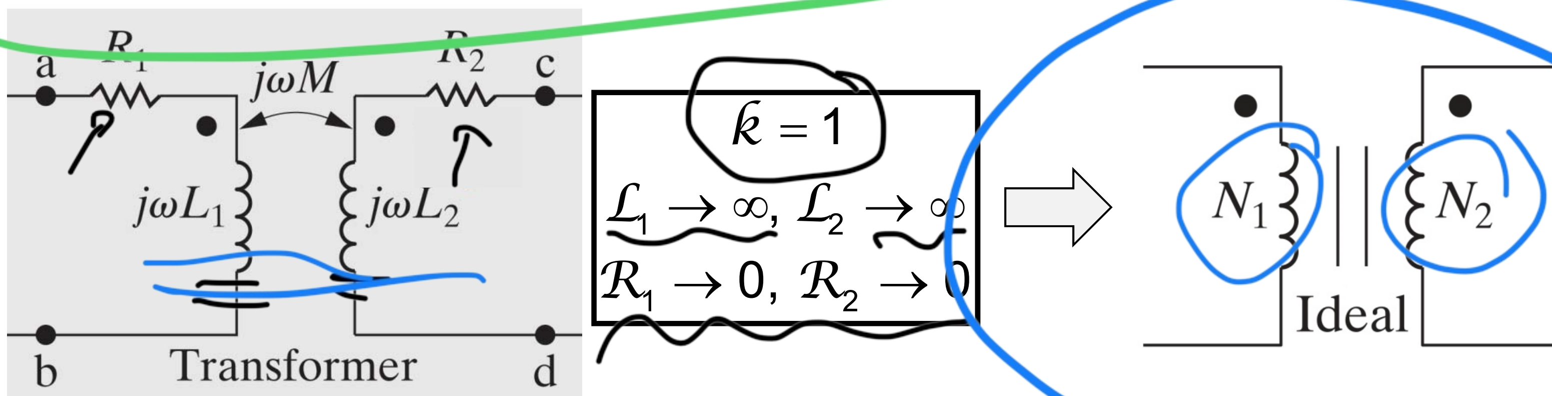
$ \qquad - \quad X_C \to C$



$$V_{th} = \frac{j1200}{500 + j100 + 200 + j3600} \times 300$$

$$= 95{,}6 \angle 10{,}713$$

The Ideal Transformer



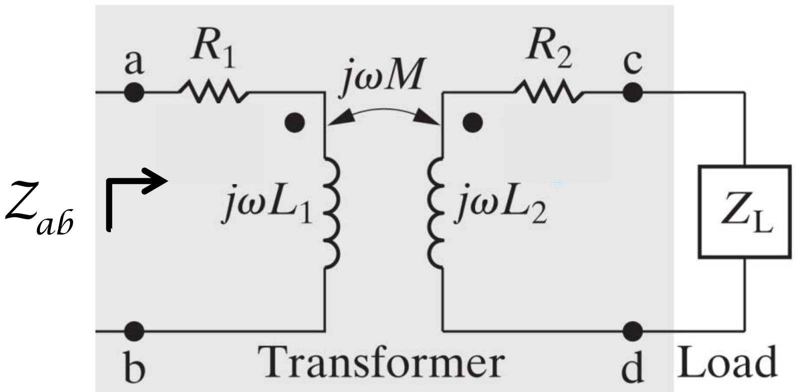
➢ An ideal transformer consists of two magnetically coupled coils having N_1 and N_2 turns respectively, and exhibiting the following three properties:

❑ The coefficient of coupling is unity. $=1$

❑ The self-inductance of each coil is infinite.

❑ The coil losses, due to parasitic resistances, are negligible. $R = 0$

Analysis of the Limit Values



$$Z_{ab} = \underbrace{\mathcal{R}_1 + \frac{\omega^2 \mathcal{M}^2 (\mathcal{R}_2 + \mathcal{R}_L)}{(\mathcal{R}_2 + \mathcal{R}_L)^2 + (\omega \mathcal{L}_2 + \mathcal{X}_L)^2}}_{\mathcal{R}_{ab}} + j \underbrace{\left(\omega \mathcal{L}_1 - \frac{\omega^2 \mathcal{M}^2 (\omega \mathcal{L}_2 + \mathcal{X}_L)}{(\mathcal{R}_2 + \mathcal{R}_L)^2 + (\omega \mathcal{L}_2 + \mathcal{X}_L)^2} \right)}_{\mathcal{X}_{ab}}$$

- Setting $\mathcal{M}^2 = \mathcal{L}_1 \mathcal{L}_2$ and letting $\mathcal{L}_1 \to \infty$, $\mathcal{L}_2 \to \infty$, we find:

$$\mathcal{R}_{ab} = \mathcal{R}_1 + \left(\frac{\mathcal{L}_1}{\mathcal{L}_2}\right)\left(\frac{(\mathcal{R}_2 + \mathcal{R}_L)}{(\mathcal{R}_2 + \mathcal{R}_L)^2 / (\omega \mathcal{L}_2)^2 + \left(1 + \mathcal{X}_L / (\omega \mathcal{L}_2)\right)^2}\right) \to \mathcal{R}_1 + \left(\frac{\mathcal{L}_1}{\mathcal{L}_2}\right)(\mathcal{R}_2 + \mathcal{R}_L)$$

$$\mathcal{X}_{ab} = \left(\frac{\mathcal{L}_1}{\mathcal{L}_2}\right)\left(\frac{(\mathcal{R}_2 + \mathcal{R}_L)^2 / (\omega \mathcal{L}_2) + \mathcal{X}_L + \mathcal{X}_L^2 / (\omega \mathcal{L}_2)}{(\mathcal{R}_2 + \mathcal{R}_L)^2 / (\omega \mathcal{L}_2)^2 + \left(1 + \mathcal{X}_L / (\omega \mathcal{L}_2)\right)^2}\right) \to \left(\frac{\mathcal{L}_1}{\mathcal{L}_2}\right)\mathcal{X}_L$$

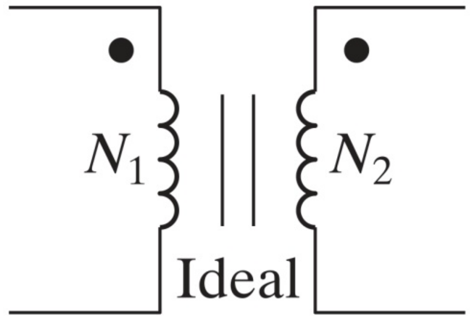
- As $k \to 1$, the two permeances $\mathcal{P}_1$, $\mathcal{P}_2$ become equal:

$$\frac{\mathcal{L}_1}{\mathcal{L}_2} = \frac{\mathcal{N}_1^2 \mathcal{P}_1}{\mathcal{N}_2^2 \mathcal{P}_2} \to \left(\frac{\mathcal{N}_2}{\mathcal{N}_1}\right)^2 = a^2;$$

a: turns ratio

$$\boxed{a = \frac{N_2}{N_1}}$$

Ideal Transformer Relations



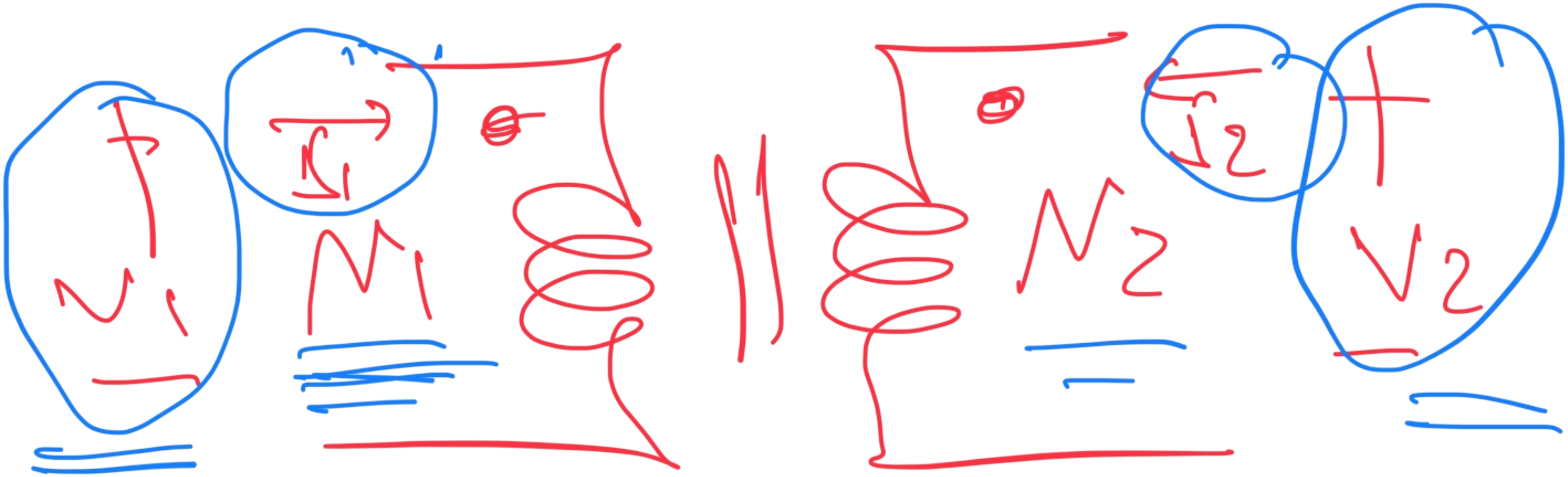
- The voltages of the primary and secondary windings of an ideal transformer are related as:

$$\left|\frac{\mathcal{V}_1}{\mathcal{N}_1}\right| = \left|\frac{\mathcal{V}_2}{\mathcal{N}_2}\right|$$

- The currents of the primary and secondary windings of an ideal transformer are related as:

$$\left|\mathcal{I}_1\mathcal{N}_1\right| = \left|\mathcal{I}_2\mathcal{N}_2\right|$$

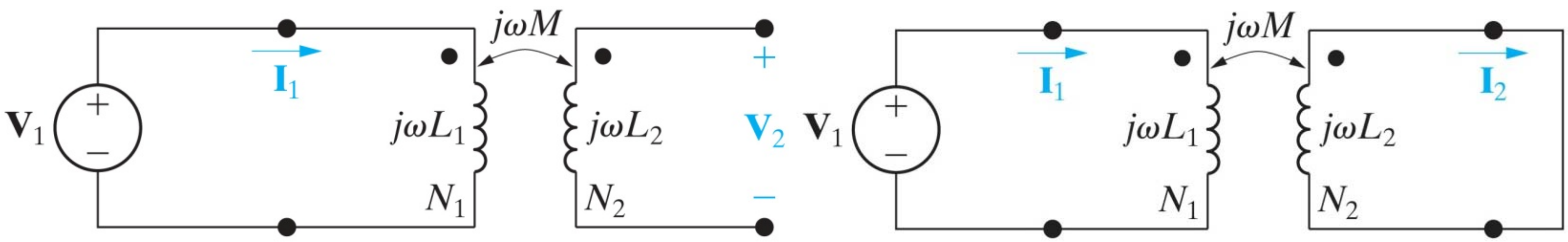
- The polarities are determined via the dot convention.



$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

Determining the Voltage-Current Ratios



- The voltage ratio is determined using the left circuit:

Since $I_2 = 0$, we have $V_2 = j\omega M I_1 = \frac{j\omega M}{j\omega L_1} V_1$.

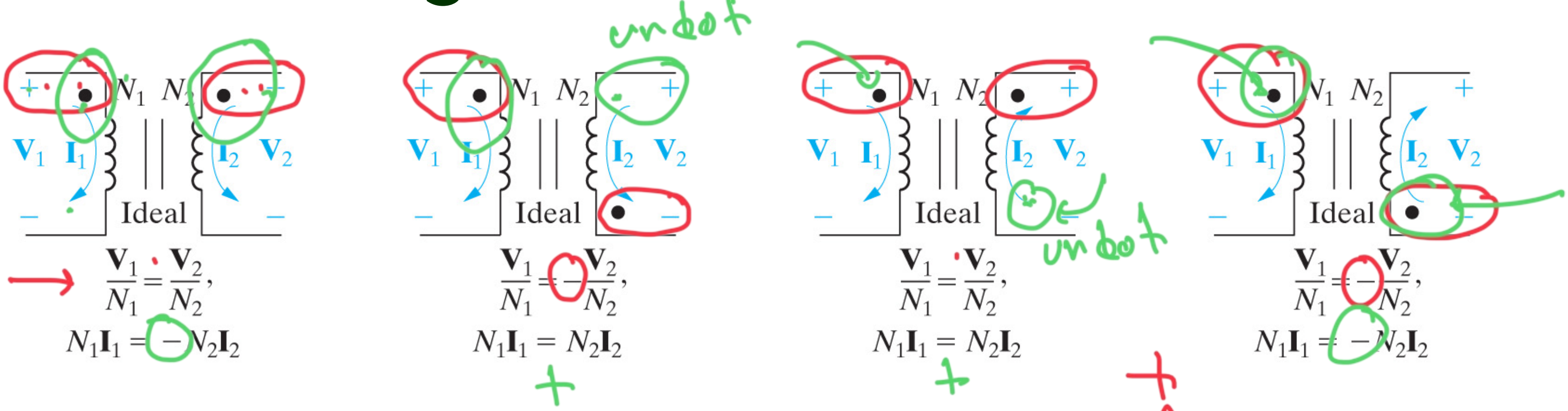
With $M = \sqrt{L_1 L_2}$, it follows that $\frac{V_1}{V_2} = \sqrt{\frac{L_1}{L_2}} = \sqrt{\frac{N_1^2 P}{N_2^2 P}} = \frac{N_1}{N_2}$.

- The current ratio is determined using the right circuit:

Since $V_2 = 0$, we have $j\omega M I_1 - j\omega L_2 I_2 = 0$.

With $M = \sqrt{L_1 L_2}$, it follows that $\frac{I_1}{I_2} = \sqrt{\frac{L_2}{L_1}} = \sqrt{\frac{N_2^2 P}{N_1^2 P}} = \frac{N_2}{N_1}$.

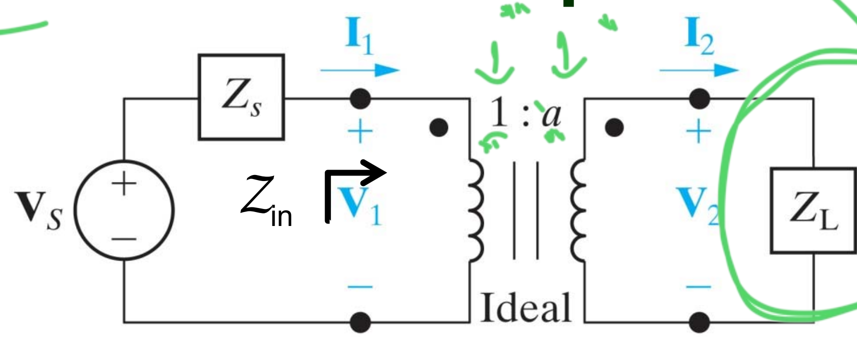
Determining the Polarities



- If the coil voltages are both positive or negative at the dot-marked terminal, use a plus sign in the voltage ratio; otherwise, use a minus sign.

- If the coil currents are both directed into or out of the dot-marked terminal, use a minus sign in the current ratio; otherwise, use a plus sign.

Ideal Transformer for Impedance Matching



- The ideal transformer relations are:

$$\frac{Z_L}{a^2}$$

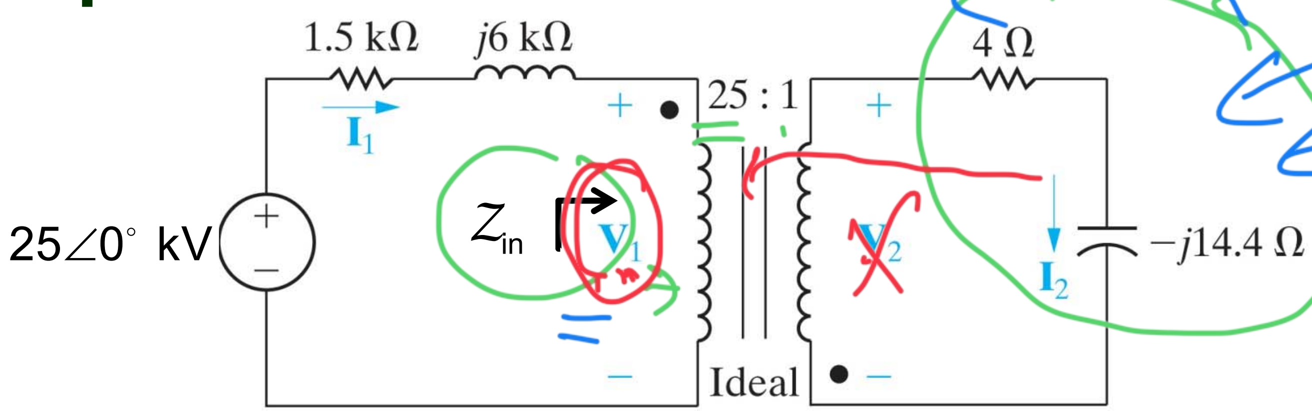
$$V_1 = \frac{V_2}{a} \quad \text{and} \quad I_1 = aI_2$$

- Then the impedance seen by the practical source is:

$$Z_{in} = \frac{V_1}{I_1} = \frac{1}{a^2}\frac{V_2}{I_2} = \frac{1}{a^2}Z_L$$

- The practical version of the ideal transformer is the ferromagnetic core transformer. It is used to match the magnitude of Z_L to the magnitude of Z_s.

Example: Ideal Transformer Circuit



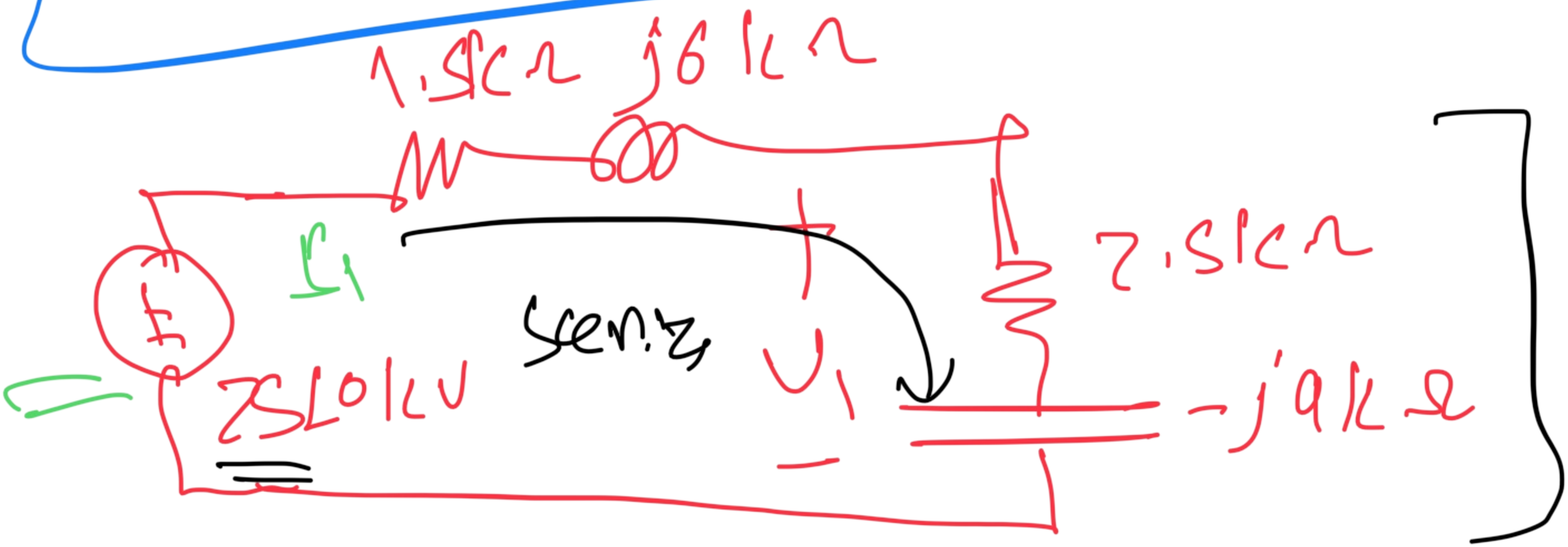
- $\mathcal{Z}_{in}$: $= (25)^2 \left(4\text{-}j14.4\right) = 2.5 - j9 \text{ k}\Omega$
- $\mathcal{V}_1$: $= \dfrac{2.5 - j9.0}{4.0 - j3.0} 25 \text{ kV} = 46\,703.85\angle -37.61^\circ$
- $\mathcal{V}_2$: $= -\mathcal{V}_1 / 25 = 1868.15\angle 142.39^\circ$
- $\mathcal{I}_2$ $= -\dfrac{25 \text{ kV}}{4 - j3} \times 25 = 125\angle 216.87^\circ kV$

$\dfrac{25}{25} : \dfrac{1}{25}$

$\boxed{1 : 0{,}04}$

$$Z_{in} = \frac{4 - j14{,}4}{(0{,}04)^2} = 2500 - j9000\ \Omega$$

$$\boxed{Z_{in} = 2{,}5 - j9\ k\Omega}$$



$\Rightarrow$ Voltage divider

$$V_1 = \frac{2.5 - j9}{1.5 + j6 + 2.5 - j9} \times 25k$$

$$V_1 = 37 - j\,28.5 \text{ kV}$$

$$V_1 = 46.7 \angle -37.61 \text{ V}$$

$$I_1 = \frac{V}{Z_{in}} = \frac{25k}{(1,5 + j6 + 2,5 - j9)k}$$

$$I_1 = 4 + j3 \text{ A}$$

$$= 5\angle 36,87 \text{ A}$$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \quad N_2$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} \quad I_2$$

$$\Rightarrow \quad V_2 = -\frac{N_2}{N_1} V_1$$

$$V_2 = -\frac{1}{25} \times 46{,}7 \angle -37{,}6$$

$$V_2 = -1{,}48 + j\,1{,}14 \text{ kV}$$

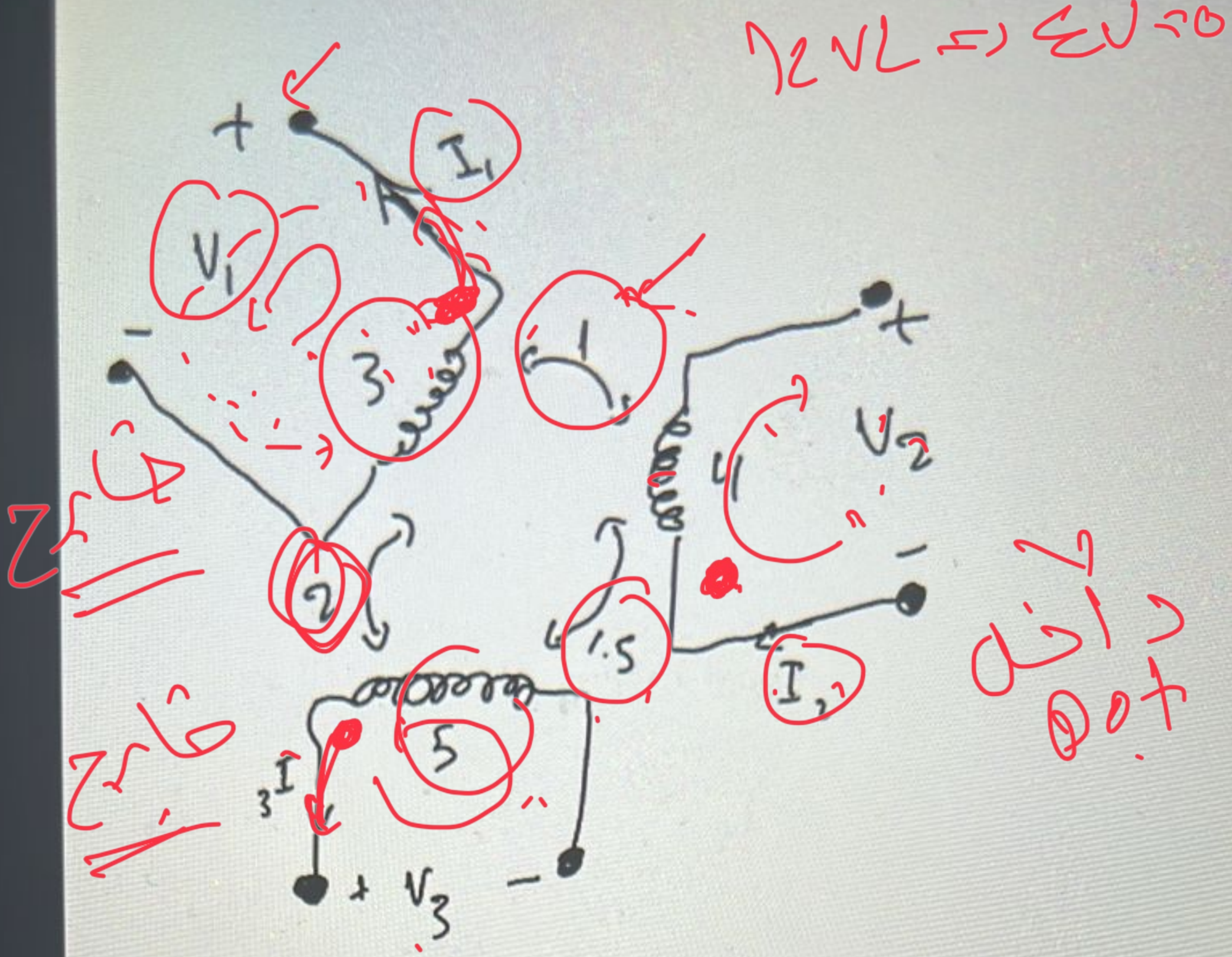
$$V_2 = 1{,}868 \angle 142{,}4 \text{ kV}$$

$$I_2 = -\frac{N_1}{N_2} I_1$$

$$I_2 = \left(-\frac{25}{1}\right)\left(5 \angle 36{,}87\right)$$

$$I_2 = -100 - j75 \text{ A}$$

$$\boxed{= 125 \angle -143{,}13}$$



$$V_1 + 3\frac{\partial I_1}{\partial t} - \frac{\partial I_2}{\partial t} + 2\frac{\partial I_3}{\partial t} = 0 \Rightarrow$$

$$V_1 = -3\frac{\partial I_1}{\partial t} + \frac{\partial I_2}{\partial t} - 2\frac{\partial I_3}{\partial t}$$

$$V_2 + 4\frac{\partial I_2}{\partial t} - \frac{\partial I_1}{\partial t} - 1.5\frac{\partial I_3}{\partial t}$$

$$V_2 = \frac{\partial I}{\partial t} - 4\frac{\partial I_2}{\partial t} + 1.5\frac{\partial I_3}{\partial t}$$

$$V_3 + 5\frac{\partial I_3}{\partial t} + 2\frac{\partial I_1}{\partial t} - 1.5\frac{\partial I_2}{\partial t}$$

$$V_3 = -2\frac{\partial I_1}{\partial t} + 1.5\frac{\partial I_2}{\partial t} - 5\frac{\partial I_3}{\partial t}$$

$$+V_1 + 3\frac{\partial c_1'}{\partial r} \overset{??}{=} 1\frac{\partial c_2'}{\partial r} \overset{??}{=} 2\frac{\partial c_3'}{\partial r} = 0$$