

EE242:Electric Circuits II
Chapter 1 Tutorial
Sinusoids and Phasors

Sinusoids and Phasors (1)

$$\omega = 2\pi f \rightarrow f = \frac{\omega}{2\pi}$$
$$f = 2 \text{ Hz} \rightarrow \omega = 4\pi \text{ rad/s}$$

- ▶ Check Example 9-1, 9-2 (Page 375)
- ▶ PP 9-1:

Given the sinusoid $30 \sin(4\pi t - 75^\circ)$, calculate its amplitude, phase, angular frequency, period, and frequency.

ω

$$f = 2$$

▶ 9-1 Solution:



Amplitude = 30,
phase = -75° ,
angular frequency = 4π rad/s,
frequency = 2 Hz,
Period = 0.5 s.



Sinusoids and Phasors (2)

► Check Example 9-1, 9-2 (Page 375)

► PP 9-2:

Find the phase angle between

$$i_1 = -4 \sin(377t + 55^\circ) \quad \text{and} \quad i_2 = 5 \cos(377t - 65^\circ)$$

Does i_1 lead or lag i_2 ?

9-2 Solution:

✓ Since $\cos \omega t = \sin(\omega t + 90^\circ)$

$$i_2 = 5 \sin(377t - 65^\circ + 90^\circ) = 5 \sin(377t + 25^\circ)$$

$$i_1 = -4 \sin(377t + 55^\circ) = 4 \sin(377t + 180^\circ + 55^\circ) = 4 \sin(377t + 235^\circ)$$

► Therefore, i_1 leads i_2 155° .



$$\begin{aligned} \sin(\omega t \pm 180^\circ) &= -\sin \omega t \\ \cos(\omega t \pm 180^\circ) &= -\cos \omega t \\ \sin(\omega t \pm 90^\circ) &= \pm \cos \omega t \\ \cos(\omega t \pm 90^\circ) &= \mp \sin \omega t \end{aligned}$$

$$i_1 = A \sin(377t + 55) + 90$$

$$\rightarrow A \cos(377t + 145) \rightarrow \theta_1 = -145$$

$$i_2 = 5 \cos(377t - 65) \rightarrow \theta_2 = 65$$



$$\therefore \text{lead } i_2 \quad 65 - (-145) = 210 \checkmark$$

Sinusoids and Phasors (3)

- ▶ Check Example 9.3, 9.4 Page 381
- ▶ PP 9.3: Evaluate the following complex numbers:

(a) $[(5 + j2)(-1 + j4) - 5\angle 60^\circ]^*$

(b) $\frac{10 + j5 + 3\angle 40^\circ}{-3 + j4} + 10\angle 30^\circ + j5$

▶ Solution



(a) $(5 + j2)(-1 + j4) = -5 + j20 - j2 - 8 = -13 + j18$
 $5\angle 60^\circ = 2.5 + j4.33$
 $(5 + j2)(-1 + j4) - 5\angle 60^\circ = -15.5 + j13.67$
 $[(5 + j2)(-1 + j4) - 5\angle 60^\circ]^* = \underline{-15.5 - j13.67} = \underline{20.67\angle 221.41^\circ}$

(b) $3\angle 40^\circ = 2.298 + j1.928$
 $10 + j5 + 3\angle 40^\circ = 12.298 + j6.928 = 14.115\angle 29.39^\circ$
 $-3 + j4 = 5\angle 126.87^\circ$
 $\frac{10 + j5 + 3\angle 40^\circ}{-3 + j4} = \frac{14.115\angle 29.39^\circ}{5\angle 126.87^\circ} = 2.823\angle -97.48^\circ$
 $2.823\angle -97.48^\circ = -0.3675 - j2.8$
 $10\angle 30^\circ = 8.66 + j5$
 $\frac{10 + j5 + 3\angle 40^\circ}{-3 + j4} + 10\angle 30^\circ = \underline{8.293 + j2.2}$



Sinusoids and Phasors (4)

► Check Example 9.3, 9.4 Page 381

► PP 9.4:

Express these sinusoids as phasors:

(a) $v = 7 \cos(2t + 40^\circ) \text{ V}$

(b) $i = -4 \sin(10t + 10^\circ) \text{ A}$

► Solution

(a). $\mathbf{V} = 7 \angle 40^\circ \text{ V}$

(b). Since $-\sin(A) = \cos(A+90^\circ)$;

$$v(t) = 4 \cos(10t + 10^\circ + 90^\circ) = 4 \cos(10t + 100^\circ)$$

Transform to phasor $\Rightarrow \mathbf{I} = 4 \angle 100^\circ \text{ A}$

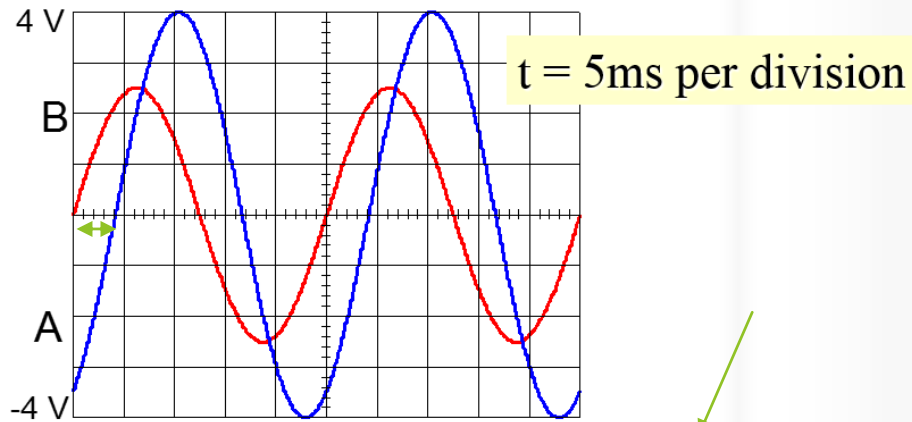


Trigonometric identities:

- $\sin(\omega t \pm 180^\circ) = -\sin \omega t$
- $\cos(\omega t \pm 180^\circ) = -\cos \omega t$
- $\sin(\omega t \pm 90^\circ) = \pm \cos \omega t$
- $\cos(\omega t \pm 90^\circ) = \mp \sin \omega t$

Sinusoids and Phasors (5)

- Write the phasors for **A** and **B**, if wave A is the reference wave.



$\theta = 57.6^\circ$ $A = 4 \angle 0^\circ V$ $B = 2.5 \angle 57.6^\circ V$

➤ Solution

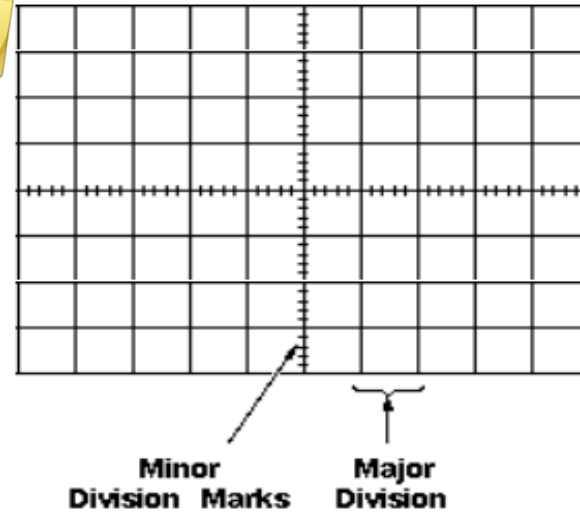


$$\theta / 360^\circ = \Delta t / T$$

$$\theta / 360^\circ = \Delta t / T$$

$$\theta / 360^\circ = 4 * 1 \text{ms (minor division)} / 5 * 5 \text{ms (major div)} = 4 / 25$$

$$\theta = 360 * 4 / 25 = 57,6^\circ$$



An Oscilloscope Graticule.

- Take a look at the oscilloscope display.
- Notice the grid markings on the screen - these markings create the graticule.
- Each vertical and horizontal line constitutes a major division. The graticule is usually laid out in an 8-by-10 division pattern.
- Labeling on the oscilloscope controls (such as volts/div and sec/div) always refers to major divisions.
- The tick marks on the center horizontal and vertical graticule lines (see Figure 1) are called minor divisions.

Sinusoids and Phasors (6)

$$v(t) = V_p \sin(\omega t + \theta)$$
$$10 \sin(100\text{krad} \cdot t + 0)$$

$$\omega = 2\pi f$$

1. What is the instantaneous voltage at $t = 3 \mu\text{s}$, if: $V_p = 10\text{V}$, $f = 50 \text{ kHz}$, $\theta = 0^\circ$?
2. What is your phasor form?

Solution

1. General sine wave equation:

$$v(t) = V_p \sin(\omega t + \theta)$$

Substitute all the values given,

$$v(t) = 10\text{V} \sin(2\pi 50\text{kHz} \cdot t)$$

At $t = 3\mu\text{s}$,

$$v(3\mu\text{s}) = 8.09\text{V}$$

2. The sine wave equation obtained:

$$v(t) = 10\text{V} \sin(2\pi 50\text{kHz} \cdot t)$$

In phasor form,

$$10\text{V} \angle 0^\circ$$

Sinusoids and Phasors (7)

Transform the sinusoids corresponding to phasors:

$$\mathbf{V} = -10 \angle 30^\circ \text{ V}$$
$$\mathbf{I} = j(5 - j12) \text{ A}$$

➤ Solution

a: Since $-\cos(A) = \cos(A+180^\circ)$;

$$v(t) = 10 \cos(\omega t + 30^\circ + 180^\circ) = 10 \cos(\omega t + 210^\circ)$$

$$v(t) = 10 \cos(\omega t + 210^\circ) \text{ V}$$

b) Since $\mathbf{I} = 12 + j5 = \sqrt{12^2 + 5^2} \angle \tan^{-1}(\frac{5}{12}) = 13 \angle 22.62^\circ$

$$i(t) = 13 \cos(\omega t + 22.62^\circ) \text{ A}$$



$$\sin(\omega t \pm 180^\circ) = -\sin \omega t$$
$$\cos(\omega t \pm 180^\circ) = -\cos \omega t$$
$$\sin(\omega t \pm 90^\circ) = \pm \cos \omega t$$
$$\cos(\omega t \pm 90^\circ) = \mp \sin \omega t$$

$$a) \rightarrow 10\text{V} < 210^\circ$$

$$b) \rightarrow 13\text{A} < 22,62^\circ$$



$\rightarrow 10 \angle 30$ math error $-180 \leq \theta \leq 180$

$\ominus r \angle \theta \rightarrow r \angle \theta + 180 \rightarrow 210$
 $\ominus r \angle \theta \rightarrow r \angle \theta - 180 \rightarrow -150 \checkmark$

9

$10 \angle -150$ or $10 \angle 210$

b) $13 \angle 22,6^\circ$

Sinusoids and Phasors (8)

If $v_1 = 10 \sin(\omega t - 30^\circ)$ V and $v_2 = 20 \cos(\omega t + 45^\circ)$ V, find $v = v_1 + v_2$.

$$10 \cos(\omega t - 30^\circ + 90^\circ)$$



Using phasor to add sinusoids

$$V_1 = 10 \angle 60^\circ, \quad V_2 = 20 \angle 45^\circ$$

$$V_1 + V_2 = (10 \angle 60^\circ) + (20 \angle 45^\circ) = 30 \angle 50^\circ$$

$$v = 30 \cos(\omega t + 50^\circ)$$

Sinusoids and Phasors (8)

If $v_1 = -10 \sin(\omega t - 30^\circ)$ V and $v_2 = 20 \cos(\omega t + 45^\circ)$ V, find $v = v_1 + v_2$.

➤ Solution

1- Since $-\sin(A) = \cos(A+90^\circ)$;

$$\rightarrow v_1(t) = 10 \cos(\omega t - 30^\circ + 90^\circ) = 10 \cos(\omega t + 60^\circ)$$

→ Transform **Phasor FORM** $\Rightarrow 10 \angle 60^\circ$

→ Transform **Rectangular FORM** $\Rightarrow 10(\cos(60^\circ) + j \sin(60^\circ)) = 5 + j8.66$



$$\begin{aligned} \sin(\omega t \pm 180^\circ) &= -\sin \omega t \\ \cos(\omega t \pm 180^\circ) &= -\cos \omega t \\ \sin(\omega t \pm 90^\circ) &= \pm \cos \omega t \\ \cos(\omega t \pm 90^\circ) &= \mp \sin \omega t \end{aligned}$$

2- $v_2(t) = 20 \cos(\omega t + 45^\circ)$

→ Transform **Phasor FORM** $\Rightarrow 20 \angle 45^\circ$

→ Transform **Rectangular FORM** $\Rightarrow 20(\cos(45^\circ) + j \sin(45^\circ)) = 14.14 + j14.14$

$$v_1(t) + v_2(t) = 5 + j8.66 + 14.14 + j14.14 = 19.14 + j22.80 = \sqrt{(19.14^2 + 22.8^2)} \angle \tan^{-1}\left(\frac{22.8}{19.14}\right)$$



Answer: $v(t) = 29.77 \cos(\omega t + 49.98^\circ)$ V.

Example 10- Phasors and Circuit Elements

► Check Example 9.8 (Page 387)

► PP 9.8:

If voltage $v = 10 \cos(100t + 30^\circ)$ is applied to a $50 \mu\text{F}$ capacitor, calculate the current through the capacitor.

► Solution

► Use the polar form



$$V = \frac{I}{j\omega C}$$

where $\omega = 100 \text{ rad/s}$ and $V = 10 \angle 30^\circ \text{ V}$.

$$C = 50 \mu\text{F} = 5 \times 10^{-5}$$

$$I = V j\omega C = 10 \angle 30^\circ \text{ V} \cdot j100 \times 5 \times 10^{-5}$$

$$I = V j\omega C = 10 \angle 30^\circ \text{ V} \times 0.005 \angle 90^\circ$$

$$I = V j\omega C = 0.05 \text{ A} \angle 120^\circ$$



Answer: $50 \cos(100t + 120^\circ) \text{ mA}$.

$$\frac{0.05}{10^{-3}}$$



$$\omega = 100 \rightarrow 10 \angle 30^\circ$$

$$V = IZ \quad Z_C = \frac{1}{j\omega C}$$

$$I = \frac{V}{Z_C}$$

Example 11- Resistors Combination

Determine the input impedance of the circuit at $\omega=10$ rad/sec.

Solution:

Let Z_1 = impedance of the 2-mF capacitor in series with the 20- Ω resistor

Z_2 = impedance of the 4-mF capacitor

Z_3 = impedance of the 2-H inductor in series with the 50- Ω resistor

$$Z_1 = 20 + \frac{1}{j\omega C} = 20 + \frac{1}{j(10)(2 \times 10^{-3})} = 20 - j50$$

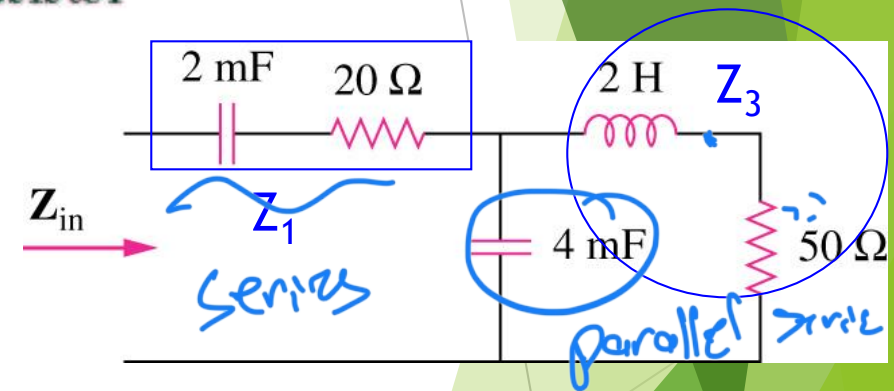
$$Z_2 = \frac{1}{j\omega C} = \frac{1}{j(10)(4 \times 10^{-3})} = -j25$$

$$Z_3 = 50 + j\omega L = 50 + j(10)(2) = 50 + j20$$

$$Z_{in} = Z_1 + Z_2 \parallel Z_3 = Z_1 + Z_2 Z_3 / (Z_2 + Z_3)$$

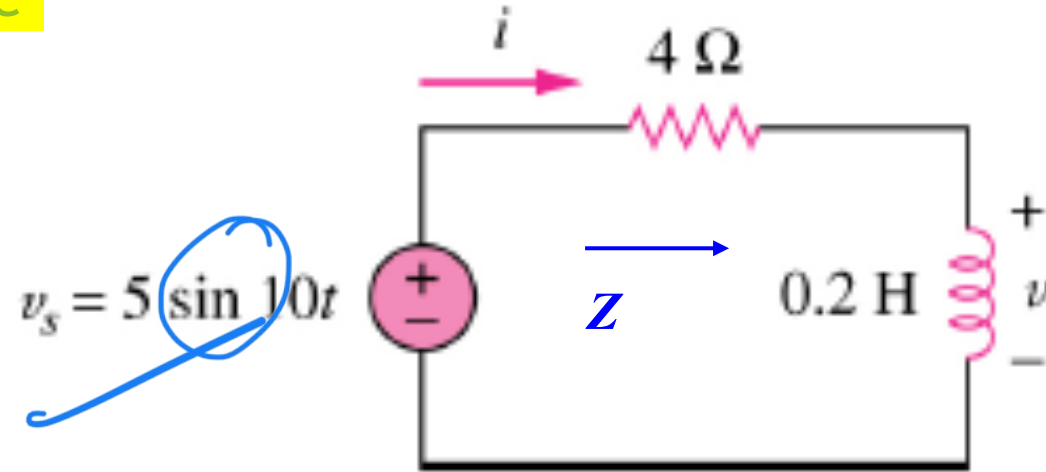
$$Z_{in} = 20 - j50 + \frac{-j25 \times (50 + j20)}{-j25 + 50 + j20} = 20 - j50 + 12.38 - j23.76$$

$$Z_{in} = \underline{\underline{32.38 - j73.76 \Omega}}$$



Example 12- If we are going back to t, we can use sin or cos as phasor reference

Determine $v(t)$ and $i(t)$.



Solution:

$$\mathbf{V}_s = 5 \angle 0^\circ, \quad \omega = 10$$

$$\mathbf{Z} = 4 + j\omega L = 4 + j2$$

$$\mathbf{I} = \mathbf{V}_s / \mathbf{Z} = \frac{5 \angle 0^\circ}{4 + j2} = \frac{5(4 - j2)}{16 + 4} = 1 - j0.5 = 1.118 \angle -26.57^\circ$$

$$\mathbf{V} = j\omega L \mathbf{I} = j2 \mathbf{I} = (2 \angle 90^\circ)(1.118 \angle -26.57^\circ) = 2.236 \angle 63.43^\circ$$

Therefore,

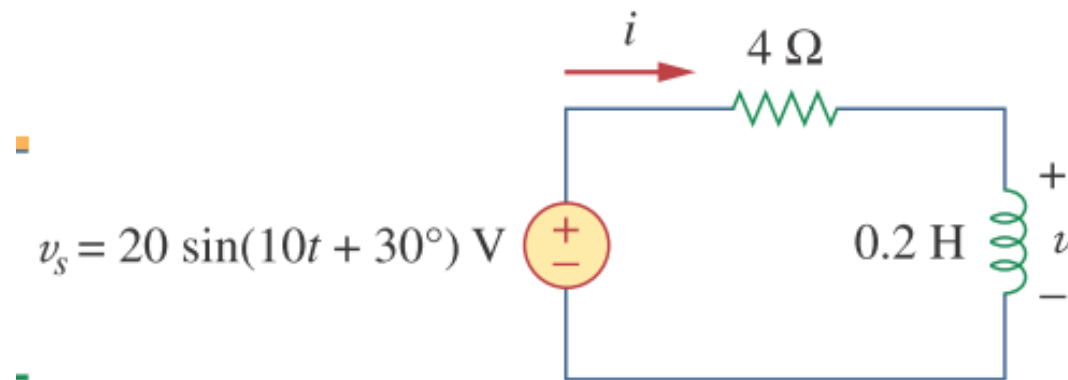
$$\begin{aligned} v(t) &= \underline{2.236 \sin(10t + 63.43^\circ) \text{ V}} \\ i(t) &= \underline{1.118 \sin(10t - 26.57^\circ) \text{ A}} \end{aligned}$$

Example 13- Circuit Analysis Using Phasors (1)

- ▶ Check Example 9.9 (Page 389)
- ▶ PP 9.9:

Refer to Fig. 9.17. Determine $v(t)$ and $i(t)$.

Answer: $8.944 \sin(10t + 93.43^\circ)$ V, $4.472 \sin(10t + 3.43^\circ)$ A.



Example 14- Circuit Analysis Using Phasors

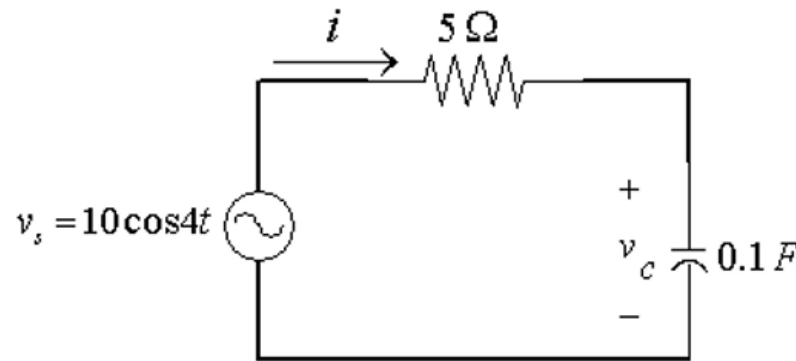
➤ Find i and v_c

➤ Solution:

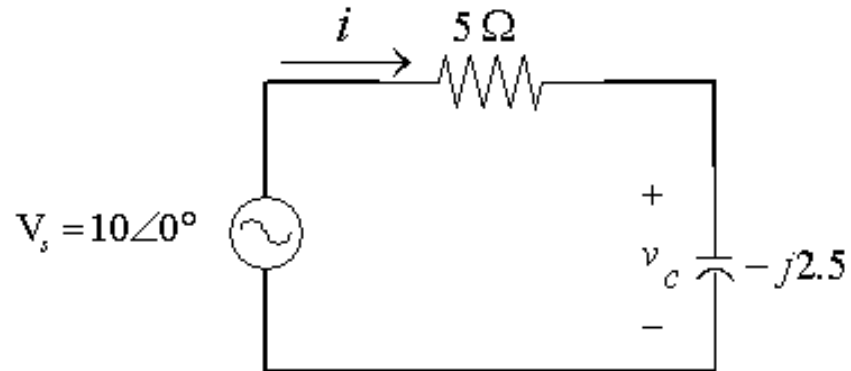
➤ Elements in Frequency domain

❖ $\omega = 4$ rad/s

Time domain	Freq domain
$v_s = 10 \cos 4t$	$V_s = 10 \angle 0^\circ$
$R = 5 \Omega$	$R = 5 \Omega$
$C = 0.1 F$	$X_C = -j\left(\frac{1}{\omega C}\right) = -j2.5$



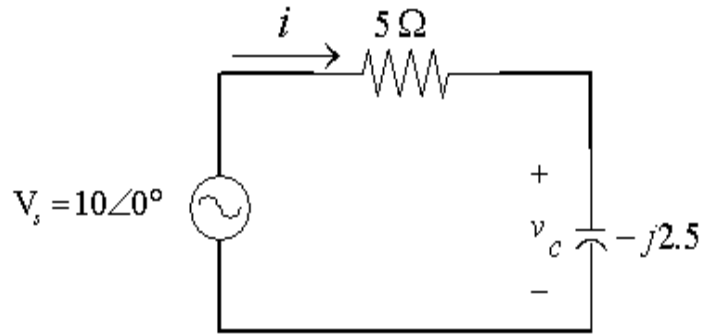
➤ Circuit in Frequency Domain



Example 14-Circuit Analysis Using Phasors

➤ Find i and v_c

➤ Solution:



a-the current, i

❖ The current in series circuit,

$$I = \frac{V_s}{Z_T}$$

❖ Total impedance is;

$$\begin{aligned} Z_T &= 5 + (-j2.5) \\ &= 5 - j2.5 = 5.59 \angle -26.56^\circ \Omega \end{aligned}$$

➤ Better performed in Rectangular form

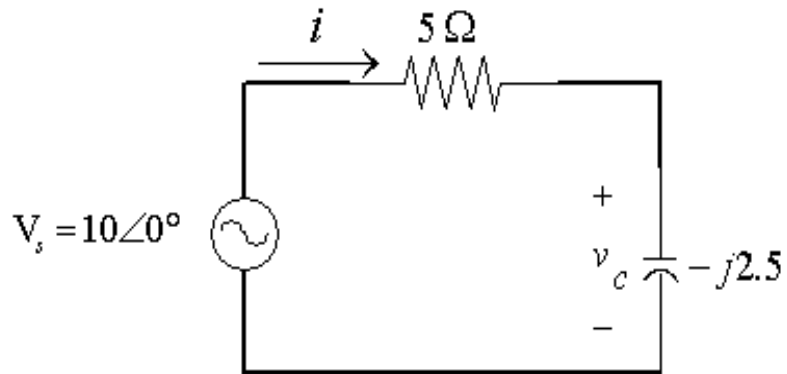
$$\begin{aligned} I &= \frac{V_s}{Z_T} = \frac{10 \angle 0^\circ}{5.59 \angle -26.56^\circ} \\ &= 1.79 \angle 26.56^\circ \\ &= 1.6 + j0.8 \text{ A} \end{aligned}$$

➤ Better performed in Polar form

Example 14-Circuit Analysis Using Phasors

➤ Find i and v_c

➤ Solution:



b-The voltage, V_C

$$\begin{aligned}V_C &= IZ_C \\&= (1.79\angle 26.56^\circ) \cdot (-j2.5) \\&= (1.79\angle 26.56^\circ) \cdot (2.5\angle -90^\circ) \\&= 4.48\angle -63.44^\circ V\end{aligned}$$

➤ Better performed
in Polar form

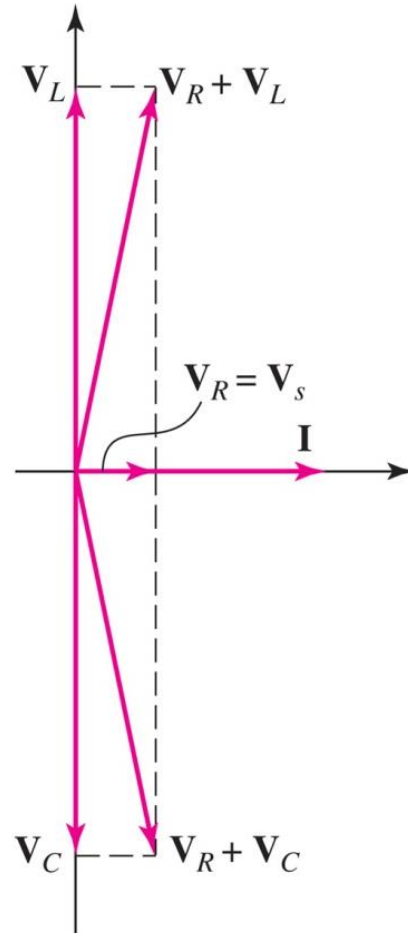
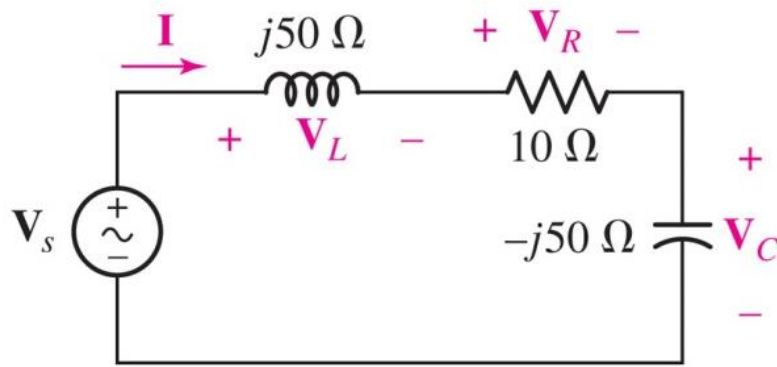
Example 16- Phasor Diagram



Impedances of passive elements	
Element	Impedance
R	$Z = R$
L	$Z = j\omega L$
C	$Z = \frac{1}{j\omega C}$

➤ **Solution:**

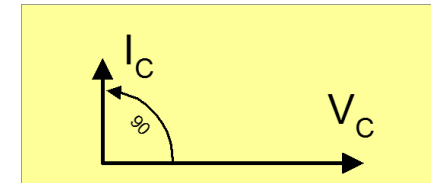
If we assume $I = 1 \angle 0^\circ \text{ A}$



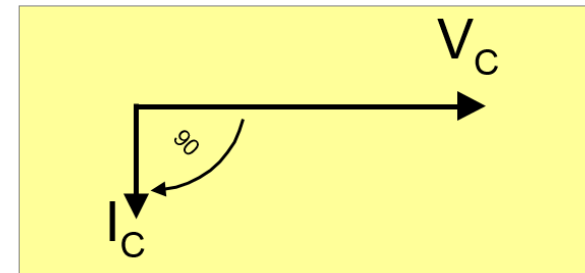
- $Z_T = Z_L + Z_R + Z_C = j50 + 10 - j50 = 10$
- V_s and V_R are in phase

❖ In the Capacitor (C), Voltage **LAGS** charging current by 90° or Charging Current (I) **LEADS** Voltage (E) by 90°

❖ **I. C. E.**



❖ In the Inductor (L), Induced Voltage **LEADS** current by 90° or Current (I) **LAGS** Induced Voltage (E) by 90° .



Example 17 - Phasor Diagram: Parallel RLC

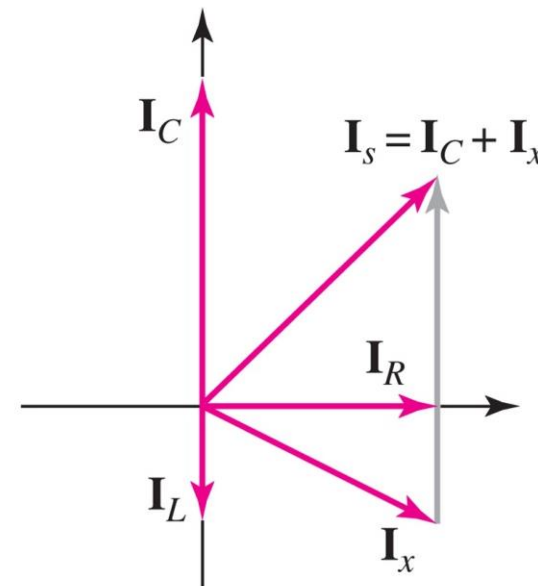
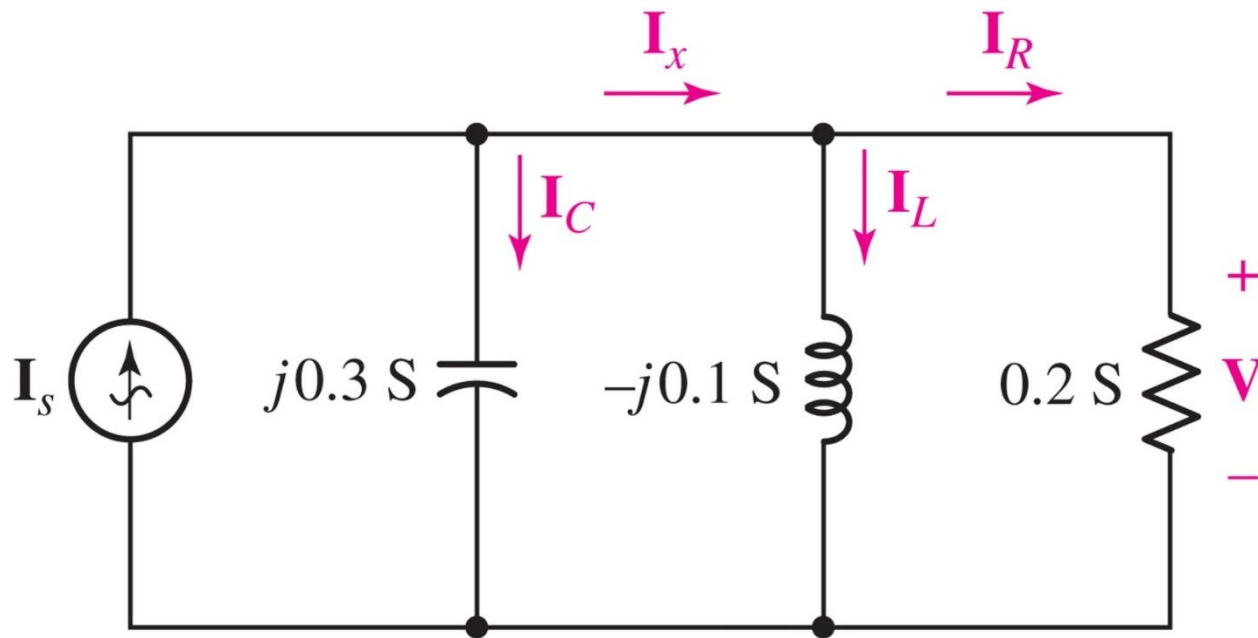
Assume $V = 1 \angle 0^\circ$ V

➤ Solution:



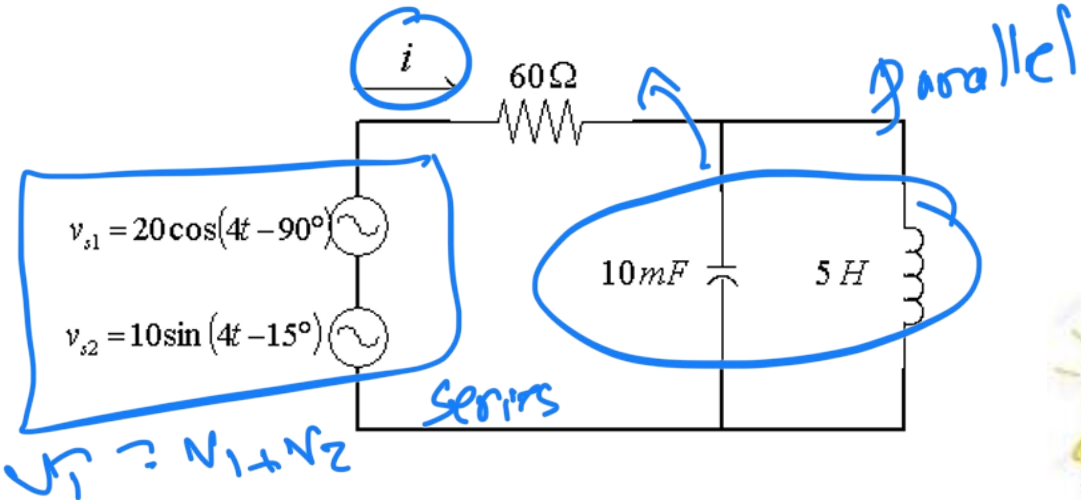
- ✓ The reciprocal of impedance.
- ✓ Symbol is Y.
- ✓ Measured in Siemens (S).

$$1/R=0,2S,..$$



Example 18 $\uparrow \Rightarrow \frac{V_T}{Z_T}$

a. Find $i(t)$



➤ Solution:

Different waveform of V_s

- V_{s1} and V_{s2} are different:

$$v_{s1} = 20 \cos(4t - 90^\circ)$$

$$v_{s2} = 10 \sin(4t - 15^\circ)$$

- When analyzing ac circuit, it is expedient to express both signals as either Sine or Cosine.

Change the waveform of V_{s1}

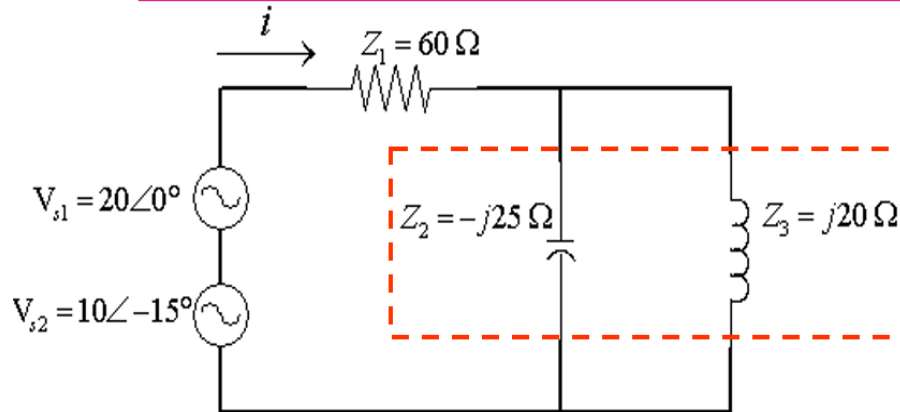
- Using identity:

$$\sin(\omega t \pm 180^\circ) = -\sin \omega t$$

$$\cos(\omega t \pm 180^\circ) = -\cos \omega t$$

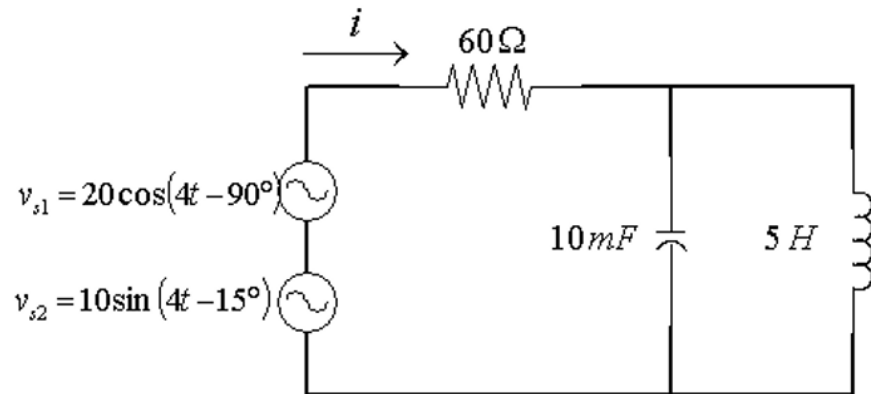
$$\sin(\omega t \pm 90^\circ) = \pm \cos \omega t$$

$$\cos(\omega t \pm 90^\circ) = \mp \sin \omega t$$

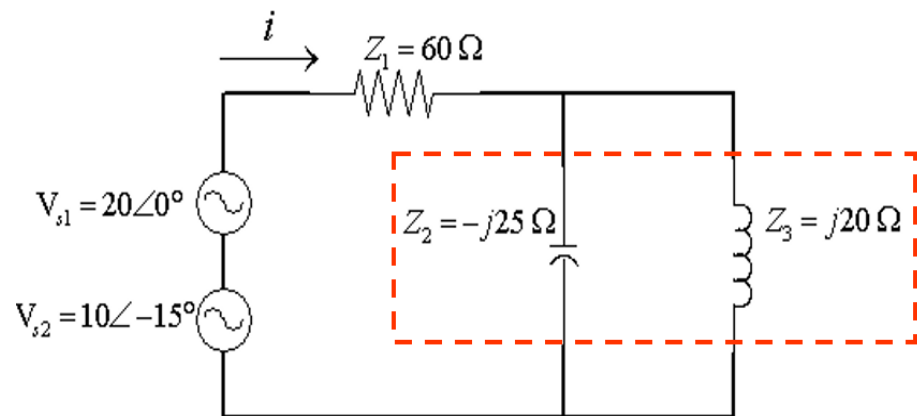


Example 18

a. Find $i(t)$



➤ Solution:



Elements in Frequency domain

• $\omega = 4 \text{ rad/s}$

Time domain	Freq domain
$v_{s1} = 20 \sin 4t$	$V_{s1} = 20 \angle 0^\circ$
$v_{s2} = 10 \sin(4t - 15^\circ)$	$V_{s2} = 10 \angle -15^\circ$
$R = 60 \Omega$	$R = 60 \Omega$
$C = 10 \text{ mF}$	$X_C = -j\left(\frac{1}{\omega C}\right) = -j25$
$L = 5 \text{ H}$	$X_L = j\omega L = j20$

Find The Current

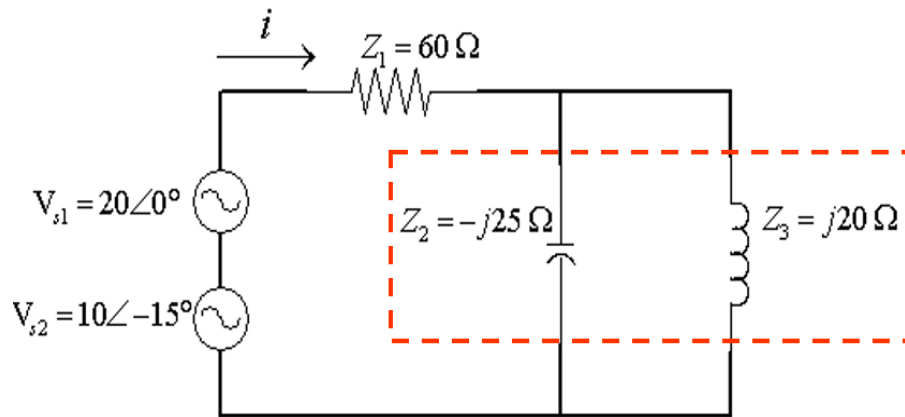
$$I = \frac{V_T}{Z_T}$$

$$V_T = v_{s1} + v_{s2}$$

$$Z_T = Z_1 + (Z_2 \parallel Z_3)$$

Example 18

➤ Solution:



Solution:

$$V_T = 29.66 - j2.59 = 29.77 \angle -4.99^\circ \text{ V}$$

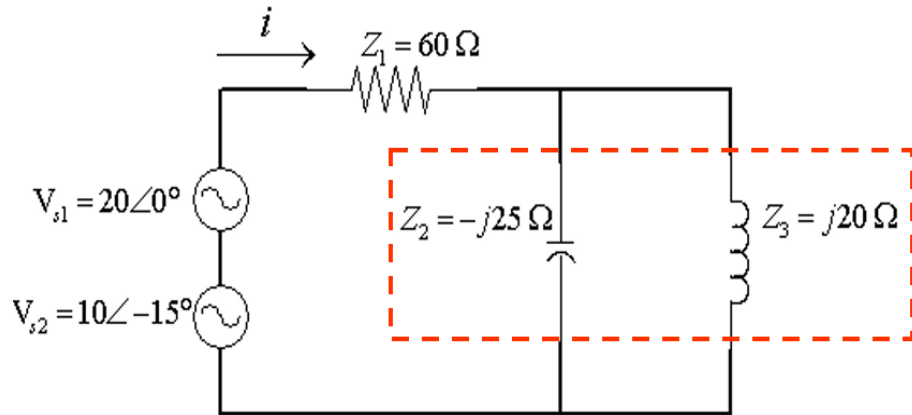
$$Z_T = 60 + j90 = 108.17 \angle 56.31^\circ \Omega$$

$$I = 0.28 \angle -61.3^\circ \text{ A}$$

$$\therefore i(t) = 0.28 \sin(4t - 61.3^\circ) \text{ A}$$

Example 18

➤ Solution:



(b) Find the $v_C(t)$, $v_L(t)$

Answer:

$$V_C = V_L = 28\angle 28.7^\circ V$$

$$\therefore v_C(t) = v_L(t) = 28\sin(4t + 28.7^\circ)V$$

(c) Find the $i_C(t)$, $i_L(t)$

Answer

$$I_C = 1.12\angle 118.7^\circ A$$

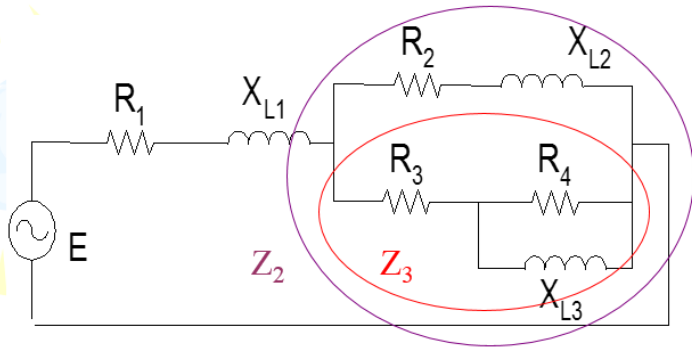
$$I_L = 1.4\angle -61.3^\circ A$$

$$\therefore i_C(t) = 1.12\sin(4t + 118.7^\circ)A$$

$$\therefore i_L(t) = 1.4\sin(4t - 61.3^\circ)A$$

Example 20

Ex: Find Z_T , I_{L3} , P



$$V_S = 10V \angle 0^\circ$$

$$R_1 = 5k\Omega$$

$$X_{L1} = 4k\Omega$$

$$R_2 = 6k\Omega$$

$$X_{L2} = 2k\Omega$$

$$R_3 = 7k\Omega$$

$$X_{L3} = 3k\Omega$$

$$R_4 = 8k\Omega$$

$$Z_2 = R_3 + (R_4 // X_{L3})$$

$$Z_3 = (R_2 + X_{L2}) // Z_2$$

$$Z_T = X_{L1} + Z_3$$

➤ Solution:

Z_2 , Z_3 , Z_T , and I_T

$$Z_2 = 3.61 k\Omega \angle 18.38^\circ$$

$$Z_3 = 8.41 k\Omega \angle 18.2^\circ$$

$$Z_T = 9.87 k\Omega \angle 31.4^\circ$$

$$I_T = \frac{V_S}{Z_T} = 1.01 mA \angle -31.4^\circ$$