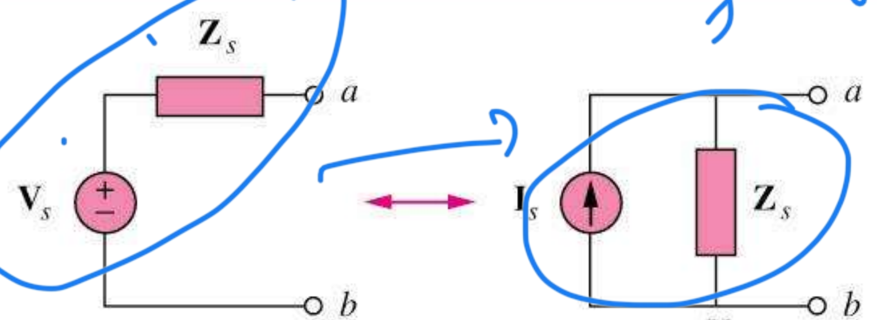
Source Transformation

series

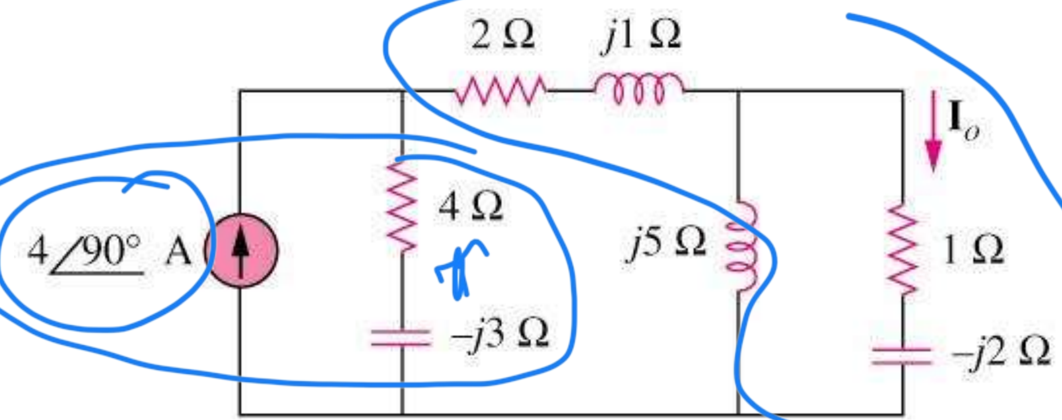
Parallel



$\mathbf{V}_s = \mathbf{Z}_s \mathbf{I}_s$ $\qquad$ $\mathbf{I}_s = \dfrac{\mathbf{V}_s}{\mathbf{Z}_s}$

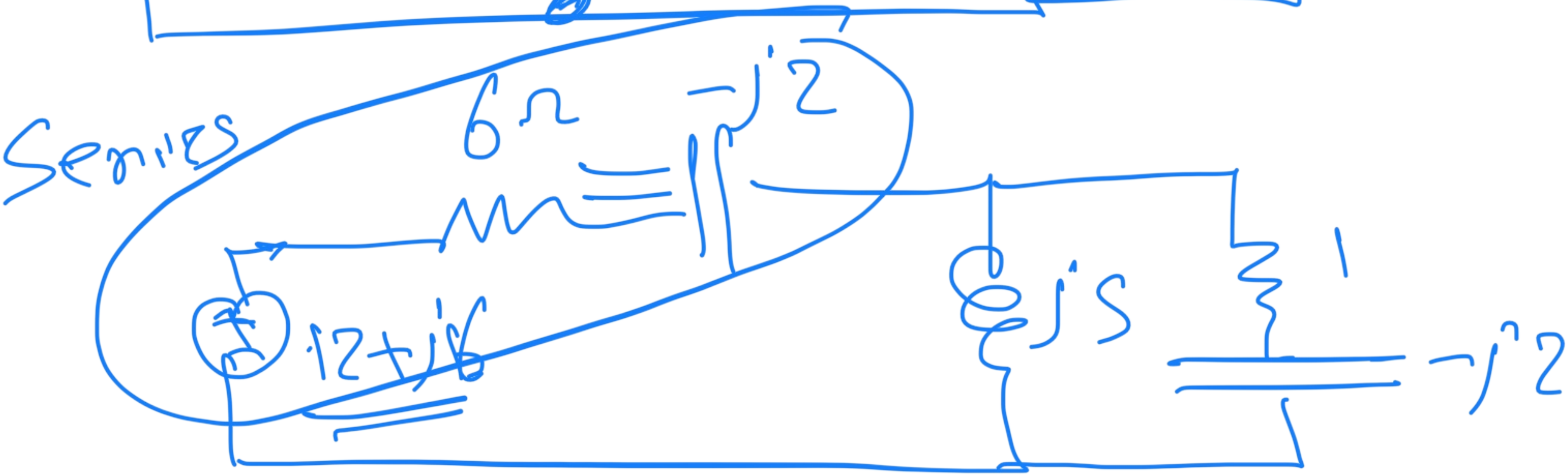
Example Find Io in the circuit of figure below using the concept of source transformation.

Parallel



Answer: : I_o = **3.288+j99.46 A** ?

$V = IR = 4\angle 90 \times (4 - j3)$



$$I = \frac{V}{Z} = \frac{12+j16}{6-j2} = 1+j3$$

current divider parallel

$$I_o = \frac{Z_{eq}}{Z_1} \times I_s$$

$$Z_{eq} = \frac{1}{\frac{1}{6-j2} + \frac{1}{j5} + \frac{1}{1-j2}}$$

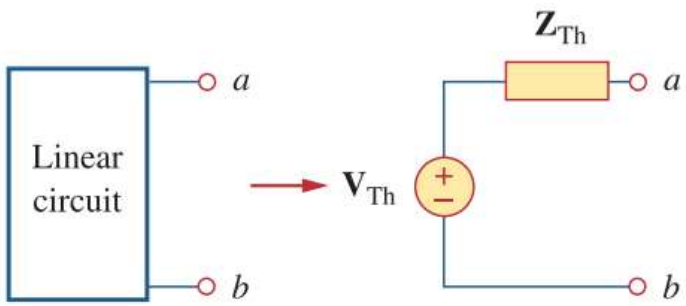
$$= \frac{70}{37} - j\frac{50}{37}\ \Omega$$

$$I_o = \frac{\quad}{1-j2} * (4+j3)$$

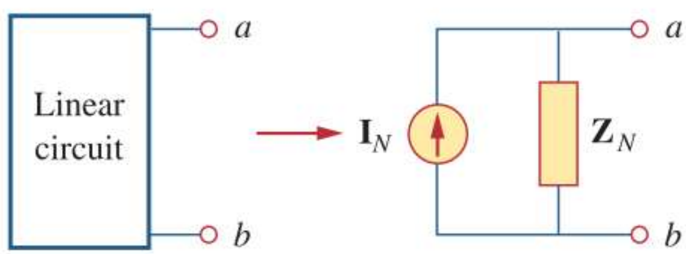
$$= \boxed{\begin{gathered} -0{,}541 + j3{,}2A\ A \\ = 3{,}29 \angle 100^\circ\ A \end{gathered}}$$

Thevenin and Norton Theorems in AC circuits

- Thevenin's and Norton theorems are applied to ac circuits in the same way as they are to DC circuits
- The ONLY additional effort is the need to manipulate complex numbers.



Thevenin equivalent.



Norton equivalent.

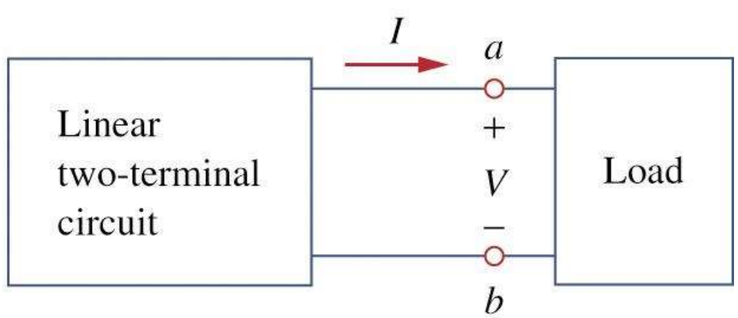
Where; V_{Th} = $V_{ab\text{-}OC}$ is the open circuit voltage (PHASOR) between terminals a-b,
I_N=$I_{ab\text{-}SC}$ is the short circuit current (PHASOR) through the terminals a-b, and
Z_{Th} is the input or equivalent IMPEDANCE at the terminals when the independent source are turn off.

How to Find Thevenin Equivalent Circuit in AC circuits

- First, **Open the circuit** (remove the load) at the **points** of interest **a-b**

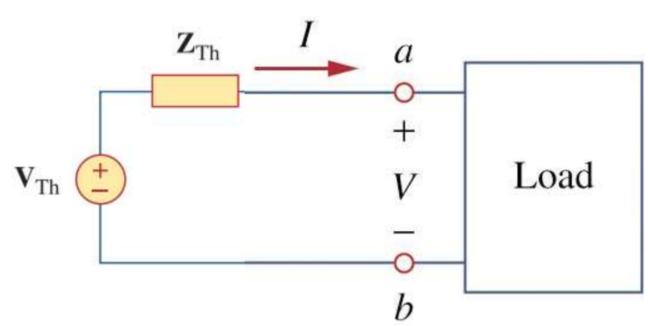
1- V_{th}= Open circuit voltage (keep all sources intact). **Note: Vth is Phasor**

2- Z_{th}= Open circuit **equivalent Impedance** appears at terminals **a-b** while all **independent sources=0** (voltage source=SC , current source=OC).



Original Circuit

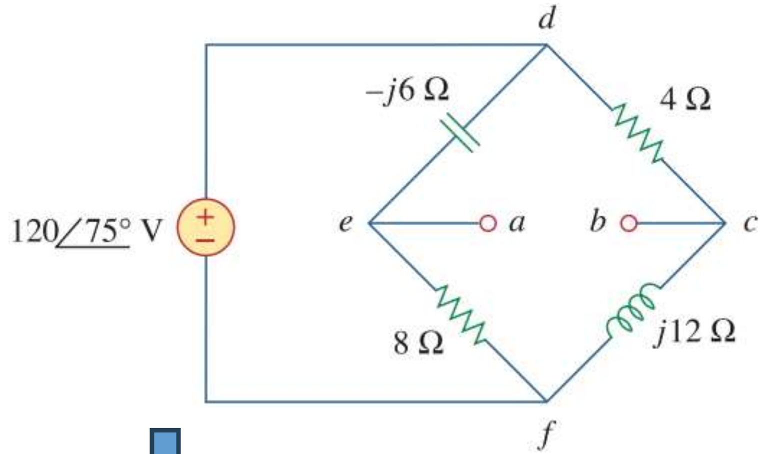
≡



Thevenin equivalent circuit

Example 10.8- Thevenin Equivalent Circuit

Obtain the Thevenin equivalent at terminals a-b of the following circuit.



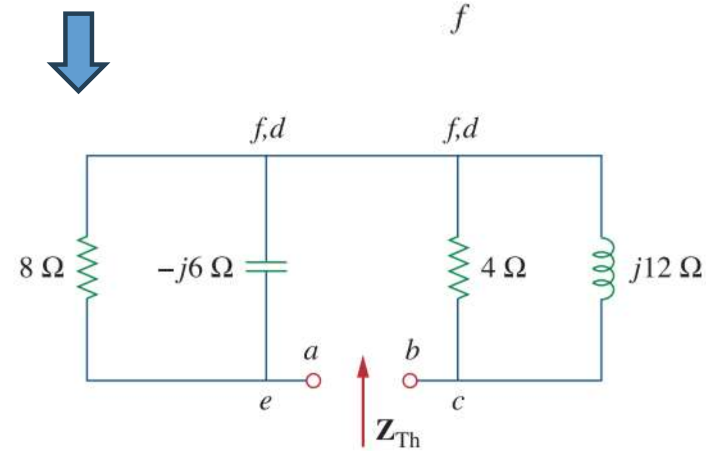
Solution

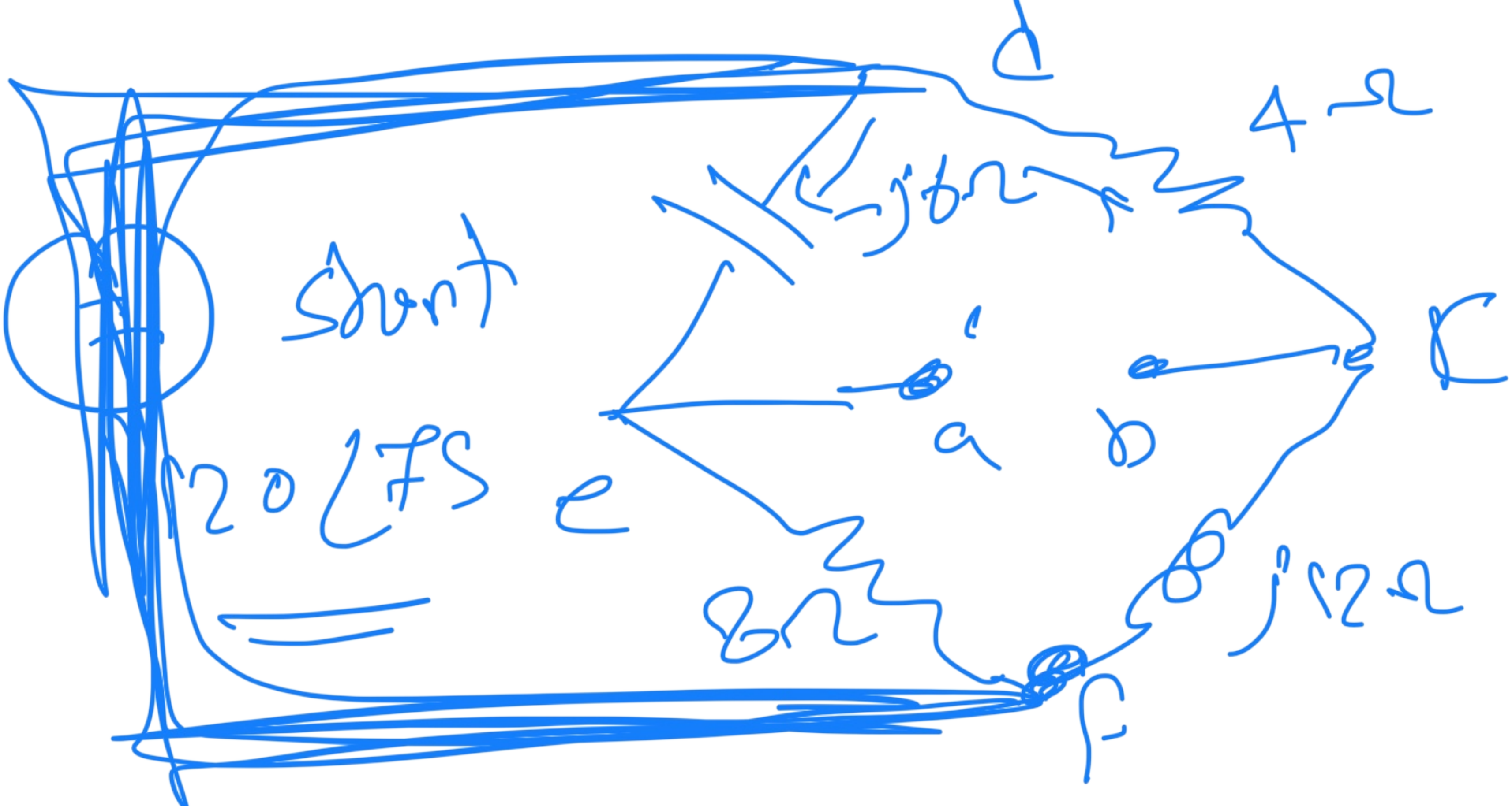
1- For Zth

$$\mathbf{Z}_1 = -j6 \parallel 8 = \frac{-j6 \times 8}{8 - j6} = 2.88 - j3.84\ \Omega$$

$$\mathbf{Z}_2 = 4 \parallel j12 = \frac{j12 \times 4}{4 + j12} = 3.6 + j1.2\ \Omega$$

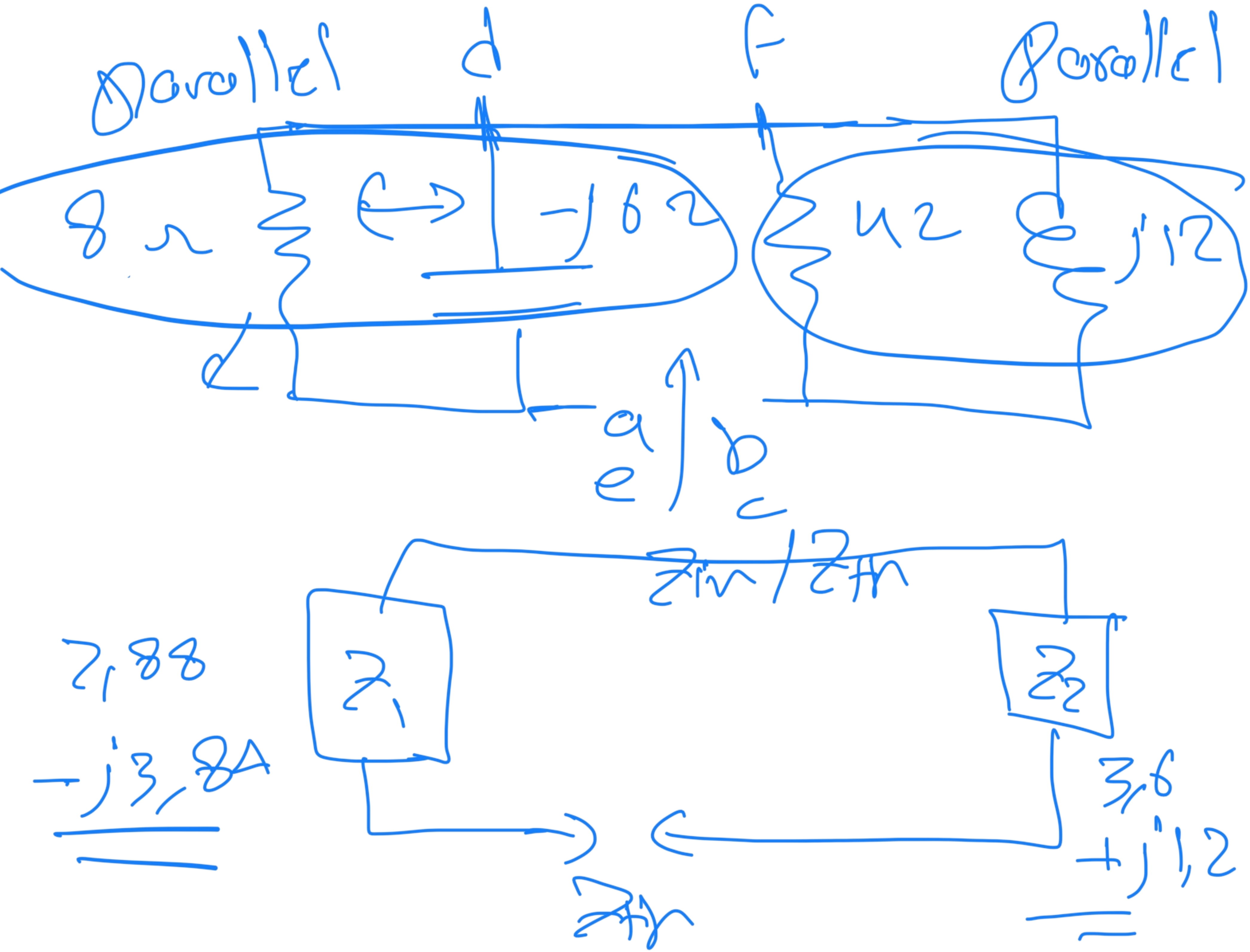
$$\boxed{\mathbf{Z}_{\text{Th}} = \mathbf{Z}_1 + \mathbf{Z}_2 = 6.48 - j2.64\ \Omega}$$





① Zth All ind source

→ Turn off all ind source


parallel
d
f
parallel
8 Ω
-j6Ω
4Ω
j12
a
b
e
c
Zin/Zth
Z1
Z2
7,88
-j3,84
3,6
+j1,2
Zin

$$Z_{th} = Z_1 + Z_2$$

$$2{,}88 - j3{,}84 + 3{,}6 + j1{,}2$$

$$\boxed{Z_{th} = 6{,}48 - j2{,}64\ \Omega}$$

Example 10.8- Thevenin Equivalent Circuit

Obtain the Thevenin equivalent at terminals a-b of the following circuit.

Solution (continue)

2- For Vth

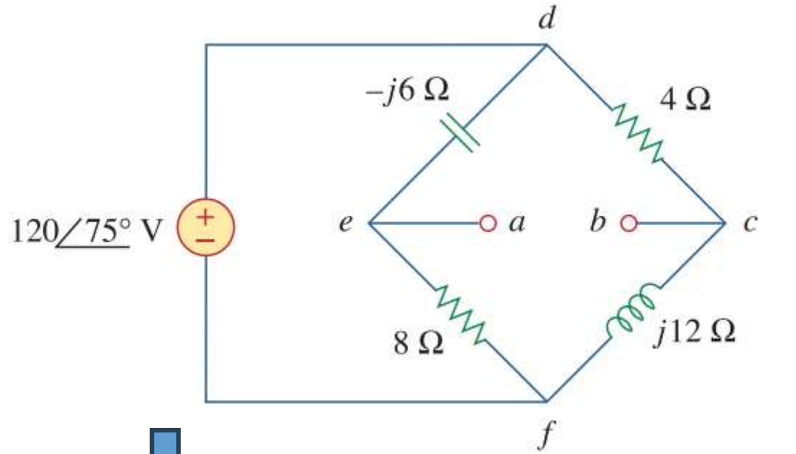
$$\mathbf{I}_1 = \frac{120\angle 75^\circ}{8 - j6}\ \text{A}, \qquad \mathbf{I}_2 = \frac{120\angle 75^\circ}{4 + j12}\ \text{A}$$

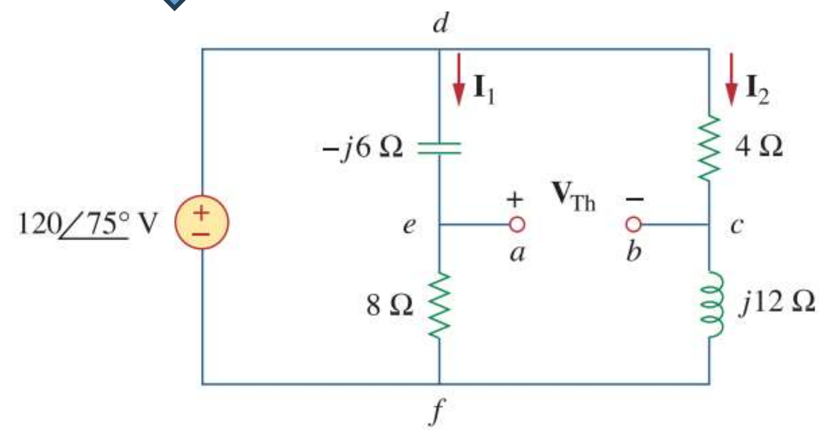
Applying KVL around loop *bcdeab*

$$\mathbf{V}_{\text{Th}} - 4\mathbf{I}_2 + (-j6)\mathbf{I}_1 = 0$$

$$\begin{aligned}\mathbf{V}_{\text{Th}} = 4\mathbf{I}_2 + j6\mathbf{I}_1 &= \frac{480\angle 75^\circ}{4 + j12} + \frac{720\angle 75^\circ + 90^\circ}{8 - j6} \\ &= 37.95\angle 3.43^\circ + 72\angle 201.87^\circ \\ &= -28.936 - j24.55 = 37.95\angle 220.31^\circ\ \text{V}\end{aligned}$$

37,95 ∠-139,69





$$I_1 = \frac{120\angle 75}{8-j6} = -4,47+j11,14$$

$$I_2 = \frac{120\angle 75}{4+j12} = 9,47+j0,568$$

$$-V_{th} - V_{R_1} + V_{R_2} = 0$$

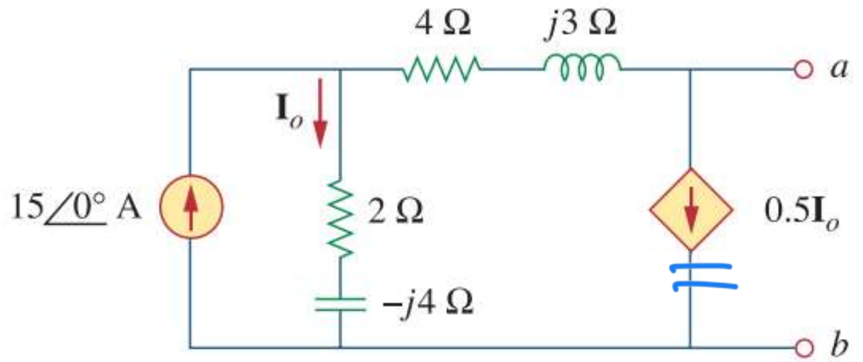
$$V_{th} = V_{R_2} - V_{R_1}$$

$$4\int_2 + j6\int_1$$

$$\boxed{V_{th} = 37{,}46 \angle -139{,}71}\ V$$

Example 10.9- Thevenin Equivalent Circuit

Obtain the Thevenin equivalent at terminals a-b of the following circuit.



Solution

1- For Vth

To find $\mathbf{V}_{Th}$, we apply KCL at node 1

$$15 = \mathbf{I}_o + 0.5\mathbf{I}_o \qquad \Rightarrow \qquad \mathbf{I}_o = 10 \text{ A}$$

Applying KVL to the loop on the right-hand side

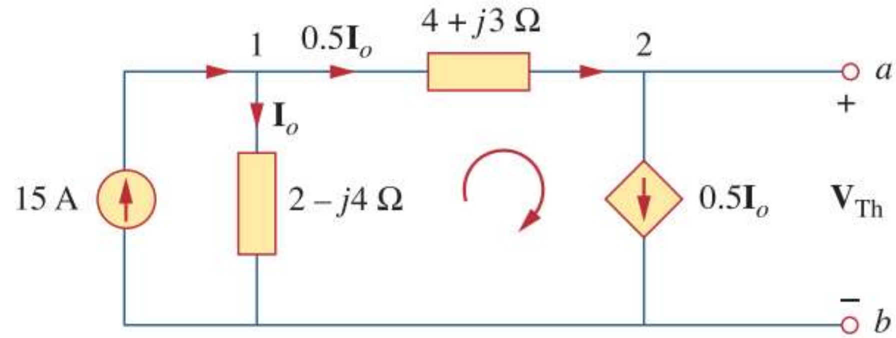
$$-\mathbf{I}_o(2 - j4) + 0.5\mathbf{I}_o(4 + j3) + \mathbf{V}_{Th} = 0$$

or

$$\mathbf{V}_{Th} = 10(2 - j4) - 5(4 + j3) = -j55$$

Thus, the Thevenin voltage is

$$\boxed{\mathbf{V}_{Th} = 55\angle{-90^\circ} \text{ V}}$$



KCL $\Rightarrow \Sigma I_{in} = \Sigma I_{out}$

$$15 = I_o + 0{,}5 I_o = 1{,}5 I_o$$

$$\frac{15}{1{,}5} = \frac{1{,}5 I_o}{1{,}5}$$

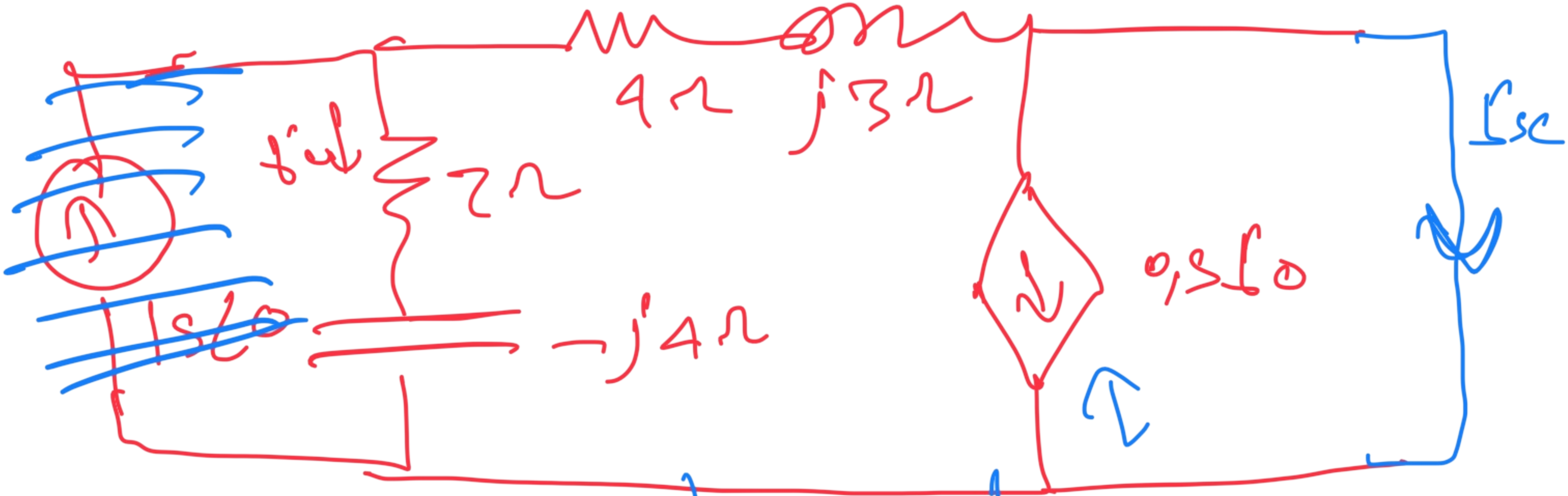
$$\Rightarrow \boxed{I_o = 10A}$$

KVL $\quad \Sigma V = 0$

$$-10[2-j4]+5[4+j3]+V_o=0$$

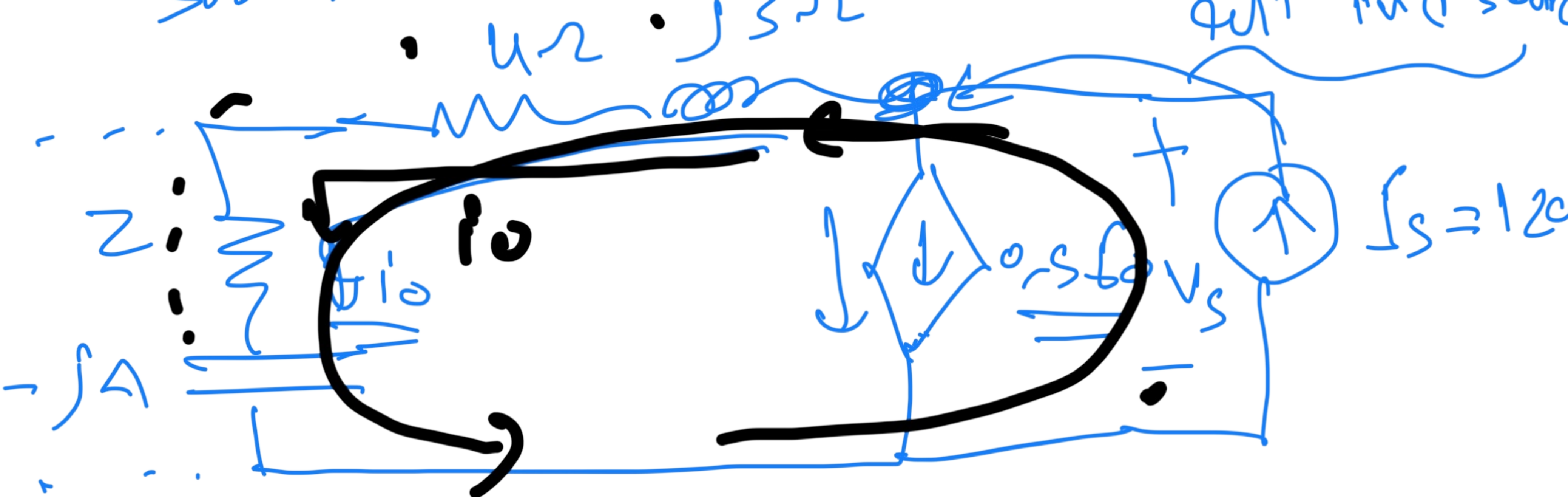
$$V_o=10[2-j4]-5[4+j3]$$

$$\boxed{V_{o,c} = -55j = 55\angle -90\ V}$$



short circuit

Turn off all ind sourc



$$Z_{th} = \frac{V_s}{I_s} =$$

KCL $\Rightarrow$ $1 = i_0 + 0.5 i_0$

$1.5 i_0 = 1$

$$i_0 = \frac{1}{1.5} = \frac{2}{3} A$$

$$-V_s + \frac{2}{3}\left[4 + j3 + 2 - j4\right] = 0$$

$=$

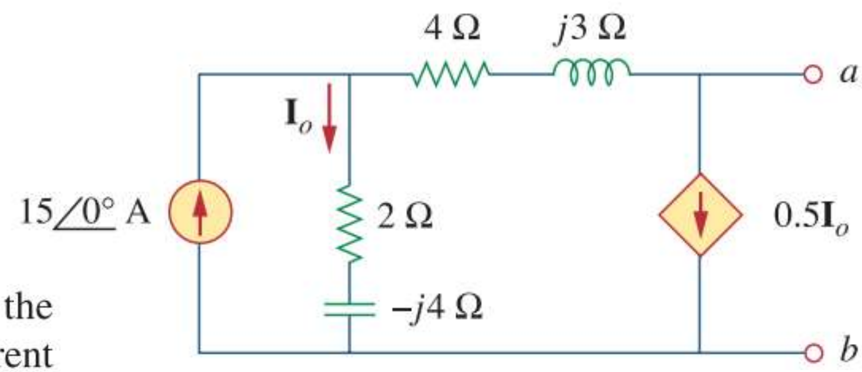
$$V_s = 4 - \frac{2}{3} j \ V$$

$$\Rightarrow Z_{in} = \frac{4 - j\frac{2}{3}}{1\angle 0}$$

$$\boxed{Z_{in} = 4 - j\frac{2}{3} \ \Omega}$$

Example 10.9- Thevenin Equivalent Circuit

Obtain the Thevenin equivalent at terminals a-b of the following circuit.



Solution (continue)

2- For Zth

To obtain $\mathbf{Z}_{\mathrm{Th}}$, we remove the independent source. Due to the presence of the dependent current source, we connect a 3-A current source (3 is an arbitrary value chosen for convenience here, a number divisible by the sum of currents leaving the node) to terminals *a*-*b*

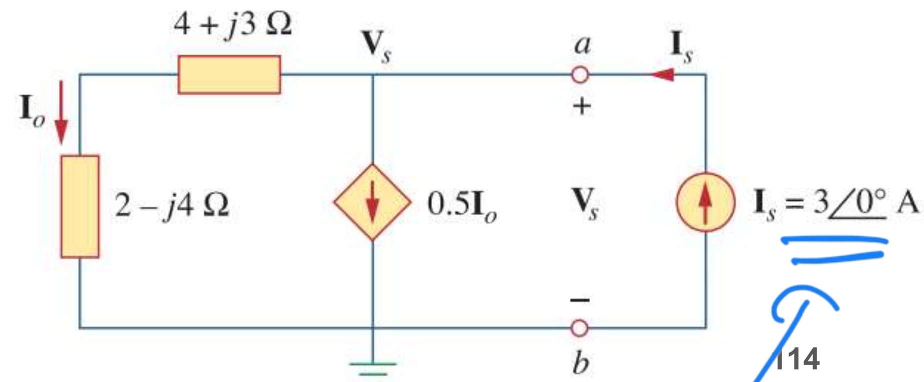
$$3 = \mathbf{I}_o + 0.5\mathbf{I}_o \qquad \Rightarrow \qquad \mathbf{I}_o = 2\text{ A}$$

Applying KVL to the outer loop in Fig. 10.26(b) gives

$$\mathbf{V}_s = \mathbf{I}_o(4 + j3 + 2 - j4) = 2(6 - j)$$

The Thevenin impedance is

$$\boxed{\mathbf{Z}_{\mathrm{Th}} = \frac{\mathbf{V}_s}{\mathbf{I}_s} = \frac{2(6 - j)}{3} = 4 - j0.6667\ \Omega}$$



Trigonometric Identities

- Sine and cosine form conversions.

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

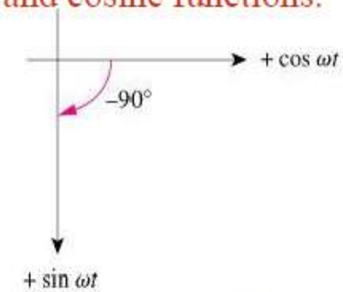
$$\sin(\omega t \pm 180°) = -\sin \omega t$$
$$\cos(\omega t \pm 180°) = -\cos \omega t$$
$$\sin(\omega t \pm 90°) = \pm \cos \omega t$$
$$\cos(\omega t \pm 90°) = \mp \sin \omega t$$

$$A \cos \omega t + B \sin \omega t = C \cos(\omega t - \theta)$$

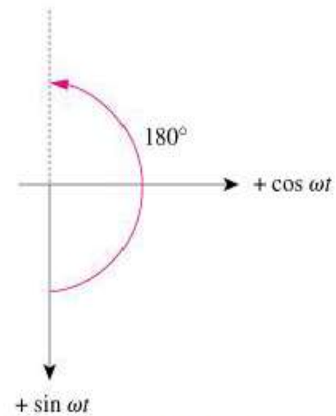
Where

$$C = \sqrt{A^2 + B^2} \quad \text{and} \quad \theta = \tan^{-1} \frac{B}{A}$$

Graphically relating sine and cosine functions.



$$\cos(\omega t - 90°) = \sin \omega t$$



$$\sin(\omega t + 180°) = -\sin \omega t$$

End of Lecture

Questions?