$$Z_3 = \frac{8 \times j4}{10} = 3{,}2j$$

Frequency Domain Analysis (Using Phasor)

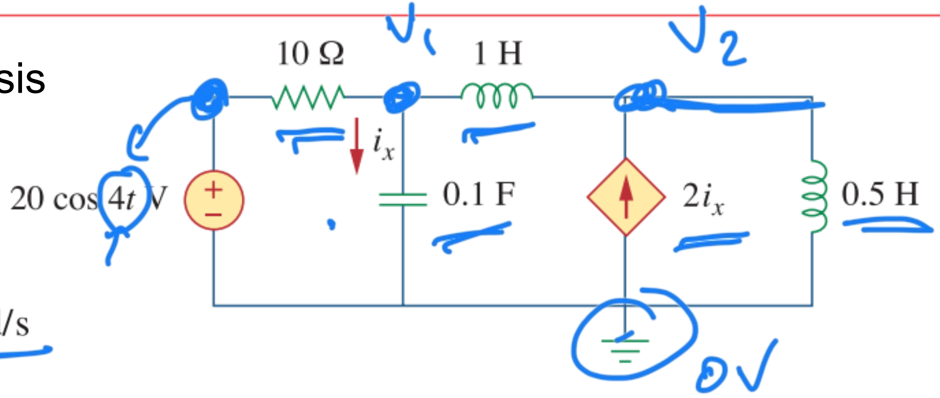
Polar
complex

- Nodal Analysis
- Mesh Analysis
- Superposition,
- Source transformation
- Thevenin theorem, and
- Norton theorem

Example 25- Nodal Analysis

Find i_x in the following circuit using nodal analysis

Solution



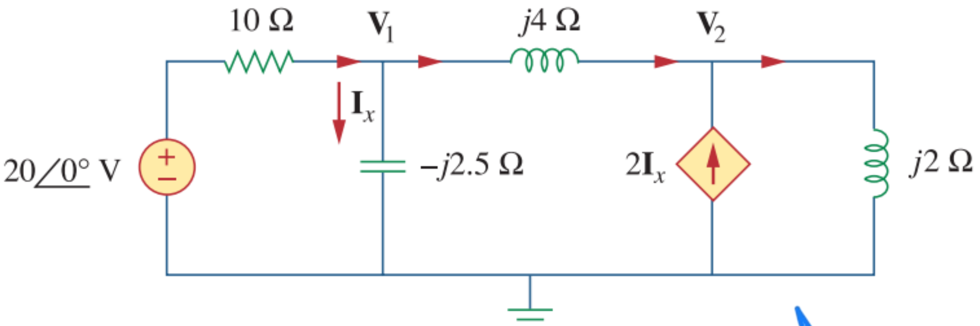
We first convert the circuit to the frequency domain:

$$20\cos 4t \quad \Rightarrow \quad 20\angle 0^\circ, \qquad \omega = 4 \text{ rad/s}$$

$$1\text{ H} \quad \Rightarrow \quad j\omega L = j4$$

$$0.5\text{ H} \quad \Rightarrow \quad j\omega L = j2$$

$$0.1\text{ F} \quad \Rightarrow \quad \frac{1}{j\omega C} = -j2.5$$



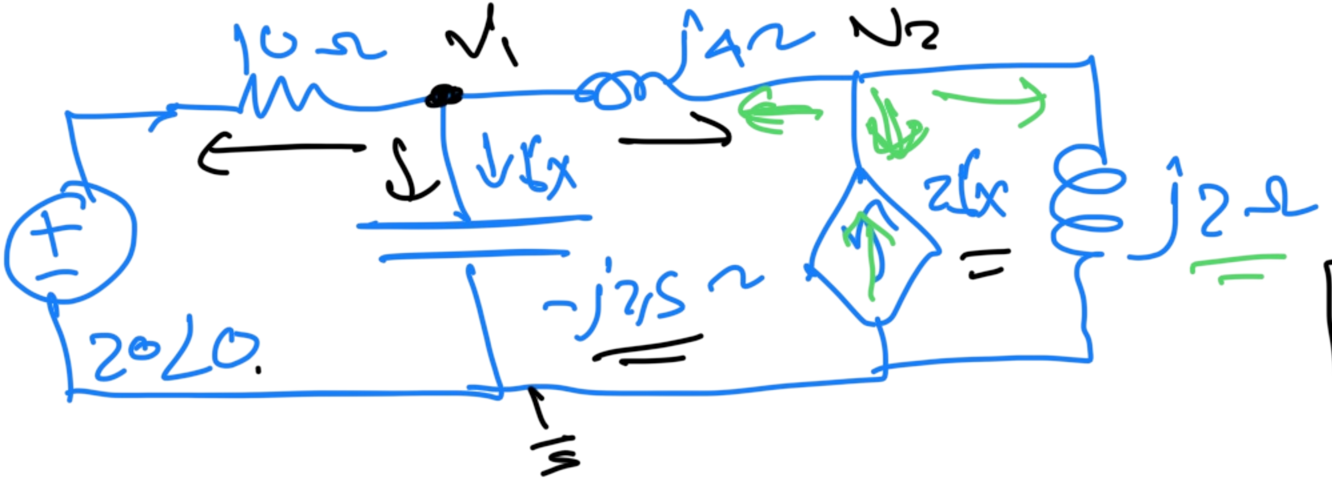
Applying KCL at node 1,

$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

or

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20$$

$Z_L = j\omega L$, $Z_C = \frac{1}{j\omega C}$



$I_x = \dfrac{V_1}{-j2.5}$

$I_x = j0.4 V_1$

node (1)

$$\frac{V_1 - 20}{10} + \frac{V_1 - 0}{-2.5j} + \frac{V_1 - V_2}{j4} = 0$$

$$\Rightarrow \underbrace{(0.1 + j0.15)}_{A} V_1 + \underbrace{(0.25j)}_{B} V_2 = \underbrace{2}_{E} \rightarrow (1)$$

node ②

$$\frac{V_2 - V_1}{j4} + \frac{V_2 - 0}{j2} + \left(-2(0{,}4j\,V_1)\right) = 0$$

$$\left(-\frac{11}{20}j\right)V_1 + \left(-\frac{3}{4}\right)j\,V_2 = 0 \longrightarrow ②$$

C, D, F

$$V_1 = \frac{ED - BF}{AD - BC} = 18 + j6\ \text{V} = 18{,}97\angle 18{,}43\ \text{V}$$

$$V_2 = \frac{AF - EC}{AD - BC} = -13{,}2 - j4{,}4\ \text{V} = 13{,}91\angle -161{,}57\ \text{V}$$

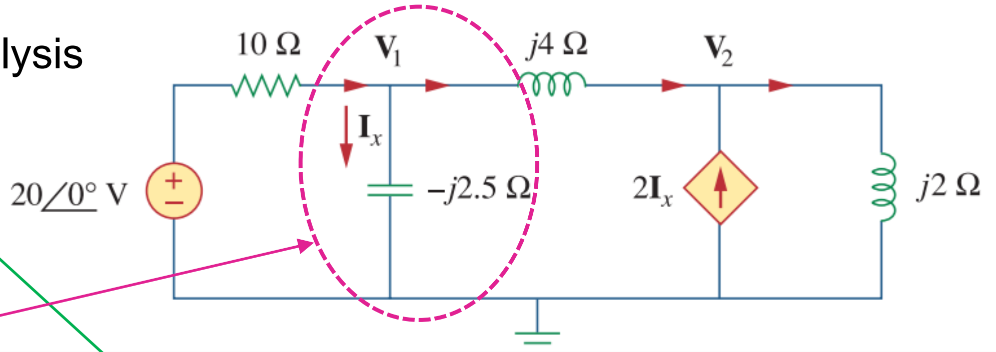
$$I_x = j\,0{,}4 \times (18 + j6) = 7{,}589 \angle 108{,}43$$

$$\boxed{I_x(t) = 7{,}589 \cos(4t + 108{,}43)\ \text{A}}$$

Example 25- Nodal Analysis

Find i_x in the following circuit using nodal analysis

Solution (continue)



$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20$$

At node 2,

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

But $\mathbf{I}_x = \mathbf{V}_1/-j2.5$. Substituting this gives

$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0$$

A B E F C D

$$\begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$\mathbf{V}_1 = 18.97\underline{/18.43^\circ}\ \text{V}$$

$$\mathbf{V}_2 = 13.91\underline{/198.3^\circ}\ \text{V}$$

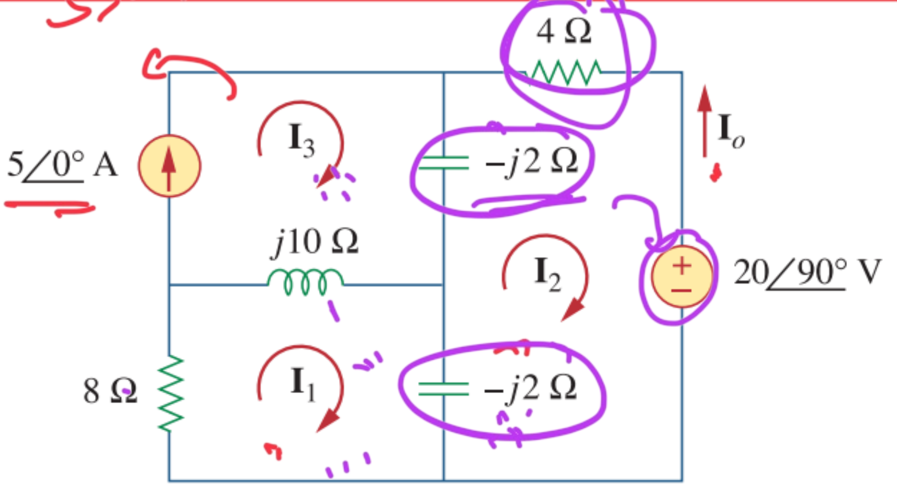
The current $\mathbf{I}_x$ is given by

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\underline{/18.43^\circ}}{2.5\underline{/-90^\circ}} = 7.59\underline{/108.4^\circ}\ \text{A}$$

$$i_x = 7.59\cos(4t + 108.4^\circ)\ \text{A}$$

Example 26- Mesh Analysis

Find I_o in the following circuit using mesh analysis



Solution

Applying KVL to mesh 1, we obtain

- 8I1+j10(I1-I3)+-2j(I1-I2)=0

$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0$$

For mesh 2,

- 20<90+ 4I2-j2(I2-I3) -j2(I2-I1) =0

$$(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\underline{/90°} = 0$$

For mesh 3, $\mathbf{I}_3 = 5$

$$\begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.17\underline{/-35.22°}}{68} = 6.12\underline{/-35.22°} \text{ A}$$

$$\mathbf{I}_o = -\mathbf{I}_2 = 6.12\underline{/144.78°} \text{ A}$$

mesh ①

$$(8+8j)I_1 + (j2)I_2 - j10(5) = 0$$

$$(8+8j)I_1 + (j2)I_2 = 50j \rightarrow ①$$

A, B, E

mesh ②

$j10$, $20j$

$$+(j2)I_1 + (4-4j)I_2 + j2(5) + 20\angle 90 = 0$$

$$\Rightarrow (j2)I_1 + (4-j4)I_2 = -j30 \rightarrow ②$$

C, D, F

$$I_1 = \frac{ED - BF}{AD - BC} = 3{,}59 \angle 55 \text{ A}$$

$$I_2 = \frac{AF - EC}{AD - BC} = 6{,}12 \angle -35{,}22 \text{ A}$$

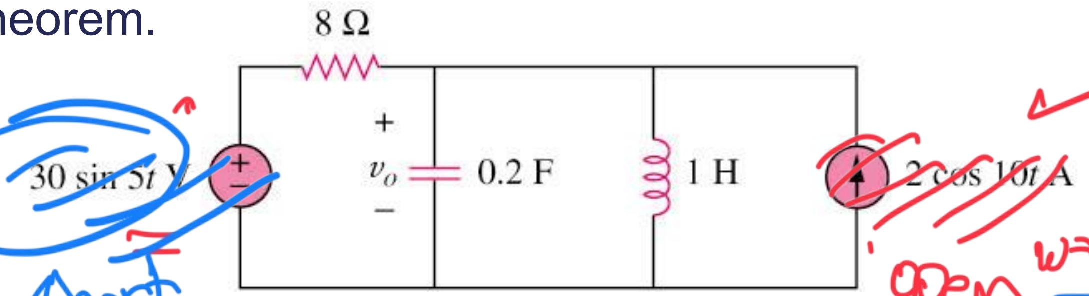
$$\boxed{I_a = -I_2 = 6{,}12 \angle 144{,}78 \text{ A}}$$

Superposition Theorem

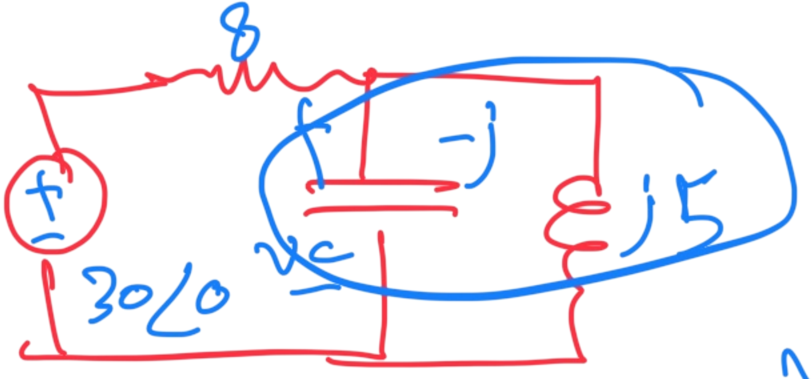
When a circuit has sources operating at different frequencies,

- The separate phasor circuit for each frequency must be solved independently, and
- The total response is the sum of time-domain responses of all the individual phasor circuits.

Example Calculate v_o in the circuit of figure shown below using the superposition theorem.



Answer: V_o = **4.631 sin(5t – 81.12) + 1.051 cos(10t – 86.24) V**

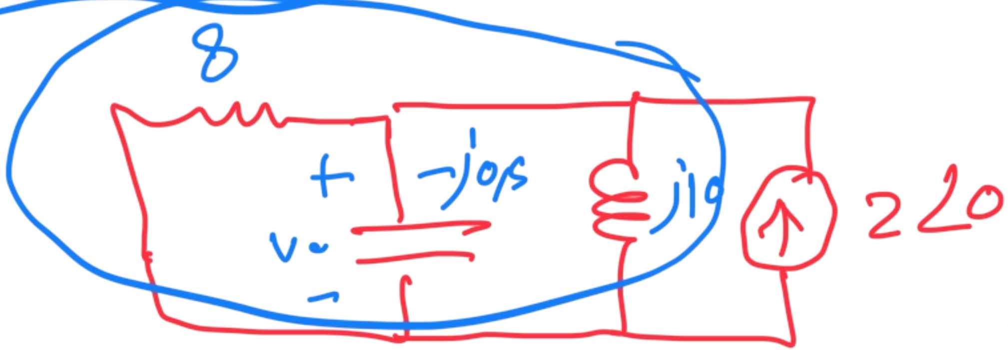


$$= -1{,}25j \ \Omega$$

Voltage divider

$$V_o' = \frac{-1{,}25j}{-1{,}25j + 8} \times 30$$

$$V_o' = 4{,}631 \angle -81{,}12 \ V$$



$$Z_T = \frac{1}{\frac{1}{8} + \frac{1}{-0{,}5j} + \frac{1}{j10}}$$

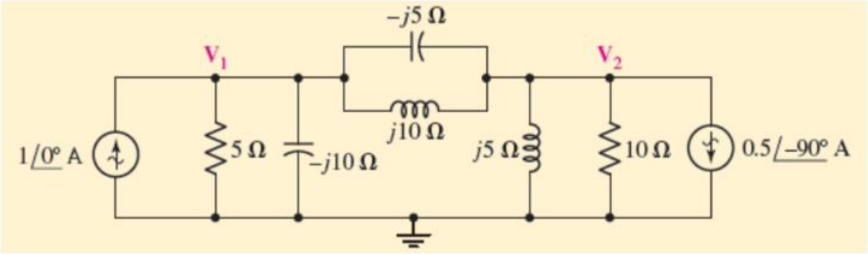
$$Z_T = 0{,}525 \angle -86{,}24 \ \Omega$$

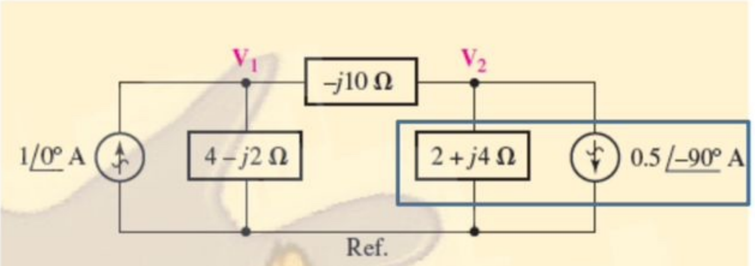
$$V_o'' = 2 * 2\Gamma = 1{,}05 \angle -86{,}24$$

$$\boxed{V_o = 4{,}631 \sin(5t - 81{,}12) + 1{,}05 \cos(10t - 86{,}24)}$$

Example 27: Superposition Theorem

- Calculate $v1$ in the circuit of figure shown below using the superposition theorem.





(b)

Answer:

First we redraw the circuit as Fig. *b* where each pair of parallel impedances is replaced by a single equivalent impedance. That is, $5 \| -j10\ \Omega$ is $4 - j2\ \Omega$; $j10 \| -j5\ \Omega$ is $-j10\ \Omega$; and $10 \| j5$ is equal to $2 + j4\ \Omega$.

To find $\mathbf{V}_1$, we first activate only the left source and find the partial response, $\mathbf{V}_{1L}$. The $1/\underline{0^\circ}$ source is in parallel with an impedance of

$(4 - j2) \| (-j10 + 2 + j4)$ and we have: $V_{1L} = (4\text{-}j2) * i1$

so that

$$\mathbf{V}_{1L} = 1/\underline{0^\circ}\, \frac{(4 - j2)(-j10 + 2 + j4)}{4 - j2 - j10 + 2 + j4} = \frac{-4 - j28}{6 - j8} = 2 - j2 \text{ V}$$

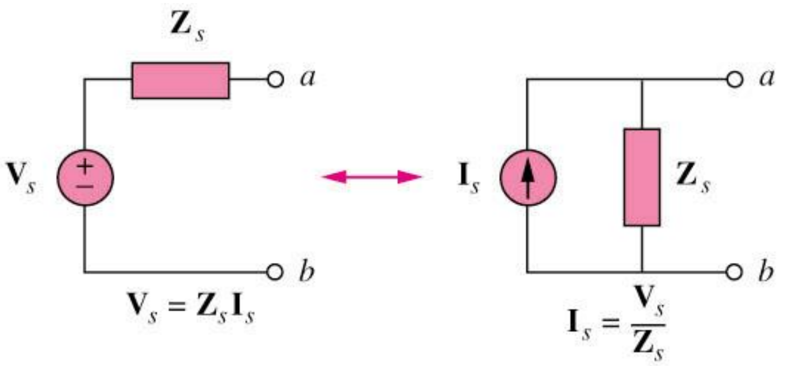
With only the right source active, current division and Ohm's law yield : $V_{1R} = (4\text{-}j2) * i1'$

$$\mathbf{V}_{1R} = (-0.5/\underline{-90^\circ}) \left(\frac{2 + j4}{4 - j2 - j10 + 2 + j4} \right) (4 - j2) = -1 \text{ V}$$

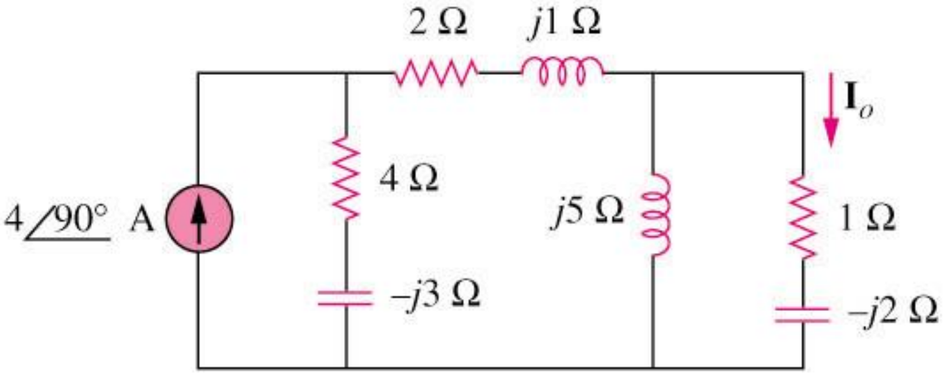
Summing, then,

$$\mathbf{V}_1 = \mathbf{V}_{1L} + \mathbf{V}_{1R} = 2 - j2 - 1 = 1 - j2 \quad \text{V}$$

Source Transformation



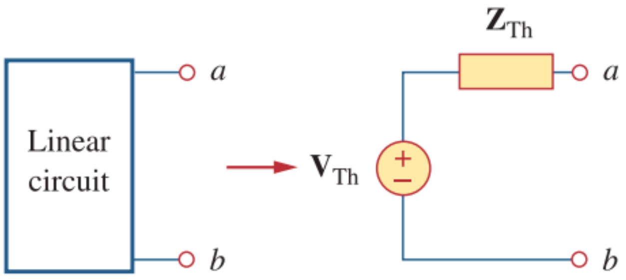
Example Find I_o in the circuit of figure below using the concept of source transformation.



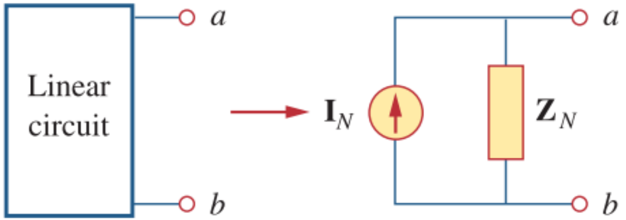
Answer: : I_o = <u>3.288+j99.46 A</u>

Thevenin and Norton Theorems in AC circuits

- Thevenin's and Norton theorems are applied to ac circuits in the same way as they are to DC circuits
- The ONLY additional effort is the need to manipulate complex numbers.



Thevenin equivalent.



Norton equivalent.

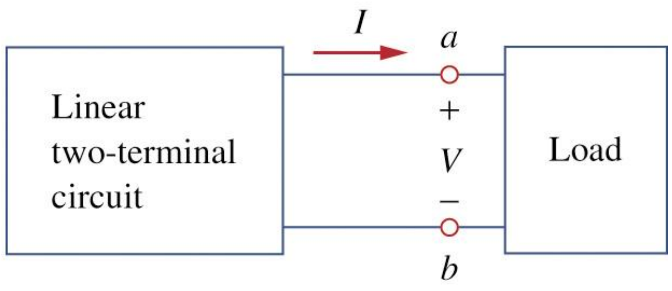
Where; $V_{Th} = V_{ab\text{-}OC}$ is the open circuit voltage (PHASOR) between terminals a-b,
$I_N = I_{ab\text{-}SC}$ is the short circuit current (PHASOR) through the terminals a-b, and
Z_{Th} is the input or equivalent IMPEDANCE at the terminals when the independent source are turn off.

How to Find Thevenin Equivalent Circuit in AC circuits

- First, **Open the circuit** (remove the load) at the **points** of interest **a-b**

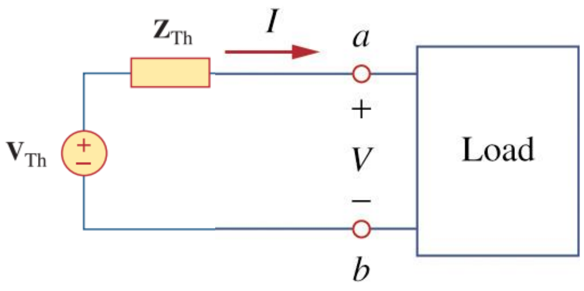
1- V_{th}= Open circuit voltage (keep all sources intact). **Note: Vth is Phasor**

2- Z_{th}= Open circuit **equivalent Impedance** appears at terminals **a-b** while all **independent sources=0** (voltage source=SC , current source=OC).



Original Circuit

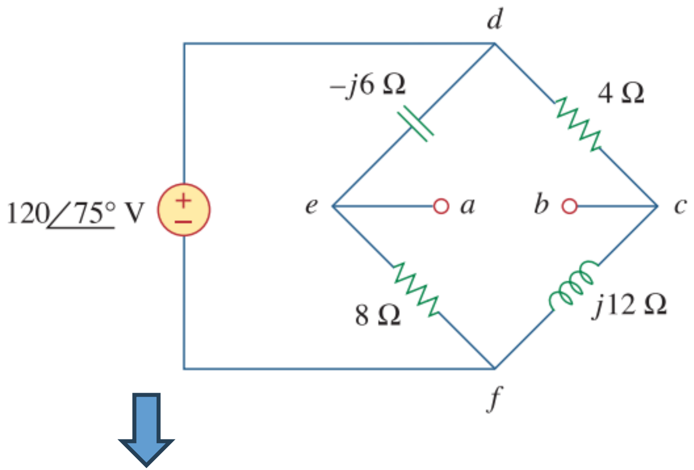
≡



Thevenin equivalent circuit

Example 10.8- Thevenin Equivalent Circuit

Obtain the Thevenin equivalent at terminals a-b of the following circuit.



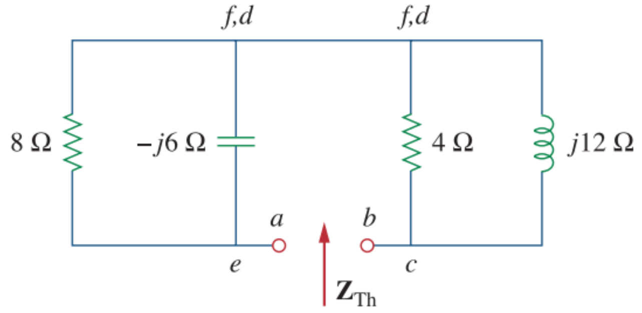
Solution

1- For Zth

$$\mathbf{Z}_1 = -j6 \parallel 8 = \frac{-j6 \times 8}{8 - j6} = 2.88 - j3.84\ \Omega$$

$$\mathbf{Z}_2 = 4 \parallel j12 = \frac{j12 \times 4}{4 + j12} = 3.6 + j1.2\ \Omega$$

$$\mathbf{Z}_{Th} = \mathbf{Z}_1 + \mathbf{Z}_2 = 6.48 - j2.64\ \Omega$$



Example 10.8- Thevenin Equivalent Circuit

Obtain the Thevenin equivalent at terminals a-b of the following circuit.

Solution (continue)

2- For Vth

$$\mathbf{I}_1 = \frac{120\angle 75^\circ}{8 - j6}\text{ A}, \qquad \mathbf{I}_2 = \frac{120\angle 75^\circ}{4 + j12}\text{ A}$$

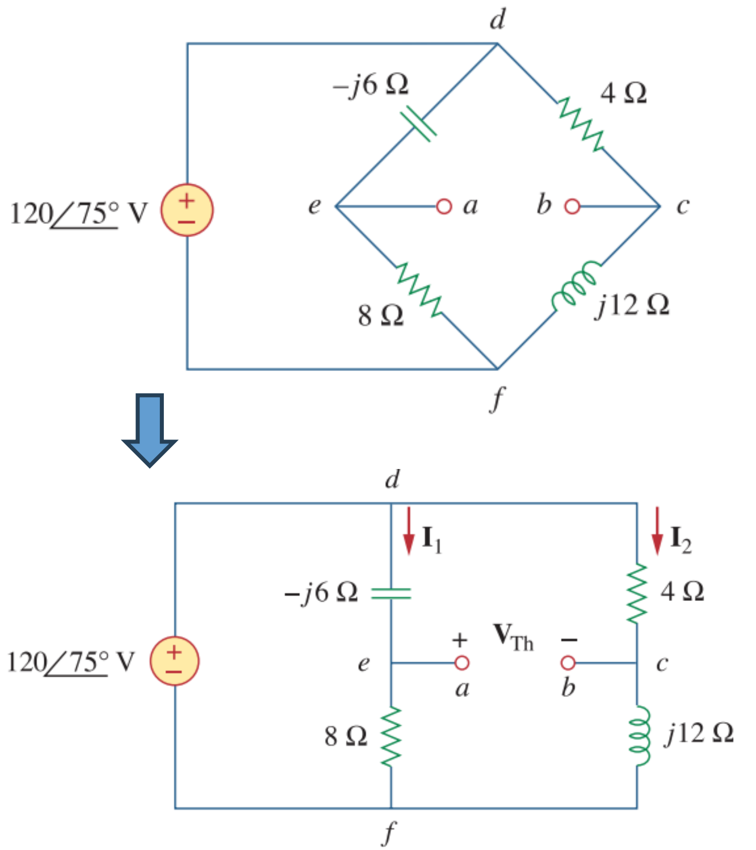
Applying KVL around loop *bcdeab*

$$\mathbf{V}_{\text{Th}} - 4\mathbf{I}_2 + (-j6)\mathbf{I}_1 = 0$$

$$\mathbf{V}_{\text{Th}} = 4\mathbf{I}_2 + j6\mathbf{I}_1 = \frac{480\angle 75^\circ}{4 + j12} + \frac{720\angle 75^\circ + 90^\circ}{8 - j6}$$

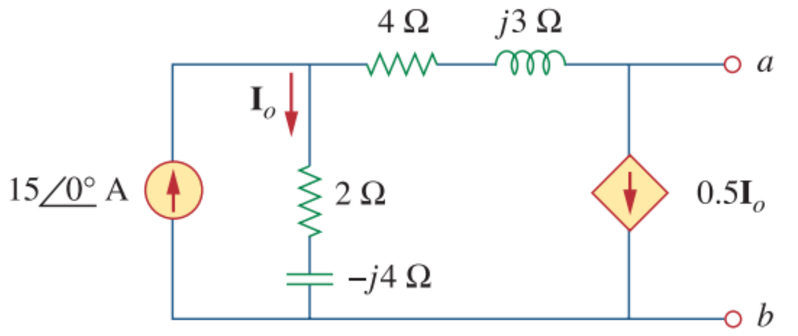
$$= 37.95\angle 3.43^\circ + 72\angle 201.87^\circ$$

$$= -28.936 - j24.55 = 37.95\angle 220.31^\circ \text{ V}$$



Example 10.9- Thevenin Equivalent Circuit

Obtain the Thevenin equivalent at terminals a-b of the following circuit.



Solution

1- For Vth

To find $\mathbf{V}_{Th}$, we apply KCL at node 1

$$15 = \mathbf{I}_o + 0.5\mathbf{I}_o \qquad \Rightarrow \qquad \mathbf{I}_o = 10 \text{ A}$$

Applying KVL to the loop on the right-hand side

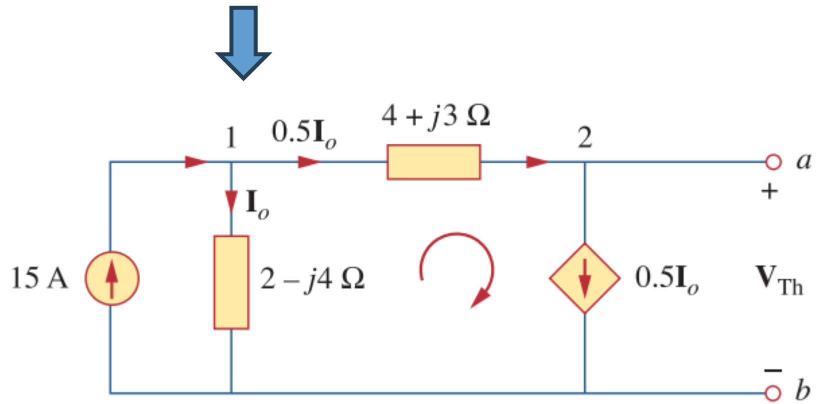
$$-\mathbf{I}_o(2 - j4) + 0.5\mathbf{I}_o(4 + j3) + \mathbf{V}_{Th} = 0$$

or

$$\mathbf{V}_{Th} = 10(2 - j4) - 5(4 + j3) = -j55$$

Thus, the Thevenin voltage is

$$\mathbf{V}_{Th} = 55\angle{-90^\circ} \text{ V}$$

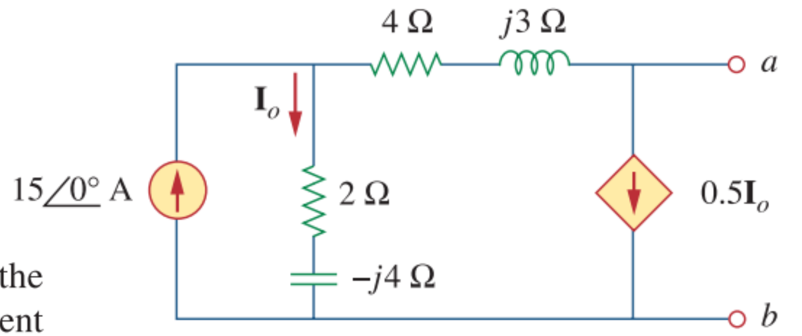


Example 10.9- Thevenin Equivalent Circuit

Obtain the Thevenin equivalent at terminals a-b of the following circuit.

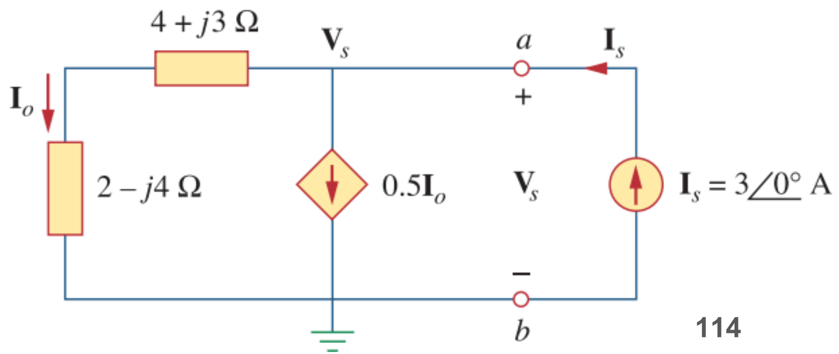
Solution (continue)

2- For Zth



To obtain $\mathbf{Z}_{Th}$, we remove the independent source. Due to the presence of the dependent current source, we connect a 3-A current source (3 is an arbitrary value chosen for convenience here, a number divisible by the sum of currents leaving the node) to terminals *a-b*

$$3 = \mathbf{I}_o + 0.5\mathbf{I}_o \qquad \Rightarrow \qquad \mathbf{I}_o = 2 \text{ A}$$



Applying KVL to the outer loop in Fig. 10.26(b) gives

$$\mathbf{V}_s = \mathbf{I}_o(4 + j3 + 2 - j4) = 2(6 - j)$$

The Thevenin impedance is

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_s}{\mathbf{I}_s} = \frac{2(6 - j)}{3} = 4 - j0.6667\ \Omega$$

Trigonometric Identities

- Sine and cosine form conversions.

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

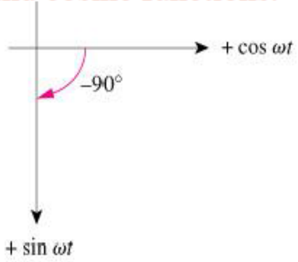
$$\sin(\omega t \pm 180°) = -\sin \omega t$$
$$\cos(\omega t \pm 180°) = -\cos \omega t$$
$$\sin(\omega t \pm 90°) = \pm \cos \omega t$$
$$\cos(\omega t \pm 90°) = \mp \sin \omega t$$

$$A \cos \omega t + B \sin \omega t = C \cos(\omega t - \theta)$$

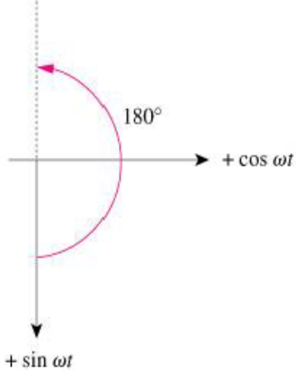
Where

$$C = \sqrt{A^2 + B^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{B}{A}$$

Graphically relating sine and cosine functions.



$$\cos(\omega t - 90°) = \sin \omega t$$



$$\sin(\omega t + 180°) = -\sin \omega t$$

End of Lecture

Questions?