C-TOTAL IMPEDANCE FOR AC CIRCUITS

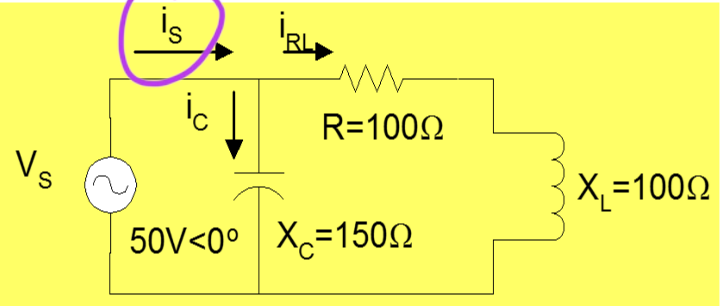
2-Example of Parallel circuit:

a- Find the total Impedance

b-Draw Impedance Triangle (Phasor Diagram)

c- Find i_s, v_R, v_C, v_L

d- Find v_R, v_C, v_L using Voltage Divider



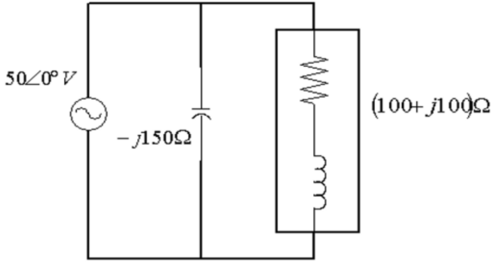
✓ **Solution**

➢ a- The total Impedance

➢ The circuit given is already in frequency domain:

$$Z_R = 100\Omega\angle 0^o \quad = 100\,\Omega$$
$$Z_C = 150\Omega\angle -90^o = -j150\,\Omega$$
$$Z_L = 100\Omega\angle 90^o \quad = j100\,\Omega$$

➢ **Circuit simplification**



$$Z_T = \frac{1}{\frac{1}{(100+100j)} + \frac{1}{(-150j)}} = 180 - j60\ \Omega$$

$R = 180$

Z_C

Z_L ✓

Z_R

~~X_L~~

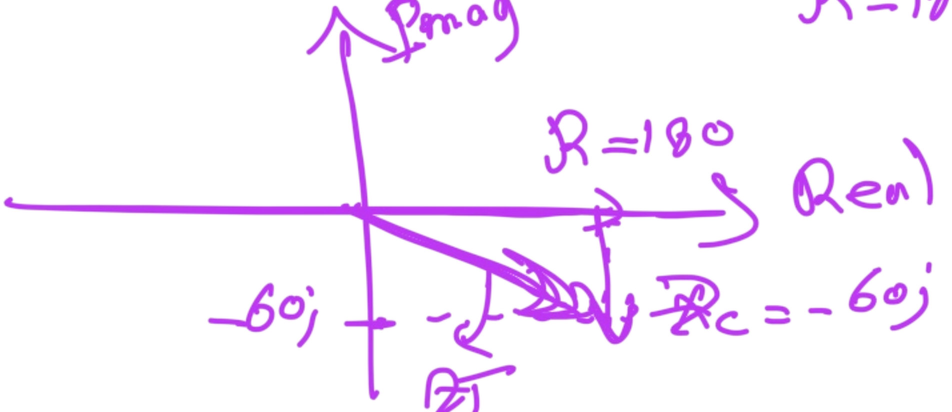
~~X_c~~

$Z_L = jX_L$

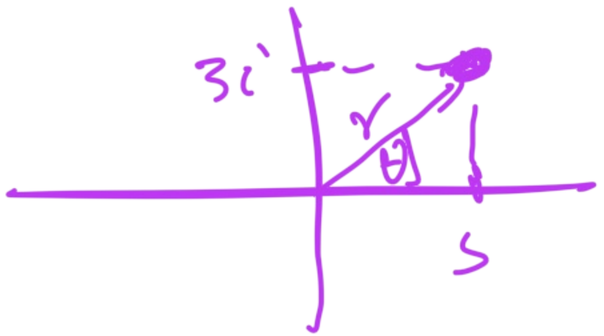
$Z_C = -jX_C$

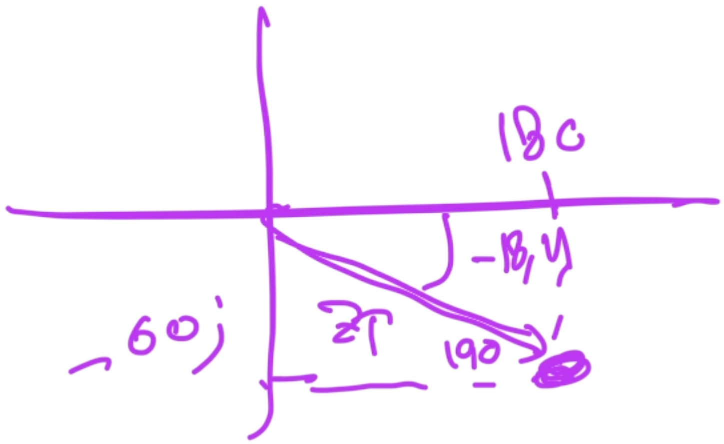
$$X_c = 60 = \frac{1}{\omega C}$$

$$C = \frac{1}{60\omega}$$



$z = 5+3i$





$z_T = 180 - j60$

$189,74 \angle -18,4$

$$I_S = \frac{V_S}{Z_T} = \frac{50\angle 0}{180 - j60} = 0{,}264\angle 18{,}43 \text{ A}$$

$$\boxed{V_C = V_S = 50\angle 0}$$

$$V_2 = \frac{j100}{100 + j100} \times 50\angle 0$$

$$= \boxed{25\sqrt{2}\angle 45 \text{ V}}$$

$$V_R = \frac{100}{100 + j100} \times 50$$

$$= \boxed{25\sqrt{2}\angle -45 \text{ V}}$$

C-TOTAL IMPEDANCE FOR AC CIRCUITS

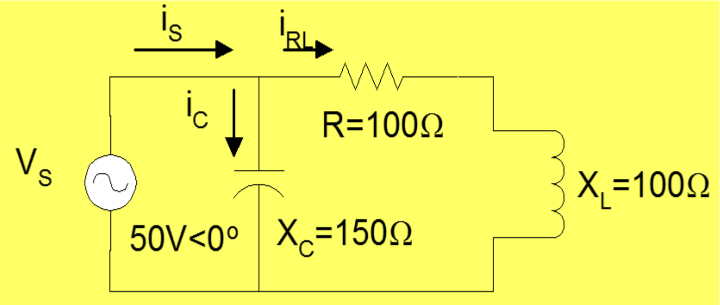
2-Example of Parallel circuit:

a- Find the total Impedance

b-Draw Impedance Triangle (Phasor Diagram)

c- Find i_s, v_R, v_C, v_L

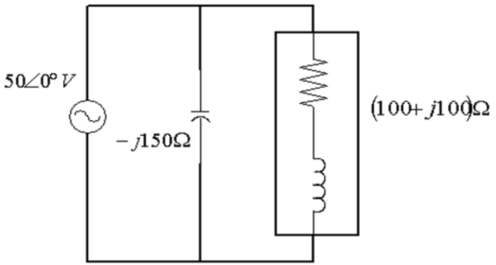
d- Find v_R, v_C, v_L using Current Divider



✓ **Solution**

➢ a- The total Impedance

➢ The circuit given is already in frequency domain:



$$Z_{total} = (Z_C) \| (Z_R + Z_L)$$
$$= (150\angle -90°) \| [(100) + (100\angle 90°)]$$
$$= (150\angle -90°) \| (100 + j100)$$
$$= (150\angle -90°) \| (141.42\angle 45°)$$
$$\therefore Z_{total} = [Y_{total}]^{-1}$$
$$= \left[\frac{1}{150\angle -90°} + \frac{1}{141.42\angle 45°}\right]^{-1}$$

$$\therefore Z_{total} = \left(\frac{1}{150}(\cos(90°) + j\sin(90°)) + \frac{1}{144.42}(\cos(-45°) + j\sin(-45°))\right)^{-1}$$

$$= (0.005 \quad +j0.0016)^{-1}$$

$$= \left(\left(\sqrt{0.005^2 + 0.0016^2}\right) < \tan^{-1}\left(\frac{0.0016}{0.005}\right)\right)^{-1}$$

$$= (0.00524 \quad < 17.744°)^{-1} \simeq 190{<}{-18°}$$

$$\therefore Z_{total} \simeq 180 - j60\,\Omega$$

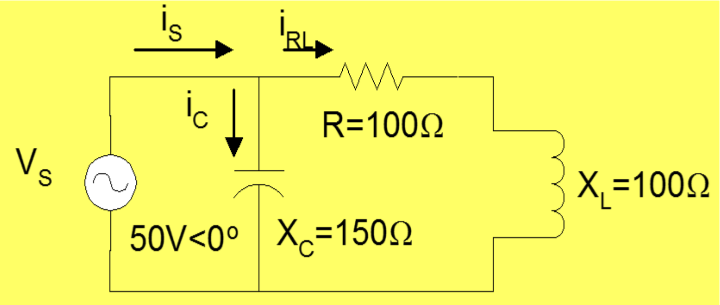
2-Example of Parallel circuit:

a- Find the total Impedance

b-Draw Impedance Triangle (Phasor Diagram)

c- Find i_s, v_R, v_C, v_L

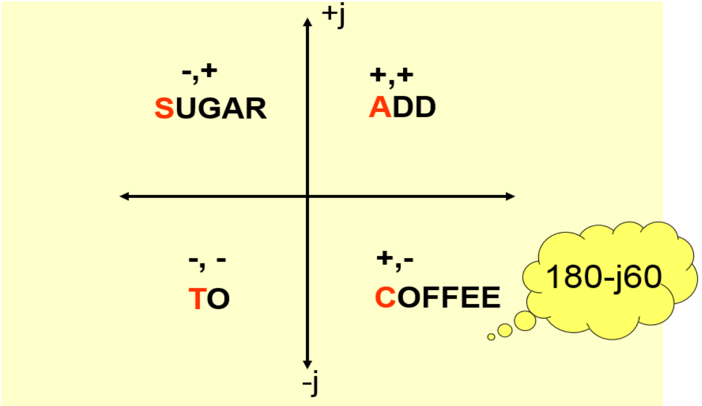
d- Find v_R, v_C, v_L using Current Divider

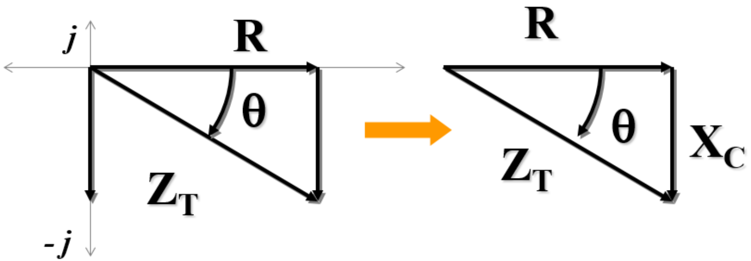


✓ **Solution**

➢ b- The Impedance Triangle (Phasor Diagram)

$$\therefore Z_{total} \simeq 180 - j60\,\Omega$$





C-TOTAL IMPEDANCE FOR AC CIRCUITS

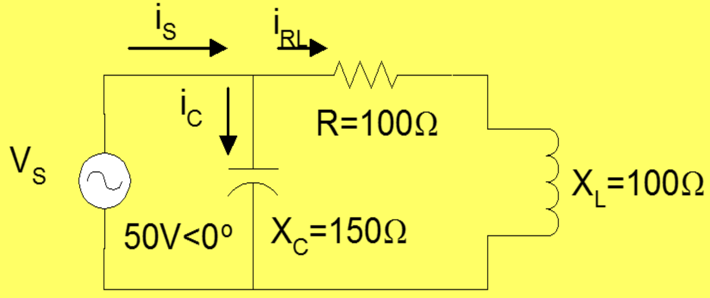
2-Example of Parallel circuit:

a- Find the total Impedance

b-Draw Impedance Triangle (Phasor Diagram)

c- Find i_s, v_R, v_C, v_L

d- Find v_R, v_C, v_L using Current Divider



✓ **Solution**

➢ c- i_s, v_R, v_C, v_L :

$$\therefore Z_{total} \simeq 180 - j60\,\Omega$$

$$i_s = \frac{v_s}{Z_T} = \frac{50\angle 0^\circ}{190\angle -18^\circ} = 263\angle 18^\circ\ mA$$

$$v_S = v_C = (v_R + v_L) = 50\angle 0^\circ\ V$$

$$\Rightarrow i_C = \frac{v_S}{Z_C} = \frac{50\angle 0^\circ}{150\angle -90^\circ} = 333\angle 90^\circ\ mA$$

$$\Rightarrow i_{RL} = \frac{v_S}{Z_R + Z_L} = \frac{50\angle 0^\circ}{100 + (100\angle -90^\circ)} = 353\angle -45^\circ\ mA$$

C-TOTAL IMPEDANCE FOR AC CIRCUITS

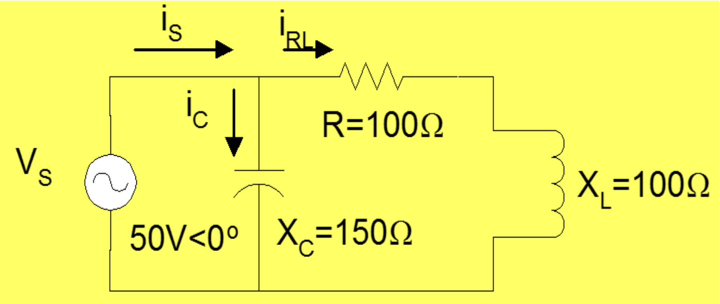
2-Example of Parallel circuit:

a- Find the total Impedance

b-Draw Impedance Triangle (Phasor Diagram)

c- Find i_s, v_R, v_C, v_L

d- Find v_R, v_C, v_L using Current Divider



✓ **Solution** ➢ d- Find v_R, v_C, v_L using Current Divider

$$i_c = \frac{Z_T}{Z_C} i_S = \frac{(190\angle -18°)}{(150\angle -90°)} \cdot (263\angle 18°\ mA)$$
$$= 333\angle 90°\ mA$$

$$i_{RL} = \frac{Z_T}{Z_R + Z_L} i_S = \frac{(190\angle -18°)}{(100)(100\angle 90°)} \cdot (263\angle 18°\ mA)$$
$$= 353\angle -45°\ mA$$

D-Impedance as a Function of Frequency

Element	Impedance
L	$Z = j\omega L$
C	$Z = \frac{1}{j\omega C}$

- The Impedance Z of a circuit is a function of the frequency.

- Inductor is SHORT CIRCUIT at DC and OPEN CIRCUIT at high frequencies. Capacitor is OPEN CIRCUIT at DC and SHORT CIRCUIT at high frequencies.

$$Z_L = j\omega L$$

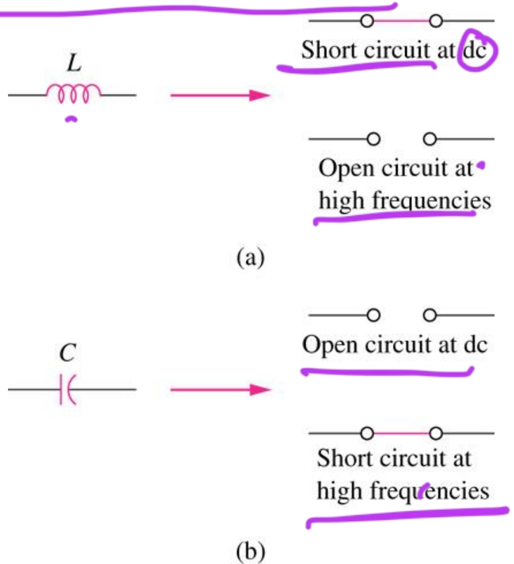
$$Z_L \rightarrow 0 \qquad \omega \rightarrow 0 \quad \text{(Short at DC)}$$

$$Z_L \rightarrow \infty \qquad \omega \rightarrow \infty \quad (\text{Open as } \omega \rightarrow \infty)$$

$$Z_C = \frac{1}{j\omega C}$$

$$Z_C \rightarrow \infty \qquad \omega \rightarrow 0 \quad \text{(Open at DC)}$$

$$Z_C \rightarrow 0 \qquad \omega \rightarrow \infty \quad (\text{Open as } \omega \rightarrow \infty)$$



AC Analysis Techniques

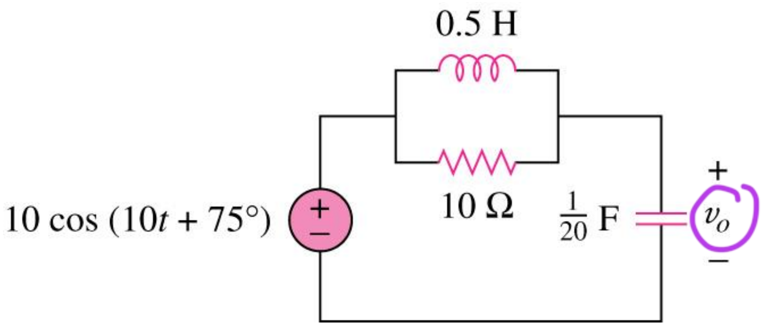
AC Analysis Techniques

- Both KVL and KCL are hold in Phasor domain or more commonly called frequency domain.
- Moreover, the variables to be handled are phasors, which are complex numbers.
- All the mathematical operations involved are now in complex domain.

- **All DC circuit analysis principles apply to AC circuits.**
    - Voltage Division
    - Current Division
    - Circuit Reduction
    - Impedance equivalence
    - Y-Delta Transformation

Example 22: Voltage Divider Rule

- Calculate the v0 in the given circuit
- Solution:



In the frequency domain,

the voltage source is $\mathbf{V}_s = 10\angle 75°$

the 0.5-H inductor is $j\omega L = j(10)(0.5) = j5$

the $\frac{1}{20}$-F capacitor is $\frac{1}{j\omega C} = \frac{1}{j(10)(1/20)} = -j2$

Let $\mathbf{Z}_1$ = impedance of the 0.5-H inductor in parallel with the 10-Ω resistor

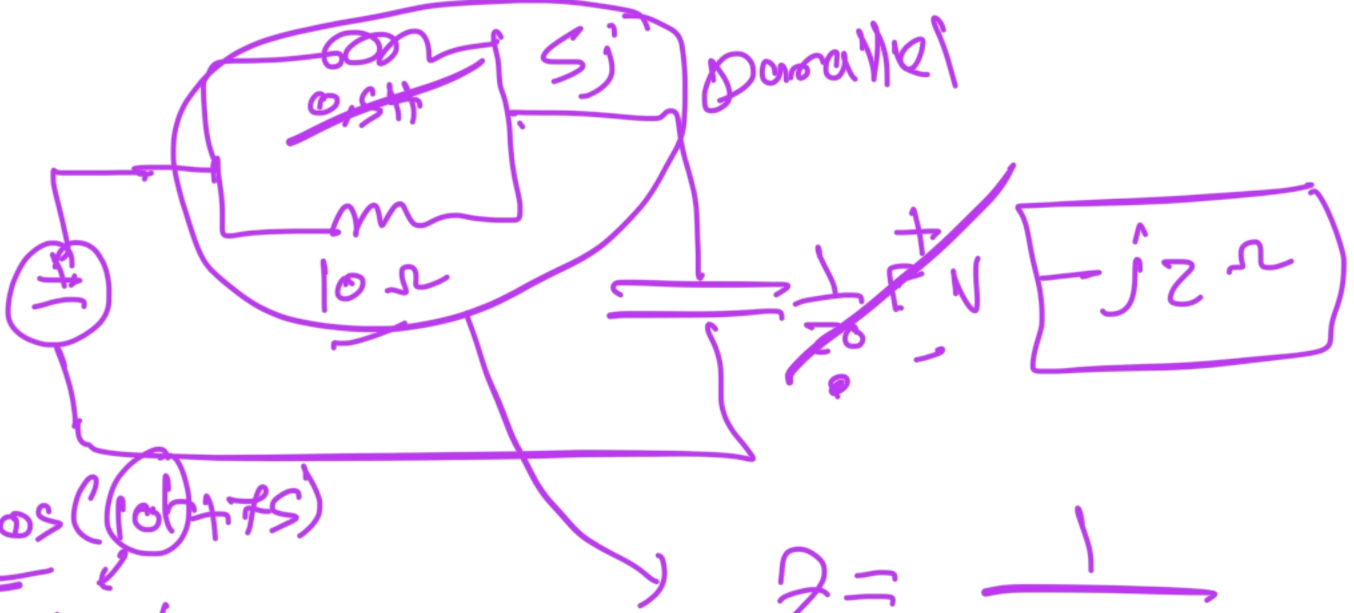
and $\mathbf{Z}_2$ = impedance of the (1/20)-F capacitor

$$\mathbf{Z}_1 = 10 \,||\, j5 = \frac{(10)(j5)}{10 + j5} = 2 + j4 \qquad \text{and} \qquad \mathbf{Z}_2 = -j2$$

$$\mathbf{V}_o = \mathbf{Z}_2 / (\mathbf{Z}_1 + \mathbf{Z}_2)\,\mathbf{V}_s$$

$$\mathbf{V}_o = \frac{-j2}{2 + j4 - j2}(10\angle 75°) = \frac{-j(10\angle 75°)}{1 + j} = \frac{10\angle(75° - 90°)}{\sqrt{2}\angle 45°} = 7.071\angle\text{-}60°$$

$v_o(t) = \underline{7.071\cos(10t - 60°)\ \text{V}}$



$10\cos(10t + 75)$

$w = 10$

$10\angle 75$

$Z_R = R$

$Z_L = jwL$

$Z_C = \frac{1}{jwC}$

$Z = \frac{1}{\frac{1}{10} + \frac{1}{j5}}$

$= 2 + j4$

Voltage divider

7.07

$$V_o = \frac{-j2}{-j2+2+j4} \times 10\angle 75 = 5\sqrt{2}\angle -60 \text{ V}$$

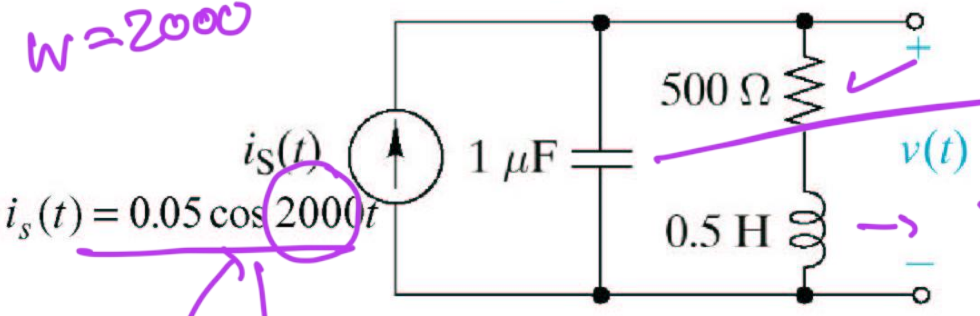
$$v_o(t) = 5\sqrt{2}\cos(10t - 60) \text{ V}$$

Steady state

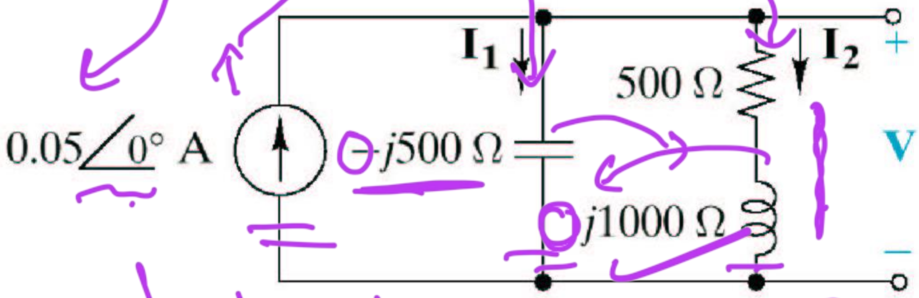
Time domain

Example 23: Current Divider Rule

Find I_1 and I_2 in phasor domain, then convert them into time domain



Solution:



$$I_1 = \frac{500 + j1000}{500 + j1000 - j500} \times 0{,}05 = 0{,}079 \angle 18{,}43 \text{ A}$$

$$I_2 = \frac{-j500}{-j500 + 500 + j1000} \times 0{,}05 = 0{,}035 \angle -135 \text{ A}$$

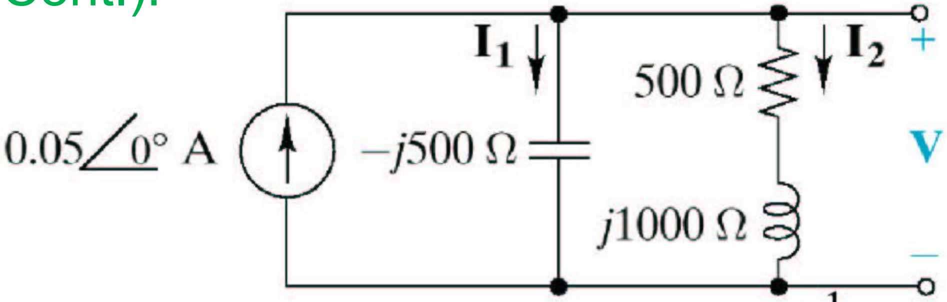
$$i_1 = 79 \cos(2000t + 18{,}43) \text{ mA}$$

$$i_2 = 35 \cos(2000t - 135) \text{ mA}$$

steady state

Example 23: Current Divider Rule

Solution (Cont.):



$$\hat{I}_1 = \frac{(R + j\omega L)}{R + j\omega L + \frac{1}{j\omega C}} \hat{I}$$

$$\hat{I}_2 = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} \hat{I}$$

$$\hat{I}_1 = \frac{(500 + j1000)}{500 + j1000 - j500} 0.05\angle 0^\circ$$

$$\hat{I}_2 = \frac{-j500}{500 + j1000 - j500} 0.05\angle 0^\circ$$

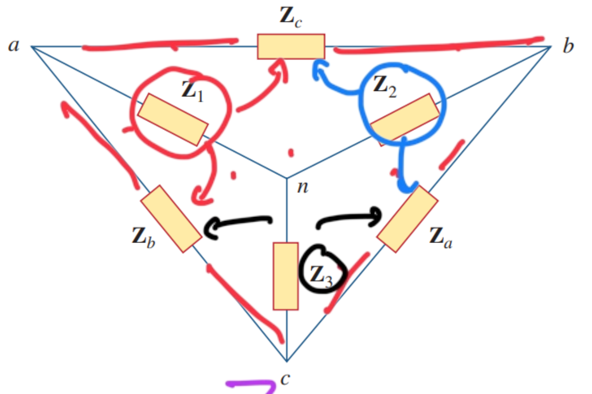
$I_1 = 0.079\angle 18.4^o$ A

$I_2 = 0.03535\angle - 135^o$ A

$i_1(t) = 79\cos(2000t + 18.4^o)\, mA$

$i_2(t) = 35.35\cos(2000t - 135^o)\, mA$

Delta-Wye ($\Delta - Y$) Transformation



Y-Δ Conversion:

$$\mathbf{Z}_a = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_1}$$

$$\mathbf{Z}_b = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_2}$$

$$\mathbf{Z}_c = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3}$$

Δ-Y Conversion:

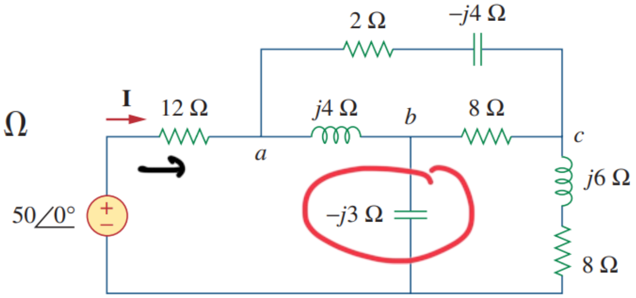
$$\mathbf{Z}_1 = \frac{\mathbf{Z}_b\mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_c\mathbf{Z}_a}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_a\mathbf{Z}_b}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

Example 24- $(\Delta - Y)$ Transformation

Find current **I** in the circuit



Solution:

$$\mathbf{Z}_{an} = \frac{j4(2 - j4)}{j4 + 2 - j4 + 8} = \frac{4(4 + j2)}{10} = (1.6 + j0.8)\ \Omega$$

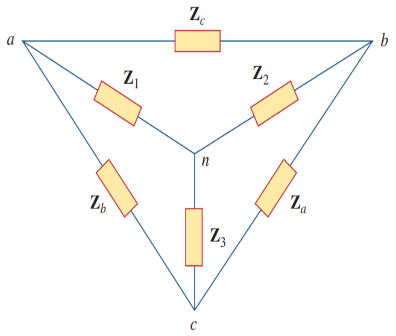
$$\mathbf{Z}_{bn} = \frac{j4(8)}{10} = j3.2\ \Omega, \qquad \mathbf{Z}_{cn} = \frac{8(2 - j4)}{10} = (1.6 - j3.2)\ \Omega$$

The total impedance at the source terminals is

$$\begin{aligned} \mathbf{Z} &= 12 + \mathbf{Z}_{an} + (\mathbf{Z}_{bn} - j3) \parallel (\mathbf{Z}_{cn} + j6 + 8) \\ &= 12 + 1.6 + j0.8 + (j0.2) \parallel (9.6 + j2.8) \\ &= 13.6 + j0.8 + \frac{j0.2(9.6 + j2.8)}{9.6 + j3} \\ &= 13.6 + j1 = 13.64\underline{/4.204^\circ}\ \Omega \end{aligned}$$

The desired current is

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{50\underline{/0^\circ}}{13.64\underline{/4.204^\circ}} = 3.666\underline{/-4.204^\circ}\ \text{A}$$

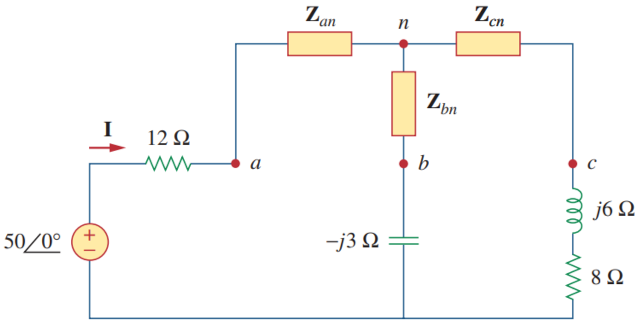


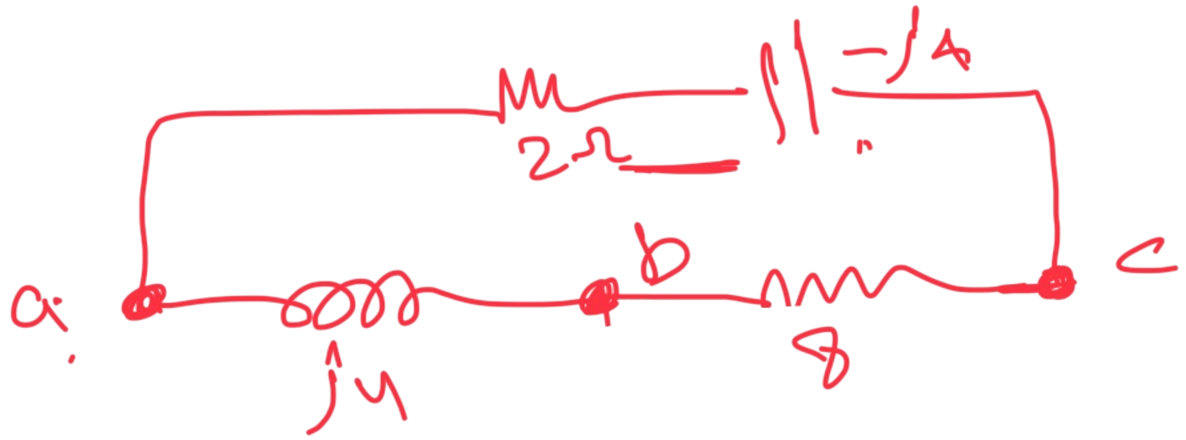
Δ-Y Conversion:

$$\mathbf{Z}_1 = \frac{\mathbf{Z}_b \mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_c \mathbf{Z}_a}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_a \mathbf{Z}_b}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

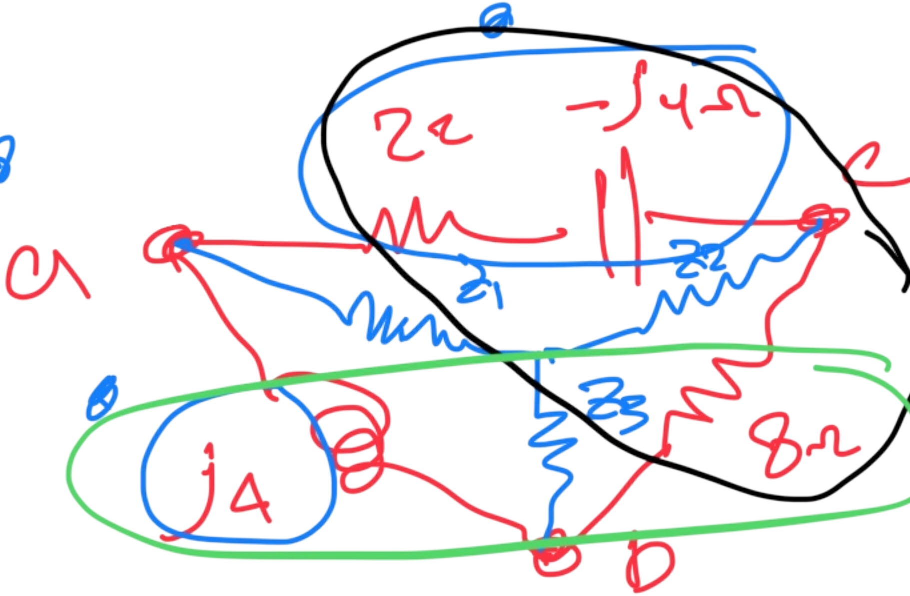


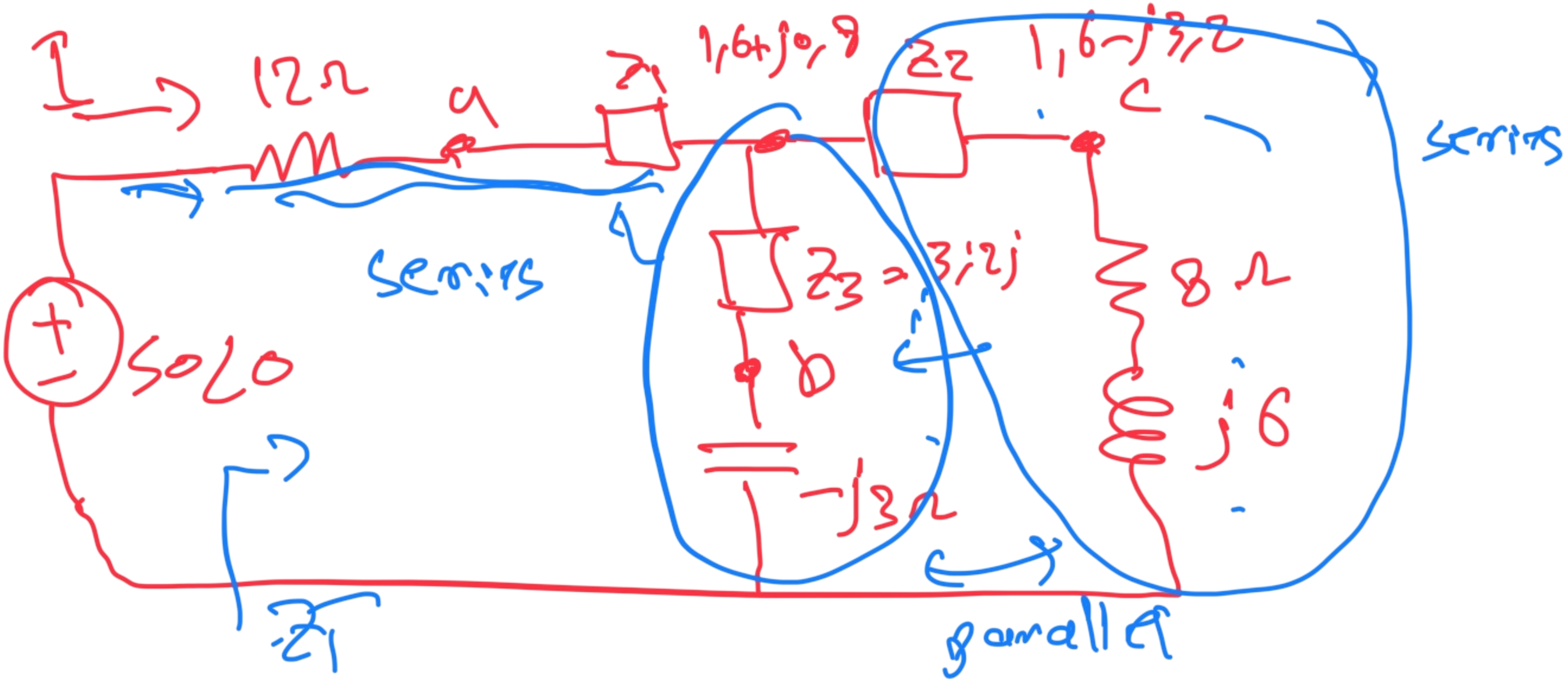


Δ → Y

$$Z_1 = \frac{(j4)(2-j4)}{\cancel{j4}+2-\cancel{j4}+8} = 1{,}6 + j0{,}8$$

$$Z_2 = \frac{8(2-j4)}{10} = 1{,}6 - j3{,}2$$





$$Z_T = 12 + 1{,}6 + j0.8 + \frac{1}{\frac{1}{(3{,}2j - j3)} + \frac{1}{8 + j6 + 1{,}6 - j3{,}2}}$$

$$Z_T = 13{,}64 \angle 4{,}2 \ \Omega$$

$$I = \frac{V}{Z_T} = \frac{50 \angle 0}{13{,}64 \angle 4{,}2} = 3{,}666 \angle -4{,}2 \ A$$

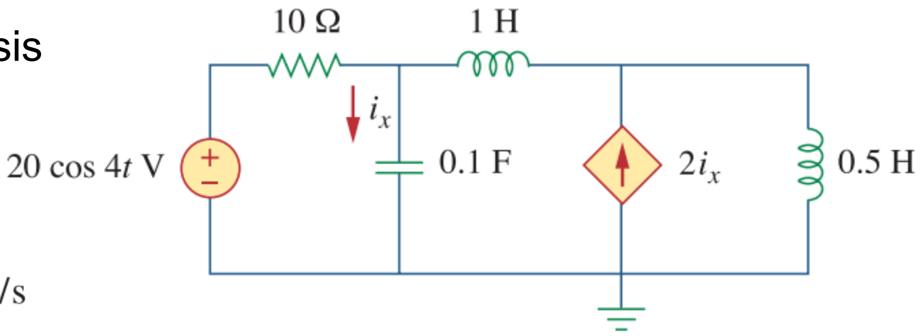
$$Z_3 = \frac{8 \times j4}{10} = 3{,}2j$$

Frequency Domain Analysis (Using Phasor)

Nodal Analysis
Mesh Analysis
Superposition,
Source transformation
Thevenin theorem, and
Norton theorem

Example 25- Nodal Analysis

Find i_x in the following circuit using nodal analysis



Solution

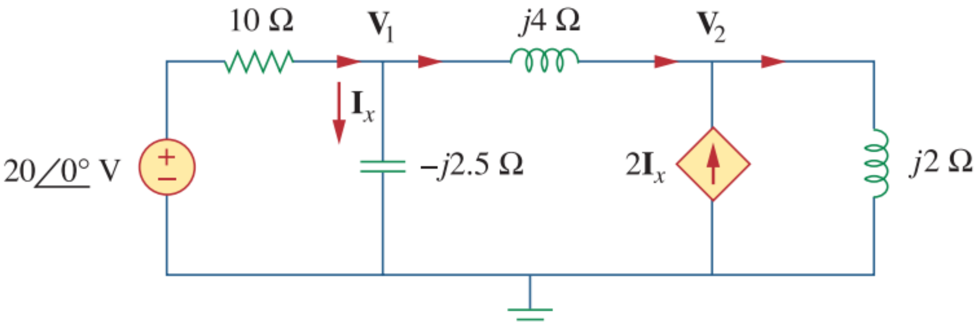
We first convert the circuit to the frequency domain:

$$20\cos 4t \quad \Rightarrow \quad 20\angle 0^\circ, \qquad \omega = 4 \text{ rad/s}$$

$$1 \text{ H} \quad \Rightarrow \quad j\omega L = j4$$

$$0.5 \text{ H} \quad \Rightarrow \quad j\omega L = j2$$

$$0.1 \text{ F} \quad \Rightarrow \quad \frac{1}{j\omega C} = -j2.5$$



Applying KCL at node 1,

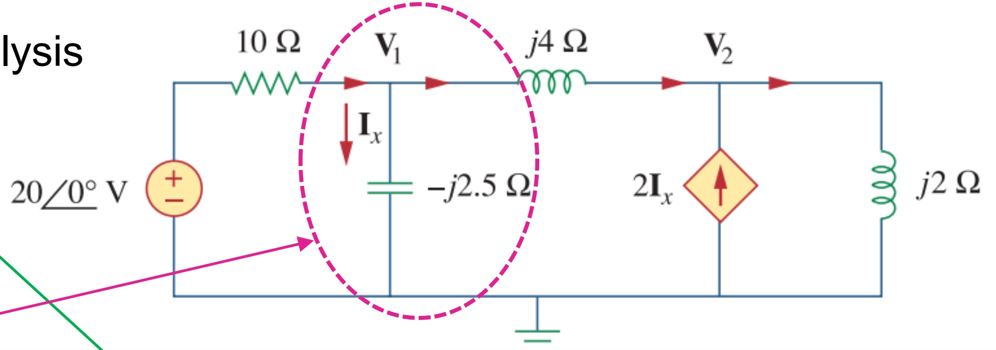
$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

or

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20$$

Example 25- Nodal Analysis

Find i_x in the following circuit using nodal analysis



Solution (continue)

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20$$

At node 2,

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

But $\mathbf{I}_x = \mathbf{V}_1/-j2.5$. Substituting this gives

$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0$$

$$\begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$\mathbf{V}_1 = 18.97\underline{/18.43^\circ} \text{ V}$$

$$\mathbf{V}_2 = 13.91\underline{/198.3^\circ} \text{ V}$$

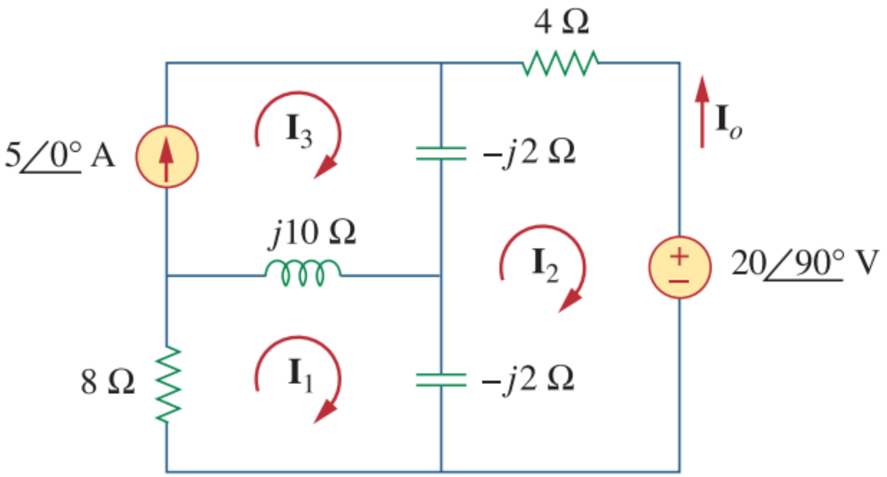
The current $\mathbf{I}_x$ is given by

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\underline{/18.43^\circ}}{2.5\underline{/-90^\circ}} = 7.59\underline{/108.4^\circ} \text{ A}$$

$$i_x = 7.59\cos(4t + 108.4^\circ) \text{ A}$$

Example 26- Mesh Analysis

Find I_o in the following circuit using mesh analysis



Solution

Applying KVL to mesh 1, we obtain

- 8I1+j10(I1-I3)+-2j(I1-I2)=0

$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0$$

For mesh 2,

- 20<90+ 4I2-j2(I2-I3) -j2(I2-I1) =0

$$(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\angle 90^\circ = 0$$

For mesh 3, $\mathbf{I}_3 = 5$

$$\begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.17\angle -35.22^\circ}{68} = 6.12\angle -35.22^\circ \text{ A}$$

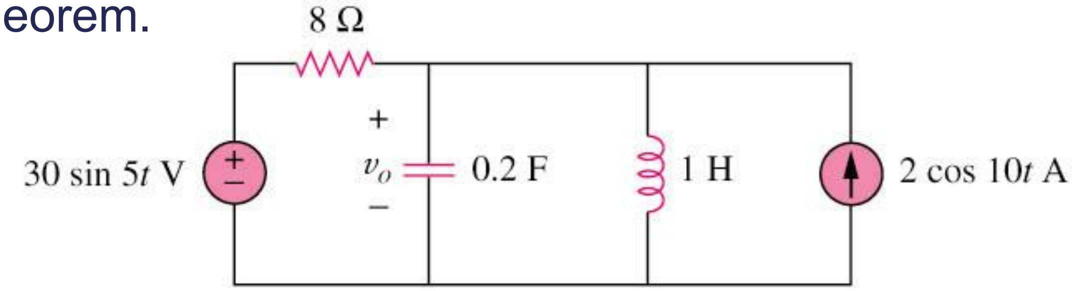
$$\boxed{\mathbf{I}_o = -\mathbf{I}_2 = 6.12\angle 144.78^\circ \text{ A}}$$

Superposition Theorem

When a circuit has sources operating at different frequencies,

- The separate phasor circuit for each frequency must be solved independently, and
- The total response is the sum of time-domain responses of all the individual phasor circuits.

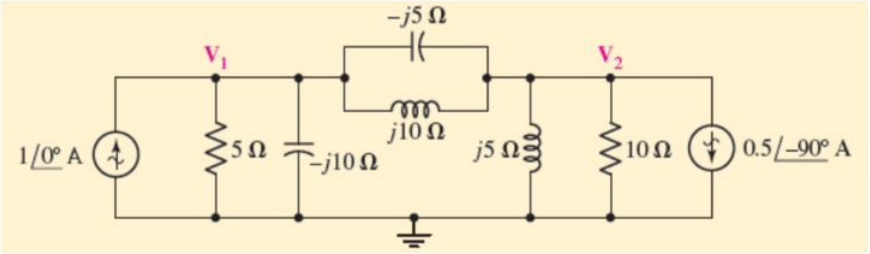
Example Calculate v_o in the circuit of figure shown below using the superposition theorem.

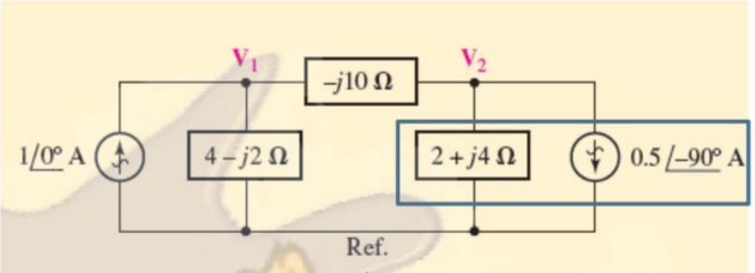


Answer: V_o = **4.631 sin(5t – 81.12) + 1.051 cos(10t – 86.24) V**

Example 27: Superposition Theorem

- Calculate $v1$ in the circuit of figure shown below using the superposition theorem.





(b)

Answer:

First we redraw the circuit as Fig. *b* where each pair of parallel impedances is replaced by a single equivalent impedance. That is, $5 \| -j10\ \Omega$ is $4 - j2\ \Omega$; $j10 \| -j5\ \Omega$ is $-j10\ \Omega$; and $10 \| j5$ is equal to $2 + j4\ \Omega$.

To find $\mathbf{V}_1$, we first activate only the left source and find the partial response, $\mathbf{V}_{1L}$. The $1\angle 0^\circ$ source is in parallel with an impedance of

$(4 - j2) \| (-j10 + 2 + j4)$ and we have: $V_{1L} = (4-j2)*i1$

so that

$$\mathbf{V}_{1L} = 1\angle 0^\circ \frac{(4 - j2)(-j10 + 2 + j4)}{4 - j2 - j10 + 2 + j4}$$

$$= \frac{-4 - j28}{6 - j8} = 2 - j2 \text{ V}$$

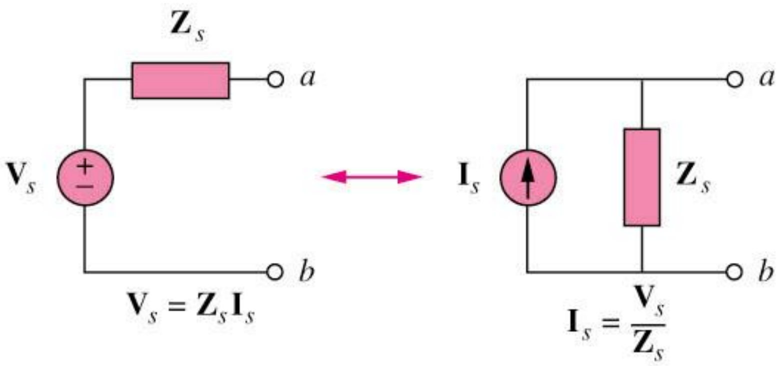
With only the right source active, current division and Ohm's law yield : $V_{1R} = (4-j2)*i1'$

$$\mathbf{V}_{1R} = (-0.5\angle -90^\circ)\left(\frac{2 + j4}{4 - j2 - j10 + 2 + j4}\right)(4 - j2) = -1 \text{ V}$$

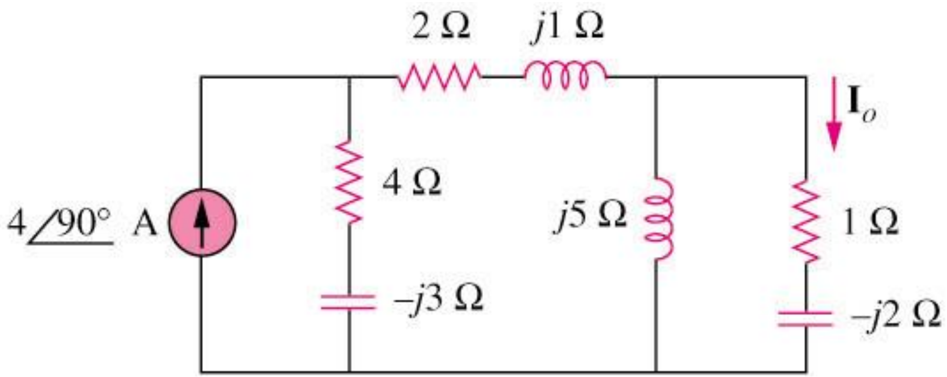
Summing, then,

$$\mathbf{V}_1 = \mathbf{V}_{1L} + \mathbf{V}_{1R} = 2 - j2 - 1 = 1 - j2 \quad \text{V}$$

Source Transformation



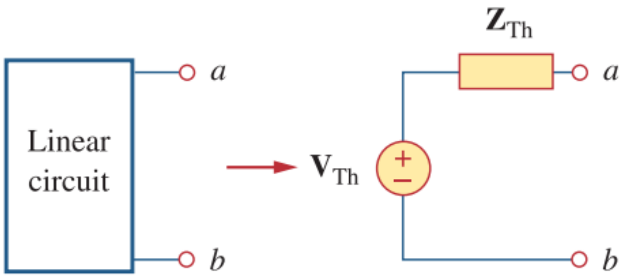
Example Find I_o in the circuit of figure below using the concept of source transformation.



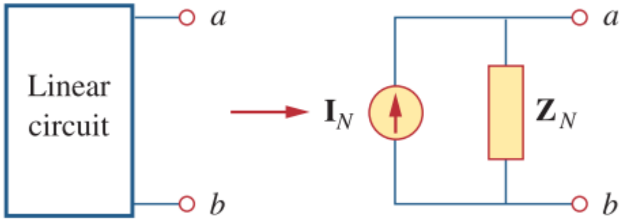
Answer: : I_o = **3.288+j99.46 A**

Thevenin and Norton Theorems in AC circuits

- Thevenin's and Norton theorems are applied to ac circuits in the same way as they are to DC circuits
- The ONLY additional effort is the need to manipulate complex numbers.



Thevenin equivalent.



Norton equivalent.

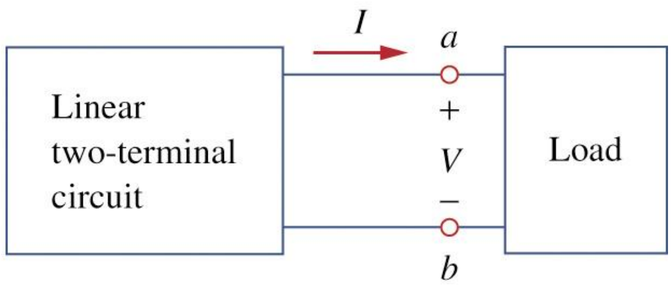
Where; $V_{Th} = V_{ab\text{-}OC}$ is the open circuit voltage (PHASOR) between terminals a-b,
$I_N = I_{ab\text{-}SC}$ is the short circuit current (PHASOR) through the terminals a-b, and
Z_{Th} is the input or equivalent IMPEDANCE at the terminals when the independent source are turn off.

How to Find Thevenin Equivalent Circuit in AC circuits

- First, **Open the circuit** (remove the load) at the **points** of interest **a-b**

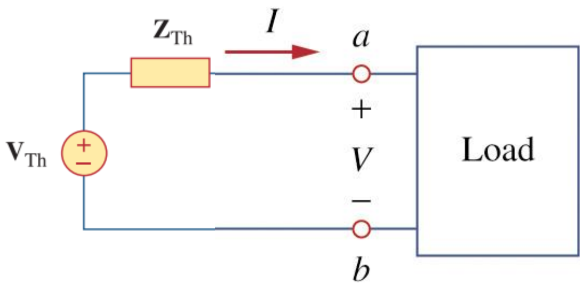
1- V_{th}= Open circuit voltage (keep all sources intact). **Note: Vth is Phasor**

2- Z_{th}= Open circuit **equivalent Impedance** appears at terminals **a-b** while all **independent sources=0** (voltage source=SC , current source=OC).



Original Circuit

≡



Thevenin equivalent circuit

Example 10.8- Thevenin Equivalent Circuit

Obtain the Thevenin equivalent at terminals a-b of the following circuit.

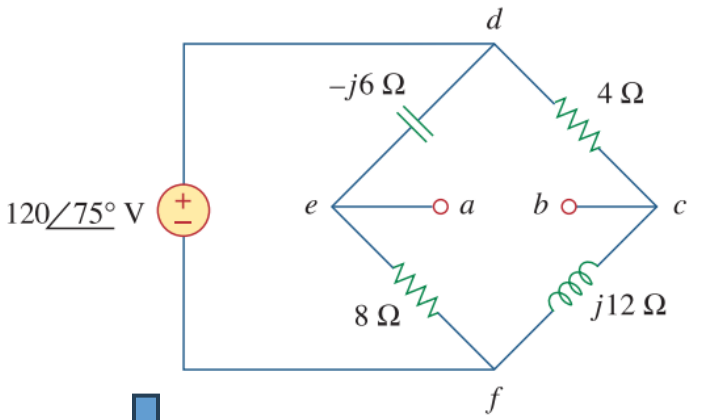
Solution

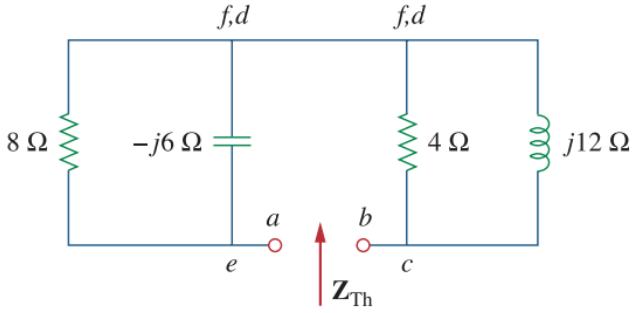
1- For Zth

$$\mathbf{Z}_1 = -j6 \parallel 8 = \frac{-j6 \times 8}{8 - j6} = 2.88 - j3.84\ \Omega$$

$$\mathbf{Z}_2 = 4 \parallel j12 = \frac{j12 \times 4}{4 + j12} = 3.6 + j1.2\ \Omega$$

$$\boxed{\mathbf{Z}_{\text{Th}} = \mathbf{Z}_1 + \mathbf{Z}_2 = 6.48 - j2.64\ \Omega}$$





Example 10.8- Thevenin Equivalent Circuit

Obtain the Thevenin equivalent at terminals a-b of the following circuit.

Solution (continue)

2- For Vth

$$\mathbf{I}_1 = \frac{120\angle 75^\circ}{8 - j6}\text{ A}, \qquad \mathbf{I}_2 = \frac{120\angle 75^\circ}{4 + j12}\text{ A}$$

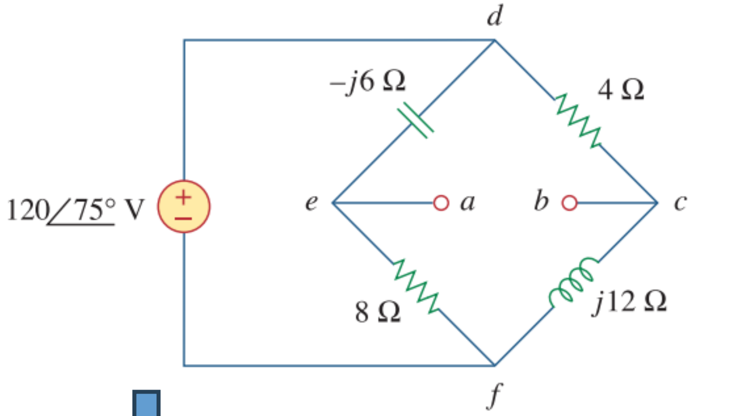
Applying KVL around loop *bcdeab*

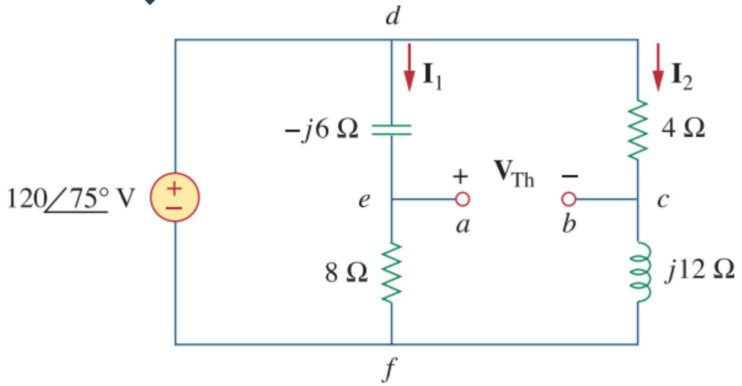
$$\mathbf{V}_{Th} - 4\mathbf{I}_2 + (-j6)\mathbf{I}_1 = 0$$

$$\mathbf{V}_{Th} = 4\mathbf{I}_2 + j6\mathbf{I}_1 = \frac{480\angle 75^\circ}{4 + j12} + \frac{720\angle 75^\circ + 90^\circ}{8 - j6}$$

$$= 37.95\angle 3.43^\circ + 72\angle 201.87^\circ$$

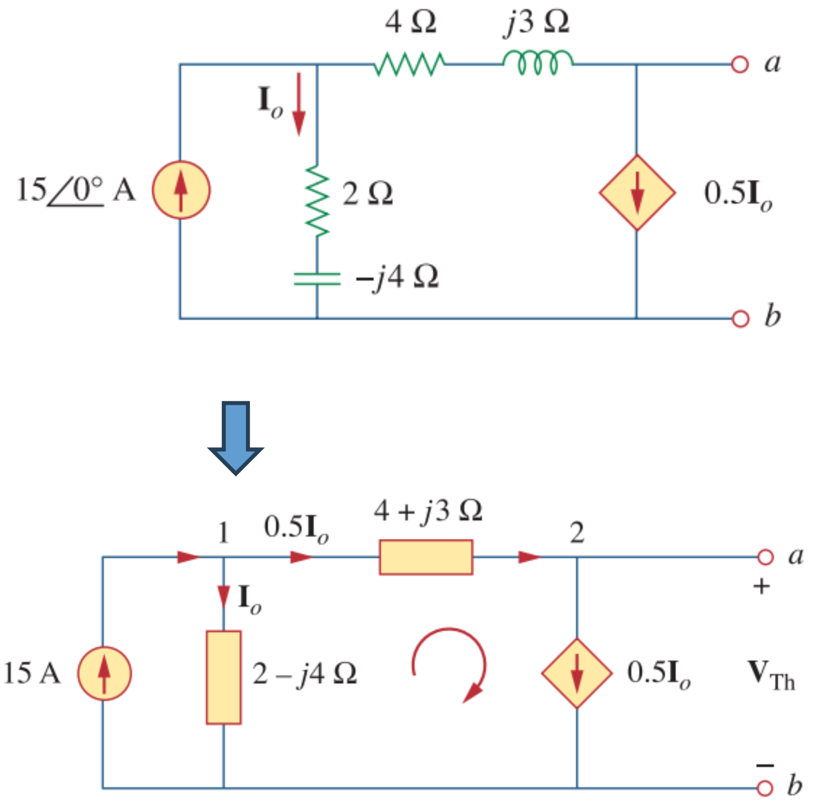
$$= -28.936 - j24.55 = 37.95\angle 220.31^\circ\text{ V}$$





Example 10.9- Thevenin Equivalent Circuit

Obtain the Thevenin equivalent at terminals a-b of the following circuit.



Solution

1- For Vth

To find $\mathbf{V}_{Th}$, we apply KCL at node 1

$$15 = \mathbf{I}_o + 0.5\mathbf{I}_o \quad \Rightarrow \quad \mathbf{I}_o = 10 \text{ A}$$

Applying KVL to the loop on the right-hand side

$$-\mathbf{I}_o(2 - j4) + 0.5\mathbf{I}_o(4 + j3) + \mathbf{V}_{Th} = 0$$

or

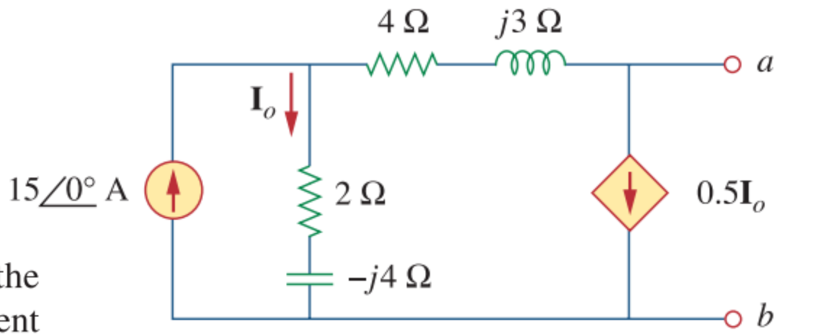
$$\mathbf{V}_{Th} = 10(2 - j4) - 5(4 + j3) = -j55$$

Thus, the Thevenin voltage is

$$\mathbf{V}_{Th} = 55\angle{-90^\circ} \text{ V}$$

Example 10.9- Thevenin Equivalent Circuit

Obtain the Thevenin equivalent at terminals a-b of the following circuit.



Solution (continue)

2- For Zth

To obtain $\mathbf{Z}_{\text{Th}}$, we remove the independent source. Due to the presence of the dependent current source, we connect a 3-A current source (3 is an arbitrary value chosen for convenience here, a number divisible by the sum of currents leaving the node) to terminals *a-b*

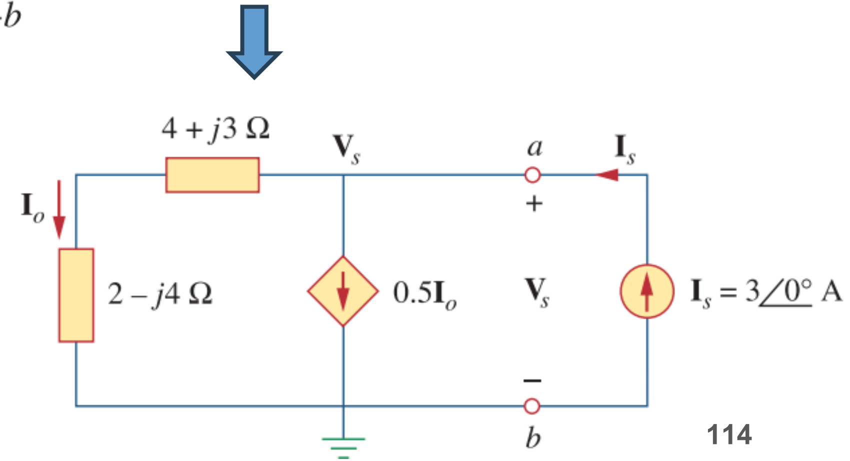
$$3 = \mathbf{I}_o + 0.5\mathbf{I}_o \qquad \Rightarrow \qquad \mathbf{I}_o = 2 \text{ A}$$

Applying KVL to the outer loop in Fig. 10.26(b) gives

$$\mathbf{V}_s = \mathbf{I}_o(4 + j3 + 2 - j4) = 2(6 - j)$$

The Thevenin impedance is

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_s}{\mathbf{I}_s} = \frac{2(6 - j)}{3} = 4 - j0.6667\ \Omega$$



Trigonometric Identities

- Sine and cosine form conversions.

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

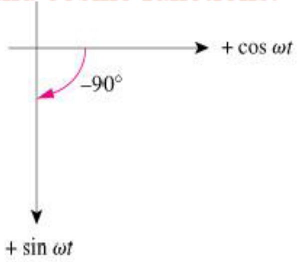
$$\sin(\omega t \pm 180°) = -\sin \omega t$$
$$\cos(\omega t \pm 180°) = -\cos \omega t$$
$$\sin(\omega t \pm 90°) = \pm \cos \omega t$$
$$\cos(\omega t \pm 90°) = \mp \sin \omega t$$

$$A \cos \omega t + B \sin \omega t = C \cos(\omega t - \theta)$$

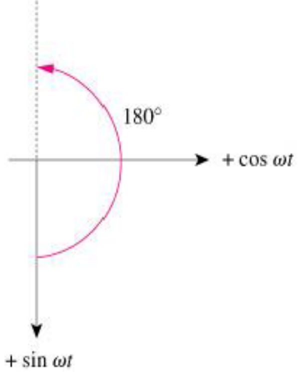
Where

$C=\sqrt{A^2 + B^2}$ and $\theta=\tan^{-1}\frac{B}{A}$

Graphically relating sine and cosine functions.



$$\cos(\omega t - 90°) = \sin \omega t$$



$$\sin(\omega t + 180°) = -\sin \omega t$$

End of Lecture

Questions?