ELECTRICAL ENGINEERING DEPARTMENT

EE242: Electric Circuits II

Lecture 4-6

AC (Sinusoidal) Analysis

Dr.Nawel AFSI

afsi.nawel@gmail.com

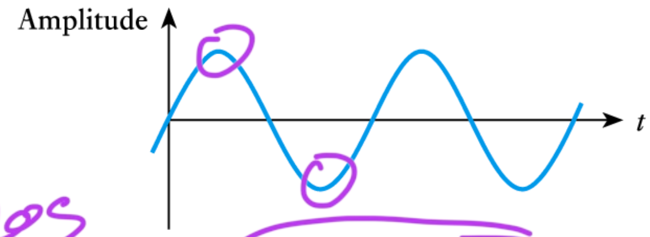
AC Analysis

1- Sinusoids and Phasor
2- Circuit elements in Phasor domain (frequency domain)
3- Impedance and admittance

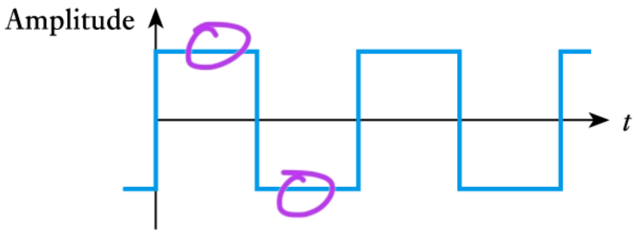
1-(a) Sinusoids

Introduction- Alternating Current (AC) waveforms

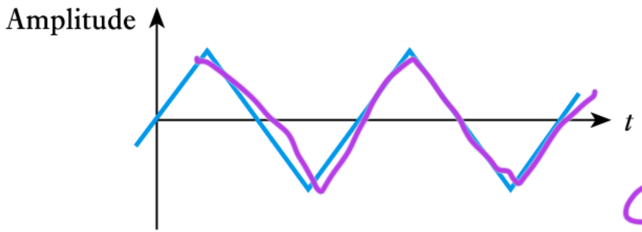
- Alternating currents and voltages vary with time and periodically change their Polarity (the magnitude alternates between "+" and "-" values).



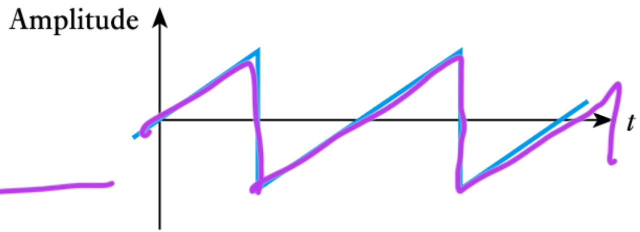
(a) A sinusoidal wave



(b) A square wave



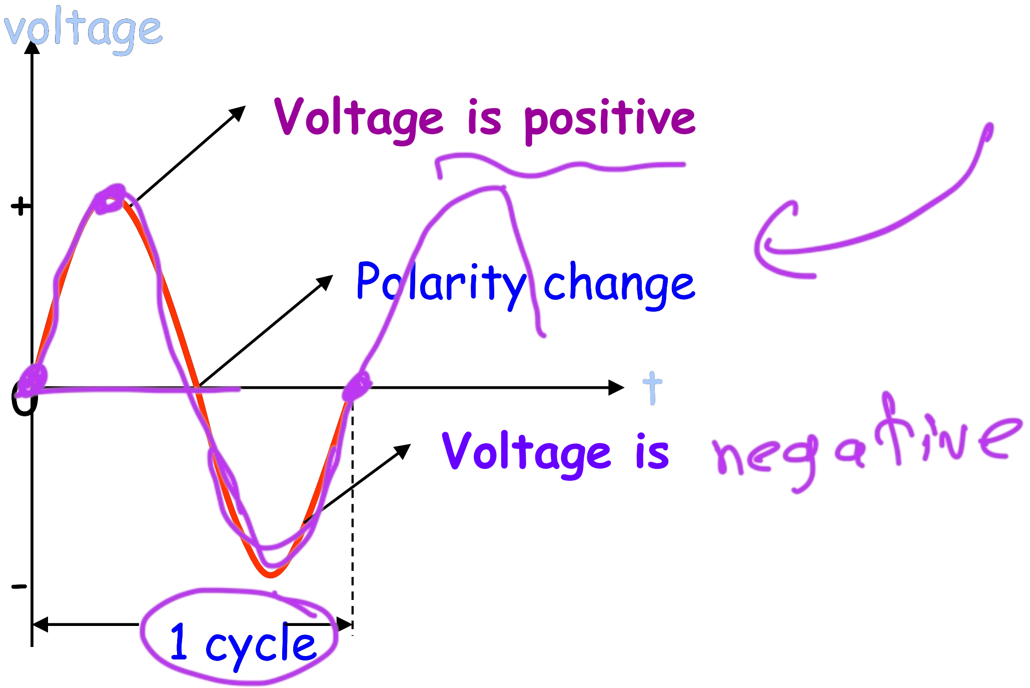
(c) A triangular wave



(d) A sawtooth wave

Introduction- SINE WAVES

- Voltage can be produced such that, over time, it follows the shape of a sine wave.
- Sine waves by far are the most important form of alternating quantity.



Introduction- GENERATING AC VOLTAGES

One way to generate ac voltage is to rotate a coil of wire at constant angular velocity in a fixed magnetic field.

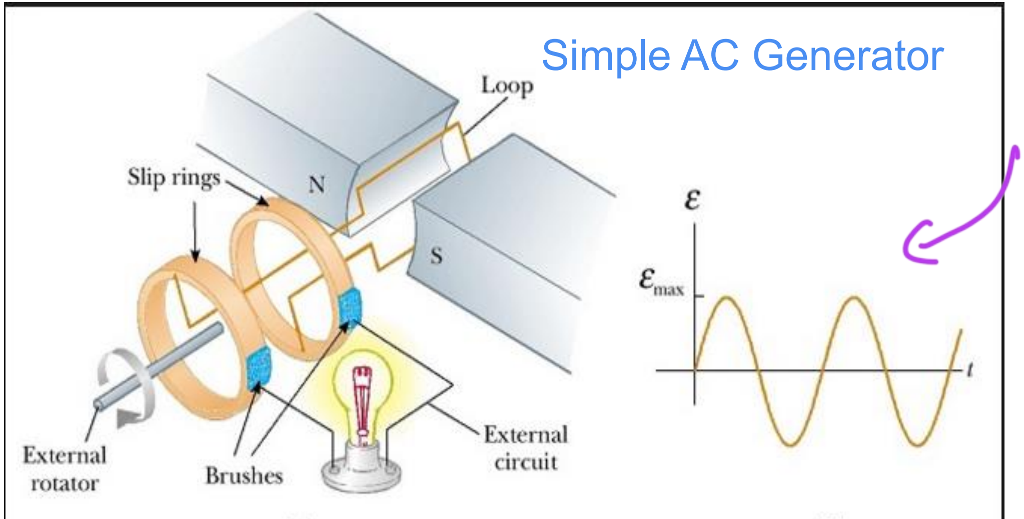
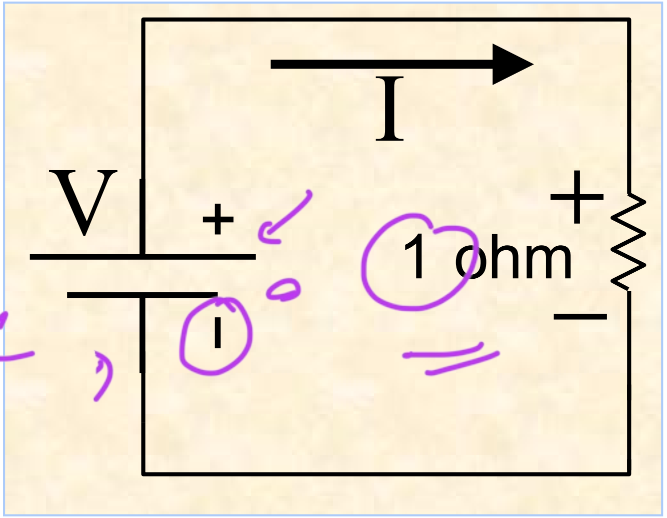


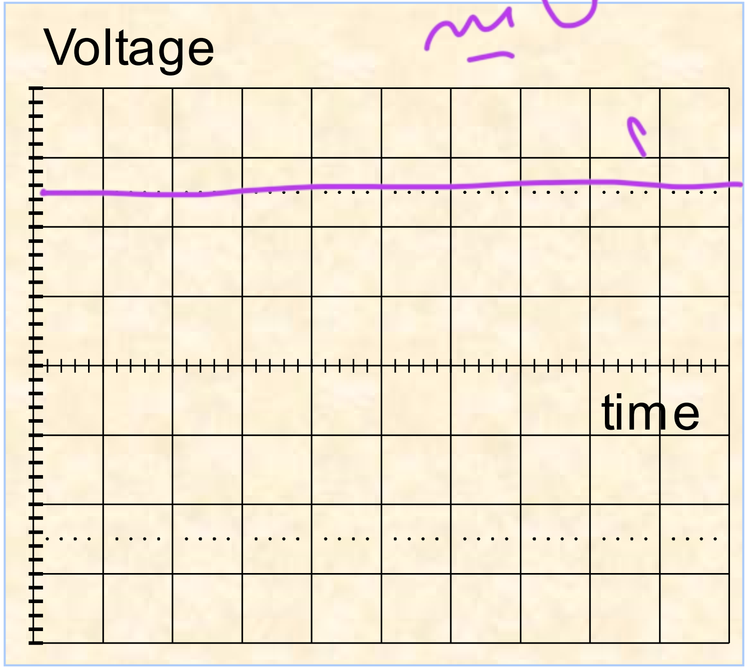
Figure 31.19 (a) Schematic diagram of an ac generator. An emf is induced in a loop that rotates in a magnetic field. (b) The alternating emf induced in the loop plotted as a function of time.

Click for more details about Working Principle of AC Generator! - YouTube!

DC vs AC

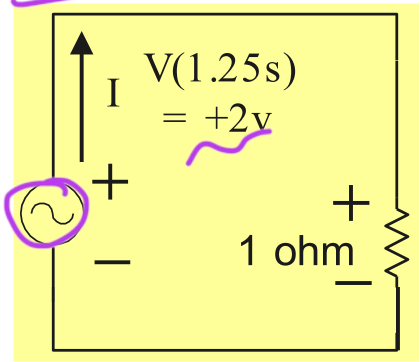
- DC Source: voltage POLARITY of the source and current DIRECTION do not change over time.

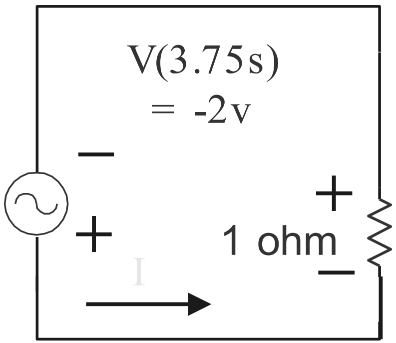


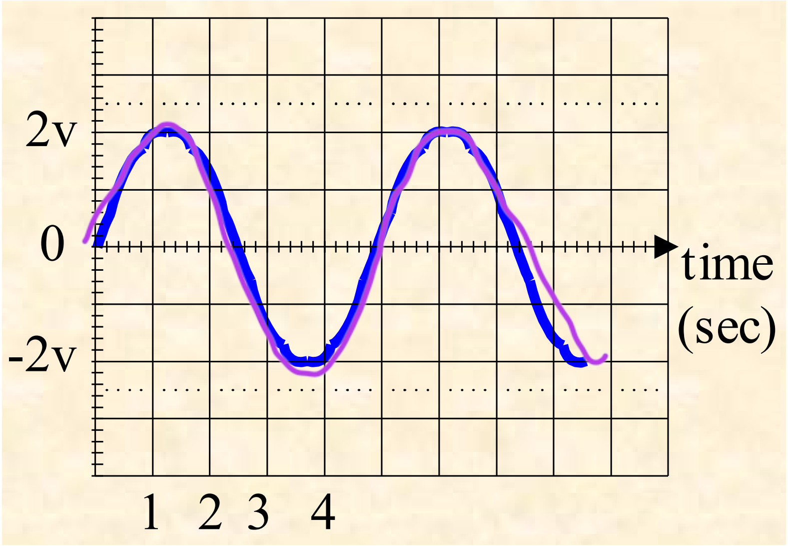


DC vs AC

- AC source: Voltage polarity changes therefore the current changes direction.

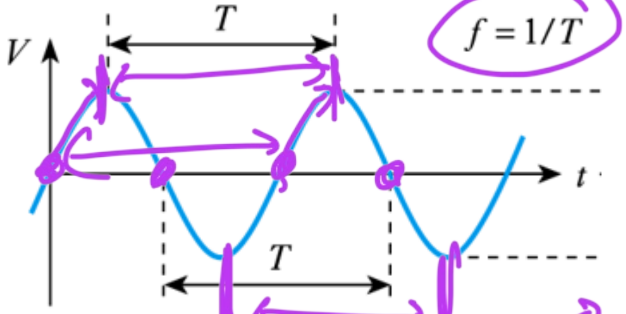






Sinusoids Characteristics: PERIOD AND FREQUENCY

- A sinusoid is a signal that has the form of the sine or cosine function.



$f = 1/T$

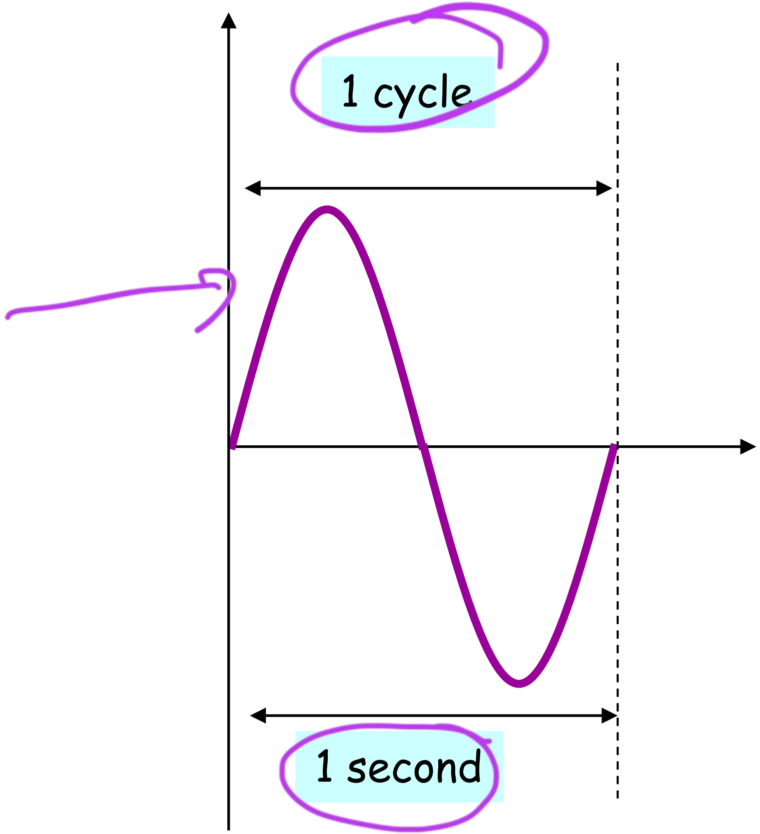
- ✓ Period: Time to complete one complete cycle
  - Symbol: T
  - Measured in second (s).
- ✓ Frequency: Number of cycles in one second
  - Symbol: f
  - Measured in hertz (Hz).

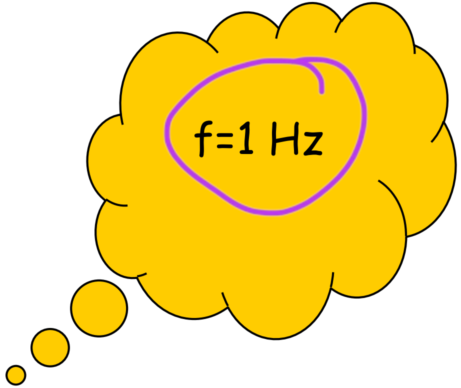
- ✓ A sinusoid is an example of a **periodic signal**.
- ✓ It repeats at regular time intervals: A section of a periodic signal between two consecutive maxima (or any other corresponding points) is called a **cycle**.
- The duration of a cycle is the **period:** Denoted by the Capital letter **T.**
- The number of cycles in one second is the **frequency**. Denoted by the lower case letter f .
- The unit of frequency is the hertz (Hz): where 1 Hz = 1 cycle per second.
- If f is the frequency in Hz and T is the period in seconds, then:

$$T = \frac{1}{f}\,(\text{s}) \text{ and } f = \frac{1}{T}\,(\text{Hz})$$

Sinusoids Characteristics: 1- PERIOD AND FREQUENCY

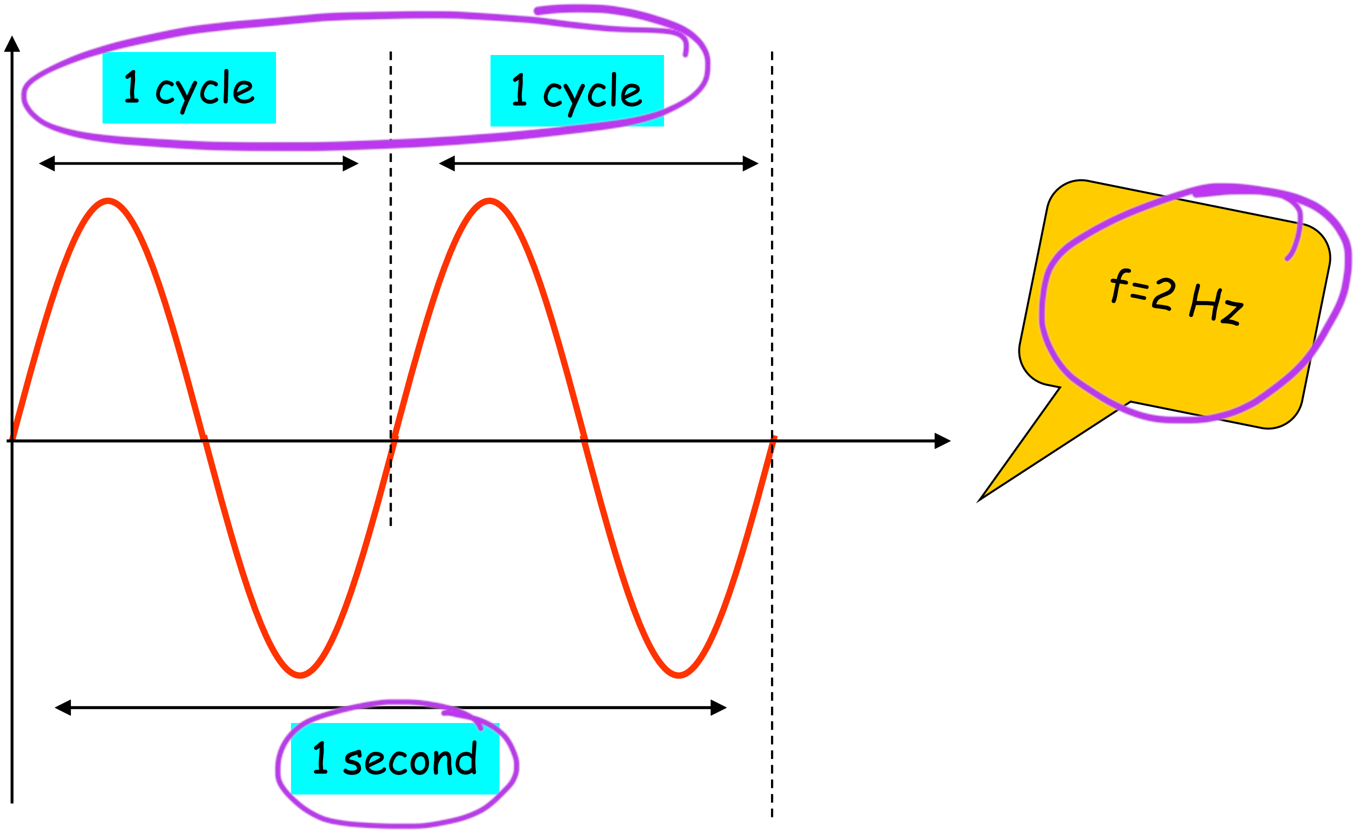
- Example:



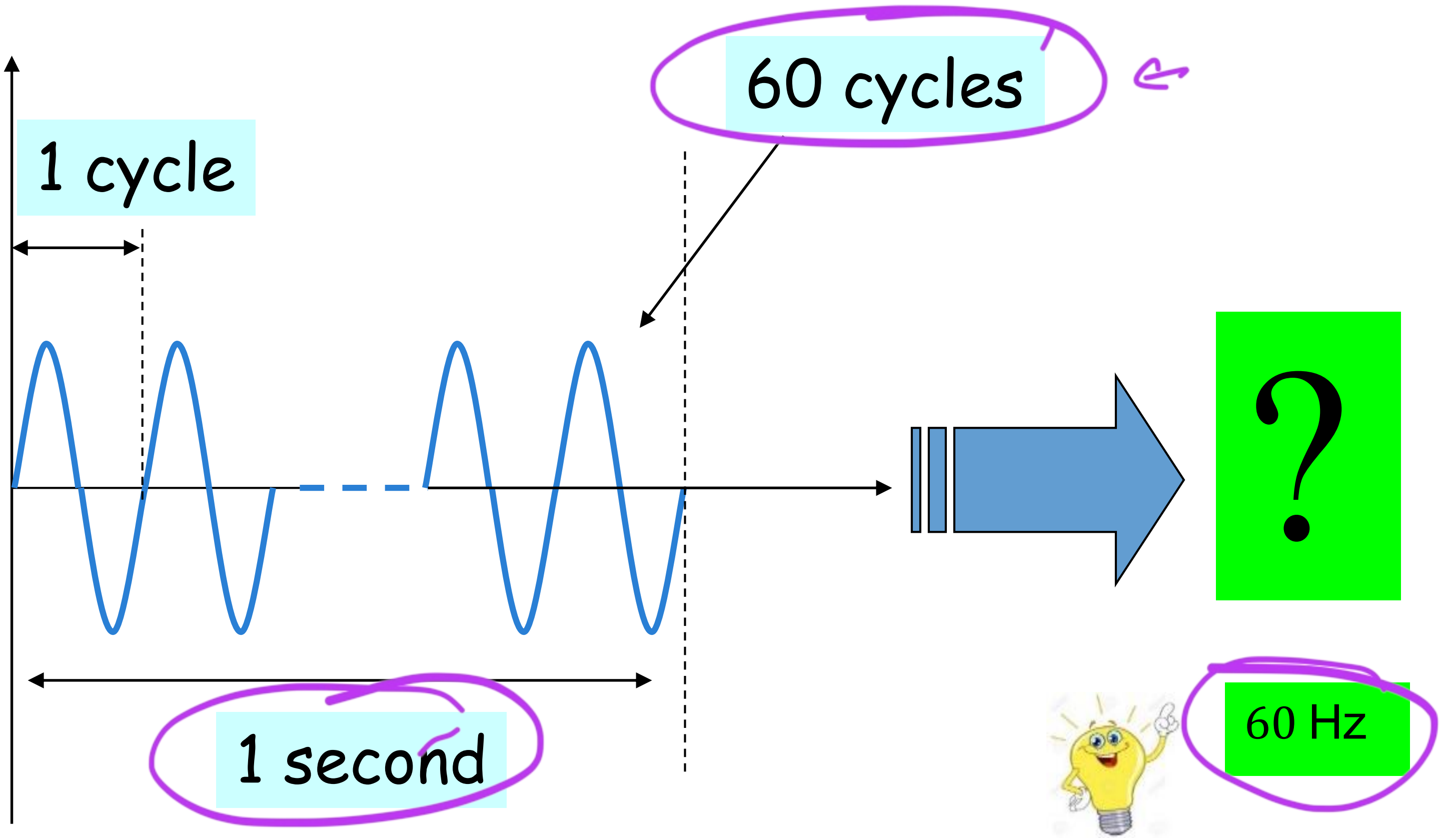


Sinusoids Characteristics: 1- PERIOD AND FREQUENCY

- Example:

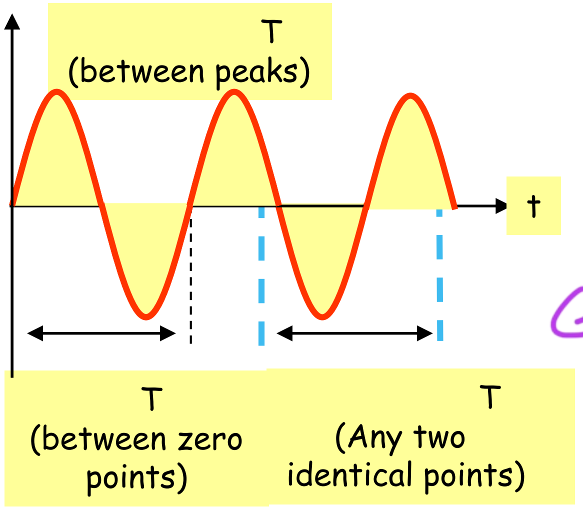


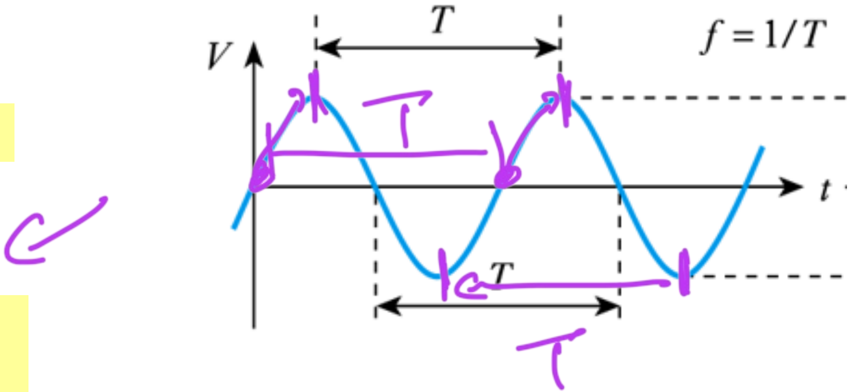
➢ Example:



Sinusoids Characteristics: 1- PERIOD AND FREQUENCY

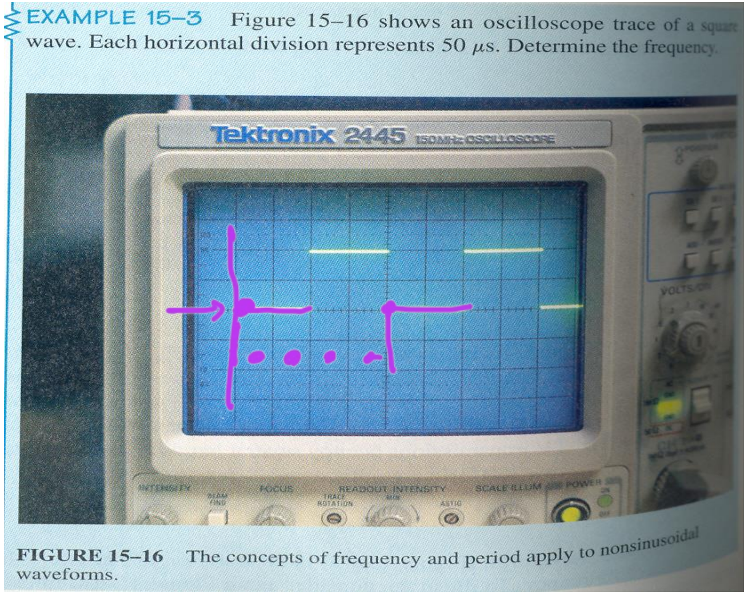
- The period of a waveform can be measured between any two corresponding points.
- Often it is measured between zero points because they are easy to establish on an oscilloscope trace.





Sinusoids Characteristics: 1- PERIOD AND FREQUENCY

➢ Example:



EXAMPLE 15–3 Figure 15–16 shows an oscilloscope trace of a square wave. Each horizontal division represents 50 μs. Determine the frequency.

FIGURE 15–16 The concepts of frequency and period apply to nonsinusoidal waveforms.

?

This Figure shows an oscilloscope trace of a square wave. Each horizontal division represents 50 μs. Determine the frequency?

4 squares × 50 μs

$T = \frac{1}{5000}$

$f = \frac{1}{T} = 5000$ Hz

Sinusoids Characteristics: 1-PERIOD AND FREQUENCY

➢ Example:

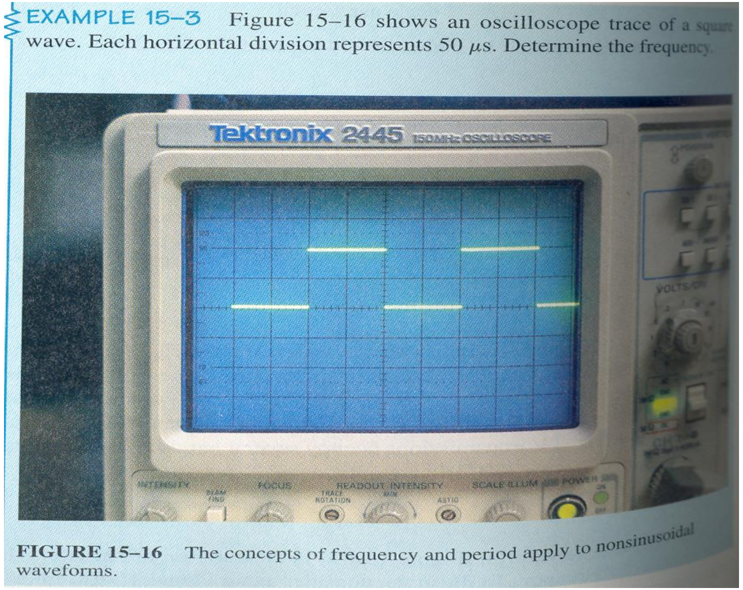

EXAMPLE 15–3 Figure 15–16 shows an oscilloscope trace of a square wave. Each horizontal division represents 50 μs. Determine the frequency.

FIGURE 15–16 The concepts of frequency and period apply to nonsinusoidal waveforms.

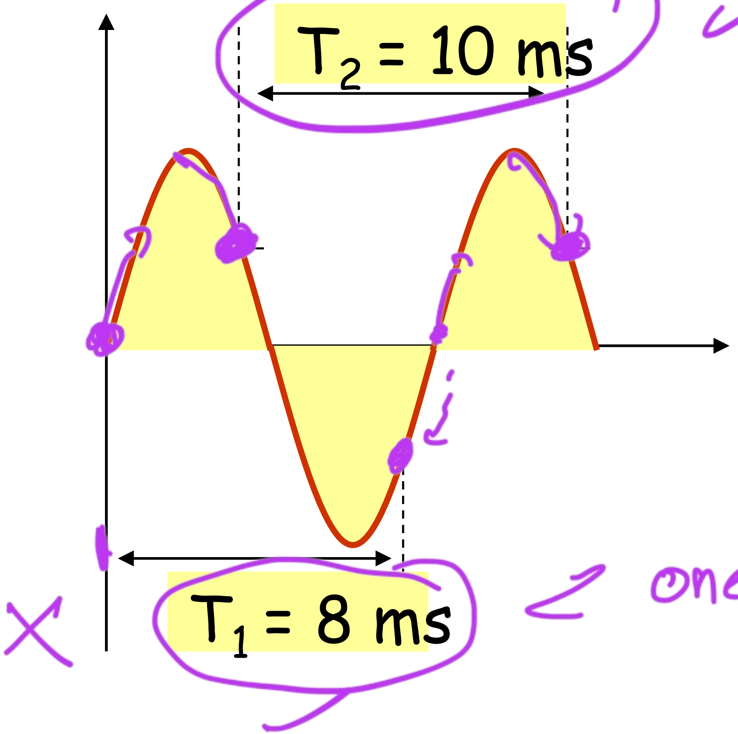
This Figure shows an oscilloscope trace of a square wave. Each horizontal division represents 50 µs. Determine the frequency?

Solution:

➢ Since the wave repeats itself every 200 µs (4*50 µs) , its period (T) is 200 µs = $200*10^{-6}$ s, and:

$$f = \frac{1}{200\times10^{-6}\,s} = 5\ \text{kHz}$$

- Example:

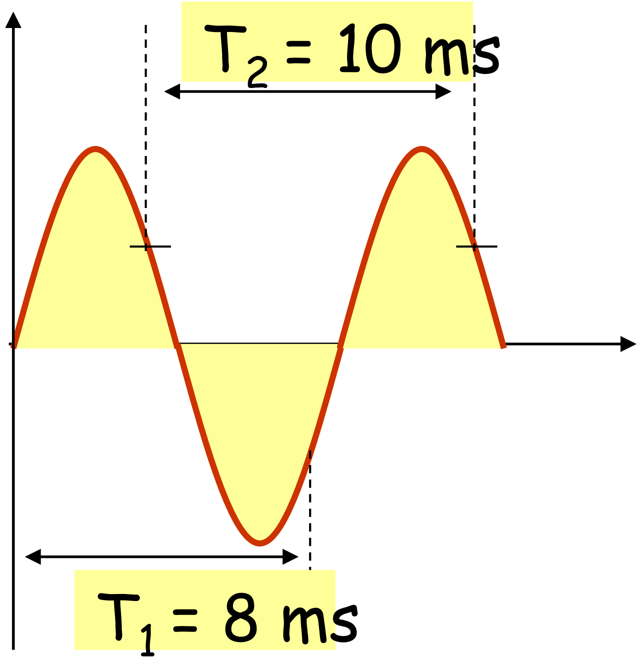


Determine the period and frequency of the waveform of the figure above.

$$f = \frac{1}{T} = \frac{1}{10 \times 10^{-3}}$$

$$= 100\ \text{Hz}$$

- Example:



Determine the period and frequency of the waveform of the figure above.

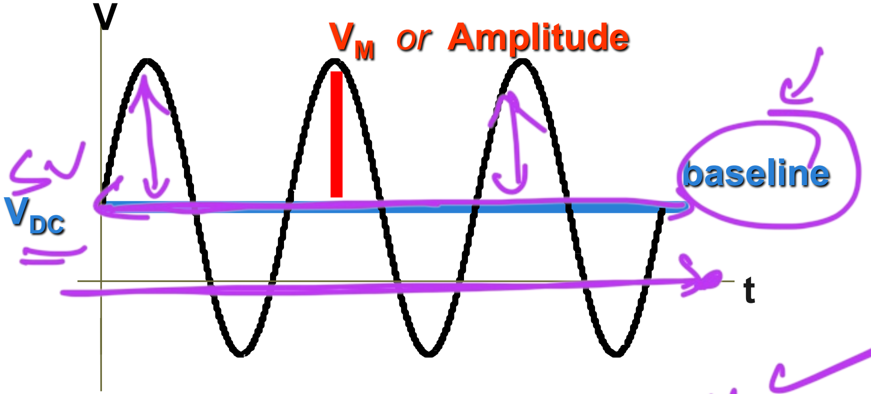
Solution:

- Time interval T_1 does not represent a period as it is not measured between corresponding points. Interval T_2, however, is. Thus, T = 10 ms and,

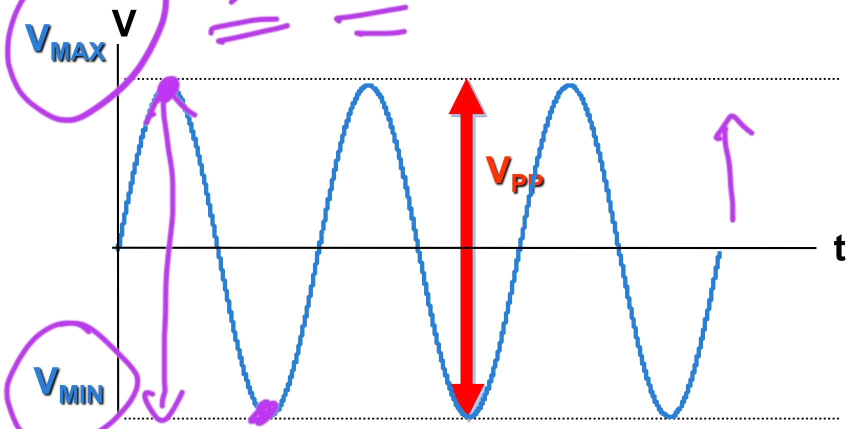
$$f = \frac{1}{10\times10^{-3}s} = 100\text{Hz}$$

Sinusoids Characteristics: 2-PEAK VALUES (V_P, I_P)

- Max Voltage (Current) : Symbol V_M (I_M).
- The maximum value of V (I) measured from the point of inflection ("baseline or DC offset").
- From the graph: $V_M - V_{DC}$.
- Also called "**Amplitude**".



Sinusoids Characteristics: 3- PEAK TO PEAK VALUES (V_{PP}, I_{PP})



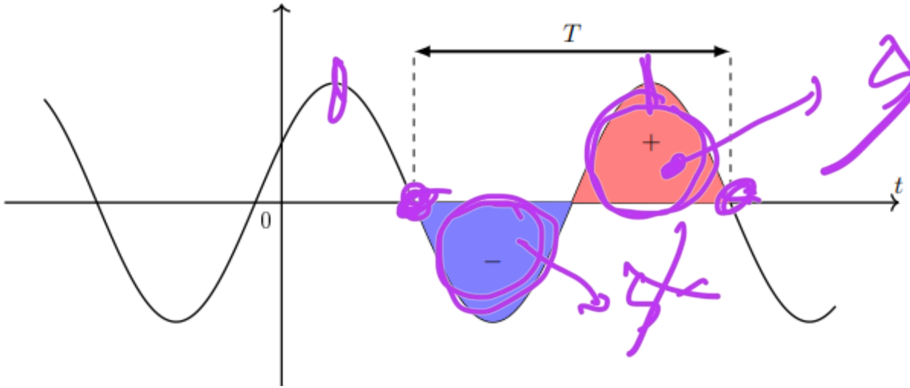
- Peak to Peak Voltage (Current): Symbol V_{PP} (I_{PP})
- The difference between the maximum value of V (I) and the minimum value of V (I)
- From the graph: $V_{MAX} - V_{MIN}$
- Equals twice peak value $V_{PP} = 2V_P$

Sinusoids Characteristics: 4- AVERAGE VALUE

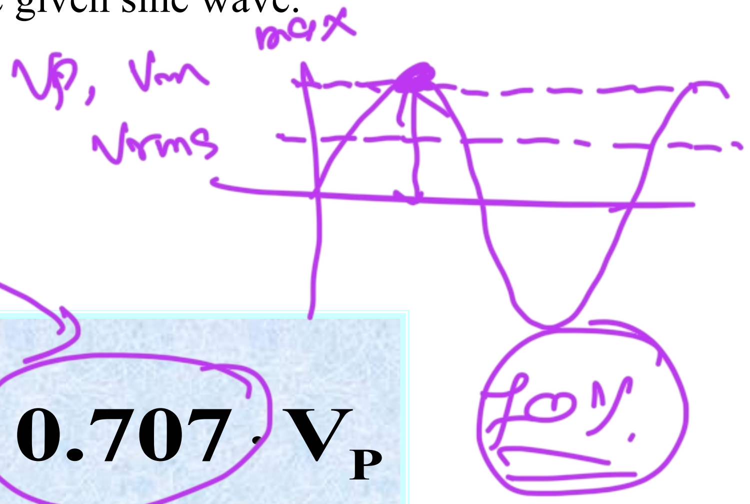
- The average value (DC value) of a periodic signal can be calculated as follows:

$$V_{av} = \frac{1}{T}\int_{0}^{T}[v(t)]dt$$

- The average value of a sinusoid signal is the integral of the sine wave over one full cycle.
- This is always equal to zero.



- ✓ Named for the mathematical process by which the value is calculated. **"Effective Voltage (V_{EFF})"**
- ✓ The RMS value of a sine wave is equal to the value of an equivalent DC circuit that would produce the same heating effect or power in a load as the given sine wave."
- ✓ Most meters read in RMS (lab DMM)
- ✓ The voltage accessed at electrical wall sockets is RMS.



$$V_{RMS} = \frac{\sqrt{2}}{2} V_P = 0.707 \cdot V_P$$

Sinusoids Characteristics: 5- ROOT MEAN SQUARE (V_{RMS}, I_{RMS})

- The effective value of a periodic wave fore is known as the root mean square value.
- This is signified by V_{RMS}.
- RMS voltage and current are used to calculate the average power associated with the voltage or current signal in one cycle.
- The RMS voltage and current are used to calculate the average power associated with the voltage or current signal in one cycle.

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T [v(t)]^2 \, dt}$$

$$V_{RMS} = \frac{V_p}{\sqrt{2}} = 0.707V_p$$

$$\mathbf{V_{RMS} = \frac{\sqrt{2}}{2} V_P = 0.707 \cdot V_P}$$

$$P_{Ave} = (V_{RMS}{}^2)/R$$

Sinusoids Characteristics:5- ROOT MEAN SQUARE (V_{RMS}, I_{RMS})

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T (V_m \sin \omega t)^2 dt}$$

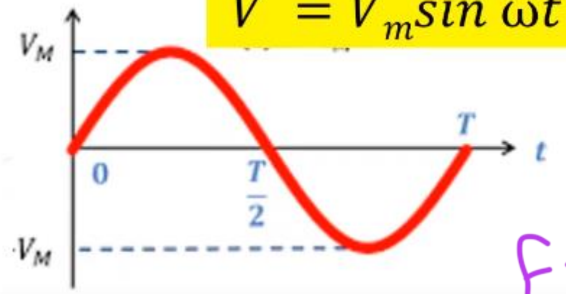
$$= Vm\sqrt{\frac{1}{T}\int_0^T \frac{(1-\cos 2\omega t)}{2} dt}$$

$$= Vm\sqrt{\frac{1}{2T}\left(t - \frac{\sin 2\omega t}{2\omega}\right)}$$

$$= Vm\sqrt{\frac{1}{2T}\left(T - \frac{\sin 2\omega T}{2\omega}\right)}$$

$$= \frac{Vm}{\sqrt{2}}$$

$$V = V_m \sin \omega t$$

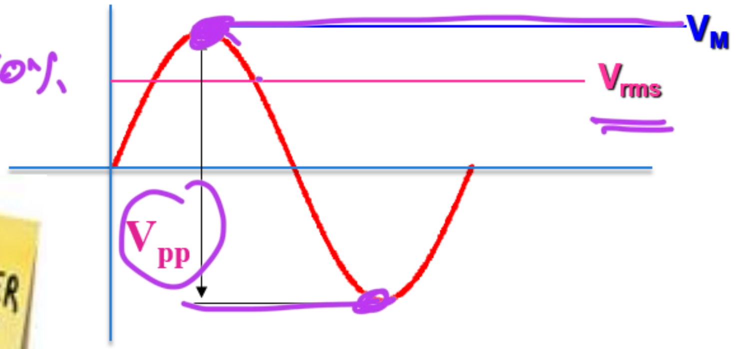


$\omega = 2\pi f = 2\pi/T$

$\omega T = 2\pi$

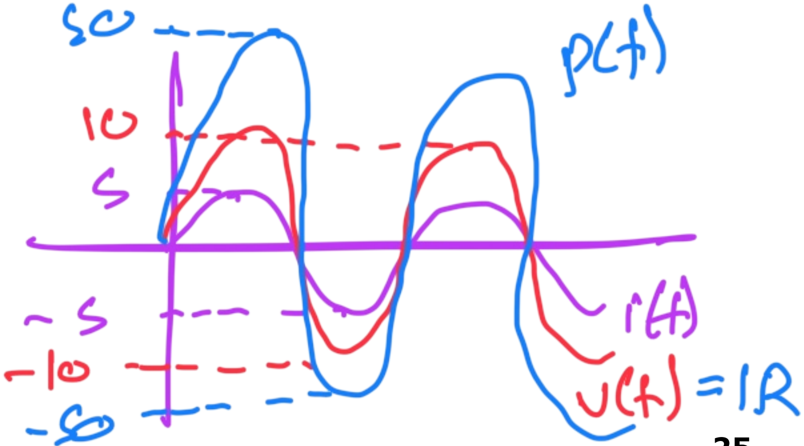
Sinusoids Characteristics: 6-COMPATIBILITY OF VALUES

- When **Peak voltages** are used as source values, current calculations will also be in Peak values.
- Likewise, an **RMS** source produces answers in RMS.
- When solving a problem make sure all values are expressed **ONE** way (peak, peak to peak, or RMS)!



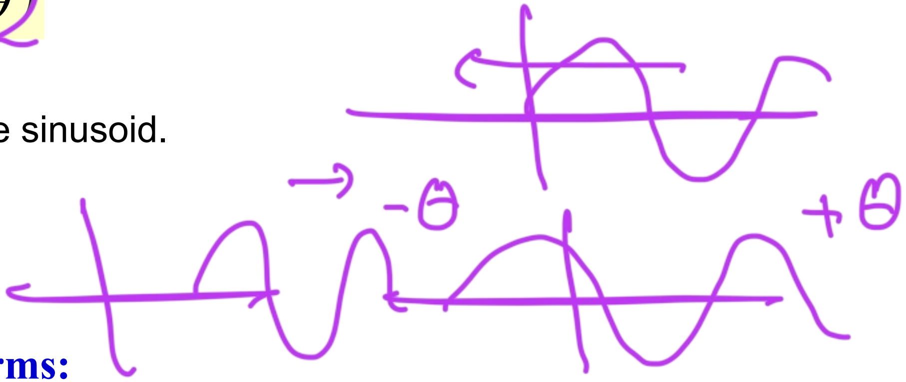
Sinusoids Characteristics: 7-VOLTAGE & CURRENT VALUES

- Ohm's Law still applies: **V=IR.**
- If current changes with time and R is a constant, voltage will also change with time.
- Voltage will be proportional to current.
- A graph of current and voltage in a resistor produces identical waveforms:
  - Peak at the same time;
  - Cross the same baseline, at the same time ;
  - Differ only in amplitude:
    - I_P is 1/R of V_P



❖ TIME DOMAIN

➢ Because the time to complete a cycle is frequency dependent, we can also identify points on the sine wave in terms of time: $v(t) = V_M \sin(2\pi ft \pm \theta)$



Where: V_m = the **amplitude** of the sinusoid.

Φ/θ = the phase

f = the **frequency**

❖ Angular Velocity of Sinusoidal Waveforms:

➢ **The quantity** $2\pi f$ is equal to the **angular frequency**, which is denoted by the Greek letter **ω** (omega).

➢ $\omega = 2\pi f = \dfrac{2\pi}{T}$

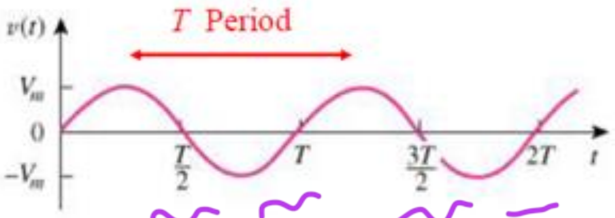
➢ $v(t) = V_m \sin(\omega t + \phi)$.

$$v(t) = V_M \sin(2\pi ft \pm \theta)$$

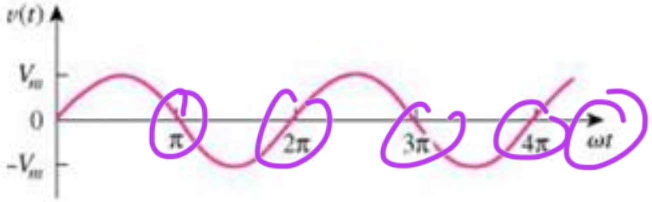
$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$v(t) = V_m \sin(\omega t + \phi)$$

- Sketch of $V_m \sin\omega t$.



(a) As a function of t.



(b) As a function of ωt.

$$\omega = \frac{2\pi}{T} \rightarrow \omega T = 2\pi$$

Sinusoids Characteristics: 8- INSTANTANEOUS VALUES

- Instantaneous Values (v, i): value of voltage and current at any:
  - Instant in time or
  - At any angle

➢ Mathematically expressed 2 ways:

$$v(t) = V_M \sin(2\pi f t \pm \theta)$$

$$v(\alpha) = V_M \sin(\alpha \pm \theta)$$

ANGULAR DOMAIN

- We can **identify points on the sine wave** in terms of an angular measurement (degrees or radians).
- The instantaneous value of the sine wave can be related to the angular rotation of the generator:

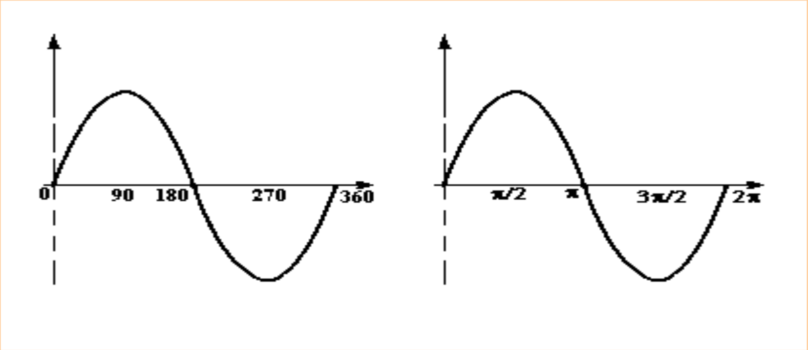
  (T=1 rotation = 360°=2π radians).

$$rad = \left(\frac{\pi}{180°}\right) deg \Leftrightarrow deg = \left(\frac{180°}{\pi}\right) rad$$

- **Sine Wave Angles: Degrees & Radians**
  - **T=2π radians = 360°**

1 radian = 57.3°

(180/π)* 2π=180*2=360°



- ANGULAR DOMAIN

$$rad = \left(\frac{\pi}{180^\circ}\right)\deg \iff \deg = \left(\frac{180^\circ}{\pi}\right)rad$$

$$30^\circ \rightarrow \text{Radians} = \frac{\pi}{180^\circ}(30^\circ) = \frac{\pi}{6}\text{ rad}$$

$$90^\circ \rightarrow \text{Radians} = \frac{\pi}{180^\circ}(90^\circ) = \frac{\pi}{2}\text{ rad}$$

$$\frac{5\pi}{4}\text{rad} \rightarrow \text{Degrees} = \frac{180^\circ}{\pi}\left(\frac{5\pi}{4}\right) = 225^\circ$$

$$\frac{3\pi}{2}\text{rad} \rightarrow \text{Degrees} = \frac{180^\circ}{\pi}\left(\frac{3\pi}{2}\right) = 270^\circ$$

$$30 \times \frac{\pi}{180} = \frac{1}{6}\pi$$

$$\frac{5\pi}{4} \times \frac{180}{\pi} = 225^\circ$$

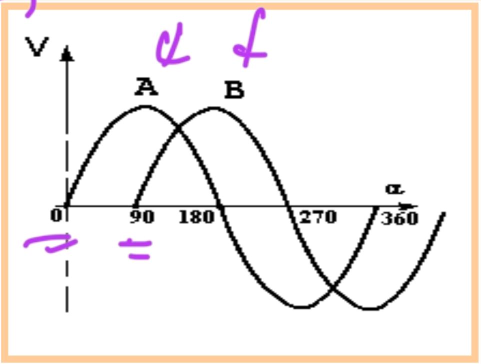
❖ PHASE ANGLE (Angular Domain)

$$v(t) = V_M \sin(2\pi ft \pm \theta)$$

- Symbol is θ (theta): It is expressed as an angle.
- Phase angle specifies the lateral shift in the position of a sine wave from a reference wave.
- Examine the same event, on each wave:
  - Two events occurring at the same angle or at the same time are in phase.
  - Events occurring at different angles or at different times are out of phase.

✓ Example:

- Wave A is the reference wave:
- Wave B is 90° out of phase.

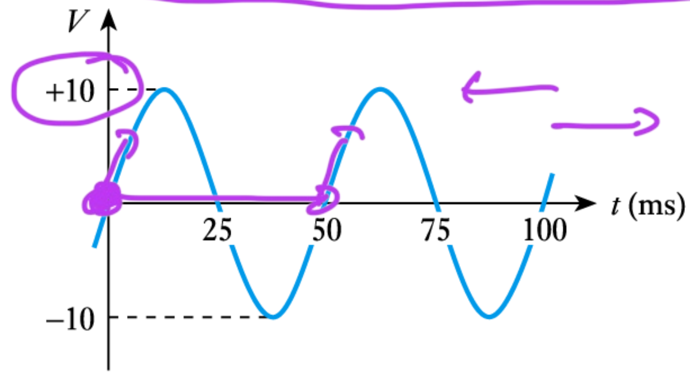


❖ PHASE ANGLE (Angular Domain)

$$v(t) = V_M \sin(2\pi ft \pm \theta)$$

Example: Determine the equation of the following voltage signal.

T = 50ms



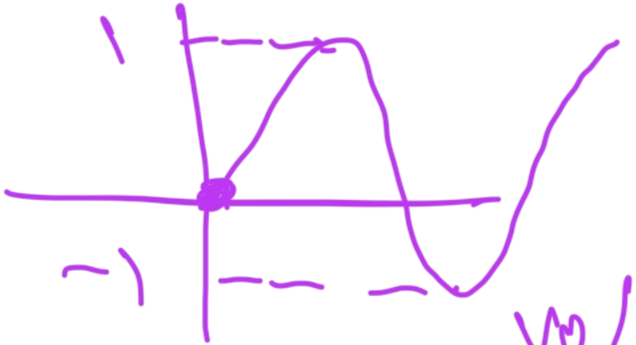
From diagram:

- Period is 50 ms = 0.05 s
- Thus $f = 1/T = 1/0.05 = 20$ Hz
- Peak voltage is 10 V
- Therefore

$$v = V_p \sin 2\pi ft$$
$$= 10 \sin 2\pi 20t$$
$$= 10 \sin 126t$$

➢ The angle of the sine wave is zero at $t = 0$.

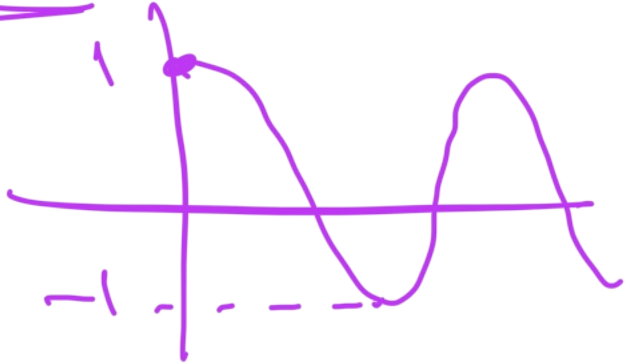
sin



$T = 50ms$

$f = \frac{1}{T} = 20Hz$

cos



Vp / Vm

$2\pi f$

$\theta \approx 0$

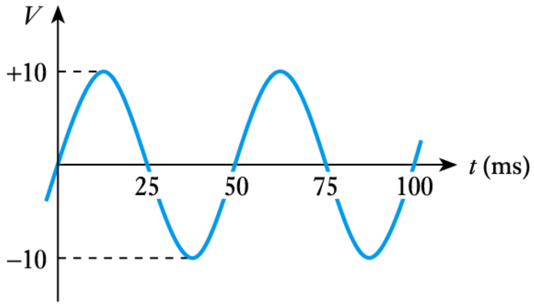
$v(t) = 10 \sin(2\pi \times 20t + \theta)$

$v(t) = 10 \sin(125,66\, t)$

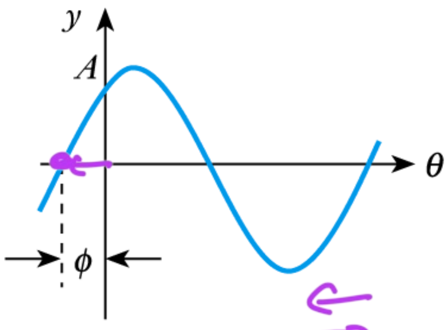
Sinusoids Characteristics: 8- INSTANTANEOUS VALUES

❖ PHASE ANGLE (Angular Domain)

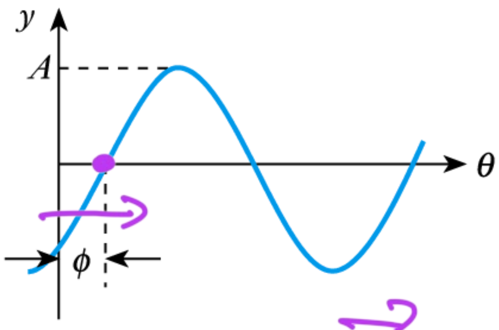
- $v(t)= V_m \sin(2\pi ft)$: The angle of the sine wave is zero at $t = 0$.



- If this is not the case the expression is modified by adding the angle at $t = 0$.



(a) $y = A\sin(\omega t + \phi)$



(b) $y = A\sin(\omega t - \phi)$

Sinusoids Characteristics: 8- INSTANTANEOUS VALUES

❖ PHASE ANGLE

✓ Example:

$$v(t) = V_M \sin(2\pi f t \pm \theta)$$

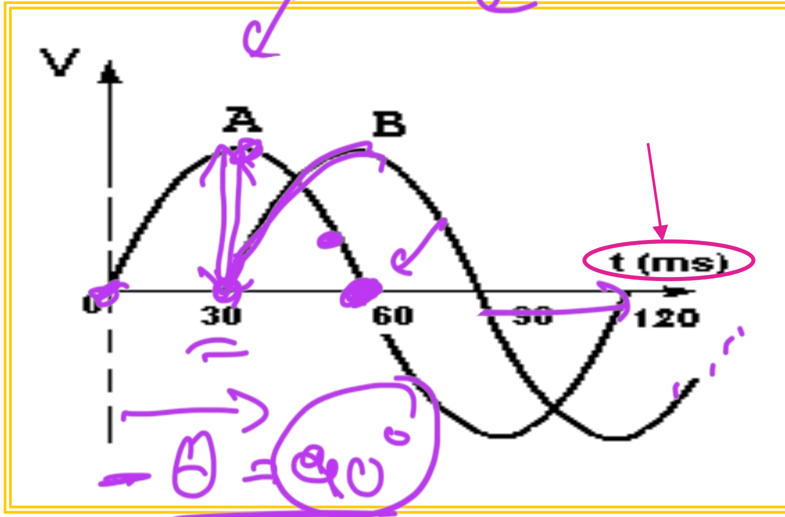
? Determine the Phase angle θ of the waveform B of the figure above?

☐ Wave A is the reference wave (θ=0).

➢ Compare the positive peak events:

- ➢ Wave A peaks at 30ms & Wave B at 60ms
- ➢ T=120ms

θ /360º = Δt/T



θ /360º = Δt/T

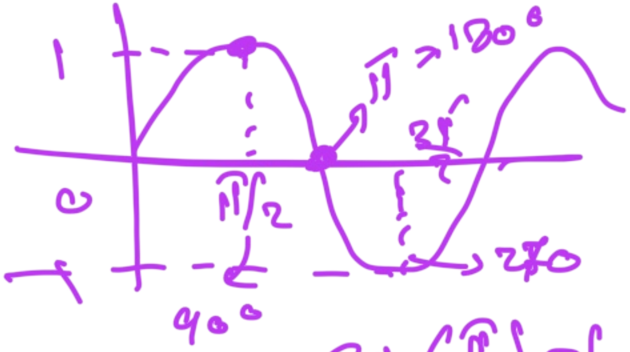
θ /360º = (30ms-0ms)/120ms=30/120

θ =360*30/120= 90º

A peak = 30 ms → T = 120 ms

B peak = 60 ms

$-1 \le \sin \le 1$



$\sin(\frac{\pi}{2}) = 1$

$$\frac{\theta}{360} = \frac{\Delta t}{T} = \frac{30 - 0}{120}$$

$\sin(\frac{3\pi}{2}) = -1$

$$360 \times \frac{\theta}{360} = \frac{30}{120} \times 360 \rightarrow \theta = 90^\circ$$

max A peak = 90°

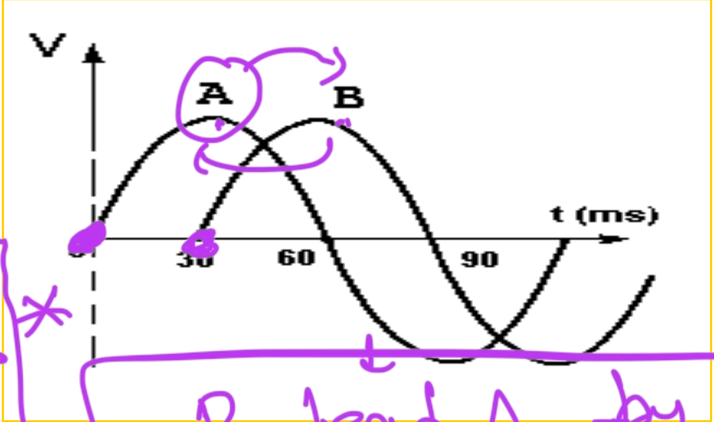
$$\frac{60}{120} \times 360 = 180^\circ$$

Sinusoids Characteristics: 9- LEADING & LAGGING

- Since wave B peaked after the reference wave peaked, we say it LAGS the reference wave by 90º : $\theta = -90º$
- If wave B was the reference, wave A would peak before the reference wave (B): we would say it LEADS the reference wave: $\theta = +90º$

NOTE

Because it is the reference wave, θ for ANY reference wave is 0º.



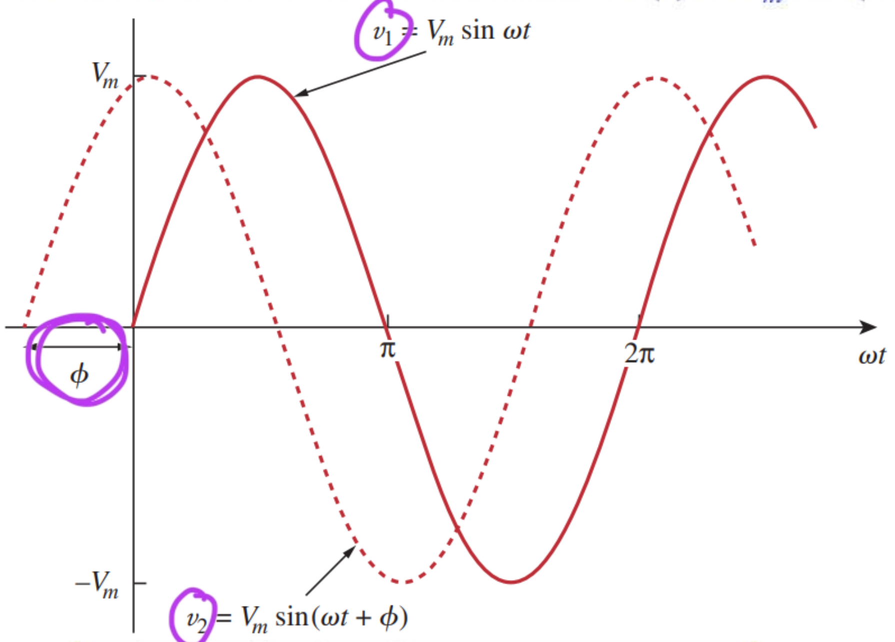
A lead B by 90°
B lag A by 90°

B lead A by -90
A lag B by -90

Sinusoids Characteristics: 9- LEADING & LAGGING

✓ Example 2:

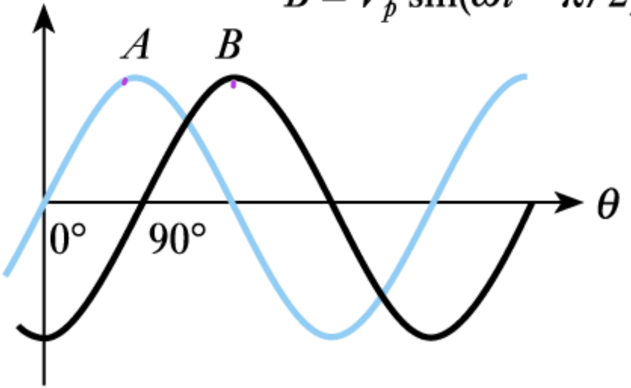
- Consider the sinusoidal voltage having phase φ, $v(t) = V_m \sin(\omega t + \phi)$



- v_2 LEADS v_1 by phase φ.
- v_1 LAGS v_2 by phase φ.
- v_1 and v_2 are out of phase.

✓ Example 3:

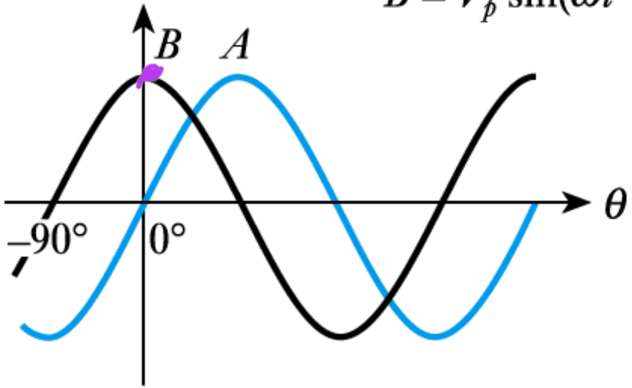
$$A = V_p \sin(\omega t)$$
$$B = V_p \sin(\omega t - \pi/2)$$



(a) *B* lags *A* by 90°

A lead B by 90

$$A = V_p \sin(\omega t)$$
$$B = V_p \sin(\omega t + \pi/2)$$

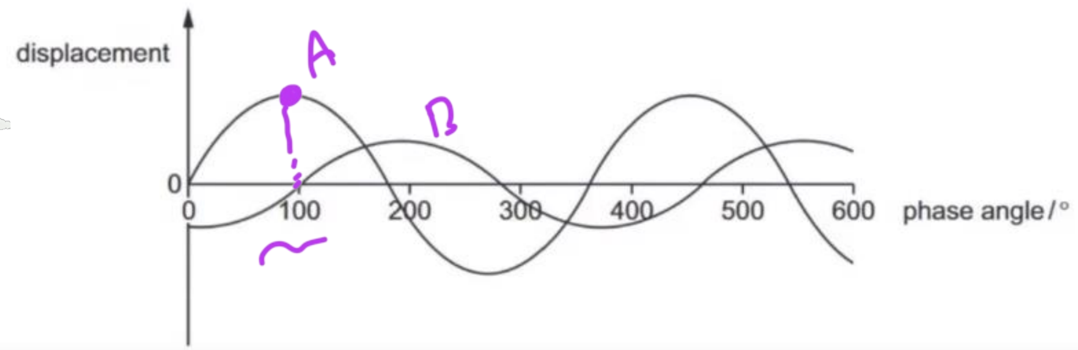


(b) *B* leads *A* by 90°

A lag B by 90

✓ Example 4:

Two light waves of the same frequency are represented by the diagram.



What could be the phase difference between the two waves?

A 150°   **B** 220°   **C** 260°   **D** 330°

- Since wave B peaked after the reference wave peaked, we say it LAGS the reference wave by 90º :

  $\theta = -100^\circ$

  $\theta = 360 - 100^\circ = 260^\circ$

Sinusoids Characteristics: 8- LEADING & LAGGING

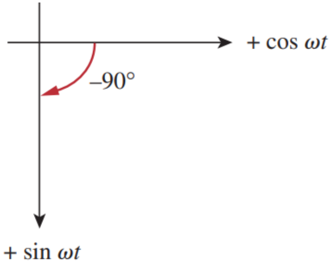
- A sinusoid can be expressed in either sine or cosine form.
- When comparing two sinusoids, it is expedient to **express both as either sine or cosine** with **positive amplitudes**.

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \qquad \textbf{(9.9)}$$

With these identities, it is easy to show that

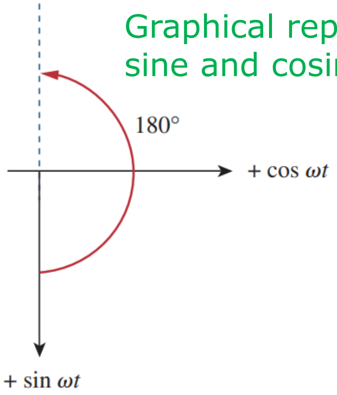
$$\sin(\omega t \pm 180^\circ) = -\sin \omega t$$
$$\cos(\omega t \pm 180^\circ) = -\cos \omega t$$
$$\sin(\omega t \pm 90^\circ) = \pm\cos \omega t \qquad \textbf{(9.10)}$$
$$\cos(\omega t \pm 90^\circ) = \mp\sin \omega t$$

Using these relationships, we can transform a sinusoid from sine form to cosine form or vice versa.



(a)

Graphical representation of sine and cosine relation



(b)

$-7\sin(\omega t) \longrightarrow 7\sin(\omega t \pm 180)$

$-8\cos(\omega t - 30) \longrightarrow +8\cos(\omega t - 30 - 180)$

-210

$+\cos(\omega t) \longrightarrow \sin(\omega t + 90)$

$-\cos(\omega t) \longrightarrow \sin(\omega t - 90)$

$+\sin(\omega t) \longrightarrow +\cos(\omega t - 90)$

$-\sin(\omega t) \longrightarrow +\cos(\omega t + 90)$

Example 5

Given a sinusoid, $5\sin(4\pi t - 60^o)$, calculate its amplitude, phase, angular frequency, period, and frequency.

Solution:

Amplitude = 5,
phase = –60°,
angular frequency = 4π rad/s,
Period = 0.5 s,
frequency = 2 Hz.

Example 6

Find the phase angle between $i_1 = -4\sin(377t + 25^o)$ and $i_2 = 5\cos(377t - 40^o)$

Does i_1 lead or lag i_2?

$$\sin(\omega t \pm 180^\circ) = -\sin\omega t$$
$$\cos(\omega t \pm 180^\circ) = -\cos\omega t$$
$$\sin(\omega t \pm 90^\circ) = \pm\cos\omega t$$
$$\cos(\omega t \pm 90^\circ) = \mp\sin\omega t$$

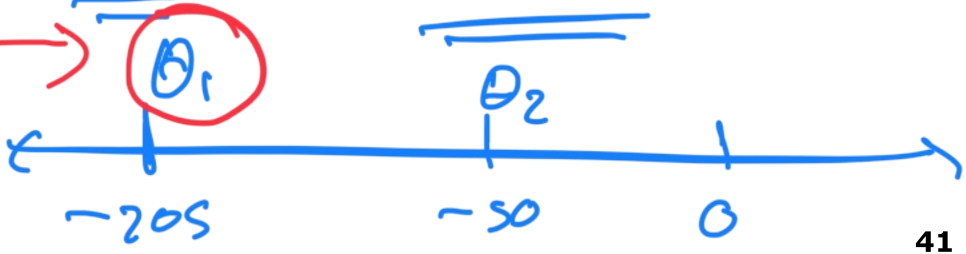
Solution:

✓ Since cos ωt= sin(ωt+90°)

$$i_2 = 5\sin(377t - 40^o + 90^o) = 5\sin(377t + 50^o)$$

$$i_1 = -4\sin(377t + 25^o) = 4\sin(377t + 180^o + 25^o) = 4\sin(377t + 205^o)$$

➢ Therefore, i_1 leads i_2 155°.



$$i_1(t) = -4\sin(377t + 25) \quad * \; 4\cos(377t + 25 + 90)$$

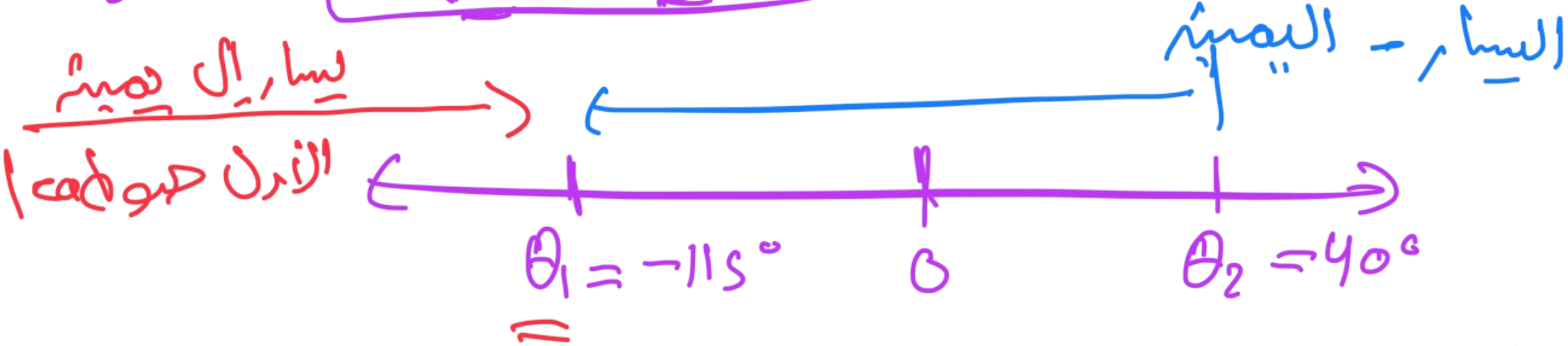
$$i_2(t) = +5\cos(377t - 40)$$

لحتى نقارن بين lead, lag لو بدي شروط

1. انه تكون موجبة
2. انه تكون كلتاهما sin أو cos
3. انه يكونو نفس w

$i_1(t) = 4\cos(377t + 115) \rightarrow \theta_1 = -115$

$i_2(t) = 5\cos(377t - 40) \rightarrow \theta_2 = +40$



i_1 lead i_2 by $40 - (-115) = 155^\circ$

Example 7

Find the phase angle between $v_1 = -10\cos(\omega t + 50^o)$ and $v_2 = 12\, sin(\omega t - 10^o)$. State which sinusoid is leading.

Solution:

$$v_1 = -10\cos(\omega t + 50^\circ) = 10\cos(\omega t + 50^\circ - 180^\circ)$$

$$v_1 = 10\cos(\omega t - 130^\circ) \quad \text{or} \quad v_1 = 10\cos(\omega t + 230^\circ)$$

and

$$v_2 = 12\sin(\omega t - 10^\circ) = 12\cos(\omega t - 10^\circ - 90^\circ)$$

$$v_2 = 12\cos(\omega t - 100^\circ)$$

$$\sin(\omega t \pm 180^\circ) = -\sin\omega t$$
$$\cos(\omega t \pm 180^\circ) = -\cos\omega t$$
$$\sin(\omega t \pm 90^\circ) = \pm\cos\omega t$$
$$\cos(\omega t \pm 90^\circ) = \mp\sin\omega t$$

v2=12 cos(wt-100+360)

v2=12 cos(wt+260)

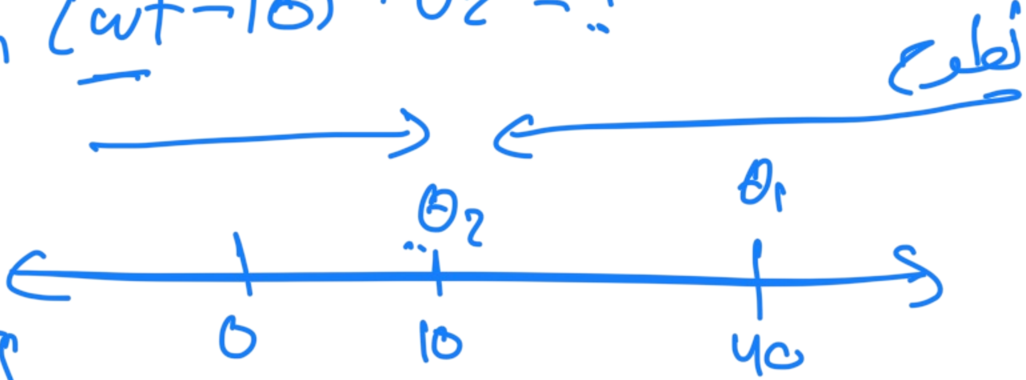
v_2 leads v_1 by 30°

$V_1 = -10\cos(\omega t + 50) \rightarrow 10\sin(\omega t + 50 - 90)$

$V_2 = +12\sin(\omega t - 10)$

$\Rightarrow V_1 = +10\sin(\omega t - 40)$, $\theta_1 = 40$

$V_2 = +12\sin(\omega t - 10)$, $\theta_2 = 10$



V_2 lead V_1

by $40 - 10 = \boxed{30}$

Example 8

Find the phase angle between $v_1 = -10\cos(\omega t + 50^o)$ and $v_2 = 12\,sin(\omega t - 10^o)$. State which sinusoid is leading.

Another Solution:

$$v_1 = -10\cos(\omega t + 50^\circ) = 10\sin(\omega t + 50^\circ - 90^\circ)$$

$$= 10\sin(\omega t - 40^\circ)$$

$$v_2 = 12\sin(\omega t - 10^\circ).$$

$$\sin(\omega t \pm 180^\circ) = -\sin\omega t$$
$$\cos(\omega t \pm 180^\circ) = -\cos\omega t$$
$$\sin(\omega t \pm 90^\circ) = \pm\cos\omega t$$
$$\cos(\omega t \pm 90^\circ) = \mp\sin\omega t$$

v_2 leads v_1 by 30°

Example 9

Let us consider the behavior of an Ironing Machine (المكواة) as a pure resistive load of 1kW power supplied by a single phase AC voltage of (220V, 50Hz) for domestic application.

a- Find R
b- Find i(t)

The supplied voltage can be represented mathematically as:

$$v(t) = V_{max}\sin(\omega t)$$

Since its rms value is 220V then its maximum value (V_{max}) is:

$$V_{rms} = \frac{V_{max}}{\sqrt{2}}$$

$$V_{max} = 220 \times \sqrt{2} = 311.127\text{V}$$

Then

$$v(t) = 311.127\sin(\omega t)$$

Since,

$$P = \frac{(V_{rms})^2}{R}$$

$$R == \frac{(V_{rms})^2}{P} = \frac{(220)^2}{1000} = 48.4\Omega$$

Then

$$I_{rms} = \frac{V_{rms}}{R} = \frac{220}{48.4} = 4.546\text{A}$$

The time domain function of the current $i(t)$ is already known as:

- $i(t) = I_{max}\sin(\omega t) = \sqrt{2}\, I_{rms}\sin(\omega t) = 6.429\sin(\omega t)$

(220V, 50Hz)

rms value

$v(t) = V_m \sin(\omega t)$, $2\pi f$

$P = 1000\,W$ $V_m > V_{rms}$

$V_m = \sqrt{2} \times V_{rms}$

$= \sqrt{2}\,220 =$

$311,127$

$$v(t) = 311,127 \sin(314,16\,t)$$

AC source → rms

$P = \frac{V_{rms}^2}{R}$

$R = \frac{V_{rms}^2}{P} = \frac{220^2}{1000} = 48,4\,\Omega$

$$V = IR \longrightarrow I(t) = \frac{V}{R}$$

$$I(t) = \frac{311,127}{48,4} \sin(314,16t)$$

$$\boxed{I(t) = 6,429 \sin(314,16t)}$$

SUPERIMPOSED DC & AC

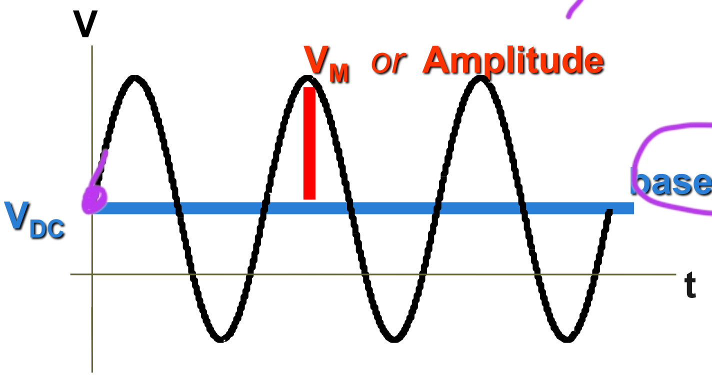
- A circuit can have both a DC voltage source and an AC
- We say that the "AC rides on the DC"
- The graph of the voltage is displaced vertically from 0, to the DC voltage level. Algebraically:

DC + AC

$$v(t) = V_{dc} + V_M \sin(2\pi ft \pm \theta)$$

$$v(\alpha) = V_{dc} + V_M \sin(\alpha \pm \theta)$$

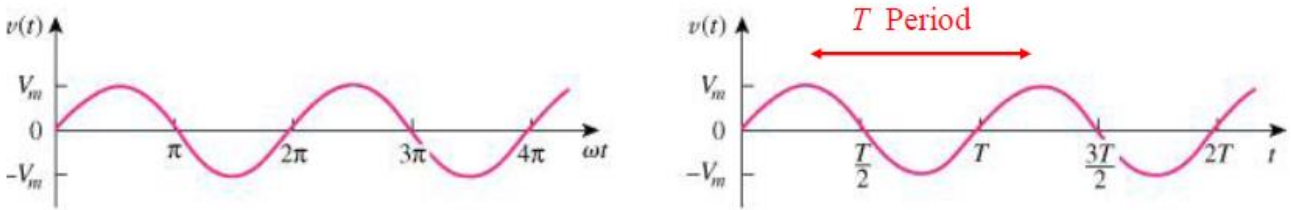
$\alpha = 2\pi ft$



AC Circuits and AC Analysis

- **A Sinusoid** is a signal that has the form of **sine** of **cosine** function
- The **sinusoidal** current is referred to as **Alternating Current (AC)**
- **AC** circuits are energized with **AC independent sources** (current or voltage)

- Sketch of $V_m \sin\omega t$.



(a) As a function of ωt.

(b) As a function of t.

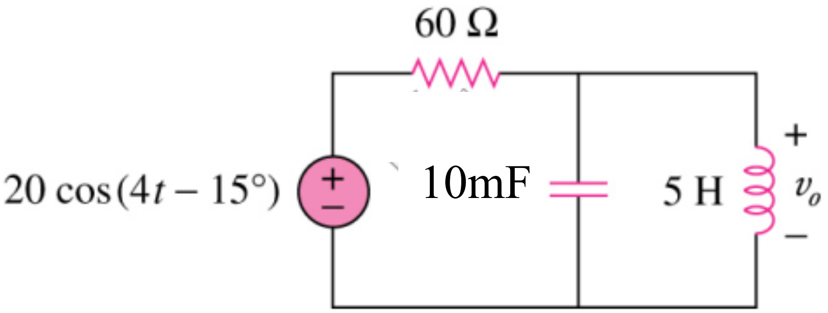
- V_m is the AMPLITUDE of the sinusoid.
- ω is the ANGULAR FREQUENCY in radians/s.

$$\omega = 2\pi f \quad \text{and} \quad f = \frac{1}{T}$$

- f is the FREQUENCY in Hertz.
- T is the period in seconds.

AC Circuits and AC Analysis

- **Solving AC circuits** in time domain involves solving **differential equations**.
- Example, for the given circuit below, what is Vo(t)?



- Writing KCL at Vo node results in: $$\frac{d^2v_o}{dt^2}+\frac{5}{3}\frac{dv_0}{dt}+20v_0=-\frac{400}{3}\sin(4t-15^o)$$

- **However,** forming and solving the DE in time domain are sometimes very tedious.
- **Therefore,** AC circuits analysis is done in **Phasor (frequency) domain.**

1-(b) Phasor

1-(b) Phasors

- A **phasor** is a **complex number** that represents the amplitude and phase of a sinusoid (**Polar Form**).
- Phasor is the mathematical equivalent of a sinusoid with **time variable dropped**.
- Phasor representation is based on **Euler's identity**.

$$e^{\pm j\phi} = \cos\phi \pm j\sin\phi \quad \text{Euler's Identity}$$

$$\cos\phi = \mathrm{Re}\{e^{j\phi}\} \quad \text{Real part}$$

$$\sin\phi = \mathrm{Im}\{e^{j\phi}\} \quad \text{Imaginary part}$$

- Given a sinusoid $v(t)=V_m\cos(\omega t+\varphi)$.

$$v(t) = V_m\cos(\omega t+\phi) = \mathrm{Re}(V_m e^{j(\omega t+\phi)}) = \mathrm{Re}(V_m e^{j\phi} e^{j\omega t}) = \mathrm{Re}(\mathbf{V} e^{j\omega t})$$

$$\mathbf{V} = V_m e^{j\phi} = V_m\angle\phi = \text{PHASOR REP.}$$

$$v(t) = V_m\cos(\omega t+\phi) \Leftrightarrow \mathbf{V} = V_m\angle\phi$$

(Time Domain Repr.) (Phasor Domain Representation)

$$v(t) = \mathrm{Re}\{\mathbf{V} e^{j\omega t}\}$$

(Converting Phasor back to time)

Complex Numbers and Mathematical Foundation of Phasor

Complex Numbers

- A complex number may be written in RECTANGULAR FORM as:

RECTANGULAR FORM

$$z = x + jy \quad j=\sqrt{-1},\ x=\mathrm{Re}(z),\ y=\mathrm{Im}(z)$$

- A second way of representing the complex number is by specifying the MAGNITUDE and r and the ANGLE θ in POLAR form.

POLAR FORM

$$z = x + jy = |z|\angle\phi = r\angle\phi$$

- The third way of representing the complex number is the EXPONENTIAL form.

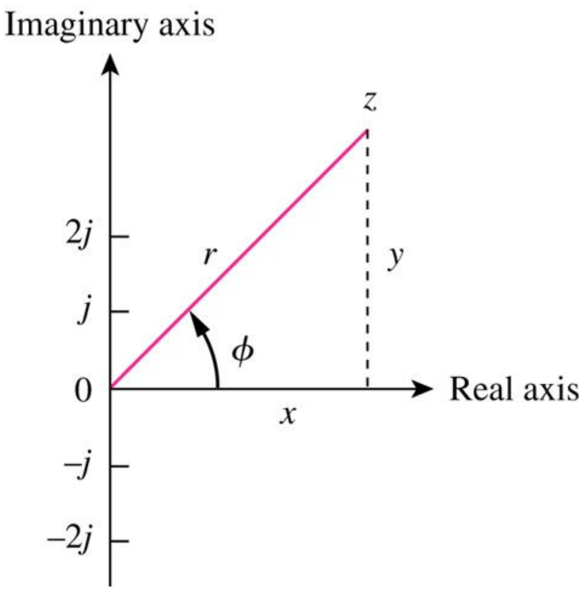
EXPONENTIAL FORM

$$z = x + jy = |z|\angle\phi = re^{j\phi}$$

$$re^{j\phi} = r\cos\phi + jr\sin\phi$$ Euler's Identity

$$r\cos\phi = \mathrm{Re}\{re^{j\phi}\}$$ Real part

$$r\sin\phi = \mathrm{Im}\{re^{j\phi}\}$$ Imaginary part



- x is the REAL part.
- y is the IMAGINARY part.
- r is the MAGNITUDE.
- φ is the ANGLE.

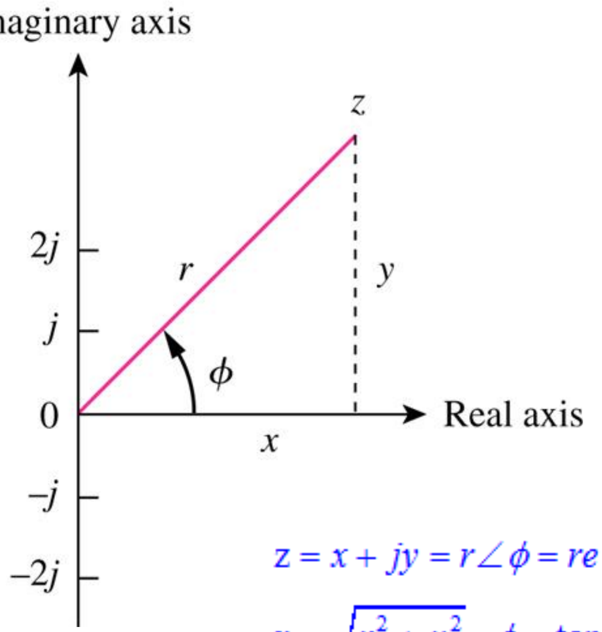
Complex Number Conversions

- We need to convert COMPLEX numbers from one form to the other form.

$$z = x + jy = r\angle\phi = re^{j\phi} = r(\cos\phi + j\sin\phi)$$

EXPONENTIAL FORM

$z = x + jy = |z|\angle\phi = re^{j\phi}$



$$z = x + jy = r\angle\phi = re^{j\phi} = r(\cos\phi + j\sin\phi)$$

$r = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1}\dfrac{y}{x}$ Rectangular to Polar

$x = r\cos\phi, \quad y = r\sin\phi$ Polar to Rectangular

Mathematical Operations of Complex Numbers

- Mathematical operations on complex numbers may require conversions from one form to other form.

ADDITION: $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$

SUBTRACTION: $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$

Better performed in Rectangular form

MULTIPLICATION: $z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$

DIVISION: $\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \phi_1 - \phi_2$

RECIPROCAL: $\frac{1}{z} = \frac{1}{r} \angle -\phi$

SQUARE ROOT: $\sqrt{z} = \sqrt{r} \angle \phi/2$

Better performed in Polar form

COMPLEX CONJUGATE: $z^* = x - jy = r\angle -\phi = re^{-j\phi}$

Complex Numbers

✓ Example 1:

Evaluate $[\,(5+j2)(-1+j4) - 5\angle 60\,]^*$ and $\dfrac{10+j5+3\angle 40^\circ}{-3+j4} + 10\angle 30^\circ$

Solution:

(a)

$$(5+j2)(-1+j4) = -5+j20-j2-8 = -13+j18$$
$$5\angle 60^\circ = 2.5+j4.33$$
$$(5+j2)(-1+j4) - 5\angle 60^\circ = -15.5+j13.67$$
$$[\,(5+j2)(-1+j4) - 5\angle 60\,]^* = \underline{\mathbf{-15.5 - j13.67}} = \underline{\mathbf{20.67\angle 221.41^\circ}}$$

(b)

$$3\angle 40^\circ = 2.298+j1.928$$
$$10+j5+3\angle 40^\circ = 12.298+j6.928 = 14.115\angle 29.39^\circ$$
$$-3+j4 = 5\angle 126.87^\circ$$
$$\frac{10+j5+3\angle 40^\circ}{-3+j4} = \frac{14.115\angle 29.39^\circ}{5\angle 126.87^\circ} = 2.823\angle -97.48^\circ$$
$$2.823\angle -97.48^\circ = -0.3675 - j2.8$$
$$10\angle 30^\circ = 8.66+j5$$
$$\frac{10+j5+3\angle 40^\circ}{-3+j4} + 10\angle 30^\circ = \underline{\mathbf{8.293 + j2.2}}$$

Complex Numbers

- ✓ Example 2:

$$(4\angle 50^\circ)(2\angle -20^\circ) = 4.2\angle\left(50^\circ + (-20^\circ)\right) = 8\angle 30^\circ$$

- ✓ Example 3:

$$\frac{4\angle 50^\circ}{2\angle -20^\circ} = \frac{4}{2}\angle\left(50^\circ - (-20^\circ)\right) = 2\angle 70^\circ$$

Back to Phasor

Sinusoids and Phasor

Circuit elements in Phasor domain (frequency domain)

Impedance and admittance

Phasors

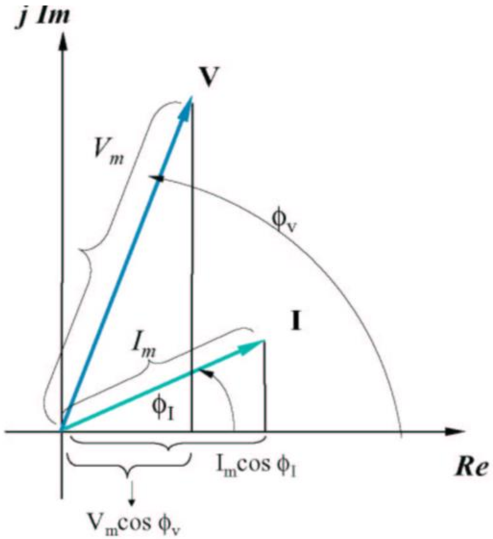
- A **phasor** is a **complex number** that represents the amplitude and phase of a sinusoid (**Polar Form**).
- Phasor is the mathematical equivalent of a sinusoid with **time variable dropped**.
- Given a sinusoid $v(t)=V_m\cos(\omega t+\varphi)$.

$$v(t) = V_m \cos(\omega t + \phi) \overset{\omega}{\longleftrightarrow} \mathbf{V} = V_m \angle \phi$$

(Time Domain Repr.) (Phasor Domain Representation)

Phasor Diagrams

- Amplitude and phase difference are two principal concerns in the study of voltage and current sinusoids.
- Phasor can be defined using either sine or cosine functions.
- By convention, source of the circuit is used as reference (zero phase angle).
- If not stated, cosine function is used as reference and any voltage or current expression is in the form of a sine is changed to a cosine:



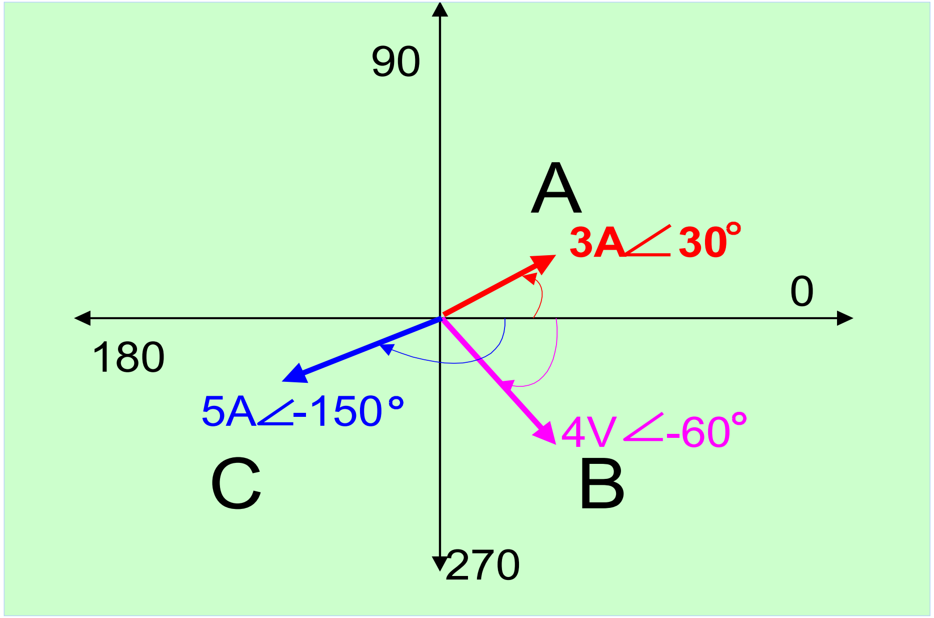
Phasor diagram

Time Domain Representation	Phasor Domain Rep.
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90°$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90°$

Phasor Diagrams

✓ Graphing phasors

- Positive phase angles are drawn counterclockwise from the axis;
- Negative phase angles are drawn clockwise from the axis.

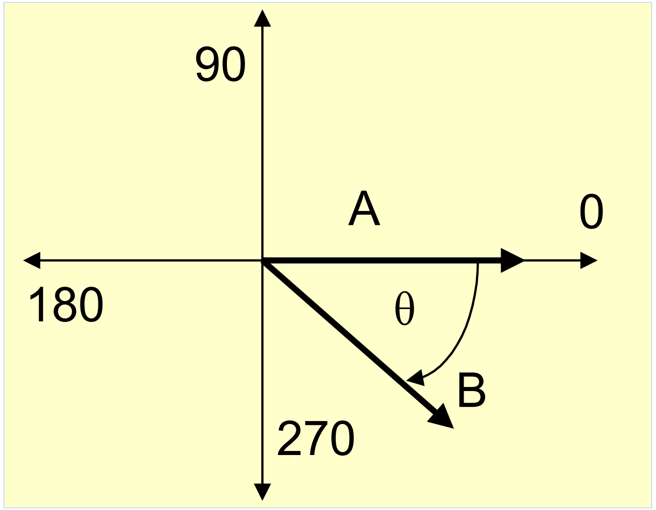


Note:
A leads B
B leads C
C lags A
etc.

Phasor Diagrams

✓ PHASOR DIAGRAM

- Represents one or more sine waves (of the same frequency) and the relationship between them.
- The arrows A and B rotate together. A leads B or B lags A.



Example 11

Transform the following sinusoids to phasors:

$i = 6\cos(50t - 40^o)$ A

$v = -4\sin(30t + 50^o)$ V

$$\sin(\omega t \pm 180^\circ) = -\sin\omega t$$
$$\cos(\omega t \pm 180^\circ) = -\cos\omega t$$
$$\sin(\omega t \pm 90^\circ) = \pm\cos\omega t$$
$$\cos(\omega t \pm 90^\circ) = \mp\sin\omega t$$

Solution:

a. $\mathrm{I} = 6\angle -40^\circ$ A

b. Since $-\sin(A) = \cos(A+90^o)$;

$v(t) = 4\cos(30t+50^o+90^o) = 4\cos(30t+140^o)$ V

Transform to phasor => $\mathrm{V} = 4\angle 140^\circ$ V

Time Domain Versus Phasor Domain

The differences between v(t) and V:

- v(t) is instantaneous or time-domain representation V is the frequency or phasor-domain representation.
- v(t) is time dependent, V is not.
- v(t) is always real with no complex term, V is generally complex.

Note: Phasor analysis applies only when frequency is constant; when it is applied to two or more sinusoid signals only if they have the same frequency.

Differentiation and Integration in Phasor Domain

- **Differentiating** a sinusoid is equivalent to **multiplying** its corresponding phasor by $j\omega$.

$$v(t) = V_m \cos(\omega t + \theta) = \text{Re}\left[\mathbf{V} e^{j\omega t}\right]$$

$$\frac{dv(t)}{dt} = -\omega V_m \sin(\omega t + \theta) = \omega V_m \cos(\omega t + \theta + 90^\circ)$$

which transforms to the phasor

$$\frac{dv(t)}{dt} = \omega V_m e^{j(\theta + 90^\circ)} = \omega V_m \angle \phi + 90^\circ$$

$$\frac{dv(t)}{dt} = \omega\, e^{j\,90^\circ}\, V_m e^{j\,\theta}$$

$$\frac{dv}{dt} \quad \Leftrightarrow \quad J\omega \mathbf{V}$$

$$\sin(\omega t \pm 180^\circ) = -\sin \omega t$$
$$\cos(\omega t \pm 180^\circ) = -\cos \omega t$$
$$\sin(\omega t \pm 90^\circ) = \pm \cos \omega t$$
$$\cos(\omega t \pm 90^\circ) = \mp \sin \omega t$$

$$e^{\pm j\phi} = \cos\phi \pm j \sin\phi$$

- **Integrating** a sinusoid is equivalent to **dividing** its corresponding phasor by $j\omega$.

Differentiation and Integration in Phasor Domain

- Relationship between differential, integral operation in phasor listed as follow:

(Time Domain)		(Phasor Domain)
$v(t) = V_m \cos(\omega t + \phi)$	$\Leftrightarrow$	$\mathbf{V} = V_m \angle \phi$
$v(t) = V_m \sin(\omega t + \phi)$	$\Leftrightarrow$	$\mathbf{V} = V_m \angle \phi - 90°$
$\dfrac{dv}{dt}$	$\Leftrightarrow$	$J\omega\mathbf{V}$
$\int v dt$	$\Leftrightarrow$	$\dfrac{\mathbf{V}}{J\omega}$

Example 9.7: Using Phasor to solve integro-differential equations

Example 9.7

Use phasor approach, determine the current *i(t)* in a circuit described by the integro-differential equation.

$$4i + 8\int i\,dt - 3\frac{di}{dt} = 50\cos(2t + 75°)$$

- Solution:

$$4\mathbf{I} + \frac{8\mathbf{I}}{j\omega} - 3j\omega\mathbf{I} = 50\angle 75°$$

But $\omega = 2$, so

$$\mathbf{I}(4 - j4 - j6) = 50\angle 75°$$

$$\mathbf{I} = \frac{50\angle 75°}{4 - j10} = \frac{50\angle 75°}{10.77\angle -68.2°} = 4.642\angle 143.2° \text{ A}$$

Converting this to the time domain,

$$i(t) = 4.642\cos(2t + 143.2°) \text{ A}$$

$v(t)$	$\longleftrightarrow$	$V = V\angle\phi$
$\frac{dv}{dt}$	$\longleftrightarrow$	$j\omega V$
$\int v\,dt$	$\longleftrightarrow$	$\frac{V}{j\omega}$

Phasor Relationships for Circuit Elements

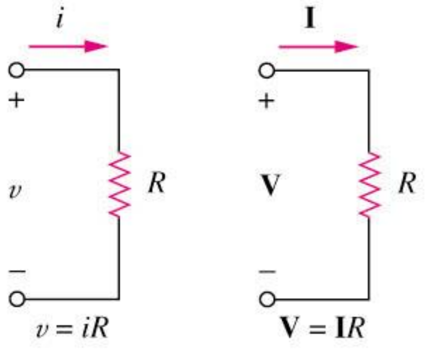
- Ohm's Law still applies even though the voltage source is AC

Terminal Equations 4

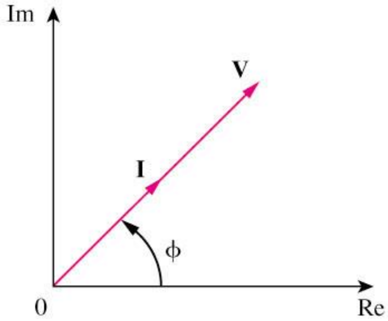
Time Domain		Frequency Domain	
R, $i(t)$, $+\ v(t)\ -$	$v(t) = Ri(t)$	R, $\hat{I}$, $+\ \hat{V}\ -$	$\hat{V} = R\hat{I}$ $\hat{I} = G\hat{V}$
L, $i(t)$, $+\ v(t)\ -$	$v(t) = L\dfrac{di(t)}{dt}$	jωL, $\hat{I}$, $+\ \hat{V}\ -$	$\hat{V} = j\omega L\hat{I}$ $\hat{I} = \dfrac{1}{j\omega L}\hat{V}$
C, $i(t)$, $+\ v(t)\ -$	$i(t) = C\dfrac{dv(t)}{dt}$ $v(t) = \dfrac{1}{C}\int i(t)dt$	C, $\hat{I}$, $+\ \hat{V}\ -$	$\hat{V} = \dfrac{1}{j\omega C}\hat{I}$ $\hat{I} = j\omega C\hat{V}$

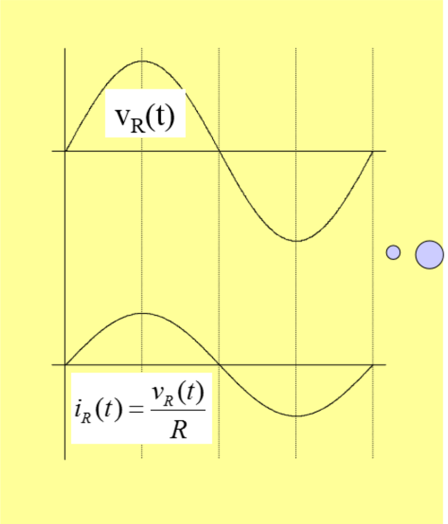
Phasor Relationships for Circuit Elements

A- Resistor:

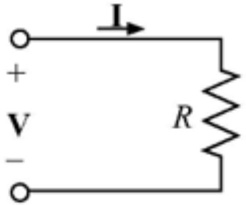


(a) (b)

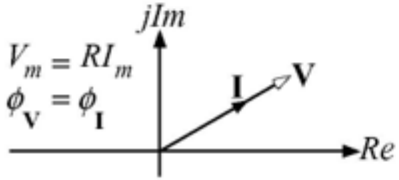




PHASE ANGLE FOR R, θ=0°



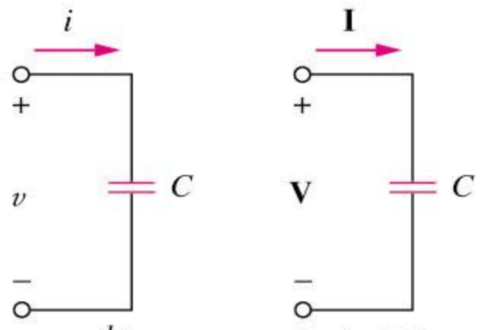
For R, I and V are in phase



(a)

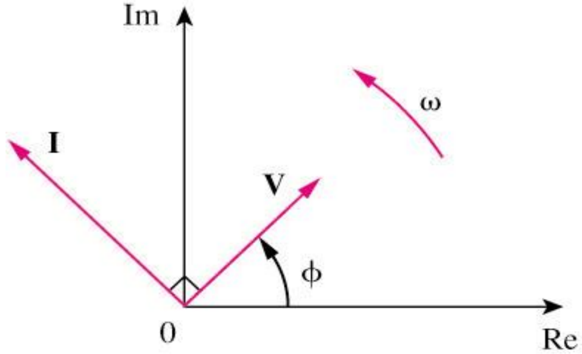
Phasor Relationships for Circuit Elements

B- Capacitor:



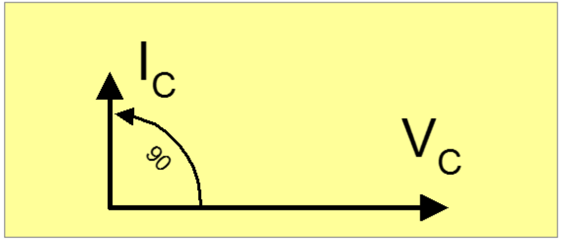
$i = C\frac{dv}{dt}$ $\mathbf{I} = j\omega C\mathbf{V}$

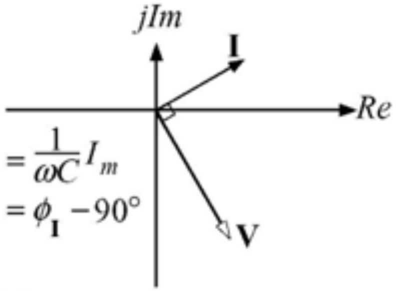
(a)   (b)



- In the Capacitor (C), Voltage **LAGS** charging current by 90º <u>or</u> Charging Current (I) **LEADS** Voltage (E) by 90º

- <u>**I. C. E.**</u>



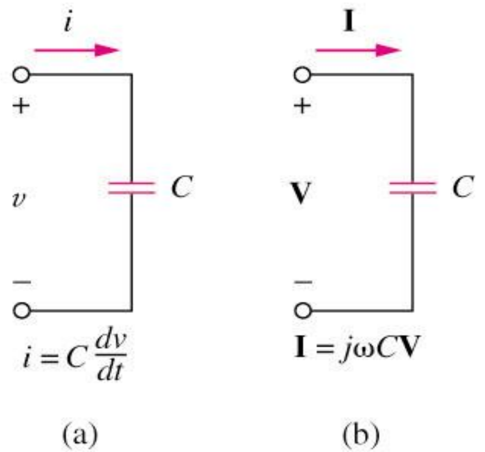


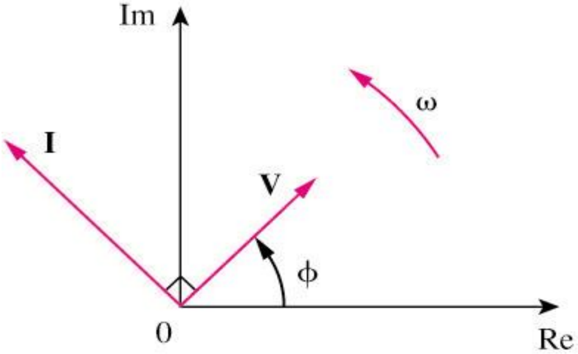
$V_m = \frac{1}{\omega C} I_m$

$\phi_V = \phi_I - 90°$

Phasor Relationships for Circuit Elements

B- Capacitor:





❖ **CAPACITIVE REACTANCE**

- In resistor, the Ohm's Law is V=IR, where R is the opposition to current.
- We will define Capacitive Reactance, X_C, as the opposition to current in a capacitor.

$$V = I\,X_C$$

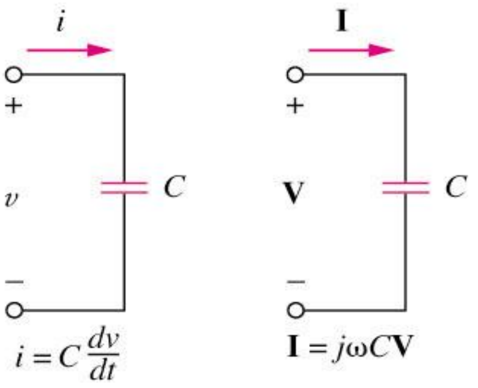
- X_C will have units of Ohms.
- Note inverse proportionality to *f* and C.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{\omega C}$$

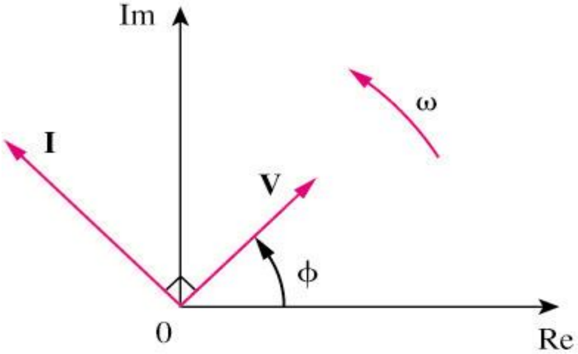
Magnitude of X_C

Phasor Relationships for Circuit Elements

B- Capacitor:

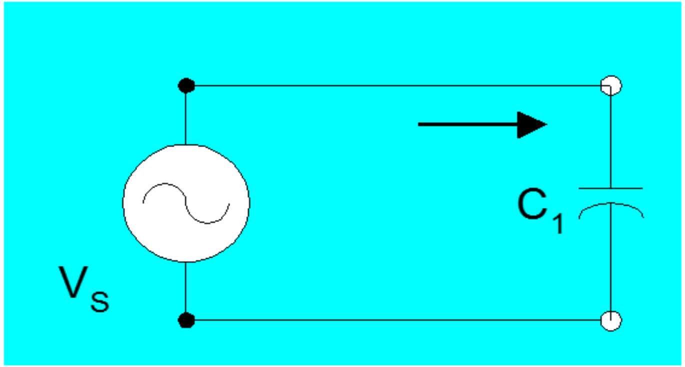


(a) (b)



❖ **CAPACITIVE REACTANCE**

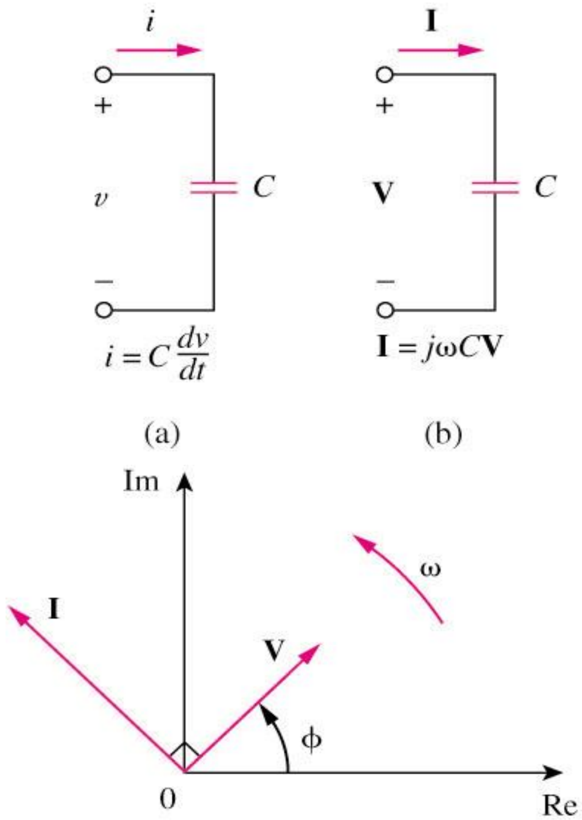
Ex: f = 500 Hz, C = 50 μF, X_C = ?



$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(500\,Hz)50\,\mu F} = 6.4\Omega$$

Phasor Relationships for Circuit Elements

B- Capacitor:



❖ PHASE ANGLE FOR X_C

- Capacitive reactance also has a phase angle associated with it.

$$X_C = \frac{\mathbf{V}}{\mathbf{I}}$$

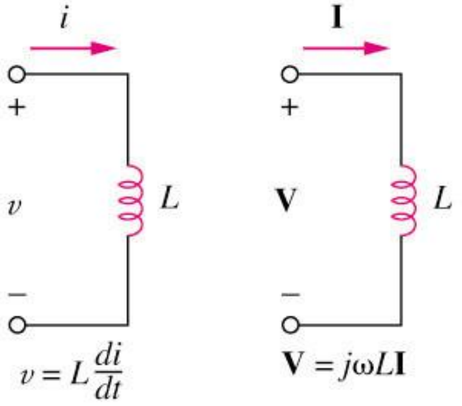
$$X_C = \frac{V\angle 0^\circ}{I\angle 90^\circ} = Z\angle -90^\circ$$

- The phase angle for Capacitive Reactance (**X_C**) will always = -90°
- **X_C** may be expressed in POLAR or RECTANGULAR form.

$$X_C\angle -90^\circ \quad \text{or} \quad -jX_C$$

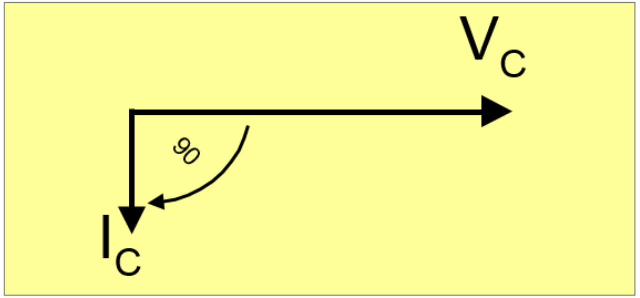
Phasor Relationships for Circuit Elements

C-Inductor:

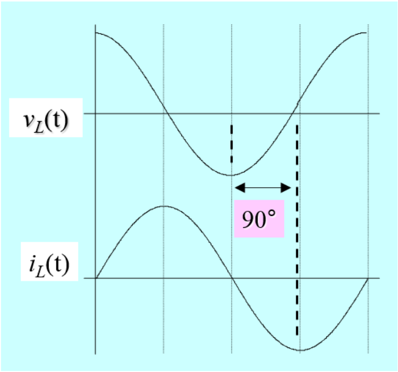


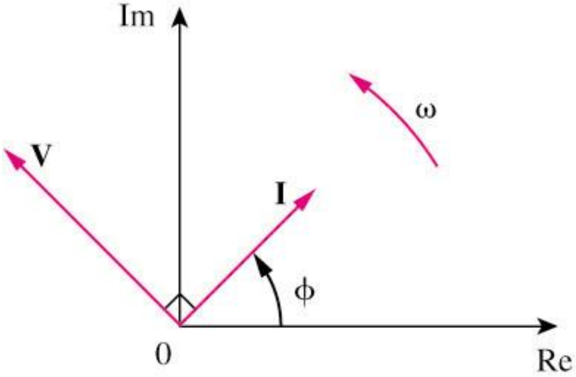
(a) (b)

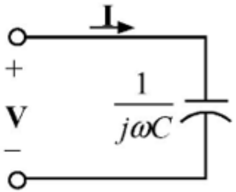
- In the Inductor (L), Induced Voltage LEADS current by 90° or Current (I) LAGS Induced Voltage (E) by 90°.



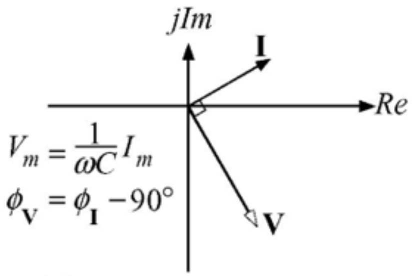
Graph $v_L(t)$ and $i_L(t)$







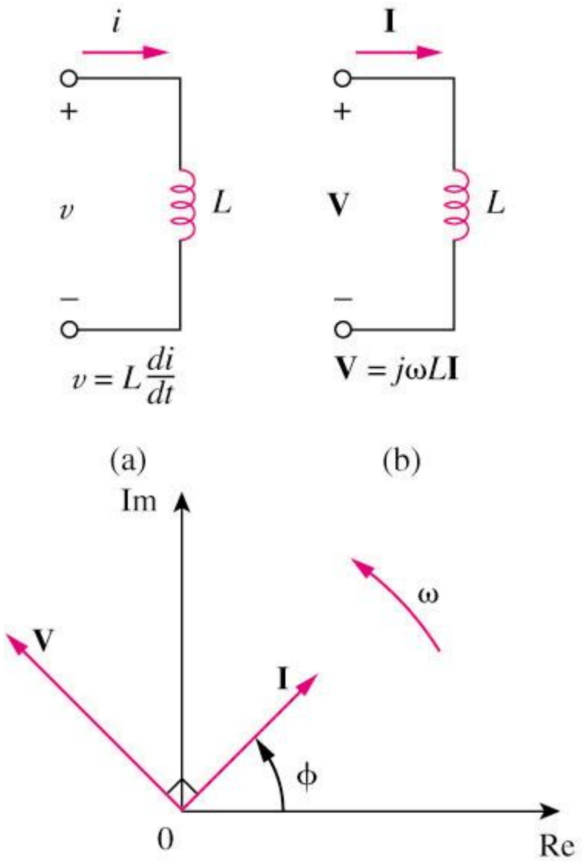
For C, I Leads V by 90°



(c)

Phasor Relationships for Circuit Elements

C-Inductor:



❖ **INDUCTIVE REACTANCE**

❖ We will define Inductive Reactance, X_L, as the opposition to current in an inductor.

$$V = I X_L$$

❖ X_L will have units of Ohms (Ω).

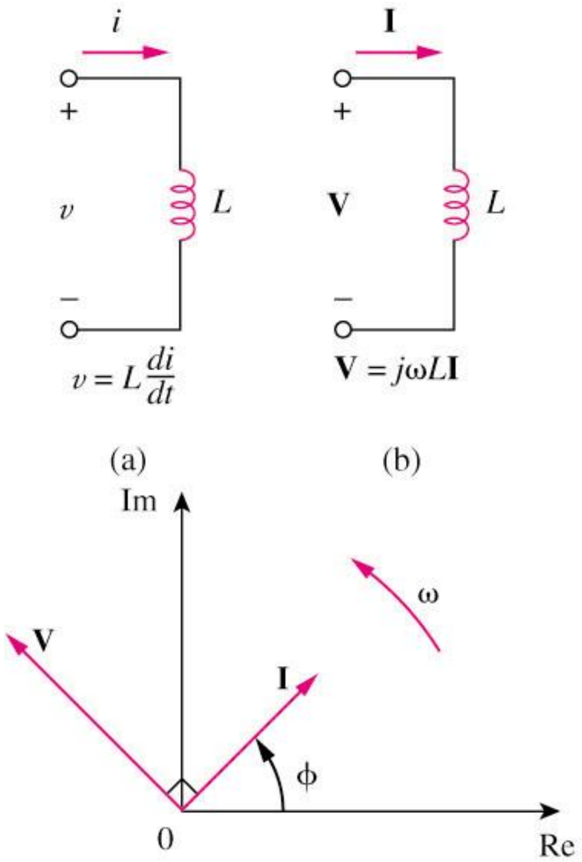
❖ Note direct proportionality to f and L.

$$X_L = 2\pi fL = \omega L$$

Magnitude of X_L

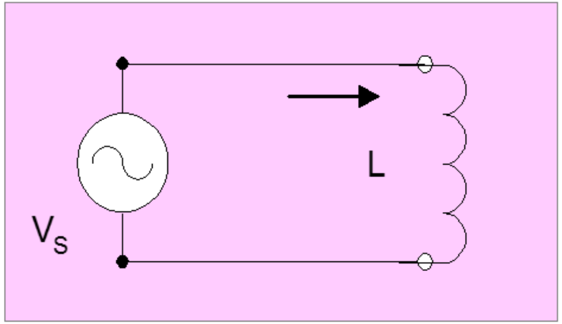
Phasor Relationships for Circuit Elements

C-Inductor:



❖ **INDUCTIVE REACTANCE**

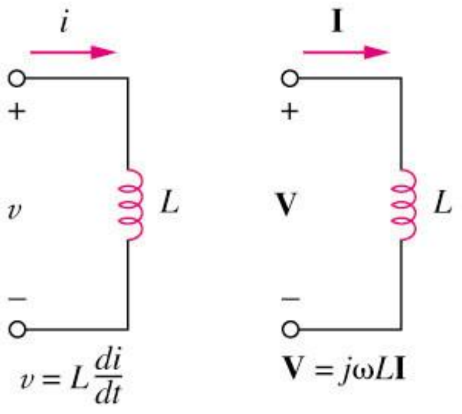
f = 500 Hz, C = 500 mH, X_L = ?



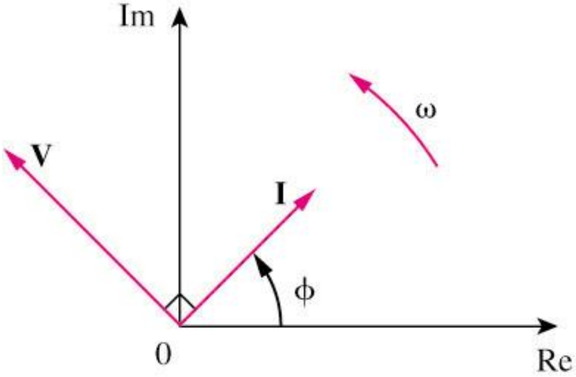
$$X_L = 2\pi fL = 2\pi(500)(.5) = 1.57k\Omega$$

Phasor Relationships for Circuit Elements

C-Inductor:



(a) (b)



❖ **PHASE ANGLE FOR X_L**

❖ If V is our reference wave:

$$\mathbf{X}_L = \frac{V\angle 0°}{I\angle -90°} = Z\angle +90°$$

❖ The phase angle for Inductive Reactance ($\mathbf{X}_L$) will always = +90°

❖ $\mathbf{X}_L$ may be expressed in POLAR or RECTANGULAR form.

$$X_L\angle 90° \quad \text{or} \quad jX_L$$

Phasor Relationships for Circuit Elements

✓ **COMPARISON OF X_L & X_C**

- X_L is directly proportional to frequency and inductance.

$$X_L = 2\pi f L = \omega L$$

- X_C is inversely proportional to frequency and capacitance.

$$X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C}$$

Phasor Relationships for Circuit Elements

❖ Frequency effects

- Using the reactances of an inductor and a capacitor you can show the effects of low and high frequencies on them.

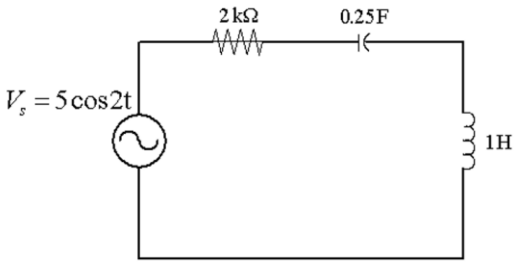
$$X_C = \frac{1}{2\pi fC} = \frac{1}{\omega C}$$

$$X_L = 2\pi fL = \omega L$$

- At low frequencys (f=0):
  - an inductor acts like a short circuit.
  - a capacitor acts like an open circuit.
- At high frequencys (f=∞):
  - an inductor acts like an open circuit.
  - a capacitor acts like a short circuit.

Example

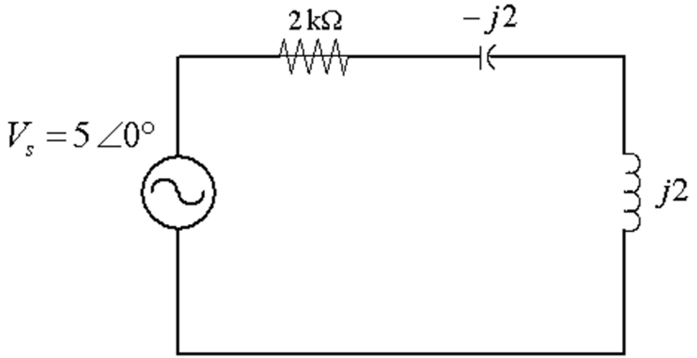
Represent the below circuit in frequency domain:



Solution:

- ω=2 rad/s:

Time domain	Freq domain
R = 2 Ω	R = 2 Ω
C = 0.25 F	X_C = -j(1/ωC) = -j2 Ω
L = 1 H	X_L = jωL = j2 Ω
V_s = 5 cos 2t	V_s = 5<0°



Example 9.8

The voltage $v = 12\cos(60t + 45°)$ is applied to a 0.1-H inductor. Find the steady-state current through the inductor.

Solution:

For the inductor, $\mathbf{V} = j\omega L\mathbf{I}$, where $\omega = 60$ rad/s and $\mathbf{V} = 12\underline{/45°}$ V. Hence,

$$\mathbf{I} = \frac{\mathbf{V}}{j\omega L} = \frac{12\underline{/45°}}{j60 \times 0.1} = \frac{12\underline{/45°}}{6\underline{/90°}} = 2\underline{/-45°}\text{ A}$$

Converting this to the time domain,

$$i(t) = 2\cos(60t - 45°)\text{ A}$$

Using phasor to add sinusoids- Example 9.6

Given $i_1(t) = 4\cos(\omega t + 30^\circ)$ A and $i_2(t) = 5\sin(\omega t - 20^\circ)$ A, find their sum.

Solution:

Here is an important use of phasors—for summing sinusoids of the same frequency. Current $i_1(t)$ is in the standard form. Its phasor is

$$\mathbf{I}_1 = 4\underline{/30^\circ}$$

We need to express $i_2(t)$ in cosine form. The rule for converting sine to cosine is to subtract 90°. Hence,

$$i_2(t) = 5\sin(\omega t - 20^\circ) = 5\cos(\omega t - 20^\circ - 90^\circ) = 5\cos(\omega t - 110^\circ)$$

and its phasor is

$$\mathbf{I}_2 = 5\underline{/-110^\circ}$$

If we let $i = i_1 + i_2$, then

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 4\underline{/30^\circ} + 5\underline{/-110^\circ}$$
$$= 3.464 + j2 - 1.71 - j4.698 = 1.754 - j2.698$$
$$= 3.218\underline{/-56.97^\circ} \text{ A}$$

$$\sin(\omega t \pm 180^\circ) = -\sin\omega t$$
$$\cos(\omega t \pm 180^\circ) = -\cos\omega t$$
$$\sin(\omega t \pm 90^\circ) = \pm\cos\omega t$$
$$\cos(\omega t \pm 90^\circ) = \mp\sin\omega t$$

Impedance and Admittance

A-Impedance

- The impedance Z of a circuit is the ratio of the phasor voltage V to the phasor current I, measured in ohms Ω.

$$Z = \frac{V}{I} = R + jX$$

➢ Impedance is a complex quantity:

- R = Real part of Z = Resistance
- X = Imaginary part of Z = Reactance

Impedances of passive elements	
Element	**Impedance**
R	$Z = R$
L	$Z = j\omega L$
C	$Z = \frac{1}{j\omega C}$

- Impedance in polar form:

$$Z = \frac{V}{I} = R + jX = |Z|\angle\theta$$

$$|Z| = \sqrt{R^2 + X^2} \qquad \theta = \tan^{-1}\frac{X}{R}$$

$$R = |Z|\cos\theta \qquad X = |Z|\sin\theta$$

➢ The Reactance may be positive or negative:

- It is Inductive if X is positive.
- It is Capacitive if X is negative.

IMPEDANCES SUMMARY

Impedance	Phasor form:	Rectangular form
Z_R	$R\angle 0^o$	R+j0
Z_L	$X_L\angle 90^o$	$0+jX_L$
Z_C	$X_C\angle -90^o$	$0-jX_C$

B- ADMITTANCE

- ✓ The reciprocal of impedance.
- ✓ Symbol is Y.
- ✓ Measured in Siemens (S).

$$Y = \frac{1}{Z} = \frac{I}{V}$$

➢ Admittance is a complex quantity:

$$\mathrm{Y} = \mathrm{G} \pm j\mathrm{B}$$

- G = Real part of Y : Conductance
- B = Imaginary part of Y : Susceptance

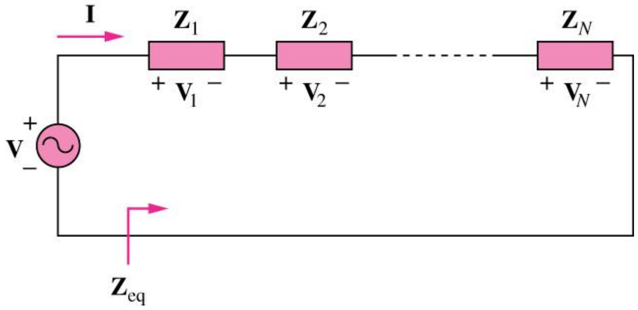
Z AND Y OF PASSIVE ELEMENTS

ELEMENT	IMPEDANCE	ADMITTANCE
R	$Z = R$	$Y = \frac{1}{R}$
L	$Z = j\omega L$	$Y = \frac{1}{j\omega L}$
C	$Z = -j\frac{1}{\omega C}$	$Y = j\omega C$

C-TOTAL IMPEDANCE FOR AC CIRCUITS

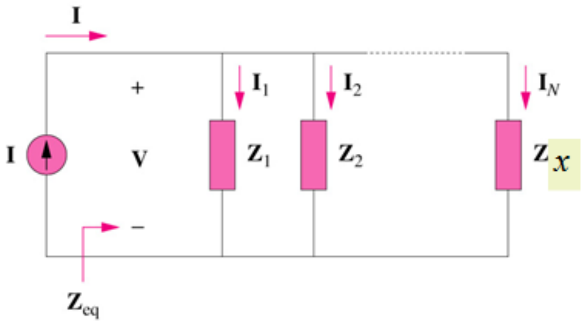
- To compute total circuit impedance in AC circuits, use the same techniques as in DC.
- The only difference is that instead of using resistors, you now have to use **complex impedance, Z.**

❖ For series connected impedances:



$$Z_{eq} = \frac{V}{I} = Z_1 + Z_2 + \cdots + Z_N$$ (Equivalent Impedance)

❖ Total impedance for parallel circuit:



$$\frac{1}{Z_{total}} = \sum \frac{1}{Z_x} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_x} \Rightarrow$$

$$Z_{total} = \left(\sum \frac{1}{Z_x} \right)^{-1} = \left(\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_x} \right)^{-1}$$

- The impedance can be easily computed from the admittance:

$$Z_{total} = \frac{1}{Y_{total}} = [Y_{total}]^{-1}$$

$$\therefore Y_{total} = (Y_1 + Y_2 + \ldots + Y_x)$$

C-TOTAL IMPEDANCE FOR AC CIRCUITS

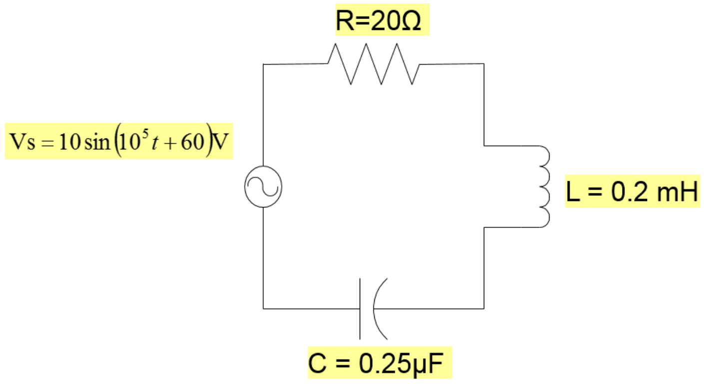
1-Example of series circuit:

a- Find Total Impedance

b- Draw Impedance Triangle (Phasor Diagram).

c- Find i_s, v_R, v_C, v_L

d- Find v_R, v_C, v_L using Voltage Divider

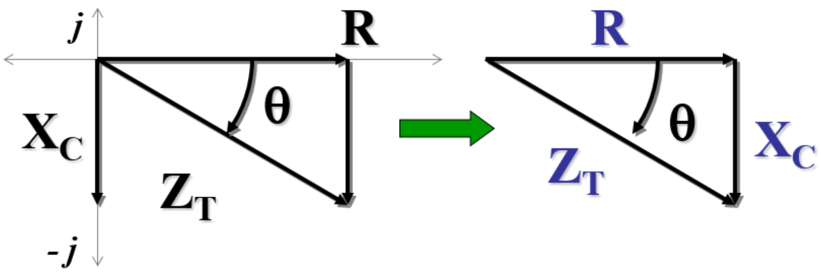


✓ **Solution**

➢ a- The total Impedance

$$
\begin{aligned}
Z_T &= \sum Z_R + Z_L + Z_C \\
&= 20 + j20 + (-j40) \\
&= (20 - j20)\Omega \\
&= 28.28\angle -45^\circ \Omega
\end{aligned}
$$

➢ b- The Impedance Triangle (Phasor Diagram)



C-TOTAL IMPEDANCE FOR AC CIRCUITS

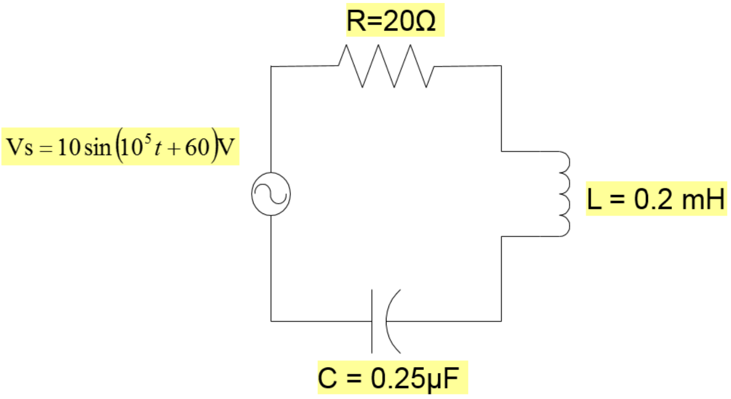
1-Example of series circuit:

a- Find Total Impedance

b-Draw Impedance Triangle (Phasor Diagram)

c- Find i_s, v_R, v_C, v_L

d- Find v_R, v_C, v_L using Voltage Divider



✓ **Solution**

- c- i_s, v_R, v_C, v_L :

$$i_s = \frac{v_s}{Z_T} = \frac{10\ \angle 60^\circ}{28.28\angle -45^\circ}$$

$$= 353\ \angle 105^\circ \quad mA$$

$$v_s = i_s \cdot Z_R = (353\angle 105^\circ\ mA)\cdot(20\ \Omega) = 7\ \angle 105^\circ\ V$$

$$v_L = i_s \cdot Z_L = (353\angle 105^\circ\ mA)\cdot(20\angle 90^\circ\ \Omega) = 7\ \angle 195^\circ\ V$$

$$v_C = i_s \cdot Z_C = (353\angle 105^\circ\ mA)\cdot(40\angle -90^\circ\ \Omega) = 7\ \angle 15^\circ\ V$$

C-TOTAL IMPEDANCE FOR AC CIRCUITS

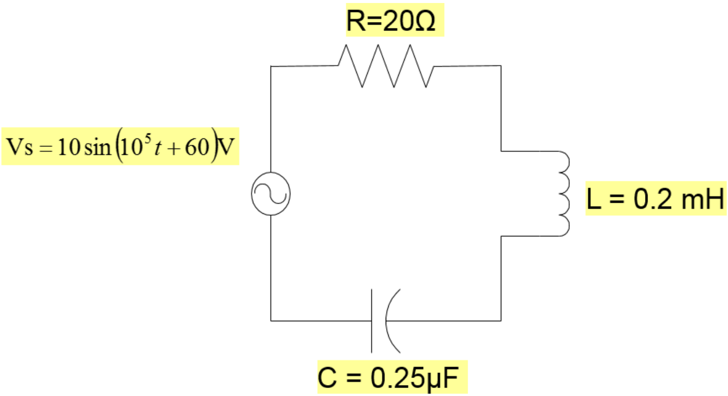
1-Example of series circuit:

a- Find Total Impedance

b-Draw Impedance Triangle (Phasor Diagram)

c- Find i_s, v_R, v_C, v_L

d- Find v_R, v_C, v_L using Voltage Divider



✓ **Solution**

- d- Find v_R, v_C, v_L using Voltage Divider
- Voltage divider still works, too.

$$v_R = \frac{Z_R}{Z_T} v_s = \frac{(20\Omega\angle 0^o)}{(28.28\ \angle -45^o)} \cdot 10\,V\angle 60^o \quad = 7\ \angle 105^o$$

$$v_L = \frac{Z_L}{Z_T} v_s = \frac{(20\Omega\angle 90^o)}{(28.28\ \angle -45^o)} \cdot 10\,V\angle 60^o \quad = 7\ \angle 195^o$$

$$v_C = \frac{Z_C}{Z_T} v_s = \frac{(20\Omega\angle -90^o)}{(28.28\ \angle -45^o)} \cdot 10\,V\angle 60^o \quad = 7\ \angle 15^o$$

C-TOTAL IMPEDANCE FOR AC CIRCUITS

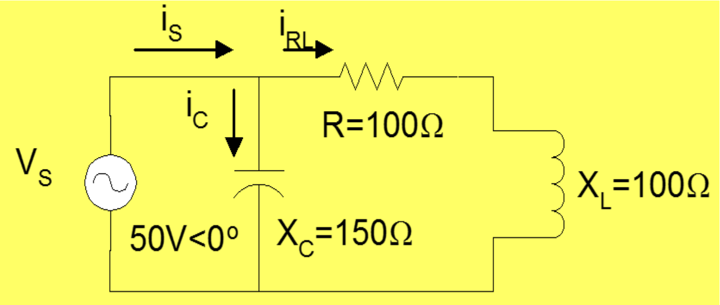
2-Example of Parallel circuit:

a- Find the total Impedance

b-Draw Impedance Triangle (Phasor Diagram)

c- Find i_s, v_R, v_C, v_L

d- Find v_R, v_C, v_L using Voltage Divider



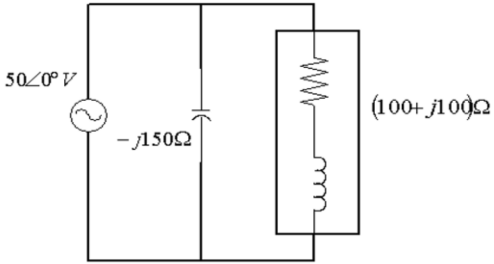
✓ **Solution**

➢ a- The total Impedance

➢ The circuit given is already in frequency domain:

$$Z_R = 100\Omega\angle 0^o \quad = 100\,\Omega$$
$$Z_C = 150\Omega\angle -90^o = -j150\,\Omega$$
$$Z_L = 100\Omega\angle 90^o \quad = j100\,\Omega$$

➢ **Circuit simplification**



C-TOTAL IMPEDANCE FOR AC CIRCUITS

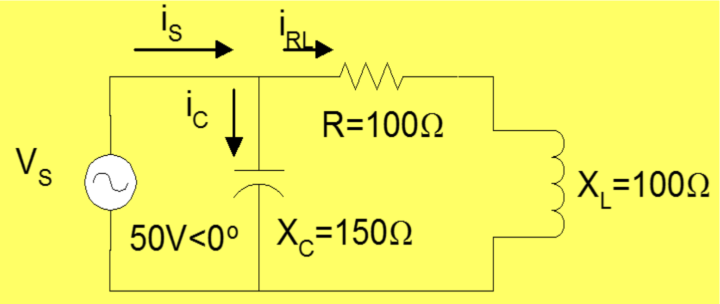
2-Example of Parallel circuit:

a- Find the total Impedance

b-Draw Impedance Triangle (Phasor Diagram)

c- Find i_s, v_R, v_C, v_L

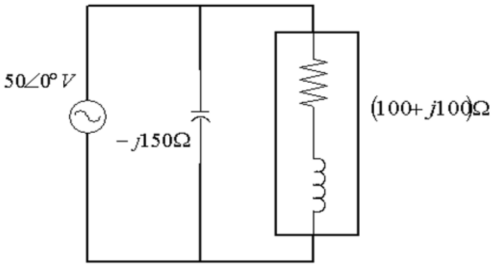
d- Find v_R, v_C, v_L using Current Divider



✓ **Solution**

➢ a- The total Impedance

➢ The circuit given is already in frequency domain:



$$Z_{total} = (Z_C)\|(Z_R + Z_L)$$
$$= (150\angle -90°)\|[(100) + (100\angle 90°)]$$
$$= (150\angle -90°)\|(100 + j100)$$
$$= (150\angle -90°)\|(141.42\angle 45°)$$
$$\therefore Z_{total} = [Y_{total}]^{-1}$$
$$= \left[\frac{1}{150\angle -90°} + \frac{1}{141.42\angle 45°}\right]^{-1}$$

$$\therefore Z_{total} = \left(\frac{1}{150}(\cos(90°) + j\sin(90°)) + \frac{1}{144.42}(\cos(-45°) + j\sin(-45°))\right)^{-1}$$

$$= (0.005 \quad +j0.0016)^{-1}$$

$$= \left(\left(\sqrt{0.005^2 + 0.0016^2}\right) < \tan^{-1}\left(\frac{0.0016}{0.005}\right)\right)^{-1}$$

$$= (0.00524 \quad < 17.744°)^{-1} \simeq 190<-18°$$

$$\therefore Z_{total} \simeq 180 - j60\,\Omega$$

C-TOTAL IMPEDANCE FOR AC CIRCUITS

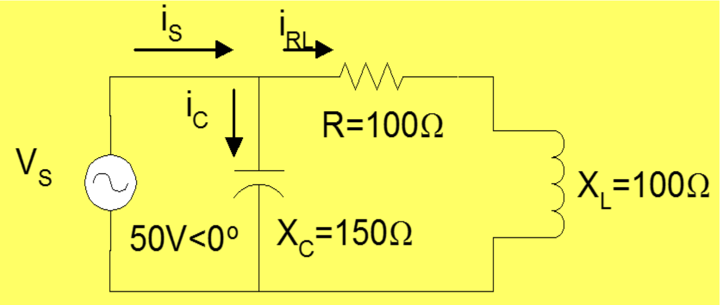
2-Example of Parallel circuit:

a- Find the total Impedance

b-Draw Impedance Triangle (Phasor Diagram)

c- Find i_s, v_R, v_C, v_L

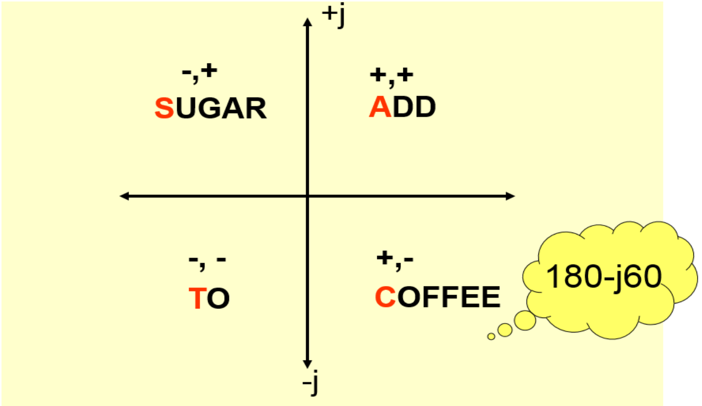
d- Find v_R, v_C, v_L using Current Divider

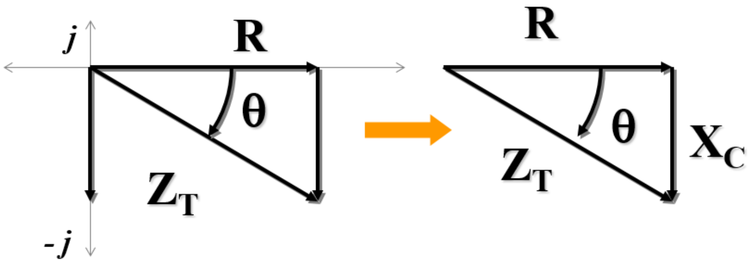


✓ **Solution**

➢ b- The Impedance Triangle (Phasor Diagram)

$$\therefore Z_{total} \simeq 180 - j60\,\Omega$$





C-TOTAL IMPEDANCE FOR AC CIRCUITS

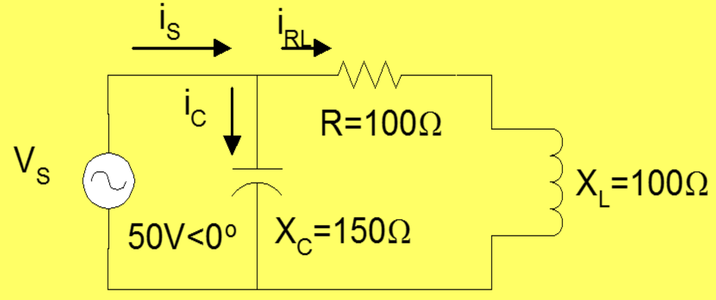
2-Example of Parallel circuit:

a- Find the total Impedance

b-Draw Impedance Triangle (Phasor Diagram)

c- Find i_s, v_R, v_C, v_L

d- Find v_R, v_C, v_L using Current Divider



✓ **Solution**

➢ c- i_s, v_R, v_C, v_L :

$$\therefore Z_{total} \simeq 180 - j60\,\Omega$$

$$i_s = \frac{v_s}{Z_T} = \frac{50\angle 0^\circ}{190\angle -18^\circ} = 263\angle 18^\circ\ mA$$

$$v_S = v_C = (v_R + v_L) = 50\angle 0^\circ\ V$$

$$\Rightarrow i_C = \frac{v_S}{Z_C} = \frac{50\angle 0^\circ}{150\angle -90^\circ} = 333\angle 90^\circ\ mA$$

$$\Rightarrow i_{RL} = \frac{v_S}{Z_R + Z_L} = \frac{50\angle 0^\circ}{100 + (100\angle -90^\circ)} = 353\angle -45^\circ\ mA$$

C-TOTAL IMPEDANCE FOR AC CIRCUITS

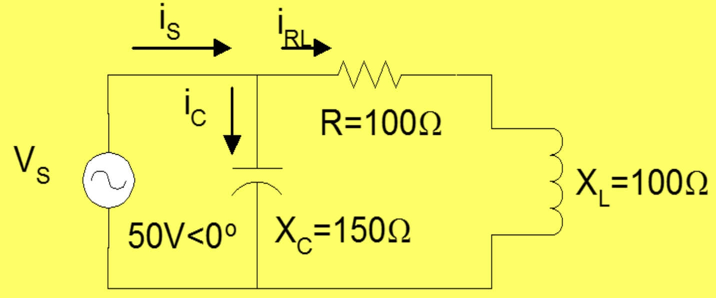
2-Example of Parallel circuit:

a- Find the total Impedance

b-Draw Impedance Triangle (Phasor Diagram)

c- Find i_s, v_R, v_C, v_L

d- Find v_R, v_C, v_L using Current Divider



✓ **Solution** ➢ d- Find v_R, v_C, v_L using Current Divider

$$i_c = \frac{Z_T}{Z_C} i_S = \frac{(190\angle -18°)}{(150\angle -90°)} \cdot (263\angle 18°\ mA)$$
$$= 333\angle 90°\ mA$$

$$i_{RL} = \frac{Z_T}{Z_R + Z_L} i_S = \frac{(190\angle -18°)}{(100)(100\angle 90°)} \cdot (263\angle 18°\ mA)$$
$$= 353\angle -45°\ mA$$

D-Impedance as a Function of Frequency

Element	Impedance
L	$Z = j\omega L$
C	$Z = \dfrac{1}{j\omega C}$

- The Impedance Z of a circuit is a function of the frequency.

- Inductor is SHORT CIRCUIT at DC and OPEN CIRCUIT at high frequencies.
  Capacitor is OPEN CIRCUIT at DC and SHORT CIRCUIT at high frequencies.

$$Z_L = j\omega L$$

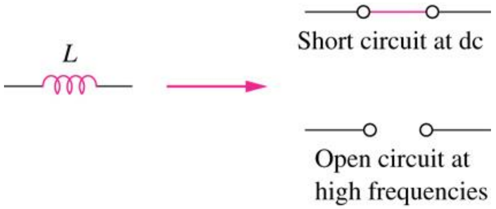
$$Z_L \to 0 \quad \omega \to 0 \quad \text{(Short at DC)}$$

$$Z_L \to \infty \quad \omega \to \infty \; (\text{Open as } \omega \to \infty)$$

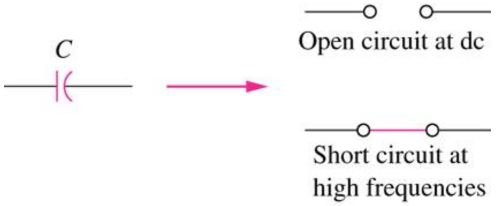
$$Z_C = \frac{1}{j\omega C}$$

$$Z_C \to \infty \quad \omega \to 0 \quad \text{(Open at DC)}$$

$$Z_C \to 0 \quad \omega \to \infty \quad (\text{Open as } \omega \to \infty)$$



(a)



(b)

AC Analysis Techniques

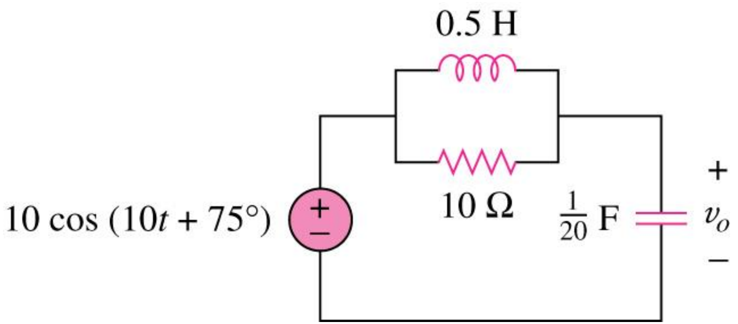
AC Analysis Techniques

- Both KVL and KCL are hold in Phasor domain or more commonly called frequency domain.
- Moreover, the variables to be handled are phasors, which are complex numbers.
- All the mathematical operations involved are now in complex domain.

- **All DC circuit analysis principles apply to AC circuits.**
  - Voltage Division
  - Current Division
  - Circuit Reduction
  - Impedance equivalence
  - Y-Delta Transformation

Example 22: Voltage Divider Rule

- Calculate the v0 in the given circuit



- Solution:

In the frequency domain,

the voltage source is $\mathbf{V}_s = 10\angle 75°$

the 0.5-H inductor is $j\omega L = j\,(10)(0.5) = j5$

the $\frac{1}{20}$-F capacitor is $\frac{1}{j\omega C} = \frac{1}{j(10)(1/20)} = -j2$

Let $\mathbf{Z}_1$ = impedance of the 0.5-H inductor in parallel with the 10-Ω resistor

and $\mathbf{Z}_2$ = impedance of the (1/20)-F capacitor

$$\mathbf{Z}_1 = 10 \,||\, j5 = \frac{(10)(j5)}{10 + j5} = 2 + j4 \qquad \text{and} \qquad \mathbf{Z}_2 = -j2$$

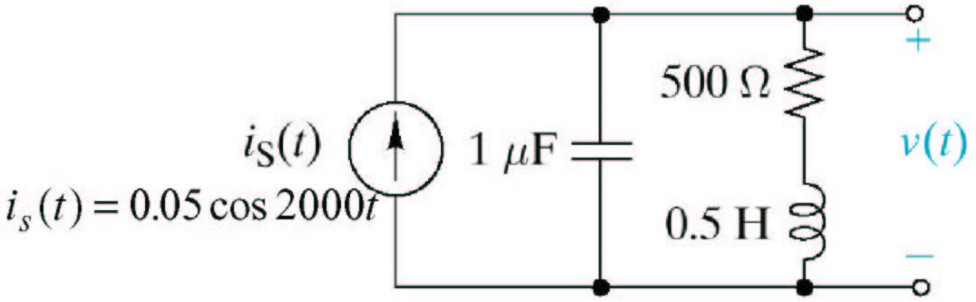
$$\mathbf{V}_o = \mathbf{Z}_2 / (\mathbf{Z}_1 + \mathbf{Z}_2)\,\mathbf{V}_s$$

$$\mathbf{V}_o = \frac{-j2}{2 + j4 - j2}(10\angle 75°) = \frac{-j(10\angle 75°)}{1 + j} = \frac{10\angle(75° - 90°)}{\sqrt{2}\angle 45°} = 7.071\angle\text{-}60°$$

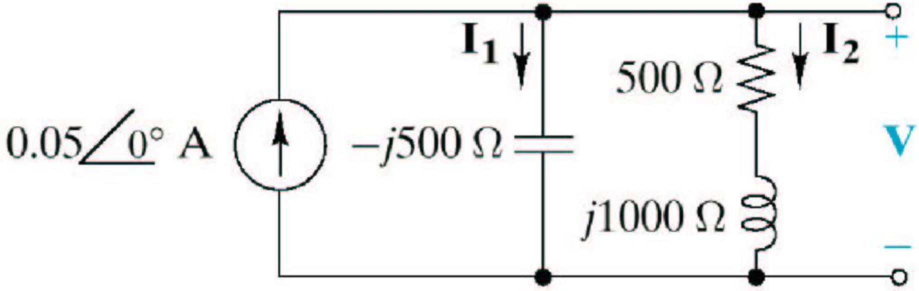
$$v_o(t) = \underline{\mathbf{7.071\cos(10t - 60°)\ V}}$$

Example 23: Current Divider Rule

Find I_1 and I_2 in phasor domain, then convert them into time domain

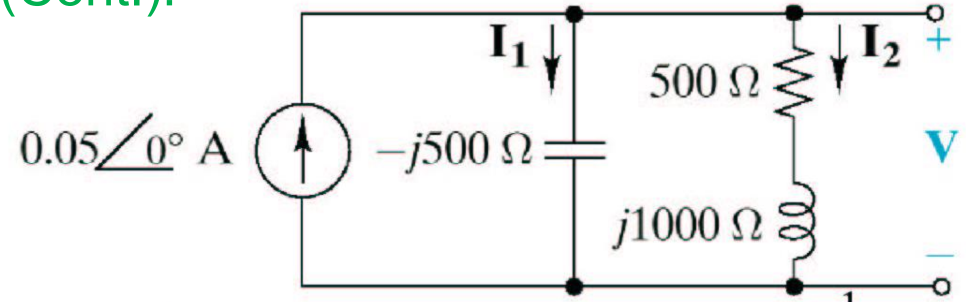


Solution:



Example 23: Current Divider Rule

Solution (Cont.):



$$\hat{I}_1 = \frac{(R + j\omega L)}{R + j\omega L + \frac{1}{j\omega C}}\hat{I}$$

$$\hat{I}_2 = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}}\hat{I}$$

$$\hat{I}_1 = \frac{(500 + j1000)}{500 + j1000 - j500}0.05\angle 0^\circ$$

$$\hat{I}_2 = \frac{-j500}{500 + j1000 - j500}0.05\angle 0^\circ$$

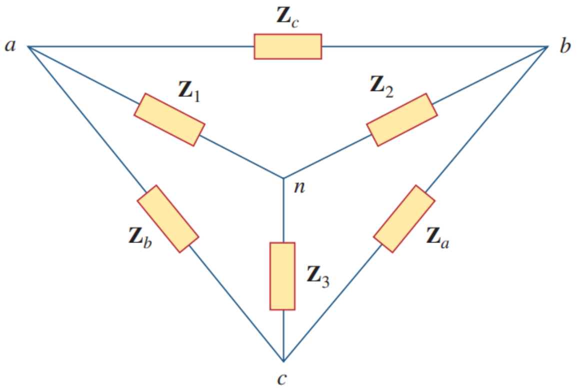
$$I_1 = 0.079\angle 18.4^o \text{ A}$$

$$I_2 = 0.03535\angle -135^o \text{ A}$$

$$i_1(t) = 79\cos(2000t + 18.4^o)\, mA$$

$$i_2(t) = 35.35\cos(2000t - 135^o)\, mA$$

Delta-Wye ($\Delta - Y$) Transformation



Y-Δ Conversion:

$$\mathbf{Z}_a = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_1}$$

$$\mathbf{Z}_b = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_2}$$

$$\mathbf{Z}_c = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3}$$

Δ-Y Conversion:

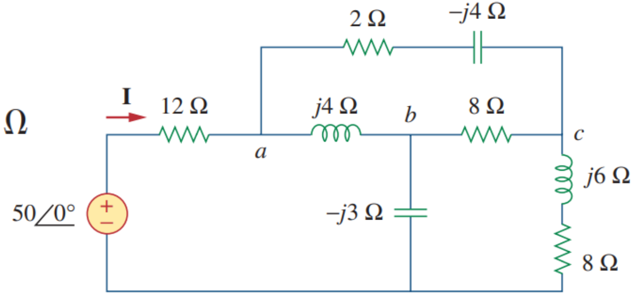
$$\mathbf{Z}_1 = \frac{\mathbf{Z}_b\mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

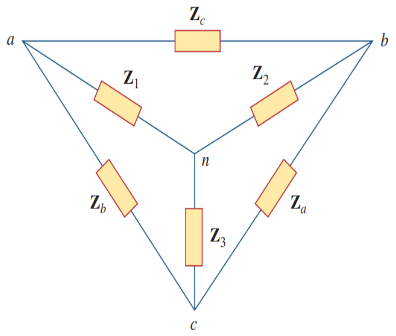
$$\mathbf{Z}_2 = \frac{\mathbf{Z}_c\mathbf{Z}_a}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_a\mathbf{Z}_b}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

Example 24- ($\Delta - Y$) Transformation

Find current **I** in the circuit





Solution:

$$\mathbf{Z}_{an} = \frac{j4(2 - j4)}{j4 + 2 - j4 + 8} = \frac{4(4 + j2)}{10} = (1.6 + j0.8)\ \Omega$$

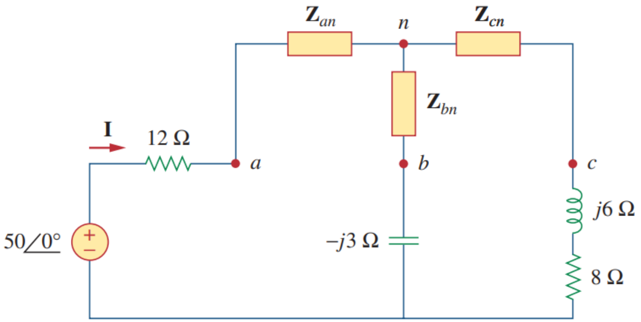
$$\mathbf{Z}_{bn} = \frac{j4(8)}{10} = j3.2\ \Omega, \qquad \mathbf{Z}_{cn} = \frac{8(2 - j4)}{10} = (1.6 - j3.2)\ \Omega$$

The total impedance at the source terminals is

$$\begin{aligned}\mathbf{Z} &= 12 + \mathbf{Z}_{an} + (\mathbf{Z}_{bn} - j3) \parallel (\mathbf{Z}_{cn} + j6 + 8)\\ &= 12 + 1.6 + j0.8 + (j0.2) \parallel (9.6 + j2.8)\\ &= 13.6 + j0.8 + \frac{j0.2(9.6 + j2.8)}{9.6 + j3}\\ &= 13.6 + j1 = 13.64\underline{/4.204^\circ}\ \Omega\end{aligned}$$

The desired current is

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{50\underline{/0^\circ}}{13.64\underline{/4.204^\circ}} = 3.666\underline{/-4.204^\circ}\ \text{A}$$



Δ-Y Conversion:

$$\mathbf{Z}_1 = \frac{\mathbf{Z}_b\mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_c\mathbf{Z}_a}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

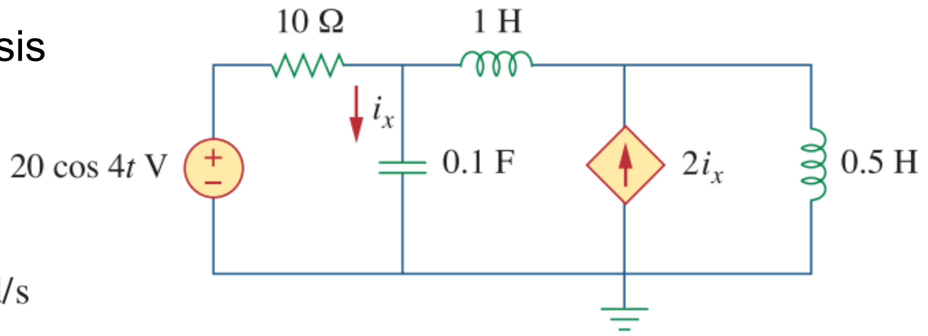
$$\mathbf{Z}_3 = \frac{\mathbf{Z}_a\mathbf{Z}_b}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

Frequency Domain Analysis (Using Phasor)

Nodal Analysis
Mesh Analysis
Superposition,
Source transformation
Thevenin theorem, and
Norton theorem

Example 25- Nodal Analysis

Find i_x in the following circuit using nodal analysis



Solution

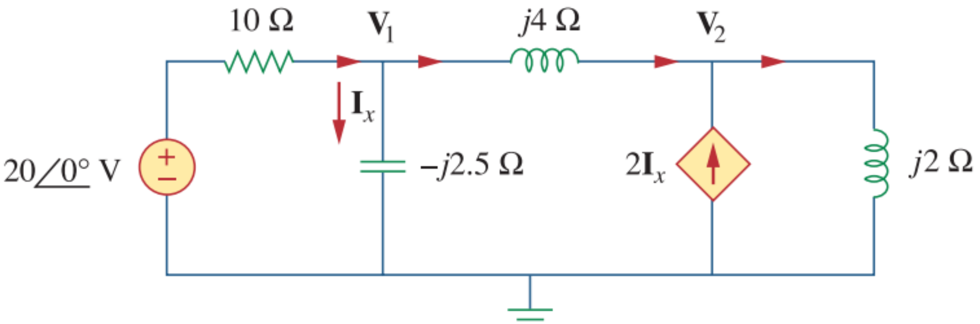
We first convert the circuit to the frequency domain:

$$20\cos 4t \quad \Rightarrow \quad 20\angle 0^\circ, \qquad \omega = 4 \text{ rad/s}$$

$$1\text{ H} \quad \Rightarrow \quad j\omega L = j4$$

$$0.5\text{ H} \quad \Rightarrow \quad j\omega L = j2$$

$$0.1\text{ F} \quad \Rightarrow \quad \frac{1}{j\omega C} = -j2.5$$



Applying KCL at node 1,

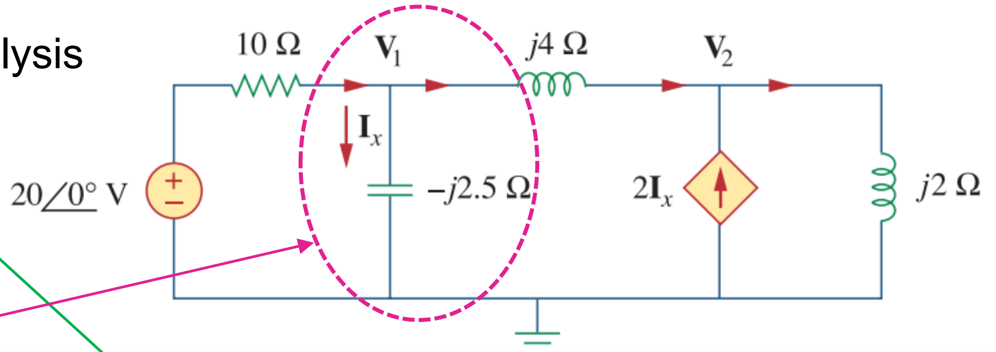
$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

or

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20$$

Example 25- Nodal Analysis

Find i_x in the following circuit using nodal analysis



Solution (continue)

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20$$

At node 2,

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

But $\mathbf{I}_x = \mathbf{V}_1/-j2.5$. Substituting this gives

$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0$$

$$\begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$\mathbf{V}_1 = 18.97\underline{/18.43^\circ} \text{ V}$$

$$\mathbf{V}_2 = 13.91\underline{/198.3^\circ} \text{ V}$$

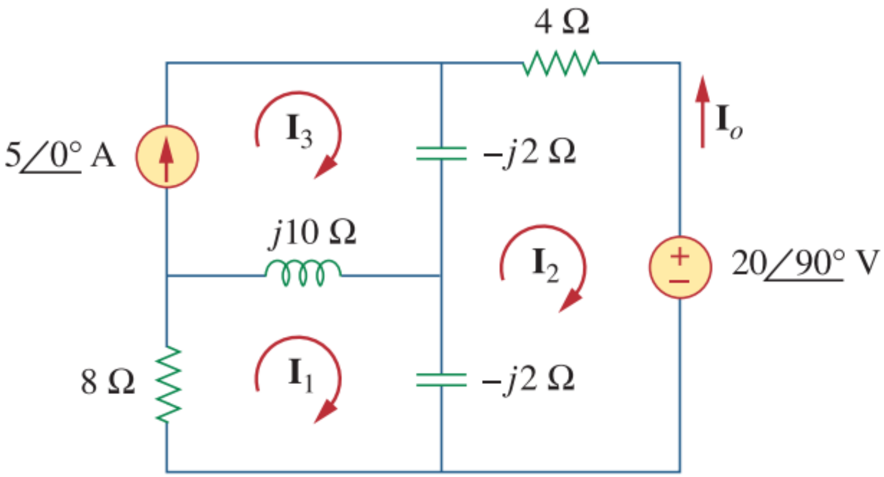
The current $\mathbf{I}_x$ is given by

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\underline{/18.43^\circ}}{2.5\underline{/-90^\circ}} = 7.59\underline{/108.4^\circ} \text{ A}$$

$$i_x = 7.59 \cos(4t + 108.4^\circ) \text{ A}$$

Example 26- Mesh Analysis

Find I_o in the following circuit using mesh analysis



Solution

Applying KVL to mesh 1, we obtain

- 8I1+j10(I1-I3)+-2j(I1-I2)=0

$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0$$

For mesh 2,

- 20<90+ 4I2-j2(I2-I3) -j2(I2-I1) =0

$$(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\angle 90^\circ = 0$$

For mesh 3, $\mathbf{I}_3 = 5$

$$\begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.17\angle -35.22^\circ}{68} = 6.12\angle -35.22^\circ \text{ A}$$

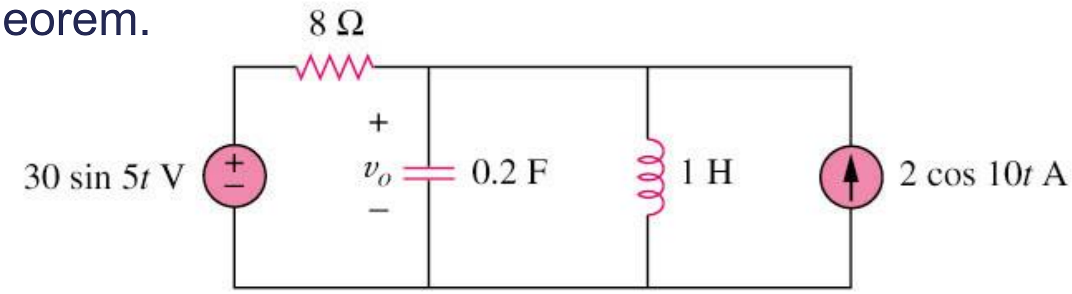
$$\mathbf{I}_o = -\mathbf{I}_2 = 6.12\angle 144.78^\circ \text{ A}$$

Superposition Theorem

When a circuit has sources operating at different frequencies,

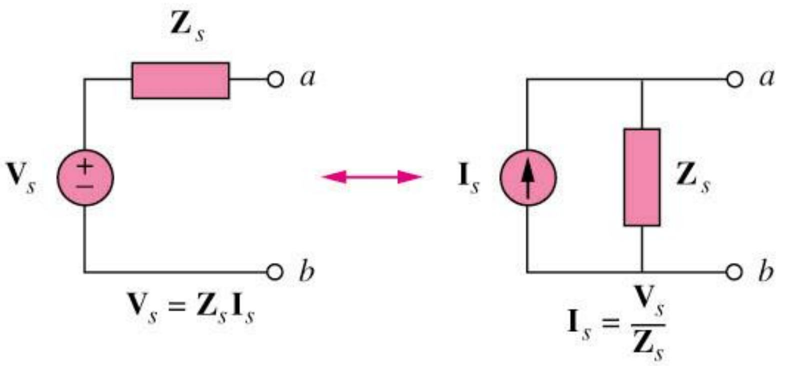
- The separate phasor circuit for each frequency must be solved independently, and
- The total response is the sum of time-domain responses of all the individual phasor circuits.

Example Calculate v_o in the circuit of figure shown below using the superposition theorem.

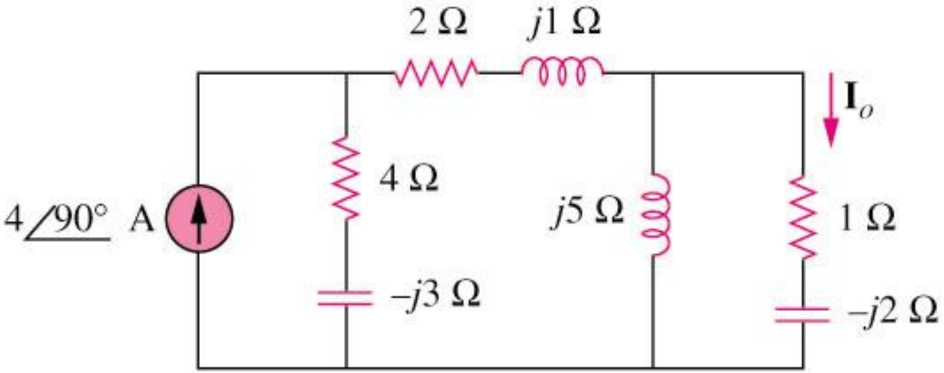


Answer: V_o = **4.631 sin(5t – 81.12) + 1.051 cos(10t – 86.24) V**

Source Transformation



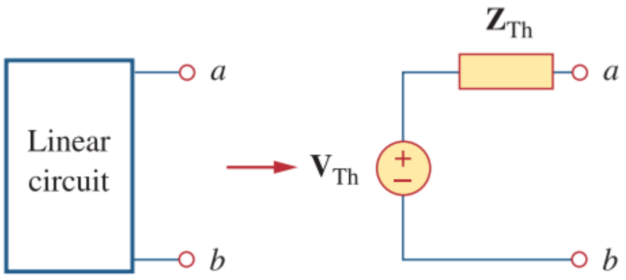
Example Find I_o in the circuit of figure below using the concept of source transformation.



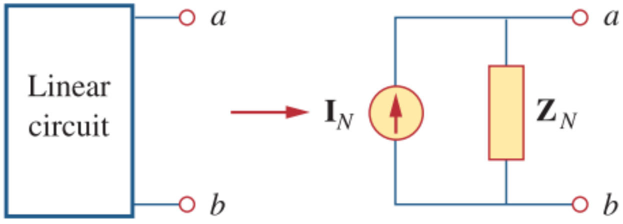
Answer: : I_o = <u>3.288+j99.46 A</u>

Thevenin and Norton Theorems in AC circuits

- Thevenin's and Norton theorems are applied to ac circuits in the same way as they are to DC circuits
- The ONLY additional effort is the need to manipulate complex numbers.



Thevenin equivalent.



Norton equivalent.

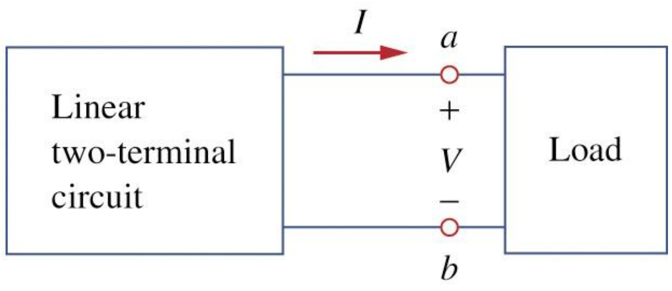
Where; $V_{Th} = V_{ab\text{-}OC}$ is the open circuit voltage (PHASOR) between terminals a-b,
$I_N = I_{ab\text{-}SC}$ is the short circuit current (PHASOR) through the terminals a-b, and
Z_{Th} is the input or equivalent IMPEDANCE at the terminals when the independent source are turn off.

How to Find Thevenin Equivalent Circuit in AC circuits

- First, **Open the circuit** (remove the load) at the **points** of interest **a-b**

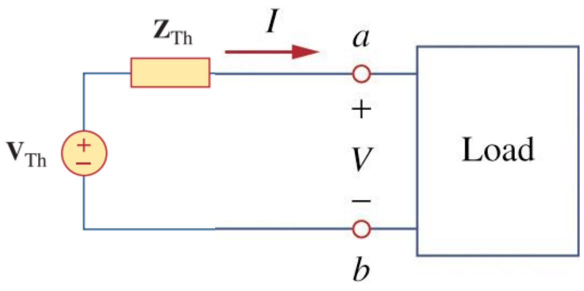
1- V_{th}= Open circuit voltage (keep all sources intact). **Note: Vth is Phasor**

2- Z_{th}= Open circuit **equivalent Impedance** appears at terminals **a-b** while all **independent sources=0** (voltage source=SC , current source=OC).



Original Circuit

≡



Thevenin equivalent circuit

Example 10.8- Thevenin Equivalent Circuit

Obtain the Thevenin equivalent at terminals a-b of the following circuit.

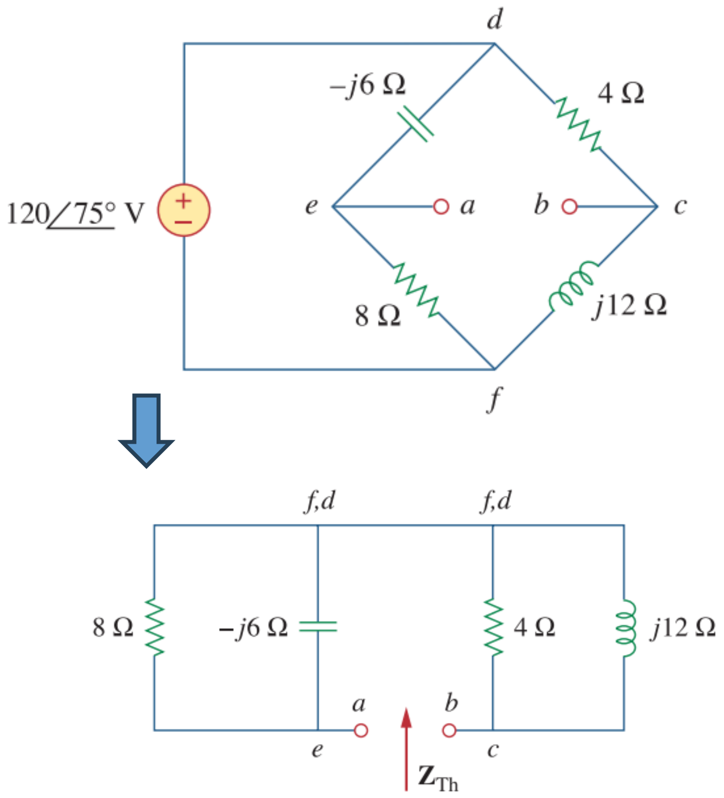
Solution

1- For Zth

$$\mathbf{Z}_1 = -j6 \parallel 8 = \frac{-j6 \times 8}{8 - j6} = 2.88 - j3.84\ \Omega$$

$$\mathbf{Z}_2 = 4 \parallel j12 = \frac{j12 \times 4}{4 + j12} = 3.6 + j1.2\ \Omega$$

$$\boxed{\mathbf{Z}_{Th} = \mathbf{Z}_1 + \mathbf{Z}_2 = 6.48 - j2.64\ \Omega}$$



Example 10.8- Thevenin Equivalent Circuit

Obtain the Thevenin equivalent at terminals a-b of the following circuit.

Solution (continue)

2- For Vth

$$\mathbf{I}_1 = \frac{120\angle 75^\circ}{8 - j6}\ \text{A}, \qquad \mathbf{I}_2 = \frac{120\angle 75^\circ}{4 + j12}\ \text{A}$$

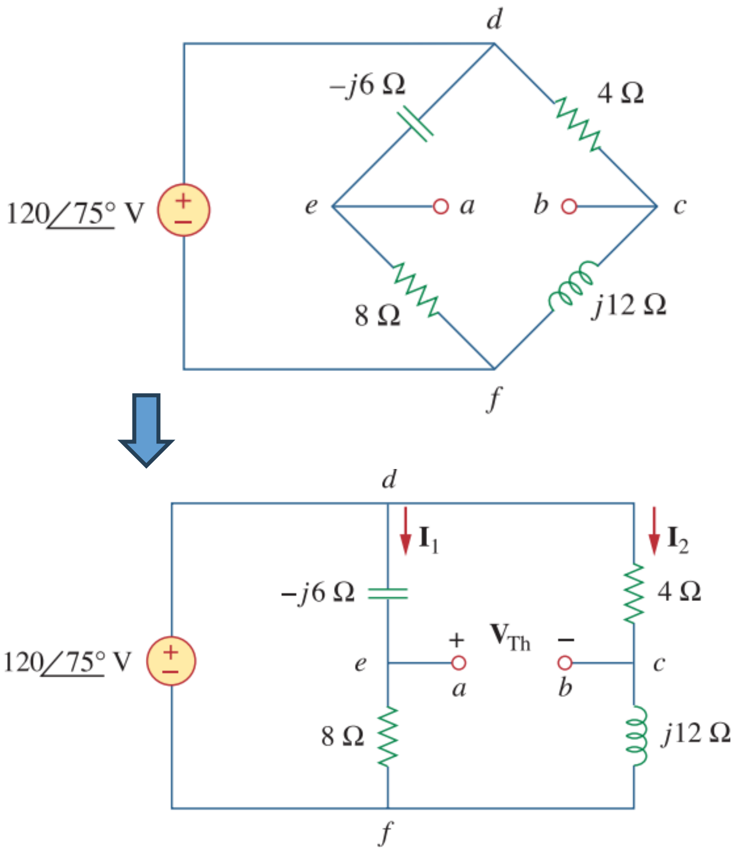
Applying KVL around loop *bcdeab*

$$\mathbf{V}_{\text{Th}} - 4\mathbf{I}_2 + (-j6)\mathbf{I}_1 = 0$$

$$\mathbf{V}_{\text{Th}} = 4\mathbf{I}_2 + j6\mathbf{I}_1 = \frac{480\angle 75^\circ}{4 + j12} + \frac{720\angle 75^\circ + 90^\circ}{8 - j6}$$

$$= 37.95\angle 3.43^\circ + 72\angle 201.87^\circ$$

$$= -28.936 - j24.55 = 37.95\angle 220.31^\circ\ \text{V}$$

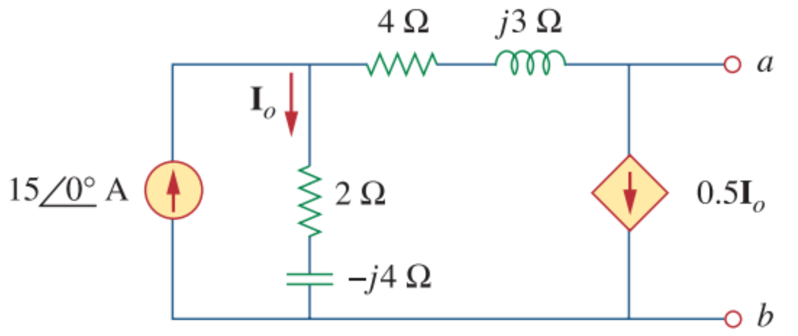


Example 10.9- Thevenin Equivalent Circuit

Obtain the Thevenin equivalent at terminals a-b of the following circuit.

Solution

1- For Vth



To find $\mathbf{V}_{Th}$, we apply KCL at node 1

$$15 = \mathbf{I}_o + 0.5\mathbf{I}_o \quad \Rightarrow \quad \mathbf{I}_o = 10 \text{ A}$$

Applying KVL to the loop on the right-hand side

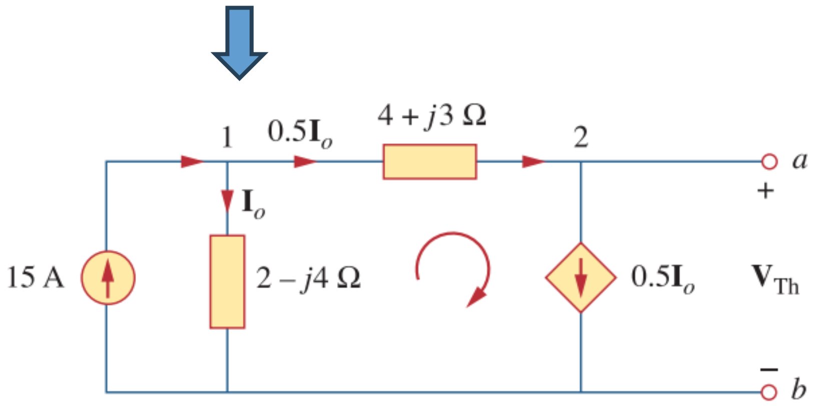
$$-\mathbf{I}_o(2 - j4) + 0.5\mathbf{I}_o(4 + j3) + \mathbf{V}_{Th} = 0$$

or

$$\mathbf{V}_{Th} = 10(2 - j4) - 5(4 + j3) = -j55$$

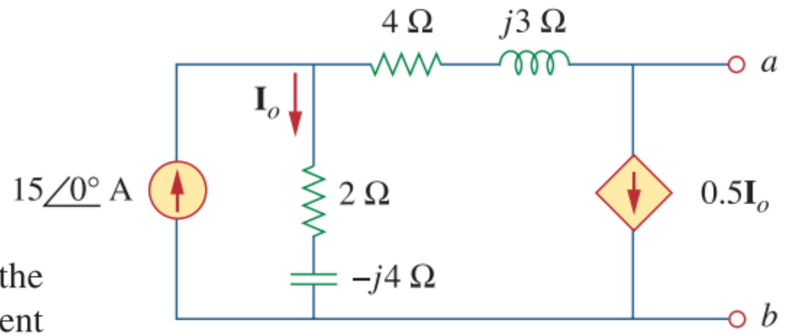
Thus, the Thevenin voltage is

$$\mathbf{V}_{Th} = 55\underline{/-90^\circ} \text{ V}$$



Example 10.9- Thevenin Equivalent Circuit

Obtain the Thevenin equivalent at terminals a-b of the following circuit.



Solution (continue)

2- For Zth

To obtain $\mathbf{Z}_{\text{Th}}$, we remove the independent source. Due to the presence of the dependent current source, we connect a 3-A current source (3 is an arbitrary value chosen for convenience here, a number divisible by the sum of currents leaving the node) to terminals *a-b*

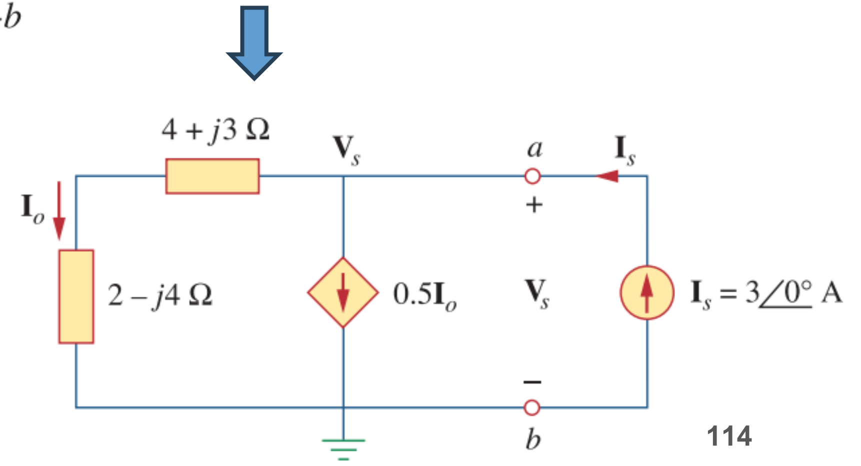
$$3 = \mathbf{I}_o + 0.5\mathbf{I}_o \qquad \Rightarrow \qquad \mathbf{I}_o = 2 \text{ A}$$

Applying KVL to the outer loop in Fig. 10.26(b) gives

$$\mathbf{V}_s = \mathbf{I}_o(4 + j3 + 2 - j4) = 2(6 - j)$$

The Thevenin impedance is

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_s}{\mathbf{I}_s} = \frac{2(6 - j)}{3} = 4 - j0.6667\ \Omega$$



Trigonometric Identities

- Sine and cosine form conversions.

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

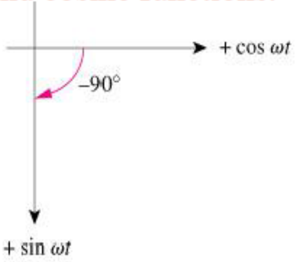
$$\sin(\omega t \pm 180°) = -\sin \omega t$$
$$\cos(\omega t \pm 180°) = -\cos \omega t$$
$$\sin(\omega t \pm 90°) = \pm \cos \omega t$$
$$\cos(\omega t \pm 90°) = \mp \sin \omega t$$

$$A \cos \omega t + B \sin \omega t = C \cos(\omega t - \theta)$$

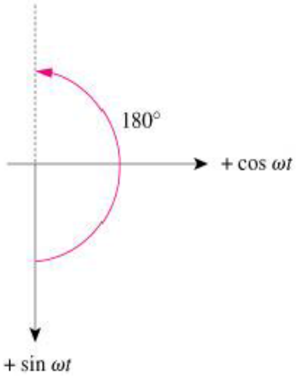
Where

$C = \sqrt{A^2 + B^2}$ and $\theta = \tan^{-1} \frac{B}{A}$

Graphically relating sine and cosine functions.



$$\cos(\omega t - 90°) = \sin \omega t$$



$$\sin(\omega t + 180°) = -\sin \omega t$$

End of Lecture

Questions?