3- Circuit Theorems

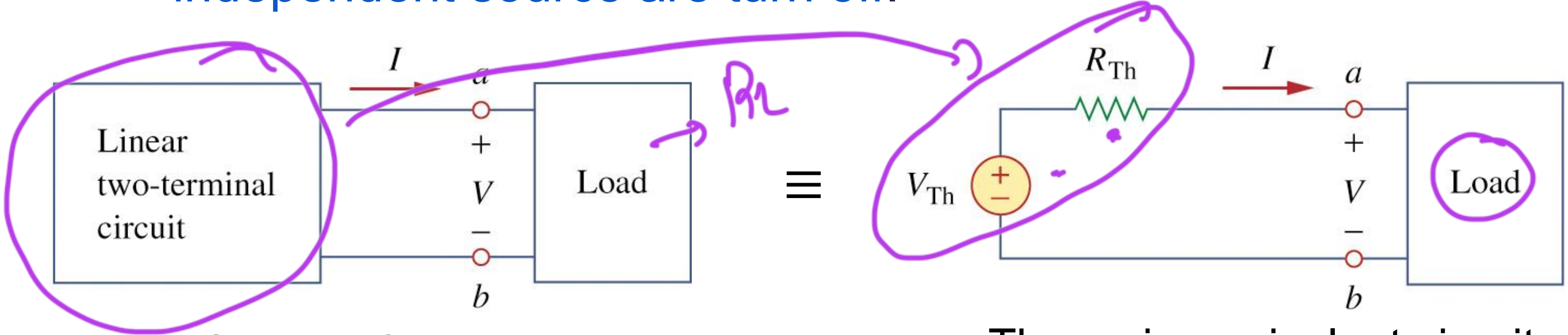
- Introduction and Linearity property
- Superposition
- Source transformations
- Thevenin's theorem
- Norton's theorem
- Maximum power transfer

Thevenin's Theorem

- Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}

Where; V_{Th} is the open circuit voltage at the terminals and

R_{Th} is the input or equivalent resistance at the terminals when the independent source are turn off.



Original Circuit

Thevenin equivalent circuit

Circuits

All ind

$+ V_{o.c} -$

$V_{th} = V_{o.c}$

$R_{th} \rightarrow$ Turn off all source

volt $\rightarrow$ short

current $\rightarrow$ open

$R_{eq} = R_{th}$

ind/dep

$+ V_{o.c} -$

$V_{th} = V_{o.c}$

I_{sc}

$R_{th} \rightarrow$

$\rightarrow$ Turn off all ind source

$R_{th} = \frac{V_{oc}}{I_{sc}}$

$R_{th} = \frac{V}{I}$

All dep

V_{th} لا يوجد

R_{th}

$R_{th} = \frac{V}{I}$

لا عادي جدًا تطلع، كما هوة سالبة

السالب " انه منطقة الدائرة تنتج power

generated power /

diss power $P = I^2 R$

$P +$ → absorbed يستهلك $= V^2/R$

$P -$ → subiled → ينتج

Thevenin's equivalent circuit

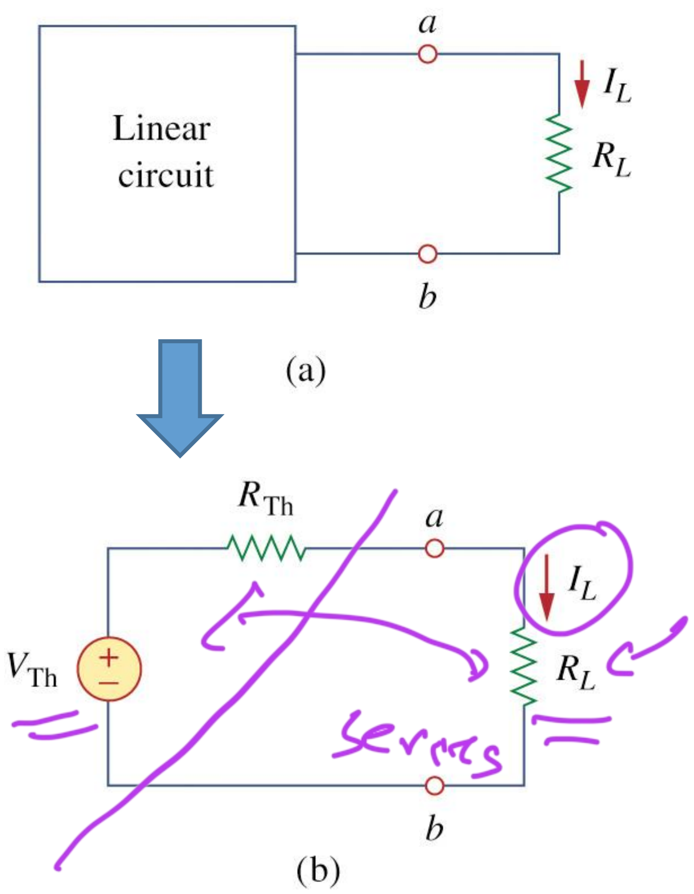
Simplified circuit

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L}$$

$$V_L = R_L I_L = \frac{R_L}{R_{\text{Th}} + R_L} V_{\text{Th}}$$

→ voltage divider

$$V_L = \frac{R_L}{R_L + R_{th}} \times V_{th}$$

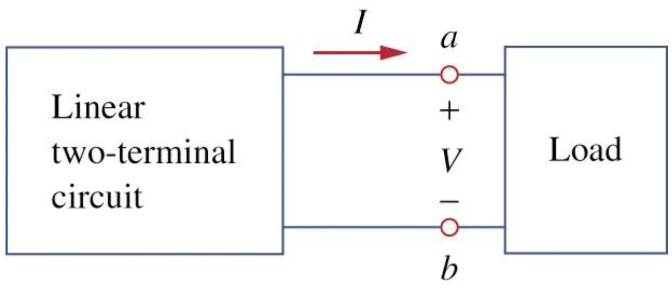


How to Find Thevenin Equivalent Circuit

- First, **Open the circuit** (remove the load) at the **points** of interest **a-b**

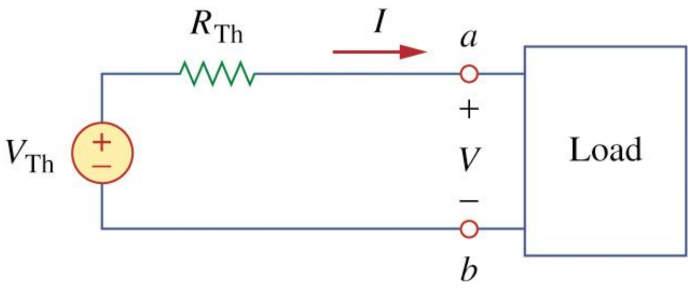
1- V_{th}= Open circuit voltage (keep all sources intact)

2- R_{th}= Open circuit **equivalent resistance** appears at terminals **a-b** while all <u>**independent sources=0**</u> (voltage source=SC , current source=OC).



Original Circuit

≡



Thevenin equivalent circuit

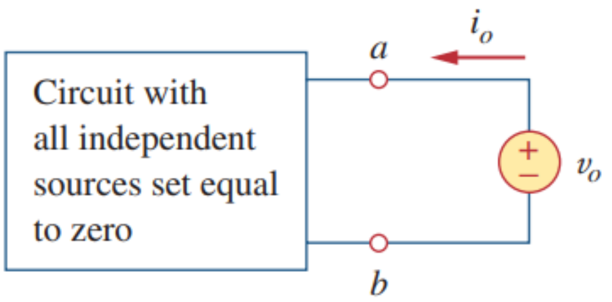
How to Find Thevenin Equivalent Circuit

- To find $\mathbf{R_{th}}$ we have to consider two cases

1- Case 1 (Circuit has No dependent sources): turn off all independent sources, then find the equivalent R_{th} using series/parallel combinations.

2- Case 2 (Circuit has dependent sources): turn off all independent sources and keep dependent sources intact (as superposition). In order to find equivalent R_{th} , apply v_o and find the produced current i_o. Then, $R_{th} = \frac{v_o}{i_o}$

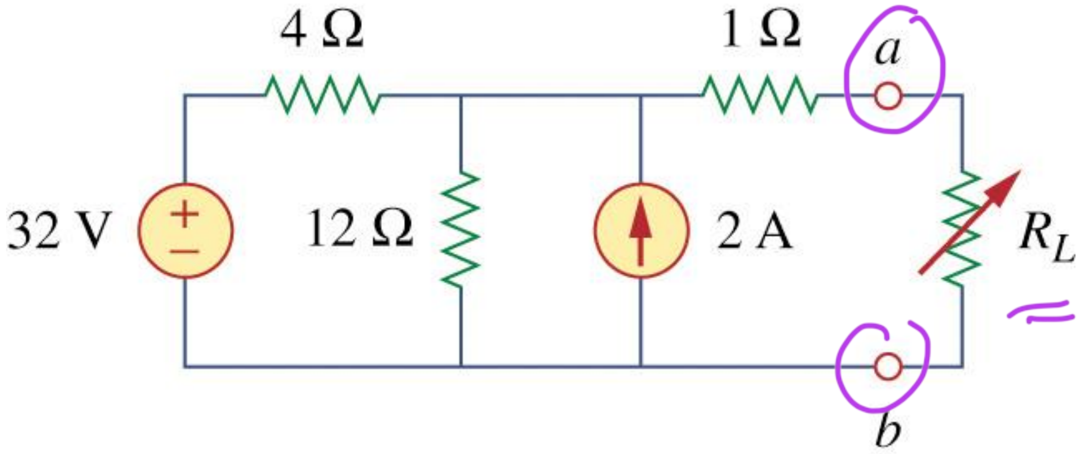
For simplicity: put $v_o = 1V$ and find the produced current i_o. Then, $R_{th} = \frac{1}{i_o}$



$$R_{Th} = \frac{v_o}{i_o}$$

Example 10

- Find the Thevenin's equivalent of the circuit shown below, to the left of the terminals *a-b*. Then find the current through R_L = 6,16,and 36 Ω.



nodel analysis

$$\frac{V-32}{4} + \frac{V}{12} - 2 = 0 \qquad 2 + \frac{32}{4}$$

$$= \cancel{3}\times\frac{1}{\cancel{3}}V = 10 \times 3 \rightarrow \boxed{V = 30V = V_{th}}$$

R_{th} Thevenin → Turn off

← $R_{th} = R_{eq}$

$$\frac{1}{\frac{1}{4}+\frac{1}{12}} = 3\,\Omega$$

series

$$R_{th} = 1+3 = \boxed{4\,\Omega}$$

Example 10

Solution:

For R_{th},

Voltage source =SC,

Current source =OC

$$\boxed{R_{th} = 1 + 4//12 = 4\Omega}$$

For V_{th},

Keep original circuit

V_{th}=V_{ac} (OC)

$$\boxed{V_{th} = 12(i_1 - i_2) = ?}$$

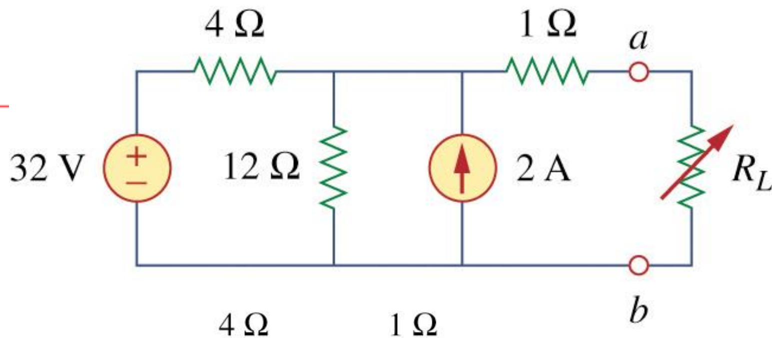
Using mesh analysis

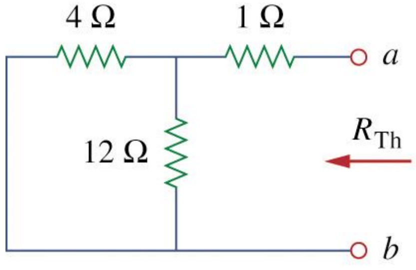
$i_2 = -2\text{A}$,

$16i_1 - 12i_2 = 32$,

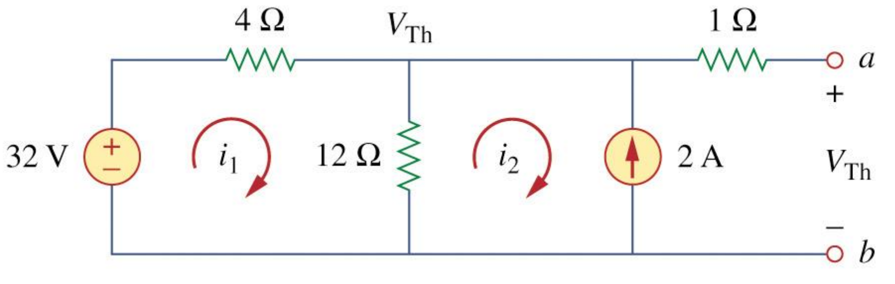
$i_1 = 0.5\text{A}$,

$$\boxed{V_{th} = 12(i_1 - i_2) = 30V}$$





(a)



(b)

Example 10

Solution:

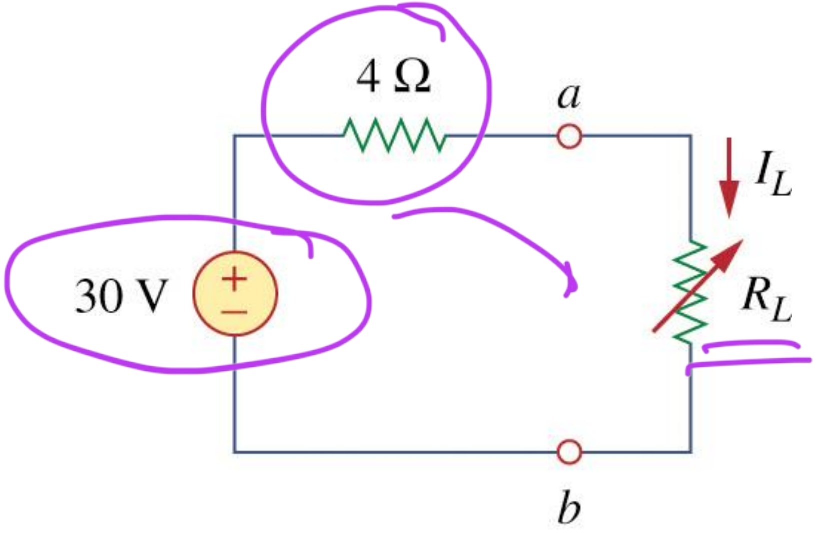
To get i_L :

$$i_L = \frac{V_{\mathrm{Th}}}{R_{\mathrm{Th}} + R_L} = \frac{30}{4 + R_L}$$

$$R_L = 6 \rightarrow I_L = 30/10 = 3\mathrm{A}$$

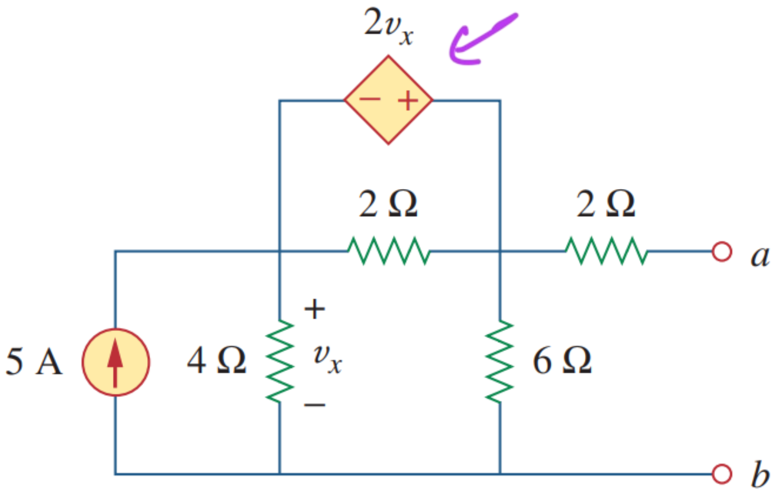
$$R_L = 16 \rightarrow I_L = 30/20 = 1.5\mathrm{A}$$

$$R_L = 36 \rightarrow I_L = 30/40 = 0.75\mathrm{A}$$



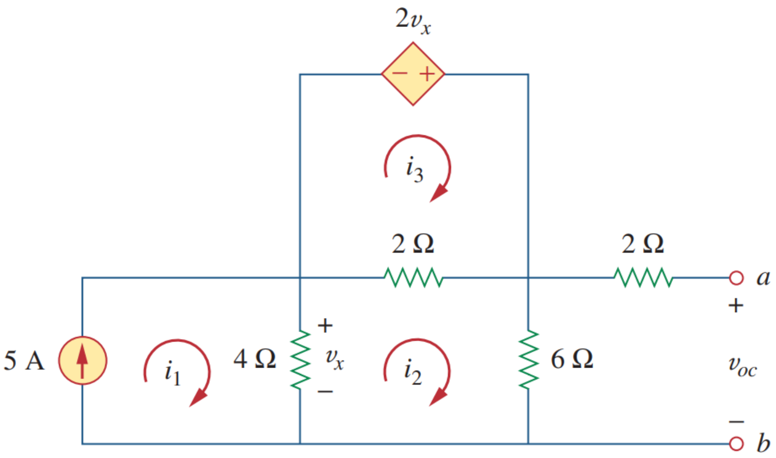
Example 11

Find the Thevenin equivalent of the following circuit at terminals a-b.



Solution:

1- For V_{th}, solve the following circuit to find v_{ab} (v_{OC}).



(b)

super node

$\boxed{V_x = V_1}$

Nodal

$V_2 - V_1 = 2V_x = 2V_1 + V_1$

$-V_2$

$\Rightarrow \boxed{3V_1 - V_2 = 0} \rightarrow 1$

node 1+2 $\quad -5 + \frac{V_1 - 0}{4} + \frac{V_2 - 0}{6} = 0$

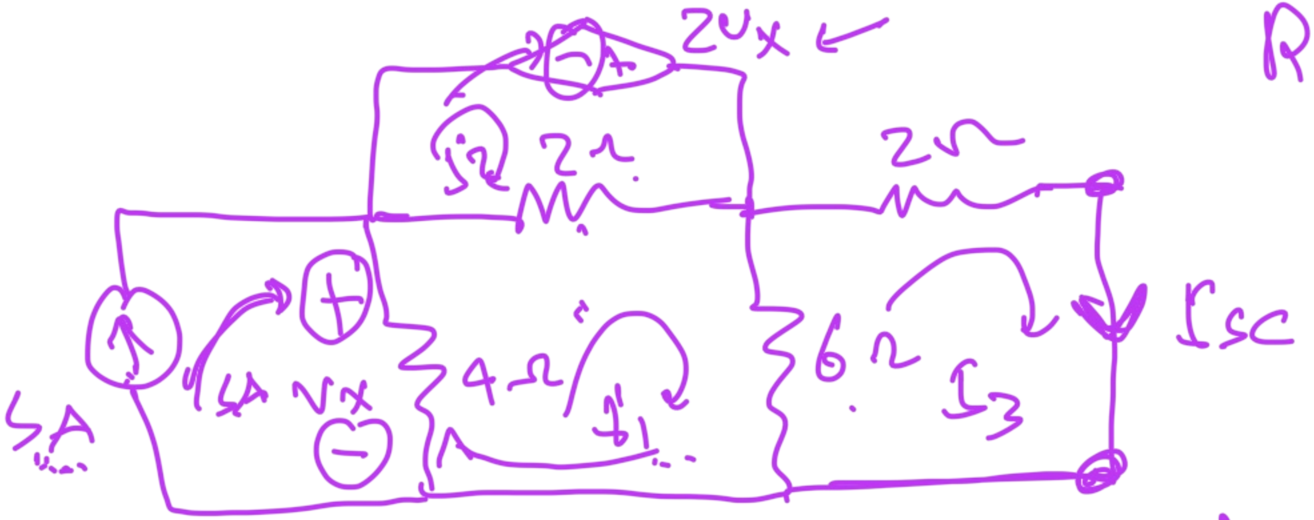
$$\boxed{\frac{V_1}{4} + \frac{V_2}{6} = 5} \rightarrow 2$$

$V_1 = \frac{20}{3}\ V$

mode $\rightarrow$ 5 $\rightarrow$ 1

$V_2 = 20V$

$$\Rightarrow \boxed{V_{o.c} = V_{th} = V_2 = 20V}$$



R_{th}

$I_3 = I_{sc}$

mesh

$V_x = 4(5 - I_1) = 20 - 4I_1$

mesh 1

$12I_1 - 2I_2 - 6I_3 - 5\times 4 = 0$

$\boxed{12I_1 - 2I_2 - 6I_3 = 20} \rightarrow 1$

mesh 2

$$-2I_1 + 2I_2 - 2(20 - 4I_1) = 0$$

$$-2I_1 + 2I_2 - 40 + 8I_1 = 0$$

$$\boxed{6I_1 + 2I_2 + 0I_3 = 40} \rightarrow 2$$

mesh 3

$$\boxed{-6I_1 + 0I_2 + 8I_3 = 0} \rightarrow 3$$

mode $\rightarrow$ 5 $\longrightarrow$ 2

$I_1 = \frac{40}{9}$, $I_2 = \frac{20}{3}$, $\boxed{I_3 = \frac{10}{3}}$

$I_{sc} = I_3 = \frac{10}{3} A$

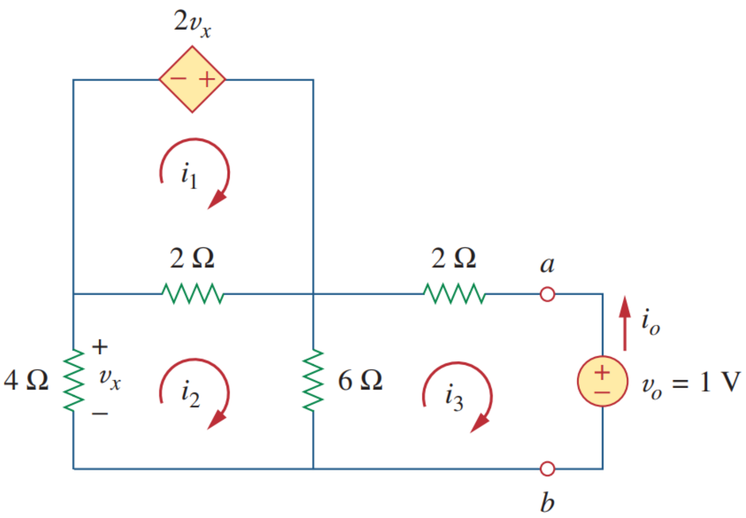
$R_{th} = \frac{V_{o.c}}{I_{sc}} = \frac{20}{10/3} = \boxed{6 \Omega}$

Example 11

Find the Thevenin equivalent of the following circuit at terminals a-b.

Solution:

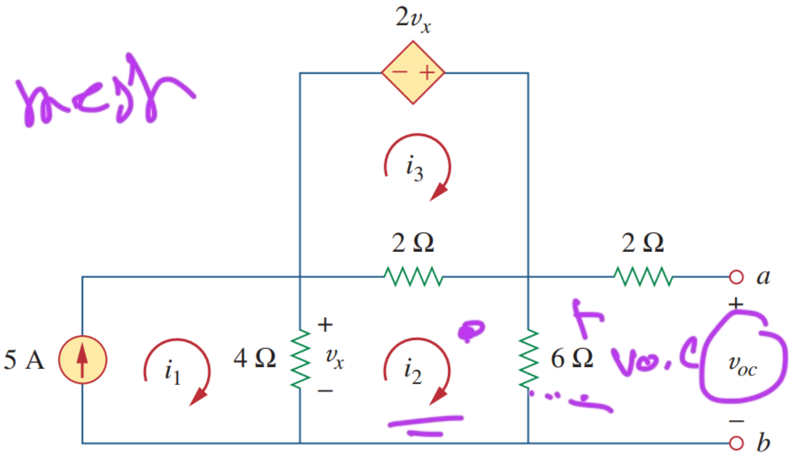
2- For R_{th}, solve the following circuit to find io. Then, $R_{th} = 1/i_o$



(a)

1- For V_{th}, solve the following circuit to find v_{ab} (v_{OC}).

mesh



(b)

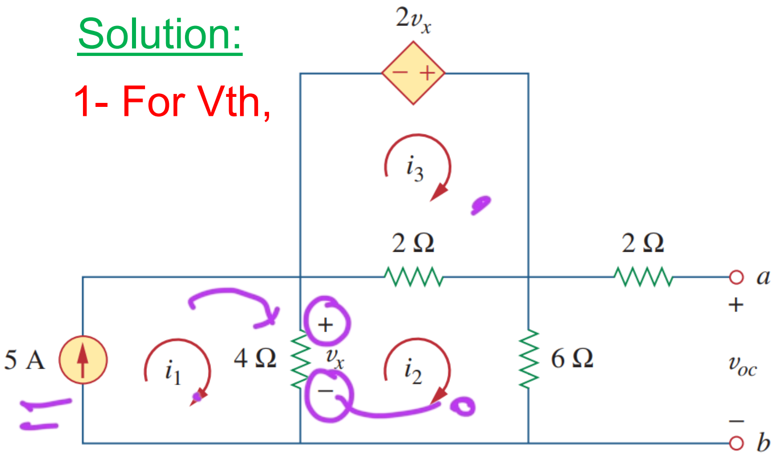
$V_{o.C} = I_2 R_{6\Omega}$

Example 11

Find the Thevenin equivalent of the following circuit at terminals a-b.

Solution:

1- For Vth,



(b)

$v_x = 4(i_1 - i_2)$

Applying mesh analysis:

Mesh 1: $\boxed{i_1 = 5}$

Mesh 2: $4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$

$\Rightarrow \boxed{12i_2 - 4i_1 - 2i_3 = 0}$

Mesh 3: $-2v_x + 2(i_3 - i_2) = 0 \quad \Rightarrow \quad v_x = i_3 - i_2$

But, $4(i_1 - i_2) = v_x. \quad \Rightarrow \boxed{4i_1 - 3i_2 - i3 = 0}$

Solving these equations leads to $\boxed{i_2 = 10/3}$

$\boxed{V_{Th} = v_{oc} = 6i_2 = 20\ \text{V}}$

Example 11

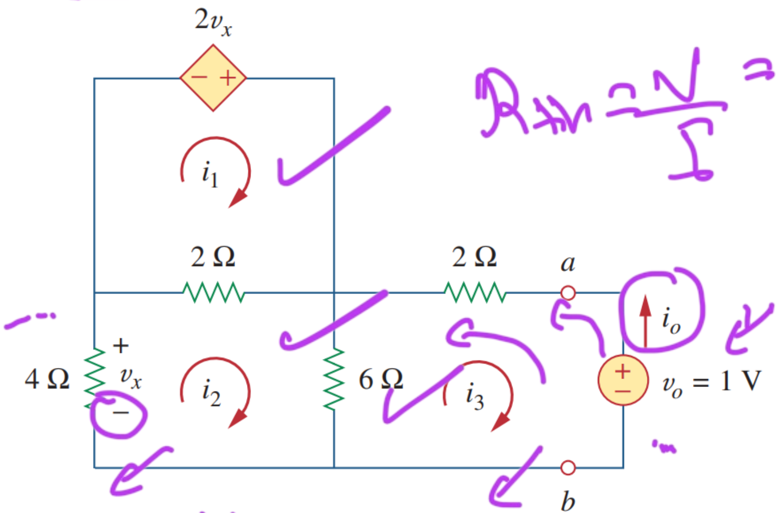
Find the Thevenin equivalent of the following circuit at terminals a-b.

Solution:

2- For Rth, Put $v_o = 1V$

and kill any independent sources

$R_{th} = \frac{V}{I} = 1$



$v_x = -4i_2$

(a)

$I_3 = -I_o$

Applying mesh analysis:

Mesh 1: $-2v_x + 2(i_1 - i_2) = 0$ or $v_x = i_1 - i_2$

But $-4i_2 = v_x = i_1 - i_2$; hence, $\Rightarrow$ $\boxed{i_1 = -3i_2}$

Mesh 2: $\boxed{4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0}$

Mesh 3: $\boxed{6(i_3 - i_2) + 2i_3 + 1 = 0}$

Solving these equations gives

$$\boxed{i_3 = -\frac{1}{6}\text{ A}}$$

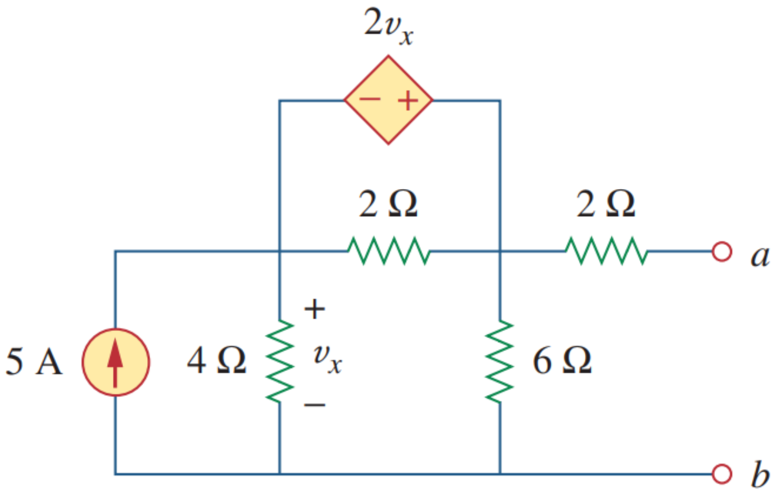
But $\boxed{i_o = -i_3 = 1/6 \text{ A}}$. Hence,

$$\boxed{R_{Th} = \frac{1\text{ V}}{i_o} = 6\ \Omega}$$

$= \frac{1}{1/6} = 6$

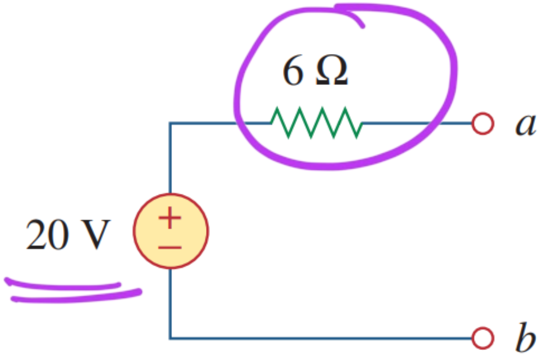
Example 11

Find the Thevenin equivalent of the following circuit at terminals a-b.



Solution:

≡



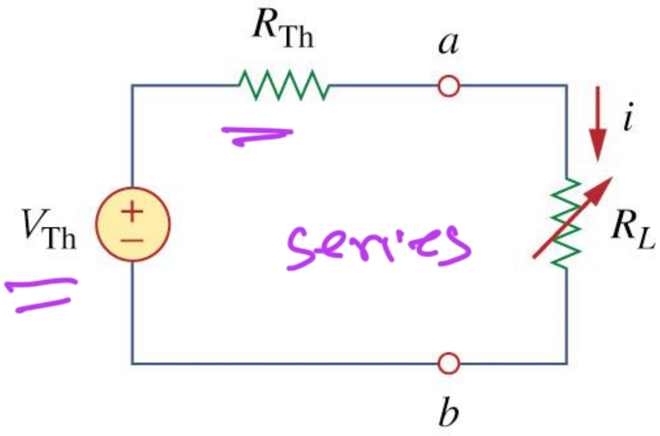
3- Circuit Theorems 2

- Introduction and Linearity property
- Superposition
- Source transformations
- Thevenin’s theorem
- Norton’s theorem
- Maximum power transfer

Maximum Power Transfer

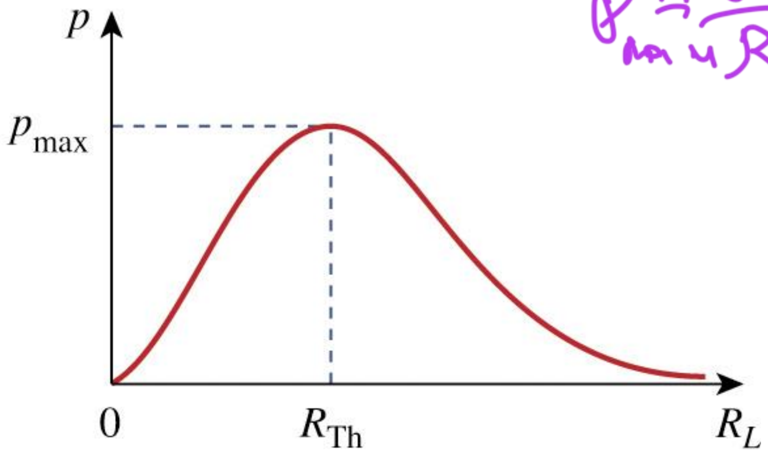
→ R_L → max power

→ find max power



$$p = i^2 R_L = \left(\frac{V_{TH}}{R_{TH} + R_L} \right)^2 R_L$$

$$P_{max} = \frac{V^2}{4R}$$



- **Maximum power** is transferred to the **load** when the load resistance equals the Thevenin resistance as seen the load (***R_L = R_{TH}***).

$$\boxed{R_L = R_{TH}}$$

$$\rightarrow \quad p_{max} = \frac{V_{TH}^2}{4R_{TH}}$$

Maximum Power Transfer: Mathematical Proof

$$\frac{dp}{dR_L} = V_{TH}^2 \left[\frac{(R_{TH}+R_L)^2 - 2R_L(R_{TH}+R_L)}{(R_{TH}+R_L)^4} \right]$$

$$= V_{TH}^2 \left[\frac{(R_{TH}+R_L-2R_L)}{(R_{TH}+R_L)^3} \right] = 0$$

$$0 = (R_{TH}+R_L-2R_L) = (R_{TH}-R_L)$$

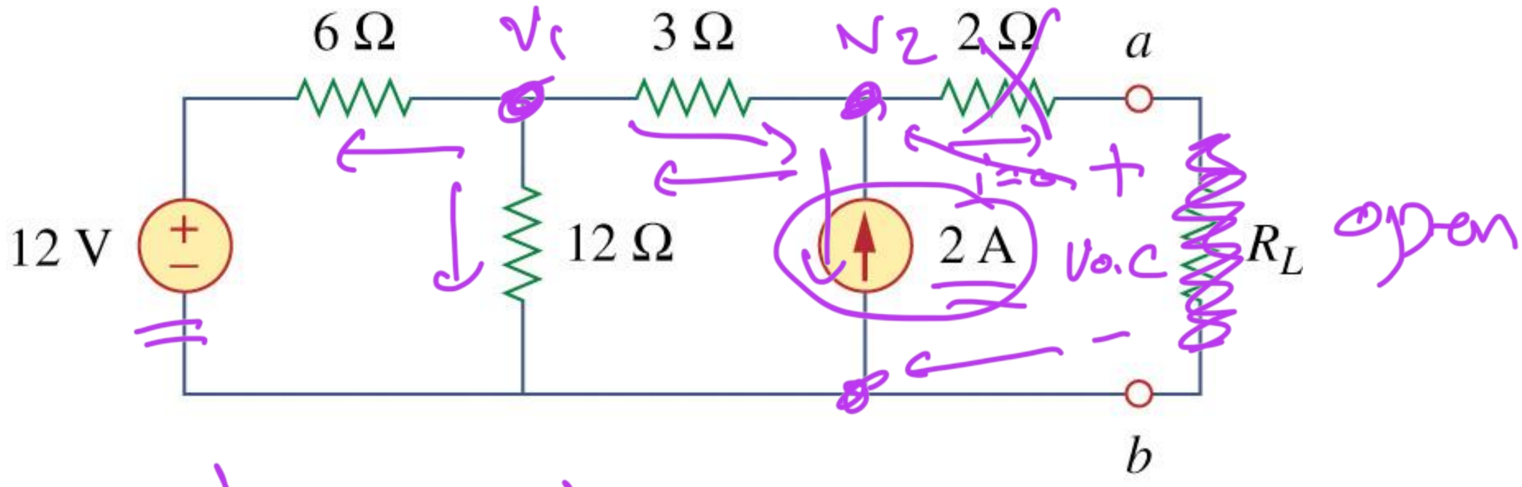
calc 1

$$\boxed{R_L = R_{TH}}$$

$$\boxed{p_{max} = \frac{V_{TH}^2}{4R_{TH}}}$$

Example 12

- Find the value of R_L for maximum power transfer in the circuit of the following figure. Find the maximum power.



Nodal Analysis

$$\frac{V_1 - 12}{6} + \frac{V_1 - 0}{12} + \frac{V_1 - V_2}{3} = 0$$

$$\frac{V_2 - V_1}{3} + -2 = 0$$

$$\rightarrow \boxed{\frac{7}{12}V_1 - \frac{V_2}{3} = 2} \rightarrow 1$$

$$\boxed{-\frac{V_1}{3} + \frac{V_2}{3} = 2} \rightarrow 2$$

$V_1 = 16\,V$ $\boxed{V_2 = 22V}$ $= V_{th}$

Example 12

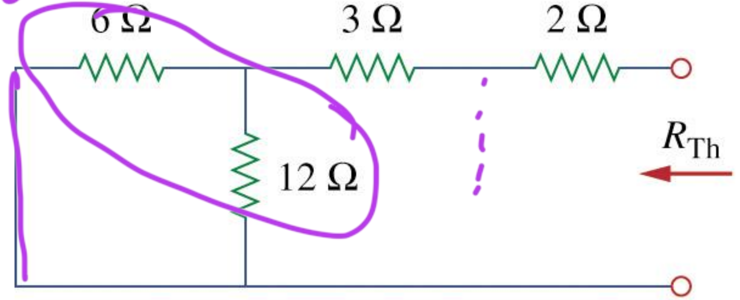
Solution:

series

1- To find R_{TH},

$$R_{TH} = 2+3+6\|12 = 5+\frac{6\times 12}{18} = \boxed{9\Omega}$$

parallel

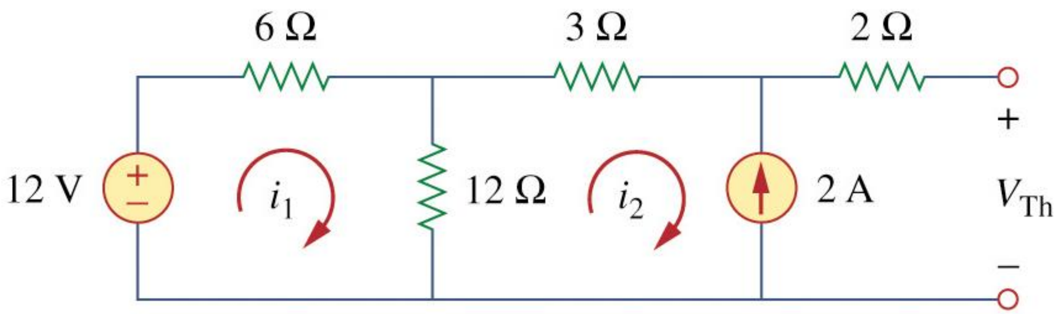


2- To find V_{TH},

Mesh: $-12+18i_1-12i_2,\quad i_2=-2A$

$$-12+6i_i1+3i_2+2(0)+V_{TH}=0$$

$$\boxed{\therefore\ V_{TH}=22V}$$



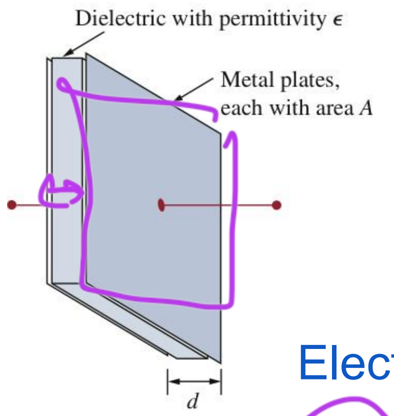
For maximum power:

$$R_L = R_{TH} = 9\Omega$$

$$p_{max} = \frac{V_{TH}^2}{4R_L} = \frac{22^2}{4\times 9} = 13.44W$$

4- Capacitors and Inductors

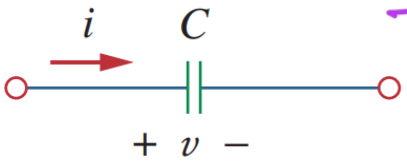
Summary of Capacitors



Physical structure

$$C = \frac{\varepsilon A}{d}$$

Electrical Characteristics



$$q = Cv$$

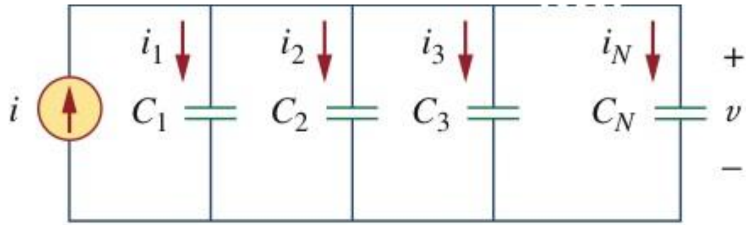
$$i = C\frac{dv}{dt}$$

O.C. in DC

Energy Stored

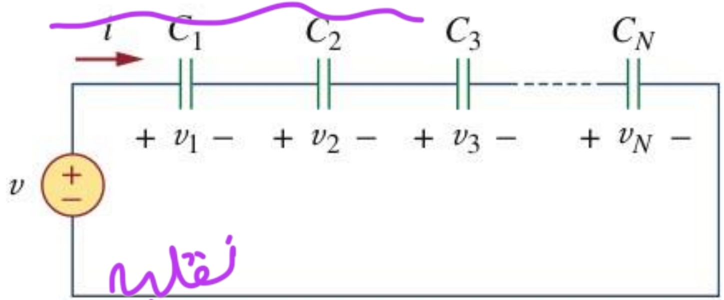
$$w = \frac{1}{2} C v^2$$

Parallel Capacitors



$$C_{eq} = C_1 + C_2 + C_3 + ... + C_N$$

Series Capacitors

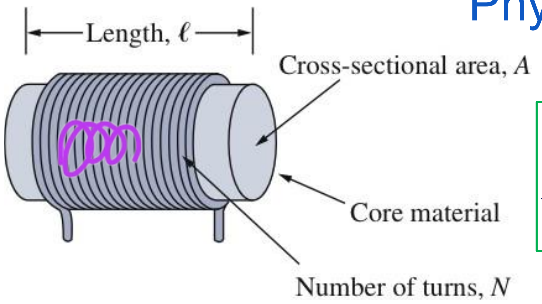


نقلبها

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_N}$$

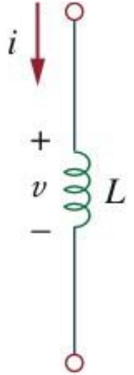
Summary of Inductors

Physical structure



$$L = \frac{N^2 \mu A}{l}$$

Electrical Characteristics



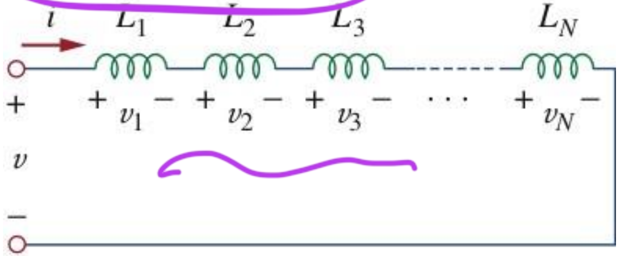
$$v = L\frac{di}{dt}$$

S.C. in DC

Energy Stored

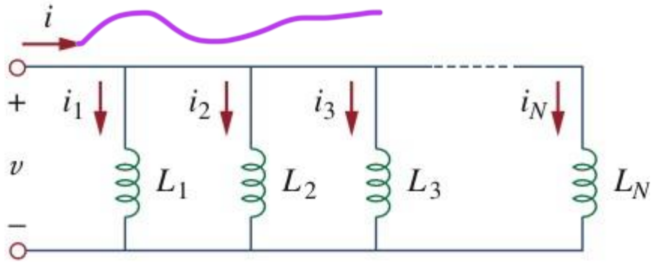
$$w(t) = \frac{1}{2}Li^2(t)$$

Series Inductors



$$L_{eq} = L_1 + L_2 + L_3 + \ldots + L_N$$

Parallel Inductors



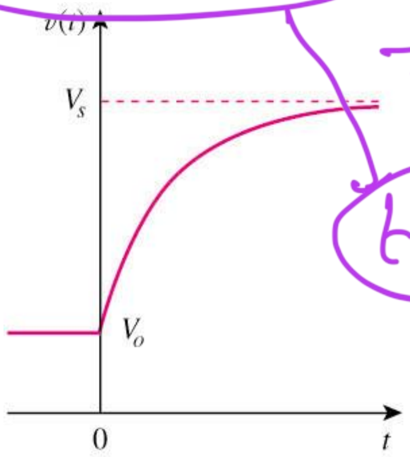
$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}$$

Remember: Important characteristics of basic elements

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

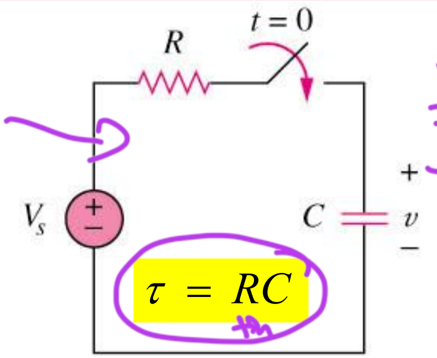
Summary of Transient-Response for RC and RL circuits

Step Response (Forced Response):



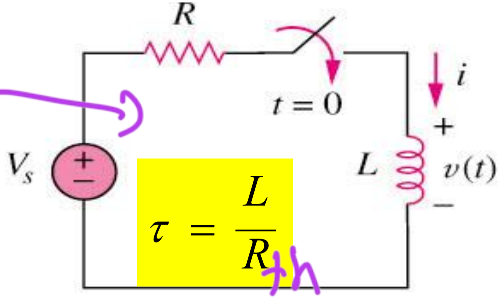
$$v(t) = v(\infty) + [v(0) - v(\infty)]\, e^{-t/\tau}$$

$t>0$



$$\tau = RC_{th}$$

$$i(t) = i(\infty) + [i(0) - i(\infty)]\, e^{-t/\tau}$$



$$\tau = \frac{L}{R_{th}}$$

Three steps to solve any transient-response problem:

1. The initial Values. $v(0), i(0) \rightarrow t<0$
2. The final value (after long time, C acts as OC and L acts as SC).
3. The time constant τ. $\rightarrow t>0$ $v(\infty)\ i(\infty)\ t>0$

Note: Natural Response (source free) is a special case where final value =0

$t > 0$

لا يوجد نظرية
أدناه

End of Lecture

Questions?