ELECTRICAL ENGINEERING DEPARTMENT

EE242: Electric Circuits II

Lecture 1-3

COURSE SYLLABUS
&
Review on DC Circuit Analysis Methods

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Course Staff

- **Instructor:**

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- **Office hours:**

  Monday/ Thursday : 11:00-12:00 AM

  Other appointments can be scheduled by email.

Course Description:

- A continuation of Electric Circuits I
- AC sinusoidal analysis and power calculations
- Balanced three phase circuits
- Laplace Transform and Circuit analysis using Laplace Transform
- Passive filters analysis and design and Bode diagram
- Two port Circuits
- Three hours lectures per week and 5 to 6 labs per semester

Course Learning Outcomes:

1. An ability to analyze AC sinusoidal circuits (1).
2. An ability to perform power calculations and analyze balanced three phase circuits (1).
3. An ability to perform circuit analysis for magnetically coupled circuits
4. An ability to perform Circuit analysis using Laplace Transform (1).
5. An ability to analyze and design passive filters (1, 3, 7).
6. An ability to plot Bode diagram (1).
7. An ability to solve two port Circuits (1).
8. A familiarity with basic electric equipment like AC power sources signal generators and measuring instruments (**3**, 5, 6).

Major Topics to be Covered and Schedule in Weeks:

No	List of Topics	Contact Hours
1	Review of DC Circuit Analysis Methods	3
2	Principals of Waveforms and Power Calculations	3
3	AC sinusoidal analysis	6
4	Balanced three phase circuits;	4.5
5	Magnetically Coupled Circuits	4.5
6	Passive Filters Analysis and design	6
7	S-Domain and Frequency Domain applications	3
8	Bode Plot and Electric circuits design and analysis using software	4.5
9	Two port Circuits	3
10	Lab experiments	7.5
	Total	45

Marking Scheme:

(1)	Quizzes and HWKs	15%
(2)	Project (Research Assignments)	10%
(3)	Labs Experiments	15%
(4)	Midterm Exam	20%
(5)	Final Exam	40%
Total		100%

Textbook and References

Essential References	Fundamentals of Electric Circuit, 5th edition, Mathew and Alexander, McGraw-Hill, 2012, ISBN-10: 0077263197
Supportive References	Electric circuits, 9th Edition, Nilsson/Riedel, Pearson Publishing Company, 2011, ISBN-10: 0137050518. Engineering circuit analysis, 9th edition, Hayt, Kemmerly and Durbin, McGraw-Hill Science/Engineering/Math, 2011), ISBN-10: 0073529575-ISBN-13: 978-0073529578
Electronic Materials	Blackboard
Other Learning Materials	Power point slides

Review on DC Circuit Analysis Methods

1- Basic Concepts

- Basics of Electric Circuits (Units, Quantities, Components, and Network Connections (series and Parallel))
- Basic Electric Lows (Ohm’s low, KCL, KVL)
- Series Resistors and Voltage Division
- Parallel Resistors and Current Division
- Resistor Combination and Wye-Delta transformation

Kirchhoff's Current Law (KCL)

$\Sigma I_{in} = \Sigma I_{out}$ single node

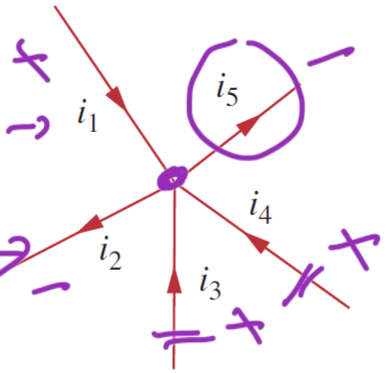
Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

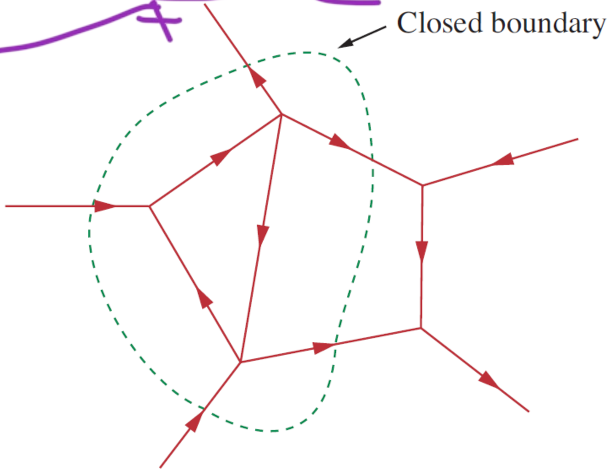
$$\sum_{n=1}^{N} i_n = 0$$

N ≜ number of branches connected to node

$i_n \triangleq n^{th}$ current entering (or leaving) the node

We might consider entering currents as positive (leaving as negative), or vice versa





$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

$$\sum I_{in} = \sum I_{out}$$

$$I_1 + I_3 + I_4 = +I_2 \pm I_5$$

$$\boxed{I_1 - i_2 + I_3 + I_4 - I_5 = 0}$$

Kirchhoff's Voltage Law - KVL

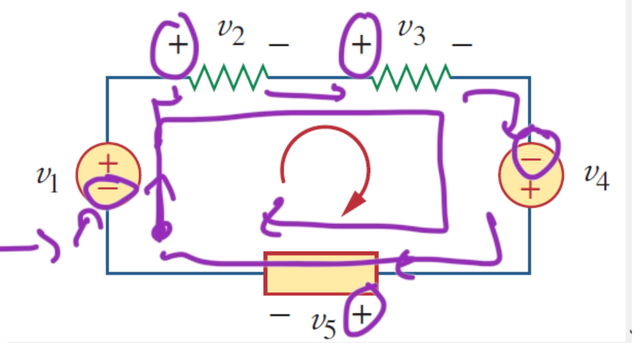
Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

$$\sum_{m=1}^{M} v_m = 0$$

M $\triangleq$ number of branches in a loop

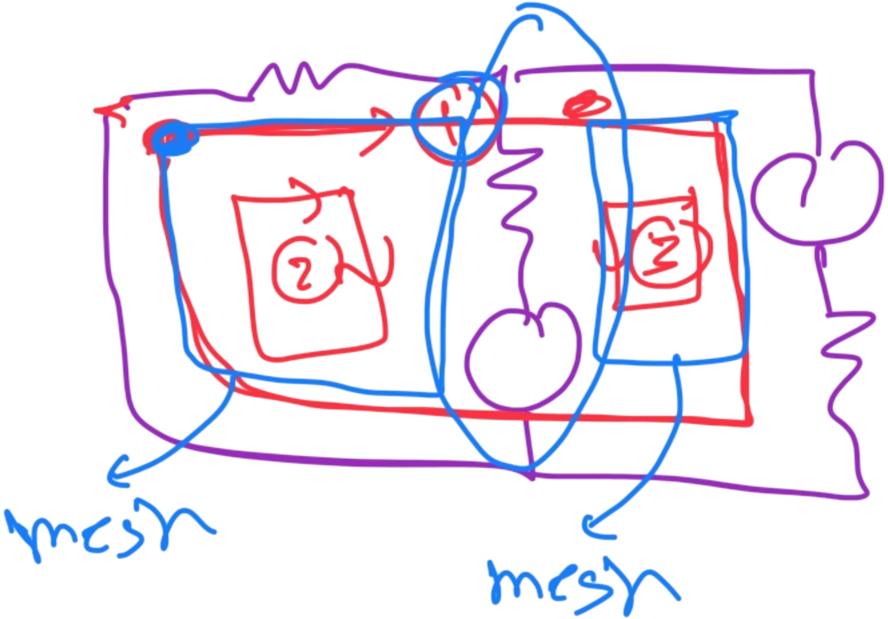
$v_m \triangleq m^{th}$ voltage (across m[th] element) in the loop

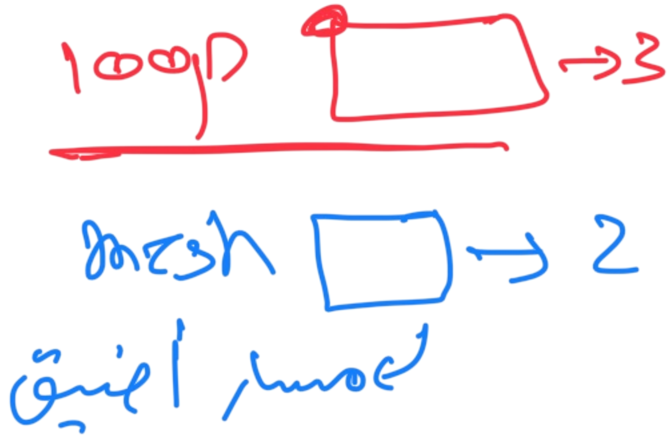
KVL can be applied clockwise or counter-clockwise around the loop. Either way, the algebraic sum is zero



Alternative Form: Sum of voltage drops = Sum of voltage rises

$$v_2 + v_3 + v_5 = v_1 + v_4$$

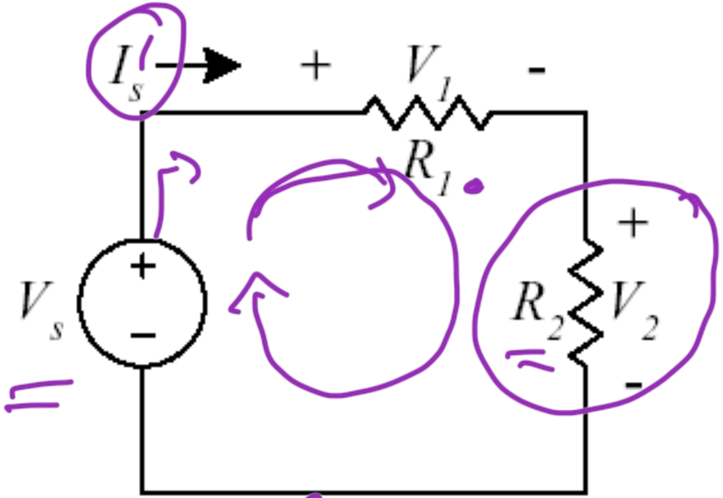




كل mesh هو loop لكن العكس غير صحيح

Voltage Divider Rule

→ series



series $I_s = I_1 = I_2$

Voltage is divided by **same ratio of resistors**

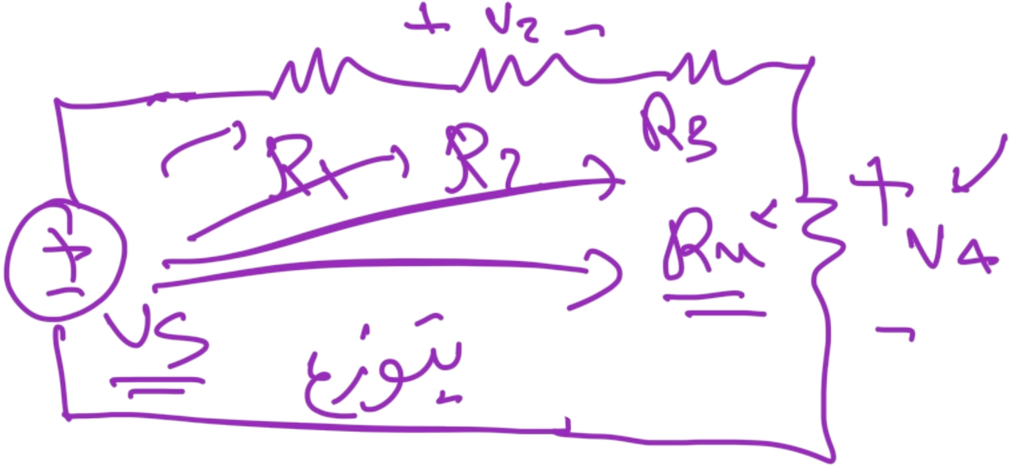
$V = IR$

$$R_{eq} = R_1 + R_2$$

$$I_s = \frac{V_s}{R_{eq}} = \frac{V_s}{R_1 + R_2}$$

$$V_2 = I_s R_2 = V_s \frac{R_2}{R_1 + R_2}$$

$$V_1 = I_s R_1 = V_s \frac{R_1}{R_1 + R_2}$$



⟹ series

Voltage divider

مجزئ الجهد

$$V_4 = \frac{R_4}{R_1 + R_2 + R_3 + R_4} \times V_s$$

$$V_2 = \frac{R_2}{R_1 + R_2 + R_3 + R_4} \times V_s$$

* Voltage divider → "series only"

$$V_N = \frac{R_N}{R_{eq}} * V_s$$

R_N → المقصودة

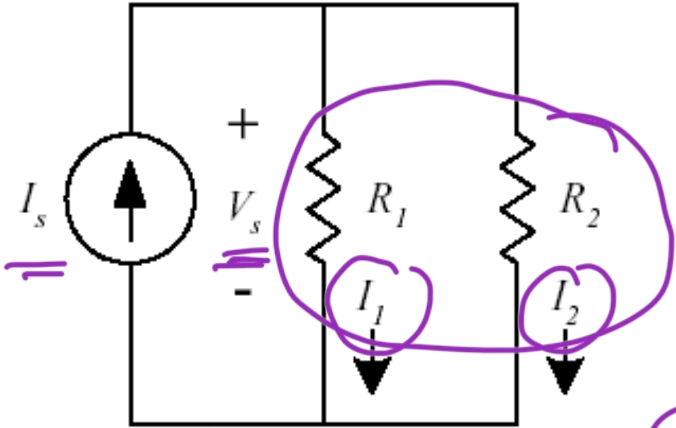
R_{eq} → $R_1 + R_2 + R_3 + \dots$

Series
V, R

Current Divider Rule

"Parallel"

I, R



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \ldots (1)$$

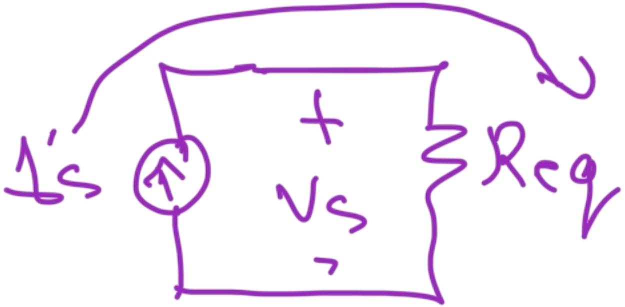
$$V_s = I_s \frac{R_1 R_2}{R_1 + R_2}$$

$$I_2 = \frac{V_s}{R_2} = I_s \frac{R_1}{R_1 + R_2}$$

$$I_1 = \frac{V_s}{R_1} = I_s \frac{R_2}{R_1 + R_2}$$

Current is divided by **opposite ratio of resistor**

parallel → $V_s = V_1 = V_2$



$$\underline{\underline{V_s}} = I_s * R_{eq} \quad (2)$$

$$I_1 = \frac{V_s}{R_1}, \quad \& \quad I_2 = \frac{V_s}{R_2} \; \ldots \quad (3)$$

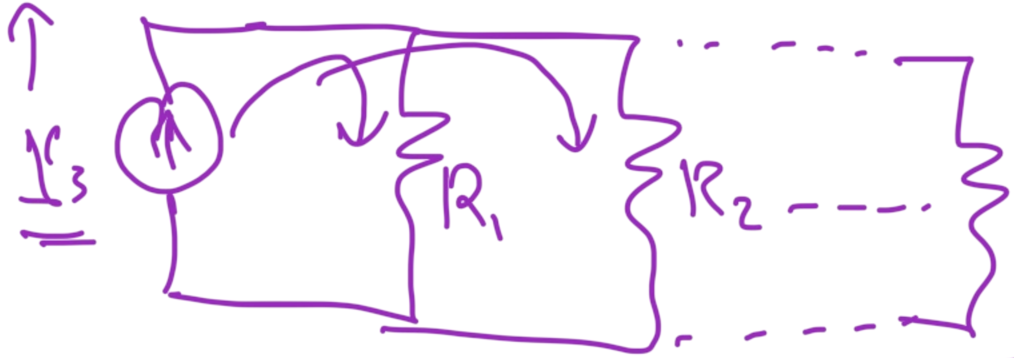
Current divider "Parallel"

$$I_2 = \frac{R_{eq}}{R_2} \times I_s$$

Parallel

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Current divider "parallel"



$$I_N = \frac{R_{eq}}{R_N} * I_s$$

مقاومة مكافئة

$$\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\cdots}$$

مقصودة

حالة خاصة

عكس

$$I_2 = \frac{R_1}{R_1+R_2} \times I_S$$

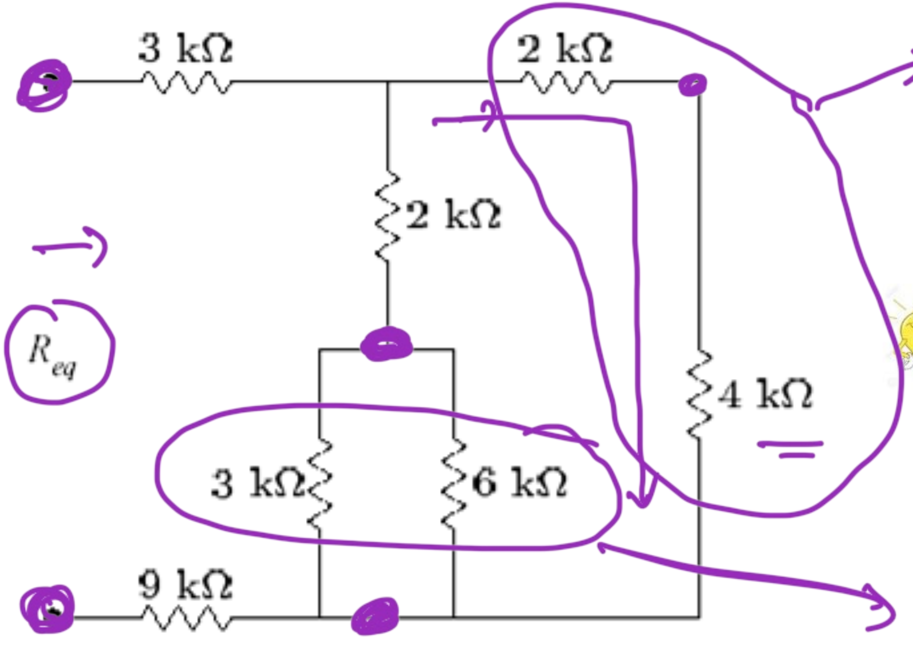
عكس

$$I_1 = \frac{R_2}{R_1+R_2} \times I_S$$

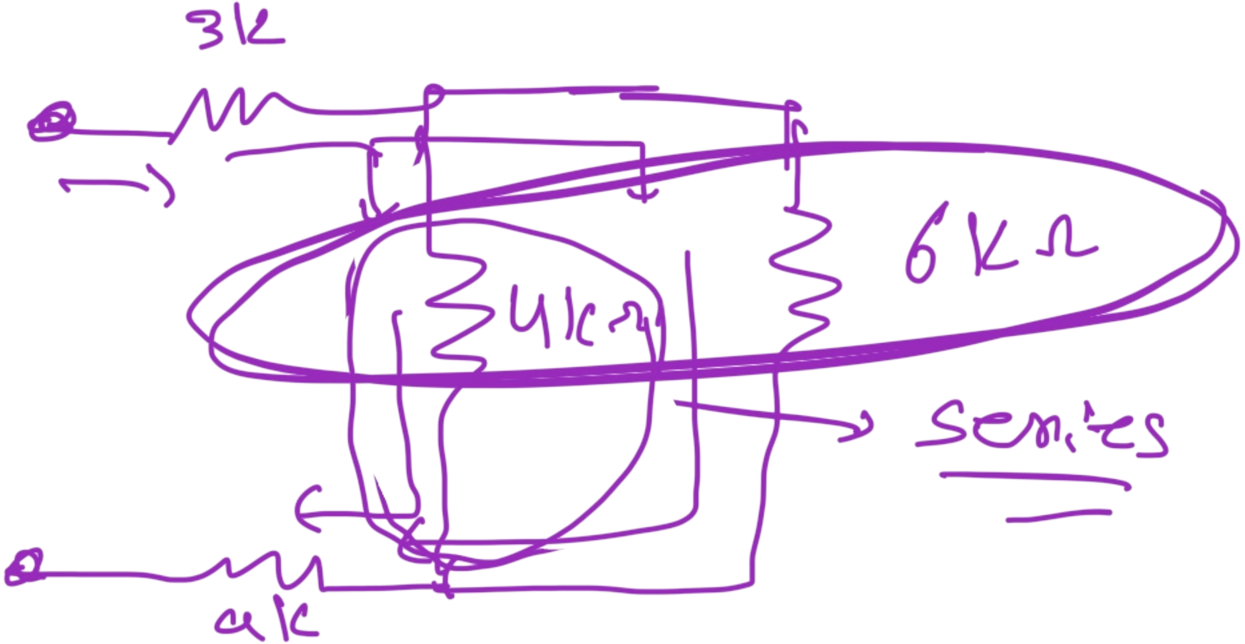
خطوة واحدة فقط

Series and Parallel Resistors

➢ Find Req in the given circuit below:



Answer: Req=3+[(2+4)//(2+3//6)+9=14.4 kΩ

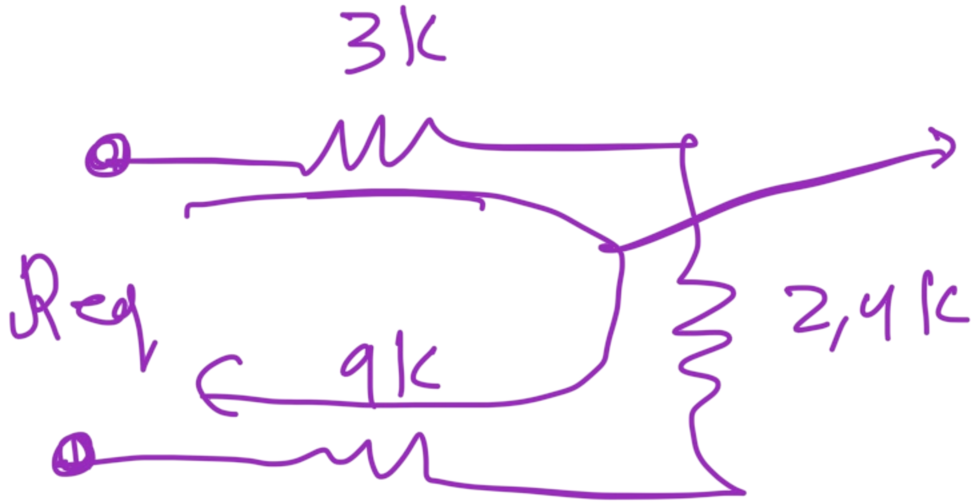


Parallel

$$\frac{1}{\frac{1}{6}+\frac{1}{4}} = 2{,}4\,k\Omega$$

Series

$2+2 = 4k$



series = 3+9+2,4

Req = 14,4 kΩ

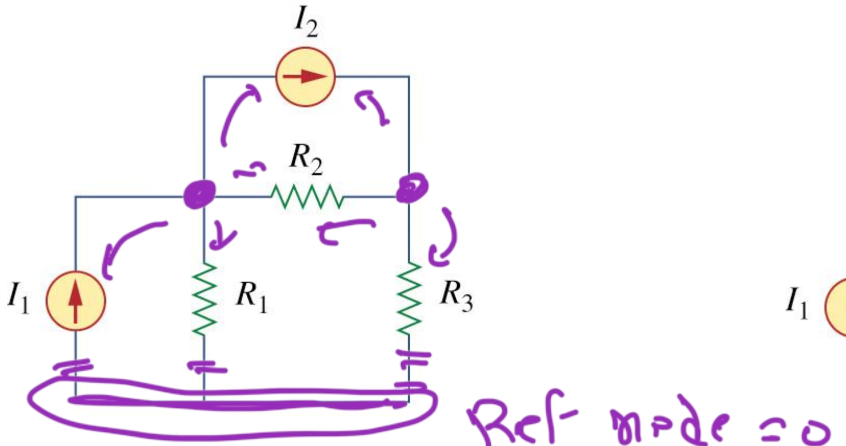
2- Circuits Methods of Analysis

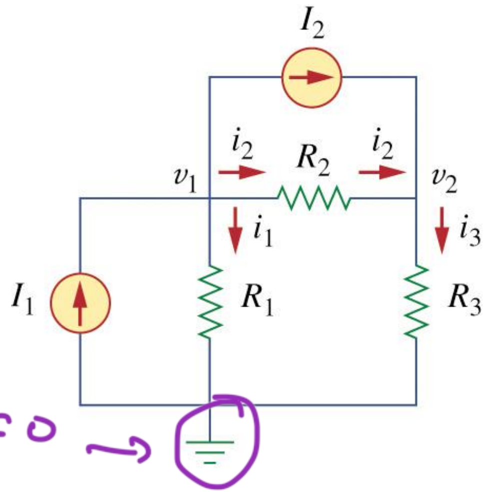
- Nodal analysis
- Nodal analysis with voltage source
- Mesh analysis
- Mesh analysis with current source
- Nodal and mesh analyses by inspection
- Nodal versus mesh analysis

Nodal Analysis

Nodal Analysis utilizes KCL to solve for unknown node voltages

Circuit Example





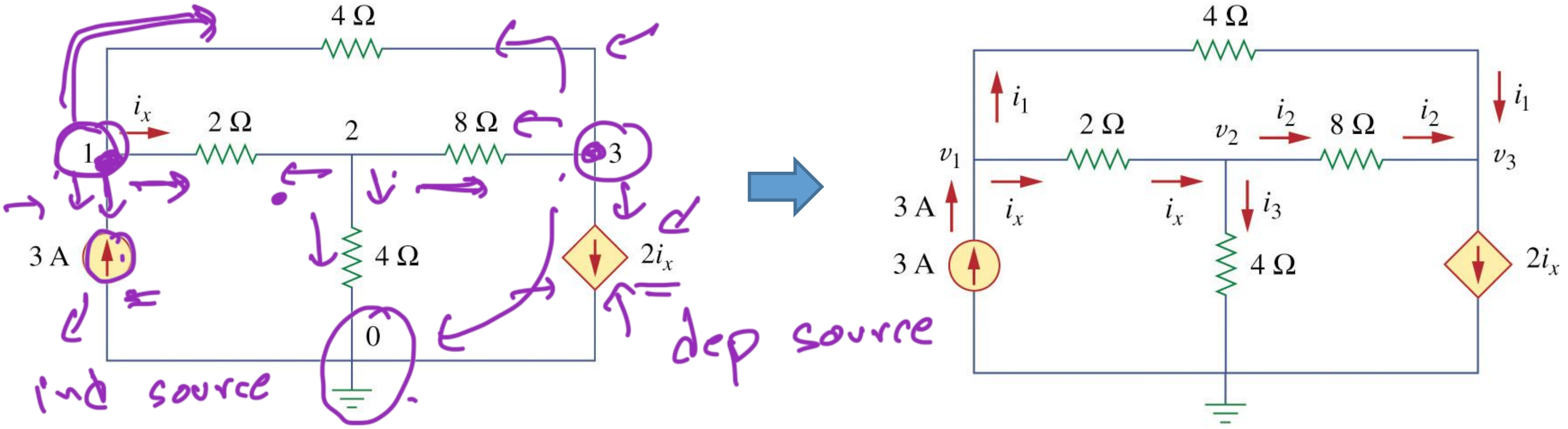
Steps to Determine Node Voltages:

1. Select a node as the reference node. Assign voltage v_1, v_2, …v_{n-1} to the remaining n-1 nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the n-1 nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

Example 2

node $\frac{V}{R} = I_e$

Determine the node voltages in the following circuit



$$I_x = \frac{V_1 - V_2}{2}$$

node ①

$$-3 + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} = 0$$

node ②

$$\frac{V_2 - V_1}{2} + \frac{V_2 - 0}{4} + \frac{V_2 - V_3}{8} = 0$$

node ③

$$\frac{V_3 - V_1}{4} + \frac{V_3 - V_2}{8} + 2\left[\frac{V_1 - V_2}{2}\right] = 0$$

$$\frac{3}{4}V_1 - \frac{V_2}{2} - \frac{V_3}{4} = 3 \longrightarrow (1)$$

$$-\frac{1}{2}V_1 + \frac{7}{8}V_2 - \frac{V_3}{8} = 0 \longrightarrow (2)$$

$$\frac{3}{4}V_1 - \frac{9}{8}V_2 + \frac{3}{8}V_3 = 0 \longrightarrow (3)$$

mode → 5 → 2

$V_1 = 4{,}8V$, $V_2 = 2{,}4V$, $V_3 = -2{,}4V$

Example 2

KCL at node 1:

$$3 = i_1 + i_x \quad \Rightarrow \quad 3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2}$$

Multiplying by 4 and rearranging terms, we get

$$\boxed{3v_1 - 2v_2 - v_3 = 12} \qquad (1)$$

KCL at node 2:

$$i_x = i_2 + i_3 \quad \Rightarrow \quad \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$

Multiplying by 8 and rearranging terms, we get

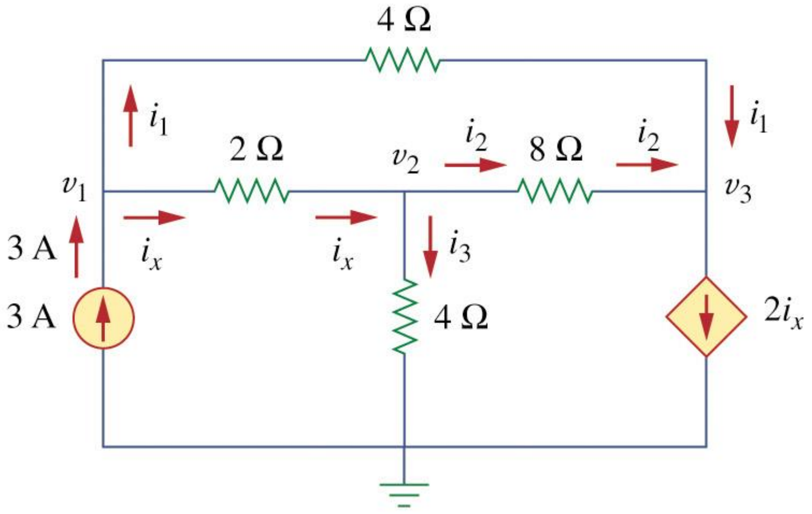
$$\boxed{-4v_1 + 7v_2 - v_3 = 0} \qquad (2)$$

KCL at node 3:

$$i_1 + i_2 = 2i_x \quad \Rightarrow \quad \frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2}$$

Multiplying by 8, rearranging terms, and dividing by 3, we get

$$\boxed{2v_1 - 3v_2 + v_3 = 0} \qquad (3)$$



Example 2

$$3v_1 - 2v_2 - v_3 = 12$$ (1),

$$-4v_1 + 7v_2 - v_3 = 0$$ (2),

$$2v_1 - 3v_2 + v_3 = 0$$ (3)

Solve using any technique

(Elimination technique, Equations reduction, Cramer's rule, or calculator)

Thus: $v_1 = 4.8\text{ V},$ $v_2 = 2.4\text{ V},$ $v_3 = -2.4\text{ V}$

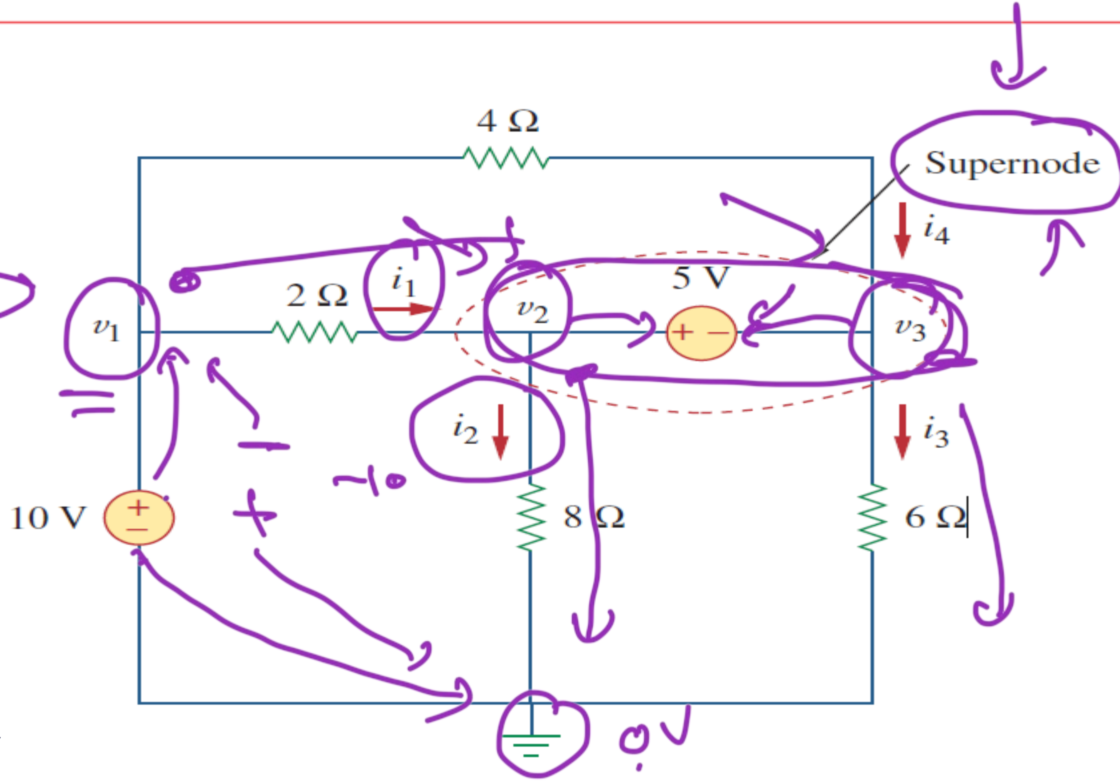
Nodal Analysis with Voltage Sources

- Case 1: The voltage source is connected between a nonreference node and the reference node:
  - The nonreference node voltage is equal to the magnitude of voltage source

    Example: in the circuit to the right:

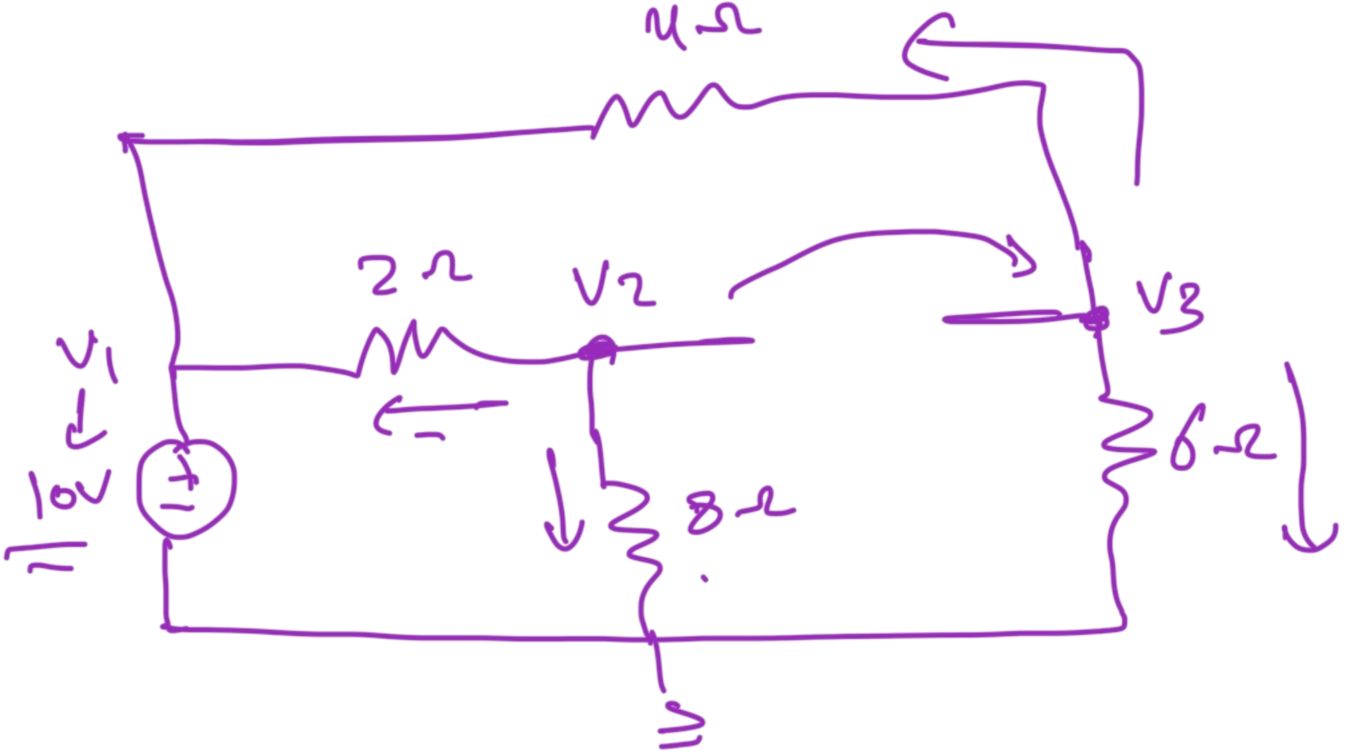
    $$v_1 = 10 \text{ V}$$

  - The number of unknown nonreference nodes is reduced by one.



$$V_2 - V_3 = 5 \rightarrow (1)$$

super node



node 2 & 3 Fusion "Zoog"

$$\frac{V_2 - 10}{2} + \frac{V_2 - 0}{8} + \frac{V_3 - 10}{4} + \frac{V_3 - 0}{6} = 0$$

$$\frac{5}{8} V_2 + \frac{5}{12} V_3 = \frac{10}{4} + \frac{10}{2} = 7,5 \rightarrow (2)$$

$$V_2 - V_3 = 5 \rightarrow (1)$$

mode → 5 → 1

$$\boxed{V_2 = 9{,}2\,V} \qquad \boxed{V_3 = 4{,}2\,V}$$

$$I_1 = \frac{V_1 - V_2}{2} = \frac{10 - 9{,}2}{2} = 0{,}4\,A$$

$$I_2 = \frac{V_2 - 0}{8} = \frac{9{,}2}{8} = 1{,}15\,A$$

$$I_3 = \frac{V_3 - 0}{6} = \frac{4{,}2}{6} = 0{,}7\,A$$

$$I_4 = \frac{V_1 - V_3}{4} = \frac{10 - 4{,}2}{4} = 1{,}45\,A$$

Nodal Analysis with Voltage Sources

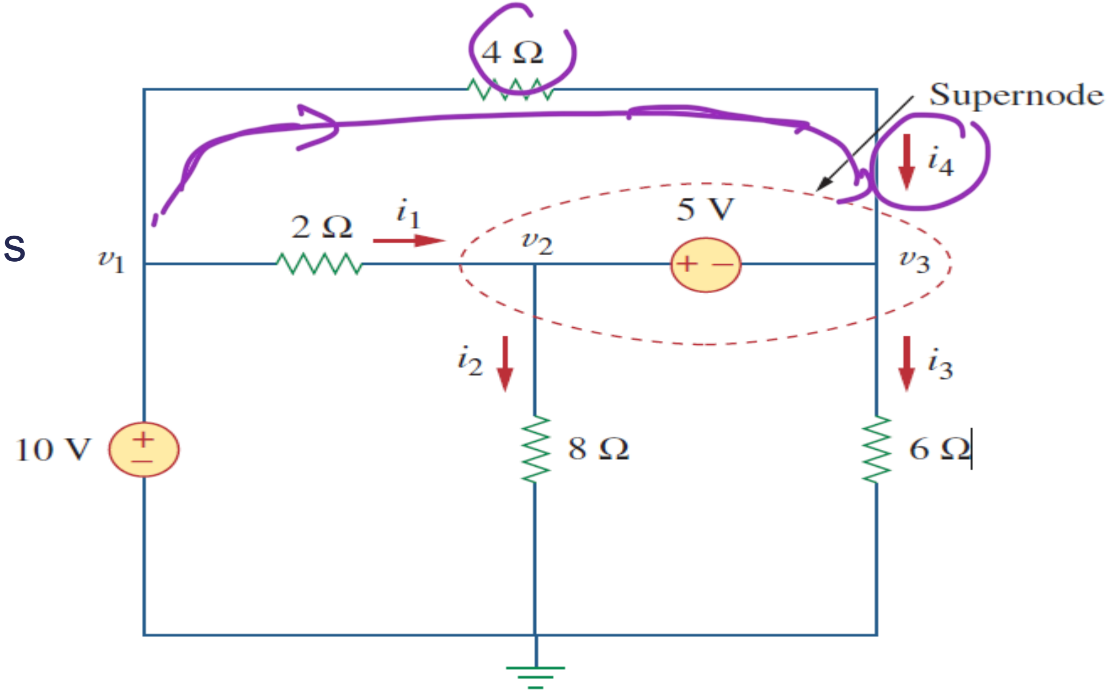
- Case 2: The voltage source is connected between two nonreferenced nodes:
  - a generalized node (supernode) is formed.
  - Write KCL of the supernode (combining node 2 and 3) as a closed surface

  Example: in the circuit to the right:

  $$i_1 + i_4 = i_2 + i_3$$

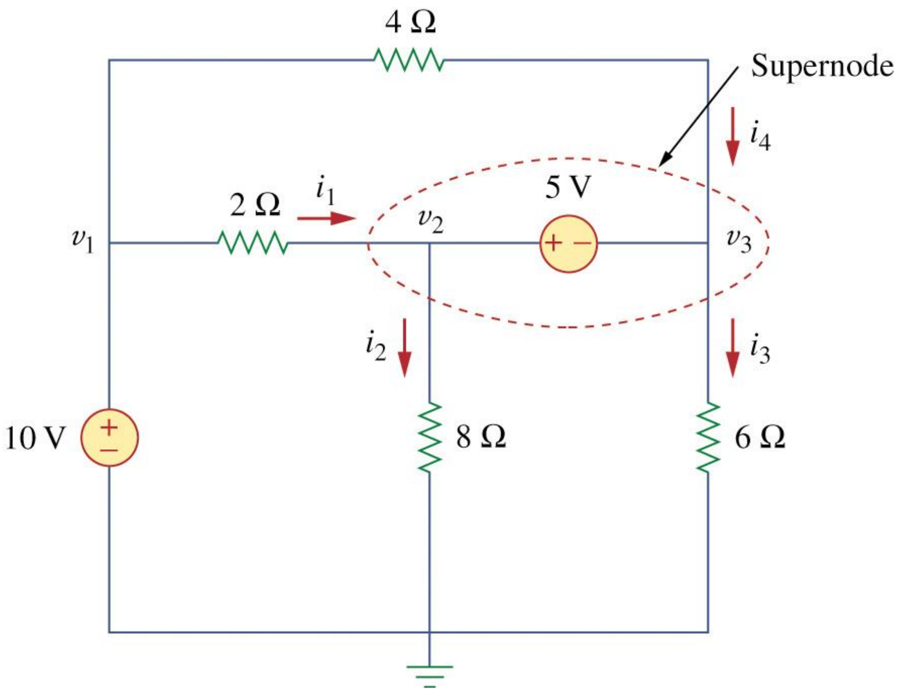
  - Use supernode KVL as one equation

  Example: in the circuit to the right: $v_2 - v_3 = 5$



Example 3

Find the node voltages in the following circuit



Solution:

The voltage at node 1 is already known

$$v_1 = 10\text{ V} \qquad (1)$$

We have only two unknown node voltages (v_2 and v_3)

We need two equations

KCL at the superndoe $\quad i_1 + i_4 = i_2 + i_3$

$$\frac{10 - v_2}{2} + \frac{10 - v_3}{4} = \frac{v_2}{8} + \frac{v_3}{6}$$

$$120 - 12v_2 + 60 - 6v_3 = 3v_2 + 4v_3$$

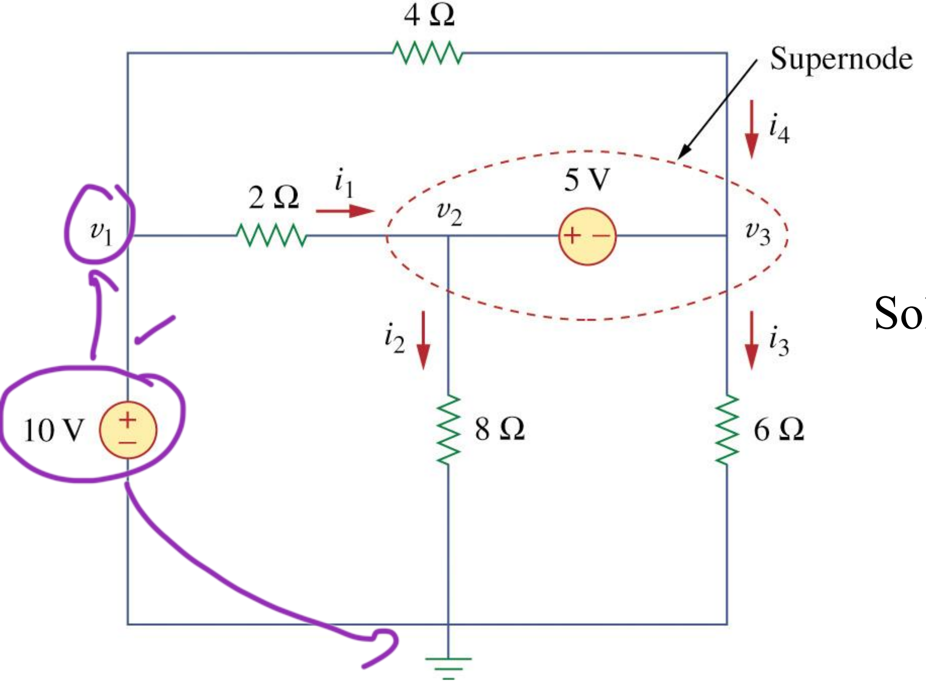
$$180 = 15v_2 + 10v_3 \qquad (2)$$

KVL at the superndoe

$$v_2 - v_3 = 5 \qquad (3)$$

Example 3

Find the node voltages in the following circuit



Solution:

$$180 = 15v_2 + 10v_3 \qquad (2)$$

$$v_2 - v_3 = 5 \qquad (3)$$

Solving equations (2) and (3) simultaneously:

$v_2 = 9.2\text{V}$ $\qquad$ $v_3 = 4.2\text{V}$

2- Methods of Analysis 2

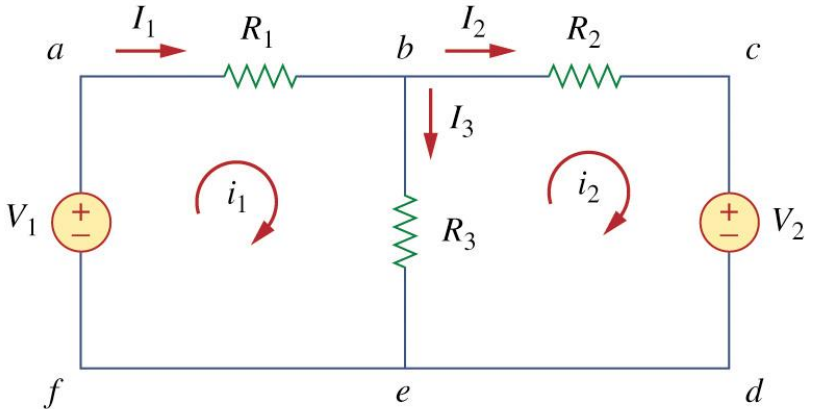
- Nodal analysis
- Nodal analysis with voltage source
- Mesh analysis
- Mesh analysis with current source
- Nodal and mesh analyses by inspection
- Nodal versus mesh analysis

Mesh Analysis

A mesh is a loop which does not contain any other loops within it.

$V = IR$

Mesh Analysis utilizes KVL to solve for unknown mesh currents

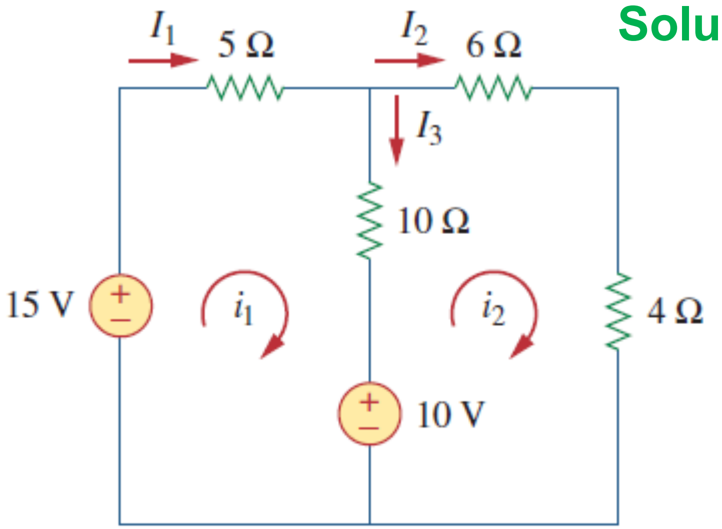


Steps to Determine mesh currents:

1. Assign mesh currents $i_1, i_2, .., i_n$ to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting *n* simultaneous equations to get the mesh currents.

Example 4

For the following circuit, use mesh analysis to find the branch currents I_1, I_2 and I_3



$$I_1 = i_1,$$
$$I_2 = i_2,$$
$$I_3 = i_1 - i_2$$

Solution: Step#1: Assign mesh currents

Step#2: Write KVL for each mesh

- KVL at mesh1: $-15 + 5I_1 + 10I_3 + 10 = 0$

$$5i_1 + 10(i_1 - i_2) = 5$$

Divide both sides by 5 and collect common terms: $3i_1 - 2i_2 = 1$ (1)

- KVL at mesh 2: $-10 - 10I_3 + 10I_2 = 0$

$$-10(i_1 - i_2) + 10i_2 = 10$$

Divide both sides by 10 and collect common terms: $-i_1 + 2i_2 = 1$ (2)

mesh → I

مع كك ب العادية

$\sum V = 0$

mesh ① ←

$$-15 + 5I_1 + 10(I_1 - I_2) + 10 = 0$$

$$\boxed{15I_1 - 10I_2 = 5} \rightarrow (1)$$

mesh ②

$6 I_2 + 4 I_2 + (-10) + 10 (I_2 - I_1) = 0$

$$-10 I_1 + 20 I_2 = 10 \rightarrow ②$$

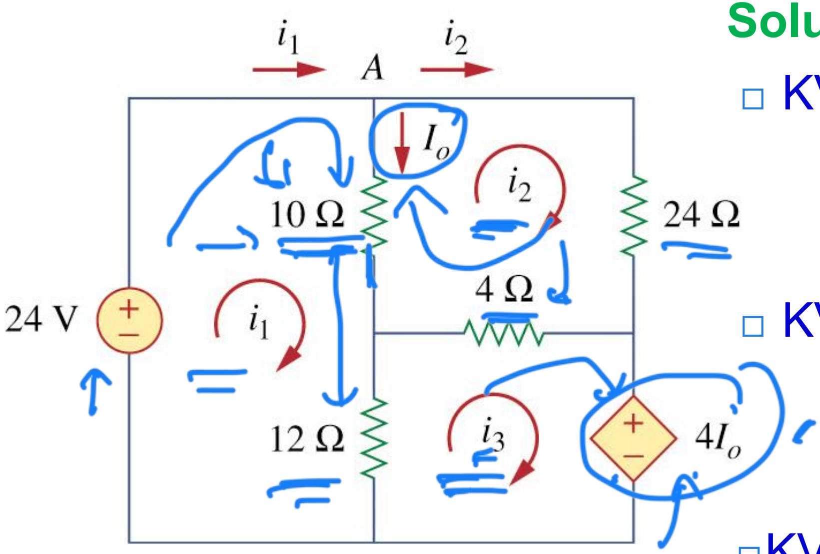
mode → 5 → ①

$$I_1 = 1A$$

$$I_2 = 1A$$

Example 5

Use mesh analysis to find the current I_0 in the following circuit



$I_o = (i_1 - i_2)$

$i_o = i_1 - i_2$

Solution:

- KVL at mesh1: $-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$

$$22i_1 - 10i_2 - 12i_3 = 24 \quad (1)$$

- KVL at mesh2: $-10(i_1 - i_2) + 24i_2 + 4(i_2 - i_3) = 0$

$$-10i_1 + 38i_2 - 4i_3 = 0 \quad (2)$$

- KVL at mesh3: $-12(i_1 - i_3) + 4(i_3 - i_2) + 4I_o = 0$

$$-12(i_1 - i_3) + 4(i_3 - i_2) + 4(i_1 - i_2) = 0$$

$$-8i_1 - 8i_2 + 16i_3 = 0 \quad (3)$$

mesh ①

$$22I_1 - 10I_2 - 12I_3 = 24 \rightarrow ①$$

mesh ②

$$-10I_1 + 38I_2 - 4I_3 = 0 \rightarrow ②$$

$$-12I_1 - 4I_2 + 16I_3 + 4I_1 - 4I_2 = 0$$

$$-8I_1 - 8I_2 + 16I_3 = 0 \rightarrow ③$$

mode → 5 → 2

$$\begin{cases} I_1 = 2,25\,A \\ I_2 = 0,75\,A \\ I_3 = 1,5\,A \end{cases}$$

Example 5

Solve using any technique

(Elimination technique, Equations reduction, Cramer's rule, or calculator)

$i_1 = 2.25\ A,$ $i_2 = 0.75\ A,$ $i_3 = 1.5\ A$

$\rightarrow I_o = (i_1 - i_2)$

Thus, $I_o = i_1 - i_2 = 1.5$ A.

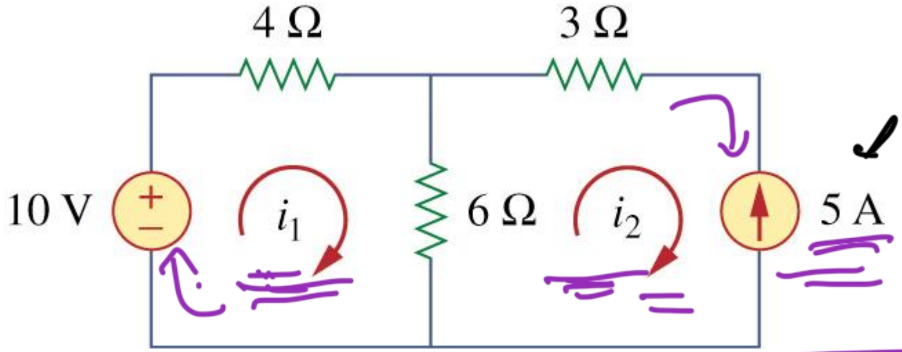
Mesh Analysis with Current Sources

- Case 1: Current source exist only in one mesh
  - The mesh current is equal to the current source

    Example: in the circuit to the right:

$$i_2 = -5A$$

  - The number of unknown currents is reduced by one.



$i_2 = -5A$

الاتجاه خاطئ

$-10 + 4i_1 + 6(i_1 - i_2) = 0$

$-10 + 4i_1 + 6i_1(-6\times-5) = 0$

$10 i_1 = 10 - 30 = -\frac{20}{10}$

$i_1 = -2A$

Mesh Analysis with Current Sources

- Case 2: Current source exists between two meshes, a super-mesh is obtained.:
  - a supermesh results when two meshes have a (dependent , independent) current source in common.
  - A supermesh has no current of its own.
    - Write KVL of the supermesh (combining Loop1 and 2) as one equation

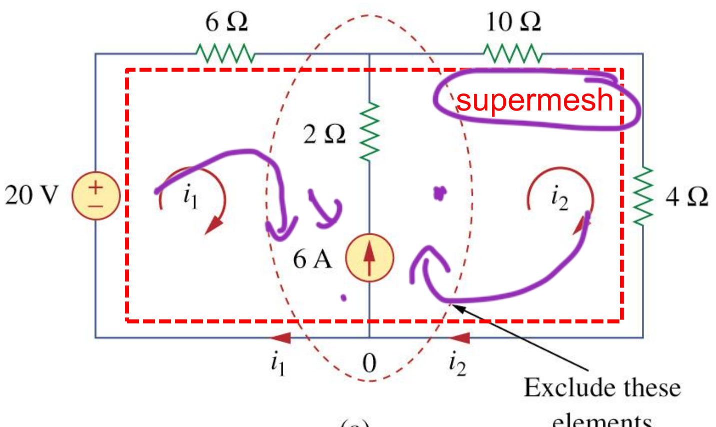
      Example: in the circuit to the right:

      $$-20 + 6i_1 + 14i_2 = 0$$

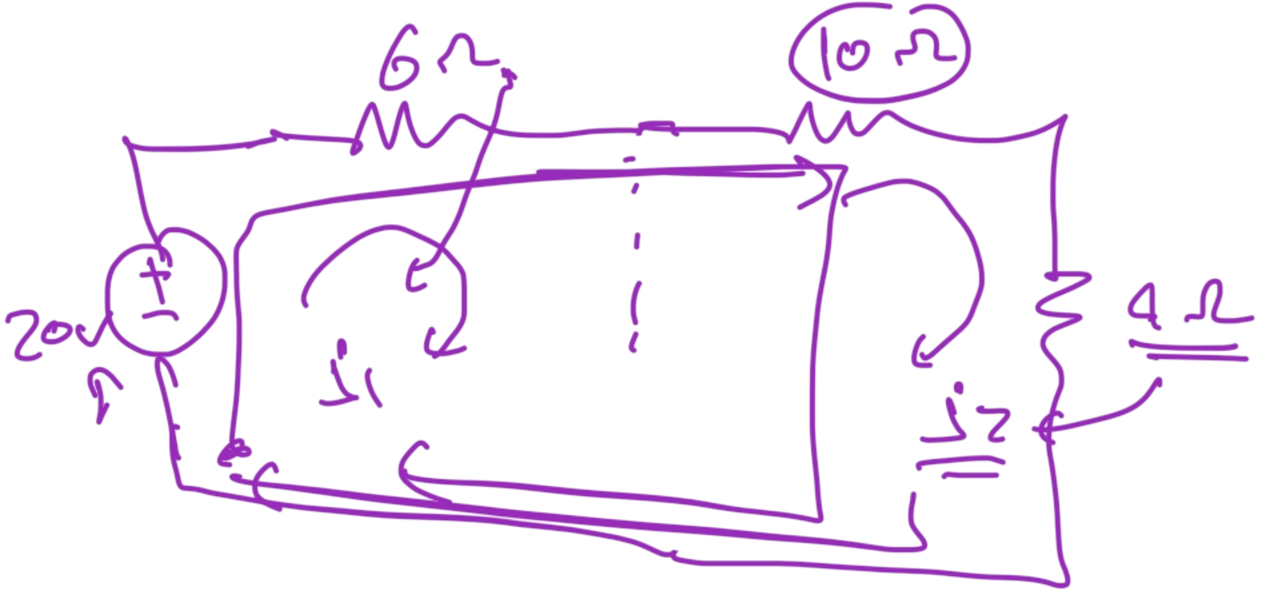
    - Use supermesh KCL as second equation

      Example: in the circuit to the right:

      $$i_2 - i_1 = 6$$



(a)



mesh 1+2

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

$$\Rightarrow \boxed{6i_1 + 14i_2 = 20}$$

$$\Rightarrow \boxed{-I_1 + I_2 = 6} \rightarrow (1)$$

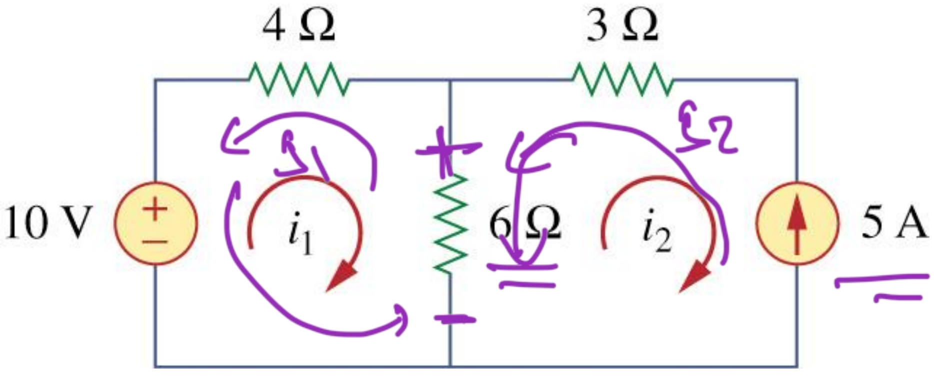
$$\boxed{6I_1 + 14I_2 = 20} \rightarrow (2)$$

mode → 5 → 1

$$\boxed{I_1 = -3{,}2\,A} \;,\; \boxed{I_2 = 2{,}8\,A}$$

Example 6

Use mesh analysis to find the voltage across the 6Ω resistor



- For mesh 2: $i_2 = -5\text{A}$

- KVL at mesh 1:

$$-10 + 4i_1 + 6(i_1 - i_2) = 0$$

$$-10 + 10i_1 - 6\,(-5) = 0$$

$$10i_1 = -20$$

$$i_1 = -2\text{A}$$

The voltage across the 6Ω resistor $= 6(i_1 - i_2) = 6(-2 + 5) = 18V$

Example 7

Find the mesh currents i_1 and i_2

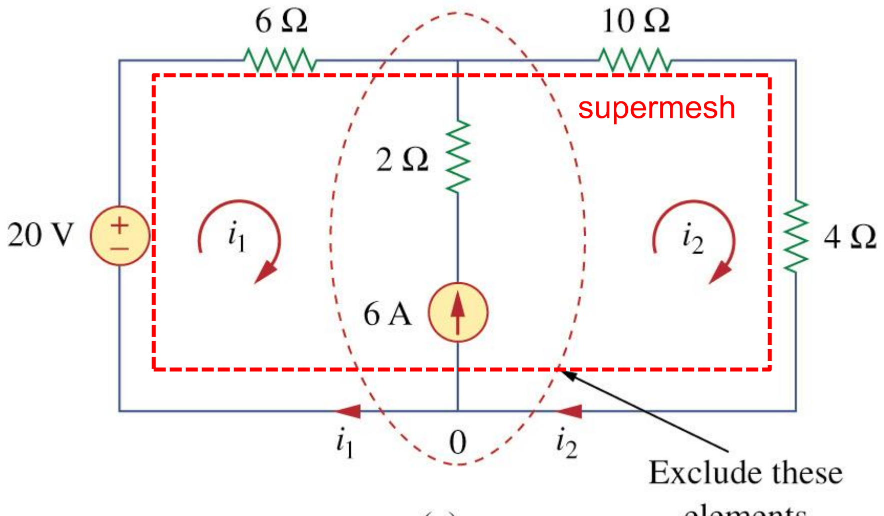
Solution:

- Supermesh consists of two meshes, two equations are needed;
  - one equation is obtained using KVL and Ohm's law to the supermesh, and

$$-20 + 6i_1 + 14i_2 = 0 \qquad \boxed{6i_1 + 14i_2 = 20} \quad (1)$$

  - the other equation is obtained by KCL at the current source.

$$\boxed{i_1 - i_2 = -6} \quad (2)$$



(a)

form (2) in (1)

$$6(i_2 - 6) + 14i_2 = 20$$

$$\boxed{i_2 = 2.8A}$$

in (2)

$$\boxed{i_1 = -3.2A}$$

2- Methods of Analysis- 3

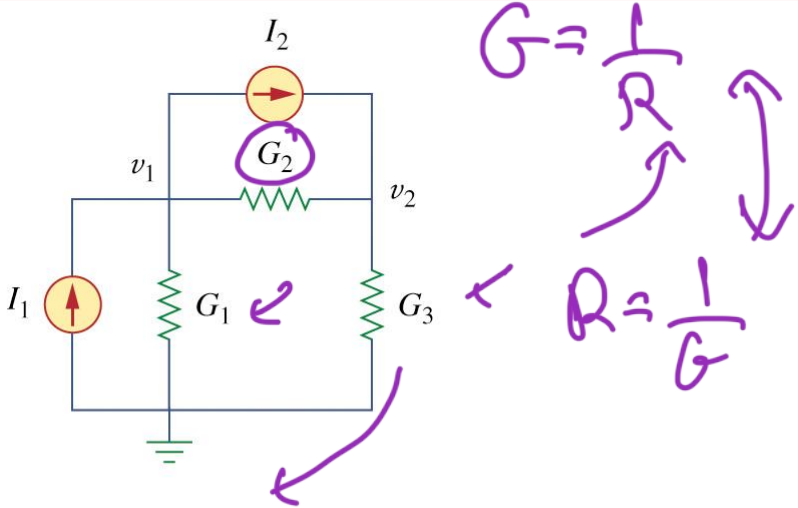
- Nodal analysis
- Nodal analysis with voltage source
- Mesh analysis
- Mesh analysis with current source
- Nodal and mesh analyses by inspection
- Nodal versus mesh analysis

Nodal and Mesh Analysis by Inspection

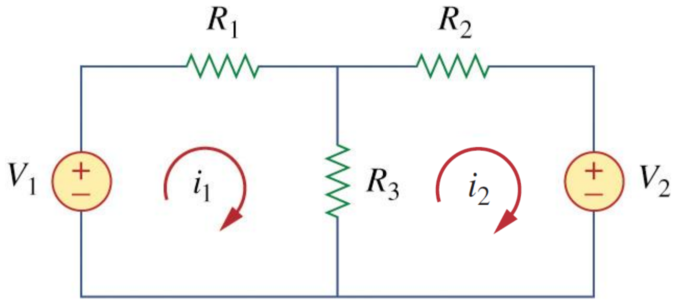
Conductance

The analysis equations can be obtained by direct inspection

(a) For circuits with only resistors and independent current sources



(b) For planar circuits with only resistors and independent voltage sources



Nodal by Inspection

For circuits with **only resistors and independent current sources**

NOTE: For consistency, while writing each node equation, use KCL form of:

$$\sum currents\ exting\ the\ node = 0 \qquad \sum i_{out} = 0$$

1) FIND A REFERENCE NODE WITH KNOWN VOLTAGE
2) ASSIGN VOLTAGES TO OTHER NODES
3) ASSIGN CONDUCTANCE TO EACH RESISTOR
4) FIND THE ELEMENTS OF THE CONDUCTANCE MATRIX
   a) FOR EACH NODE G_{ii}
   b) BETWEEN EACH 2 NODES $G_{ij} = G_{ji}$
5) SET UP THE NODE EQUATIONS IN MATRIX FORM
6) SOLVE THE NODE-VOLTAGE EQUATIONS

$G = \frac{1}{R}$

Example: Write the nodal equations for the following circuit.

KCL at node 1: $-I_1 + I_2 + G_1 v_1 + G_2(v_1 - v_2) = 0$

$$(G_1 + G_2)\, v_1 - G_2 v_2 = I_1 - I_2$$

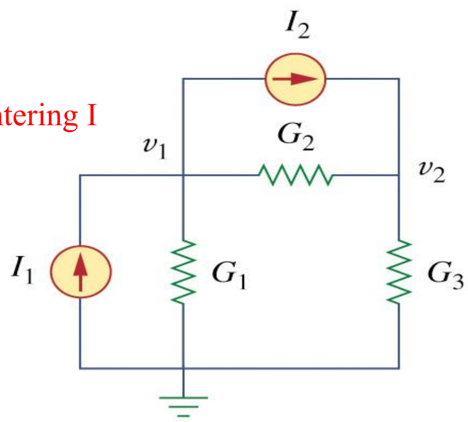
KCL at node 2: $-I_2 + G_3 v_1 + G_2(v_2 - v_1) = 0$

$$-G_2 v_1 + (G_2 + G_3) v_2 = I_2$$

← Sum of entering I

Other nodes X – sum of Common G

Same nodes X sum of connected G



Matrix form:

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

Mesh by Inspection

For circuits with **only resistors and independent voltage sources**

NOTE: For consistency, while writing each mesh equation, use **same mesh current** to provide **reference** for voltage **polarity**

Example: Write the mesh equations for the following circuit.

KVL at mesh1: $-V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0$

$$(R_1 + R_3)\, i_1 - R_3 i_2 = V_1$$

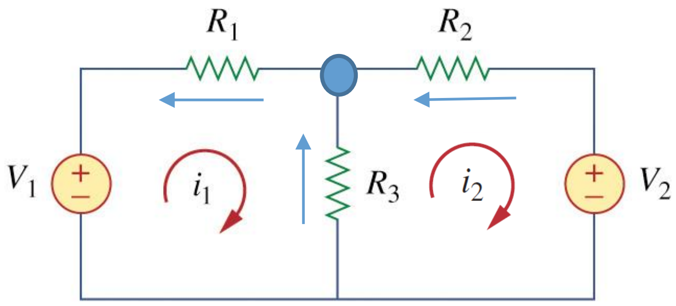
KVL at mesh2: $R_3(i_1 - i_2) - R_2 i_2 - V_2 = 0$

$$-R_3 i_1 + (R_2 + R_3) i_2 = -V_2$$

Sum of V supporting the mesh current

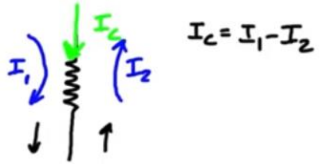
Other mesh X – sum of Common R

Same mesh X sum of loop R



Matrix form:

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$



Nodal Versus Mesh Analysis

- Both nodal and mesh analyses provide a systematic way of analyzing a complex network.
- The choice of the better method dictated by two factors.
  - First factor : nature of the particular network. The key is to select the method that results in the smaller number of equations.
  - Second factor : information required.

Nodal → 4 node → 1 super node

mesh → 2 mesh

3- Circuit Theorems

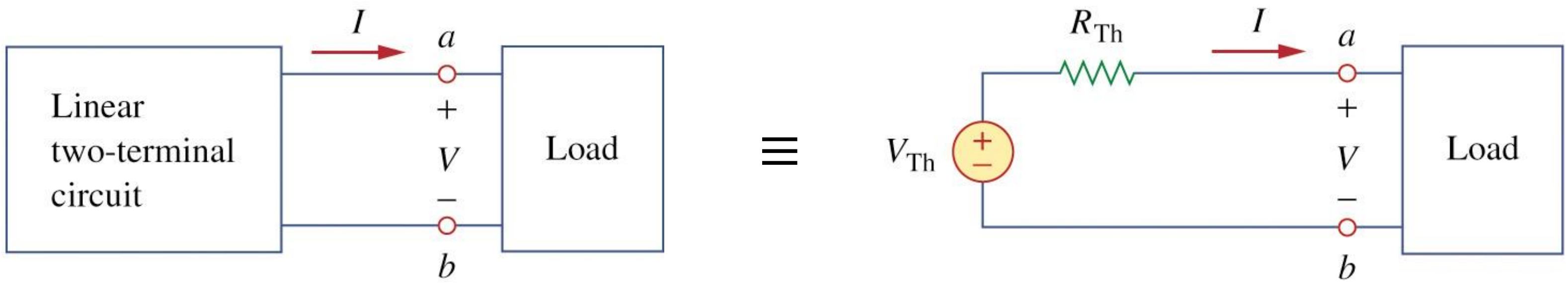
- Introduction and Linearity property
- Superposition
- Source transformations
- Thevenin's theorem
- Norton's theorem
- Maximum power transfer

Thevenin's Theorem

- Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}

Where; V_{Th} is the open circuit voltage at the terminals and

R_{Th} is the input or equivalent resistance at the terminals when the independent source are turn off.



Original Circuit

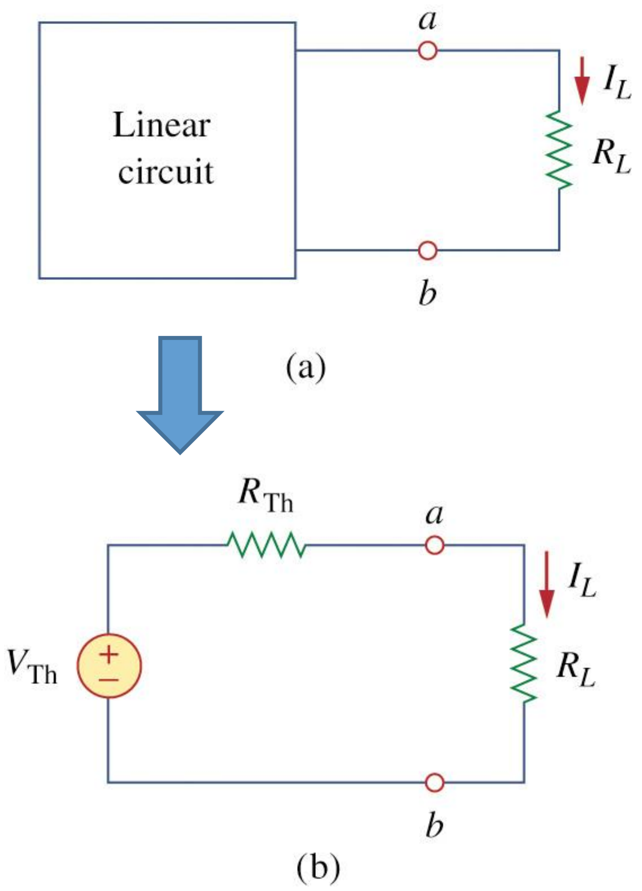
Thevenin equivalent circuit

Thevenin's equivalent circuit

Simplified circuit

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L}$$

$$V_L = R_L I_L = \frac{R_L}{R_{\text{Th}} + R_L} V_{\text{Th}}$$

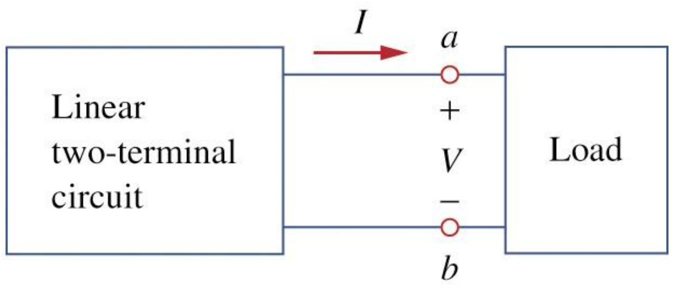


How to Find Thevenin Equivalent Circuit

- First, **Open the circuit** (remove the load) at the **points** of interest **a-b**

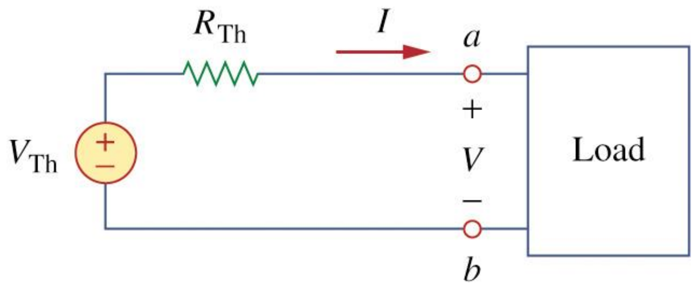
1- V_{th}= Open circuit voltage (keep all sources intact)

2- R_{th}= Open circuit **equivalent resistance** appears at terminals **a-b** while all **independent sources=0** (voltage source=SC , current source=OC).



Original Circuit

≡



Thevenin equivalent circuit

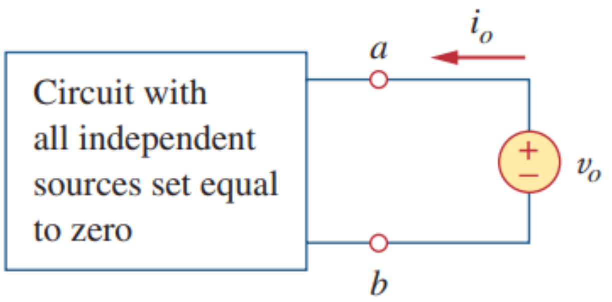
How to Find Thevenin Equivalent Circuit

- To find **R_{th}** we have to consider two cases

1- Case 1 (Circuit has No dependent sources): turn off all independent sources, then find the equivalent R_{th} using series/parallel combinations.

2- Case 2 (Circuit has dependent sources): turn off all independent sources and keep dependent sources intact (as superposition). In order to find equivalent R_{th} , apply v_o and find the produced current i_o. Then, $R_{th} = \frac{v_o}{i_o}$

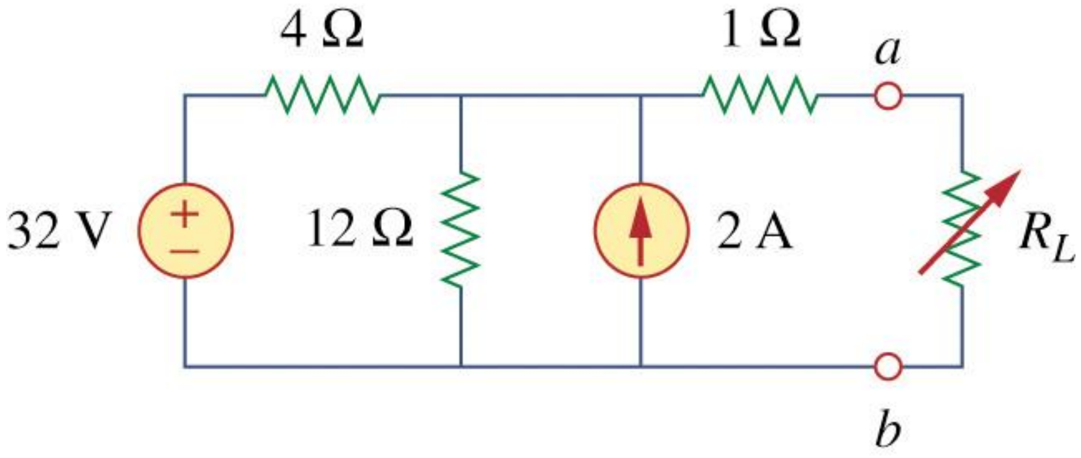
For simplicity: put $v_o = 1V$ and find the produced current i_o. Then, $R_{th} = \frac{1}{i_o}$



$$R_{Th} = \frac{v_o}{i_o}$$

Example 10

- Find the Thevenin's equivalent of the circuit shown below, to the left of the terminals *a-b*. Then find the current through R_L = 6,16,and 36 Ω.



Example 10

Solution:

For R_{th},

Voltage source =SC,

Current source =OC

$$R_{th} = 1 + 4//12 = 4\Omega$$

For V_{th},

Keep original circuit

V_{th}=V_{ac} (OC)

$$V_{th} = 12(i_1 - i_2) = ?$$

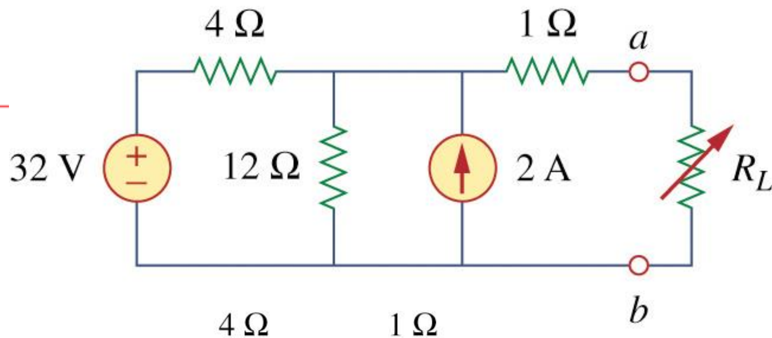
Using mesh analysis

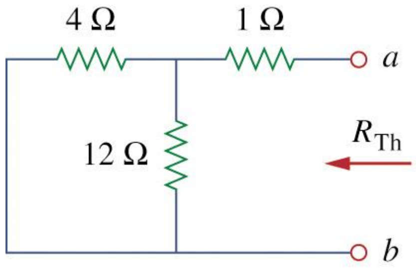
$i_2 = -2A,$

$16i_1 - 12i_2 = 32,$

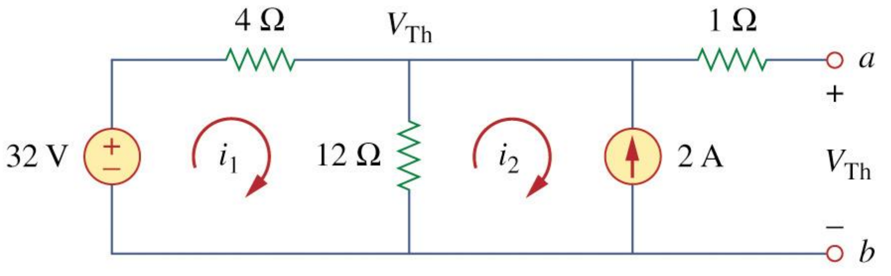
$i_1 = 0.5A,$

$$V_{th} = 12(i_1 - i_2) = 30V$$





(a)



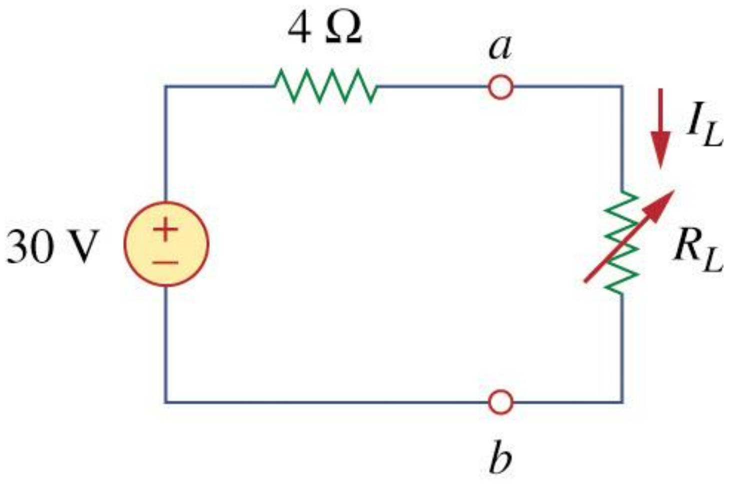
(b)

Example 10

Solution:

To get i_L :

$$i_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}$$



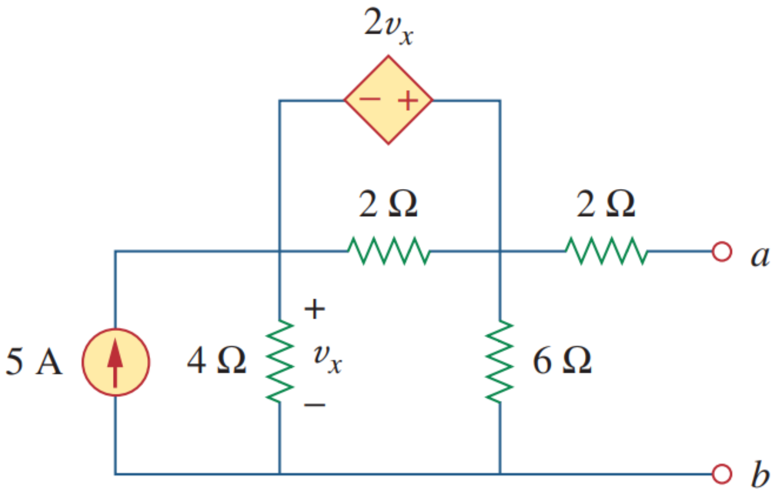
$$R_L = 6 \rightarrow I_L = 30/10 = 3\text{A}$$

$$R_L = 16 \rightarrow I_L = 30/20 = 1.5\text{A}$$

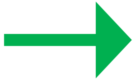
$$R_L = 36 \rightarrow I_L = 30/40 = 0.75\text{A}$$

Example 11

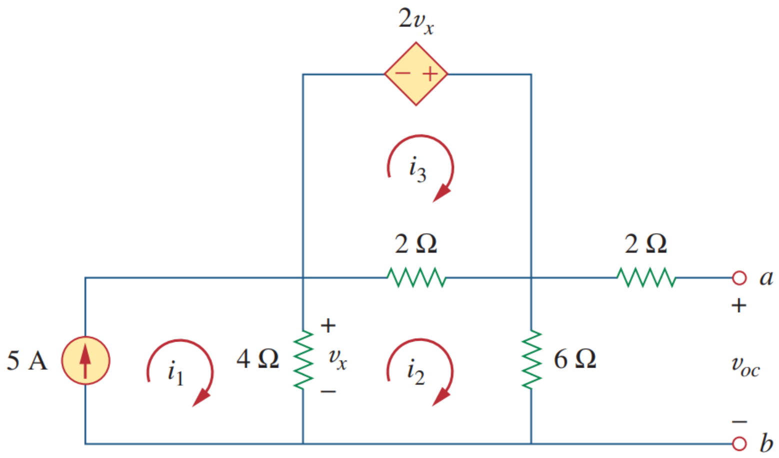
Find the Thevenin equivalent of the following circuit at terminals a-b.



Solution:



1- For V_{th}, solve the following circuit to find v_{ab} (v_{OC}).



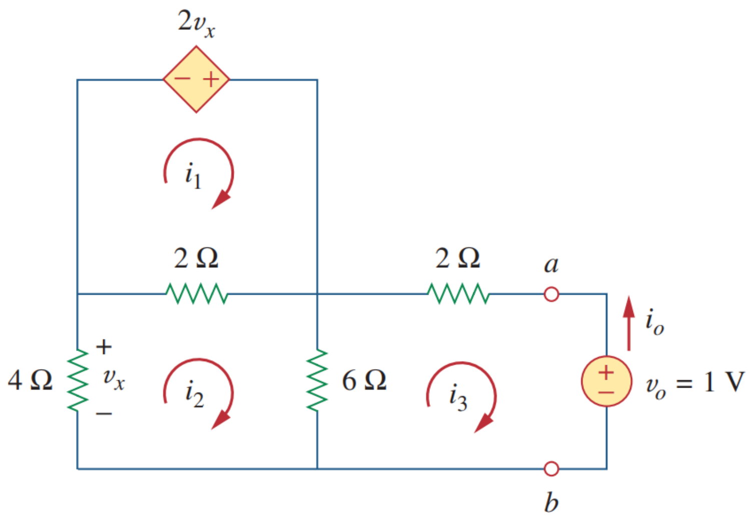
(b)

Example 11

Find the Thevenin equivalent of the following circuit at terminals a-b.

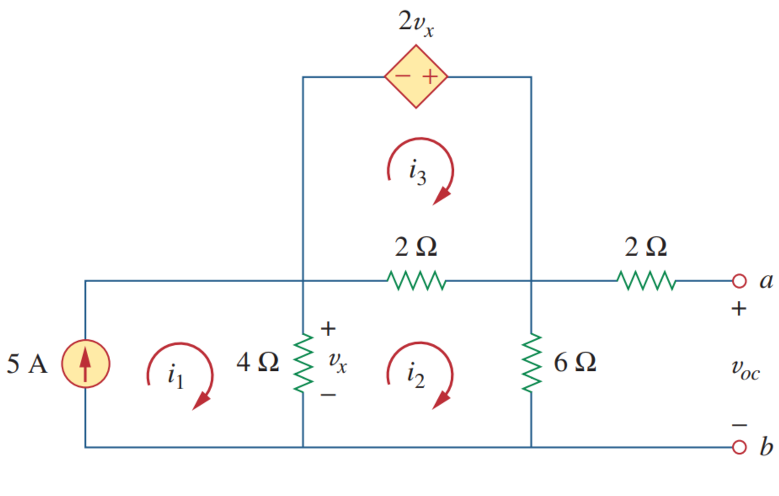
Solution:

2- For R_{th}, solve the following circuit to find io. Then, $R_{th} = 1/i_o$



(a)

1- For V_{th}, solve the following circuit to find v_{ab} (v_{OC}).



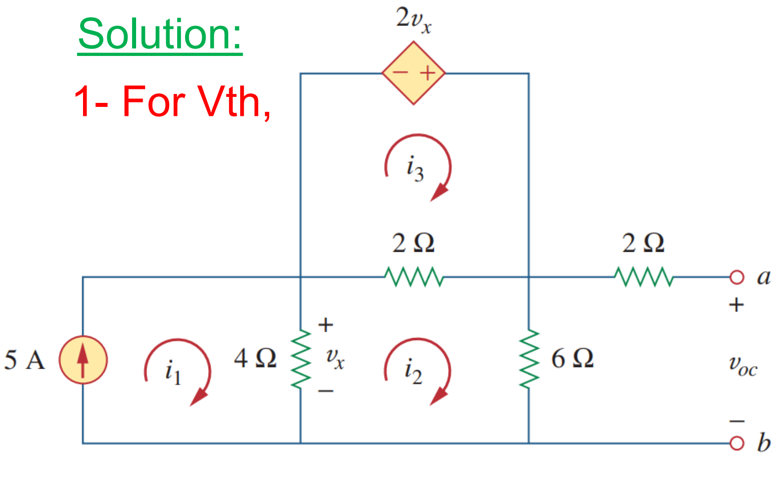
(b)

Example 11

Find the Thevenin equivalent of the following circuit at terminals a-b.

Solution:

1- For Vth,



(b)

Applying mesh analysis:

Mesh 1: $i_1 = 5$

Mesh 2: $4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$

$\Rightarrow$ $12i_2 - 4i_1 - 2i_3 = 0$

Mesh 3: $-2v_x + 2(i_3 - i_2) = 0 \quad \Rightarrow \quad v_x = i_3 - i_2$

But, $4(i_1 - i_2) = v_x$. $\Rightarrow$ $4i_1 - 3i_2 - i3 = 0$

Solving these equations leads to $i_2 = 10/3$

$V_{Th} = v_{oc} = 6i_2 = 20 \text{ V}$

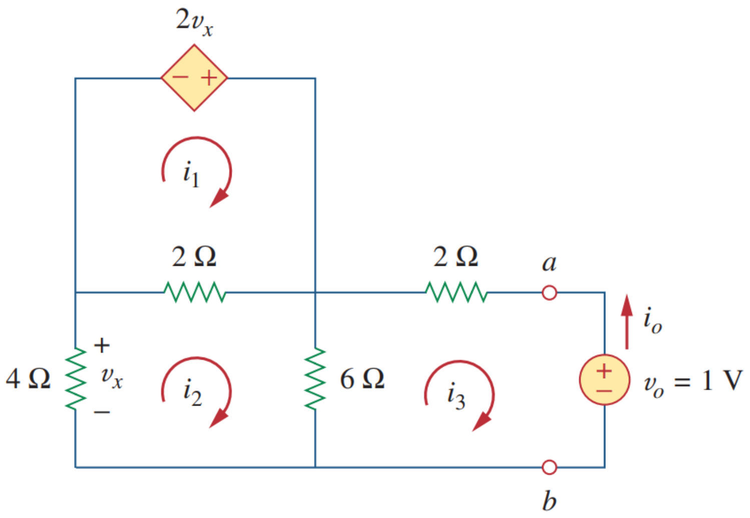
Example 11

Find the Thevenin equivalent of the following circuit at terminals a-b.

Solution:

2- For Rth, Put $v_o = 1V$

and kill any independent sources



(a)

Applying mesh analysis:

Mesh 1: $-2v_x + 2(i_1 - i_2) = 0$ or $v_x = i_1 - i_2$

But $-4i_2 = v_x = i_1 - i_2$; hence, $\Rightarrow$ $i_1 = -3i_2$

Mesh 2: $4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$

Mesh 3: $6(i_3 - i_2) + 2i_3 + 1 = 0$

Solving these equations gives

$$i_3 = -\frac{1}{6}\text{ A}$$

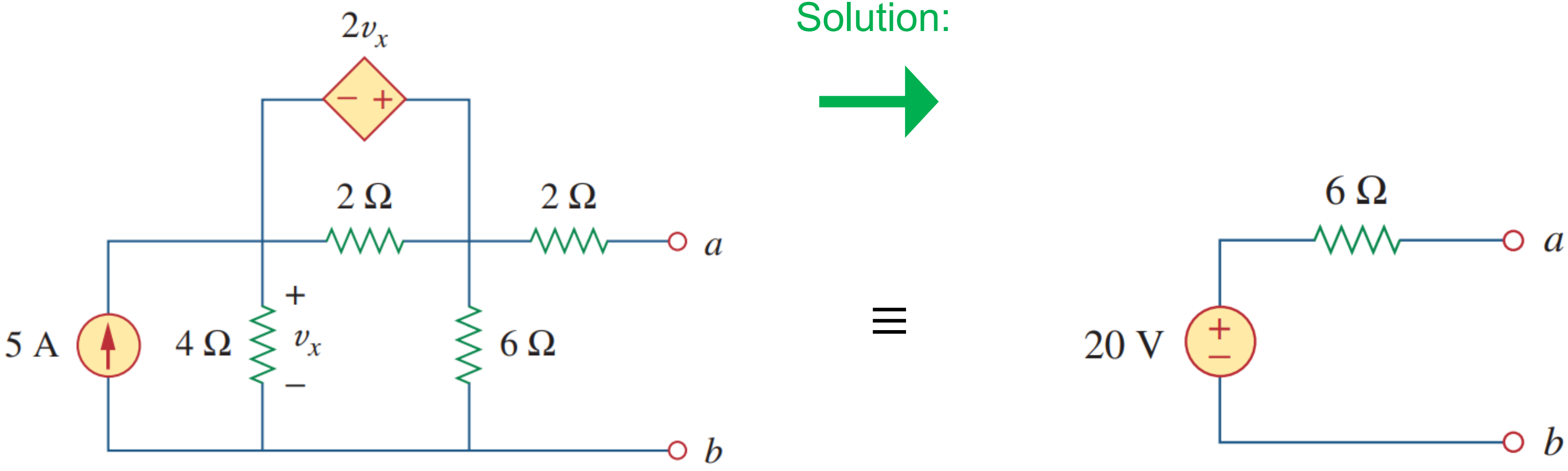
But $i_o = -i_3 = 1/6$ A. Hence,

$$R_{Th} = \frac{1\text{ V}}{i_o} = 6\ \Omega$$

Example 11

Find the Thevenin equivalent of the following circuit at terminals a-b.

Solution:



3- Circuit Theorems 2

Introduction and Linearity property

Superposition

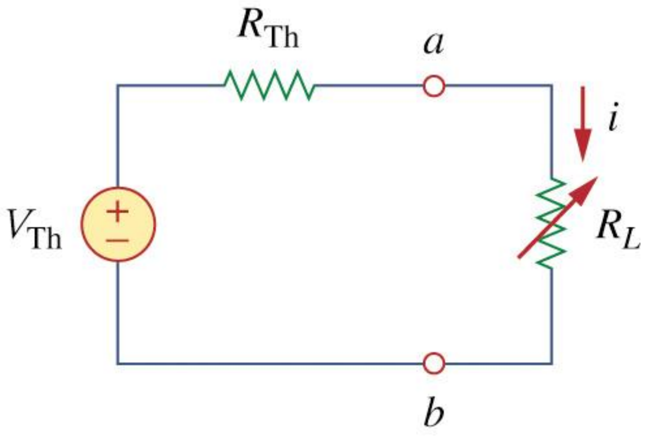
Source transformations

Thevenin’s theorem

Norton’s theorem

Maximum power transfer

Maximum Power Transfer

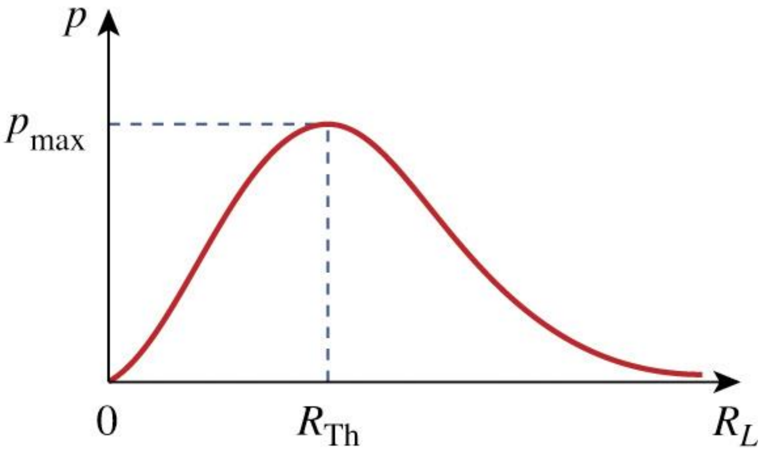


$$p = i^2 R_L = \left(\frac{V_{TH}}{R_{TH} + R_L} \right)^2 R_L$$

- **Maximum power** is transferred to the **load** when the load resistance equals the Thevenin resistance as seen the load (***R**$_L$* **=** ***R**$_{TH}$*).

$$R_L = R_{TH}$$

$$p_{max} = \frac{V_{TH}^2}{4R_{TH}}$$



Maximum Power Transfer: Mathematical Proof

$$\frac{dp}{dR_L} = V_{TH}^2 \left[\frac{(R_{TH} + R_L)^2 - 2R_L(R_{TH} + R_L)}{(R_{TH} + R_L)^4} \right]$$

$$= V_{TH}^2 \left[\frac{(R_{TH} + R_L - 2R_L)}{(R_{TH} + R_L)^3} \right] = 0$$

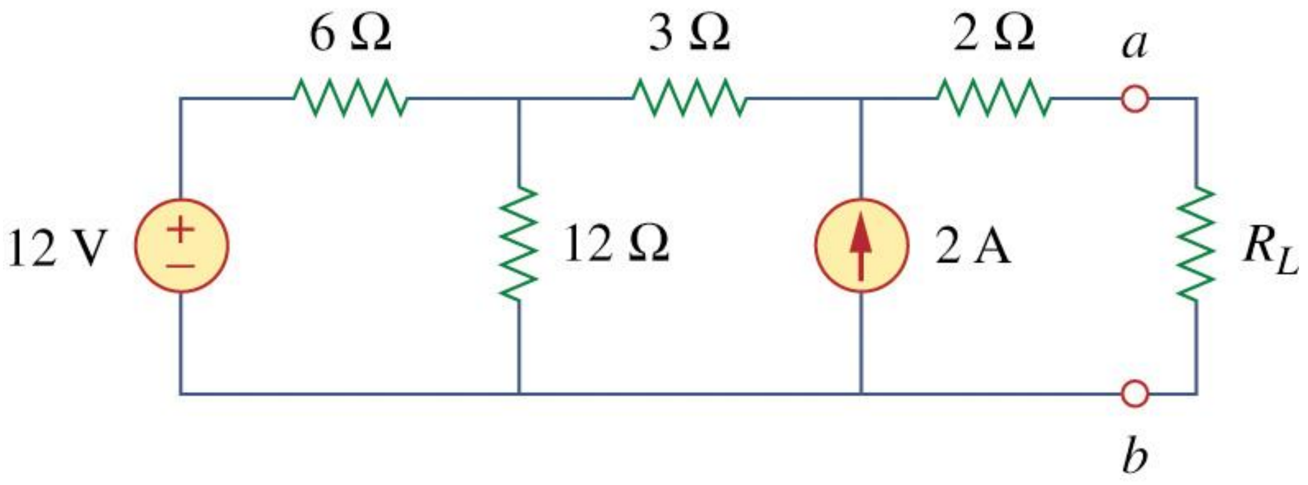
$$0 = (R_{TH} + R_L - 2R_L) = (R_{TH} - R_L)$$

$$\boxed{R_L = R_{TH}}$$

$$\boxed{p_{\max} = \frac{V_{TH}^2}{4R_{TH}}}$$

Example 12

- Find the value of R_L for maximum power transfer in the circuit of the following figure. Find the maximum power.

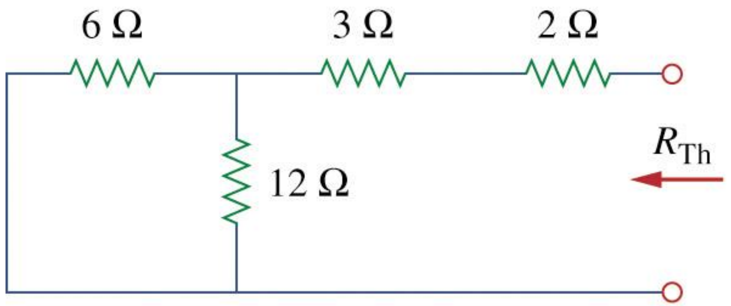


Example 12

Solution:

1- To find R_{TH},

$$R_{TH} = 2 + 3 + 6 \| 12 = 5 + \frac{6 \times 12}{18} = 9\Omega$$

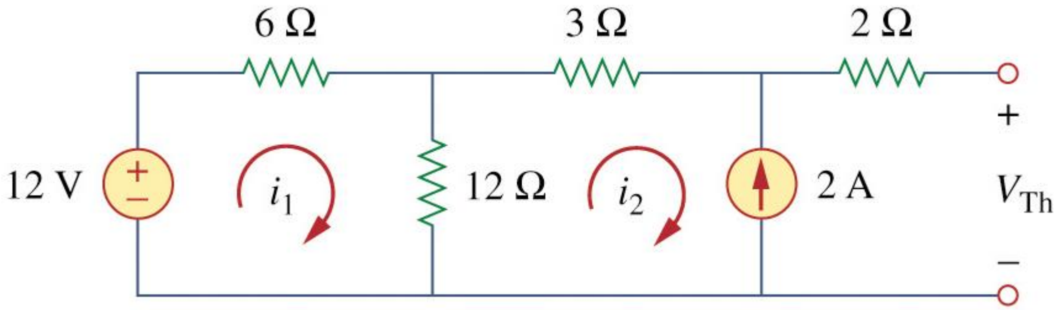


2- To find V_{TH},

Mesh: $-12 + 18i_1 - 12i_2, \quad i_2 = -2A$

$$-12 + 6i_i 1 + 3i_2 + 2(0) + V_{TH} = 0$$

$$\therefore \; V_{TH} = 22V$$



For maximum power:

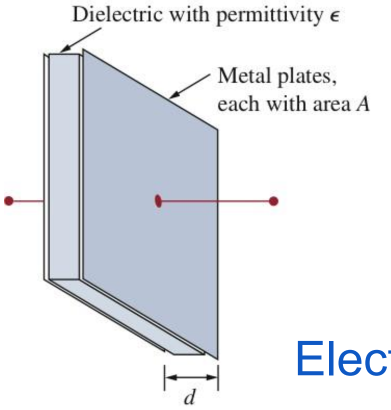
$$R_L = R_{TH} = 9\Omega$$

$$p_{\max} = \frac{V_{TH}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44W$$

4- Capacitors and Inductors

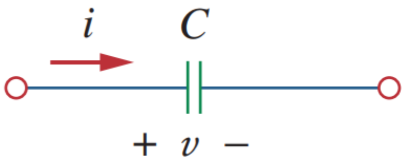
Summary of Capacitors

Physical structure



$$C = \frac{\varepsilon A}{d}$$

Electrical Characteristics



$$q = Cv$$

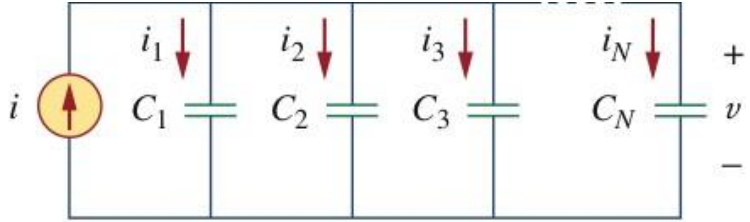
$$i = C\frac{dv}{dt}$$

O.C. in DC

Energy Stored

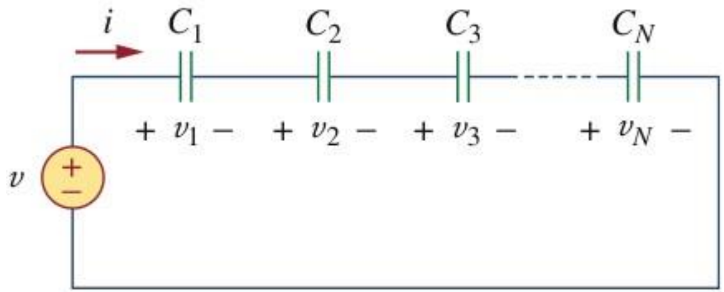
$$w = \frac{1}{2} C v^2$$

Parallel Capacitors



$$C_{eq} = C_1 + C_2 + C_3 + ... + C_N$$

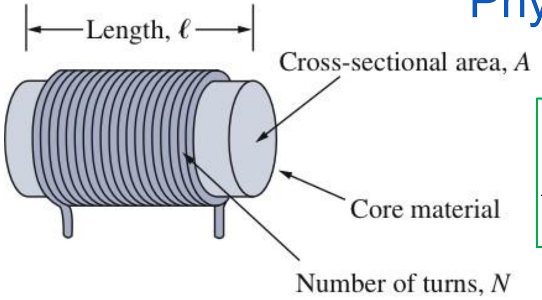
Series Capacitors



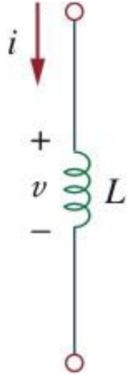
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_N}$$

Summary of Inductors

Physical structure



$$L = \frac{N^2 \mu A}{l}$$



Electrical Characteristics

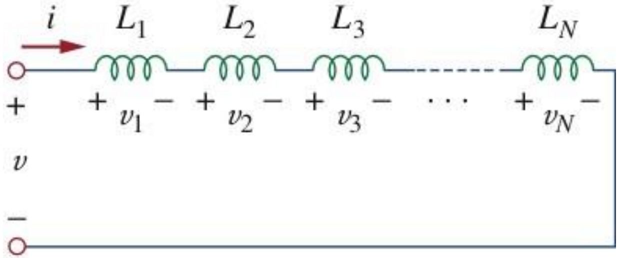
$$v = L\frac{di}{dt}$$

S.C. in DC

Energy Stored

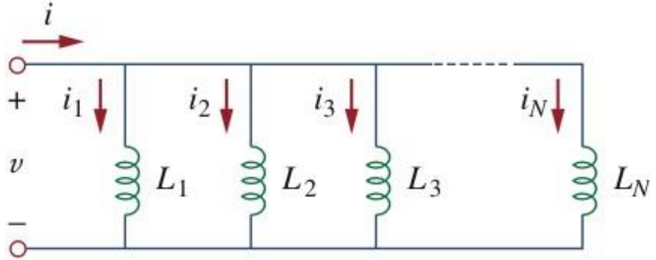
$$w(t) = \frac{1}{2} L i^2(t)$$

Series Inductors



$$L_{eq} = L_1 + L_2 + L_3 + \ldots + L_N$$

Parallel Inductors



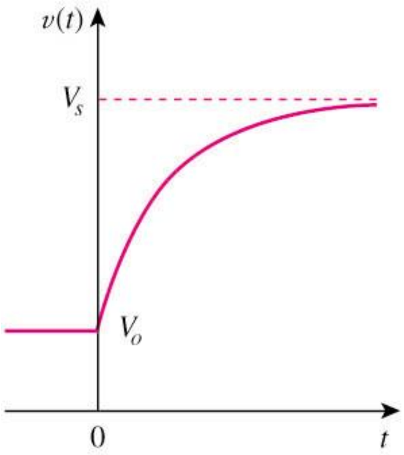
$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}$$

Remember: Important characteristics of basic elements

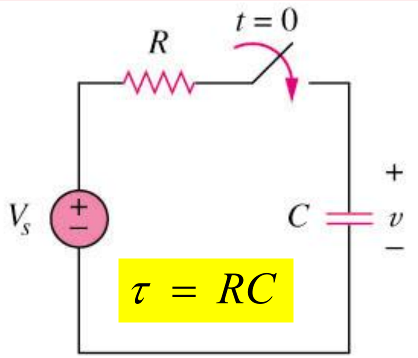
Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

Summary of Transient-Response for RC and RL circuits

Step Response (Forced Response):

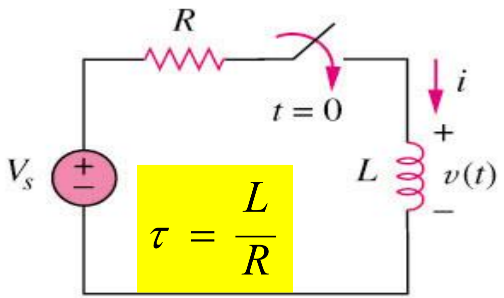


$$v(t) = v(\infty) + [v(0) - v(\infty)]\, e^{-t/\tau}$$



$$\tau = RC$$

$$i(t) = i(\infty) + [i(0) - i(\infty)]\, e^{-t/\tau}$$



$$\tau = \frac{L}{R}$$

Three steps to solve any transient-response problem:

1. The initial Values.
2. The final value (after long time, C acts as OC and L acts as SC).
3. The time constant τ.

Note: Natural Response (source free) is a special case where final value =0

End of Lecture

Questions?