EE2020-Electrical Circuits Analysis

Inductors

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Key inductor relationships:

Primary v-i equation	$v(t) = L\frac{di(t)}{dt}$
Alternate v-i equation	$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau + i(t_0)$
Initial condition	$i(t_0)$
Behavior with a constant source	If $i(t) = I, v(t) = 0$ and the inductor behaves like a short circuit
Continuity requirement	$i(t)$ is continuous for all time so $v(t)$ is finite
Power equation	$p(t) = v(t)i(t) = Li(t)\frac{di(t)}{dt}$
Energy equation	$w(t) = \frac{1}{2}Li(t)^2$
Series-connected equivalent	$L_{eq} = \sum_{j=1}^{n} L_j$ $i_{eq}(t_0) = i_j(t_0)$ for all j
Parallel-connected equivalent	$\frac{1}{L_{eq}} = \sum_{j=1}^{n} \frac{1}{L_j}$ $i_{eq}(t_0) = \sum_{j=1}^{n} i_j(t_0)$

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Problem #1

6.2 The triangular current pulse shown in Fig. P6.2 is applied to a 500 mH inductor. L

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a) Write the expressions that describe $i(t)$ in the four intervals $t < 0$, $0 \le t \le 25$ ms, 25 ms $\le t \le 50$ ms, and $t > 50$ ms.

b) Derive the expressions for the inductor voltage, power, and energy. Use the passive sign convention.

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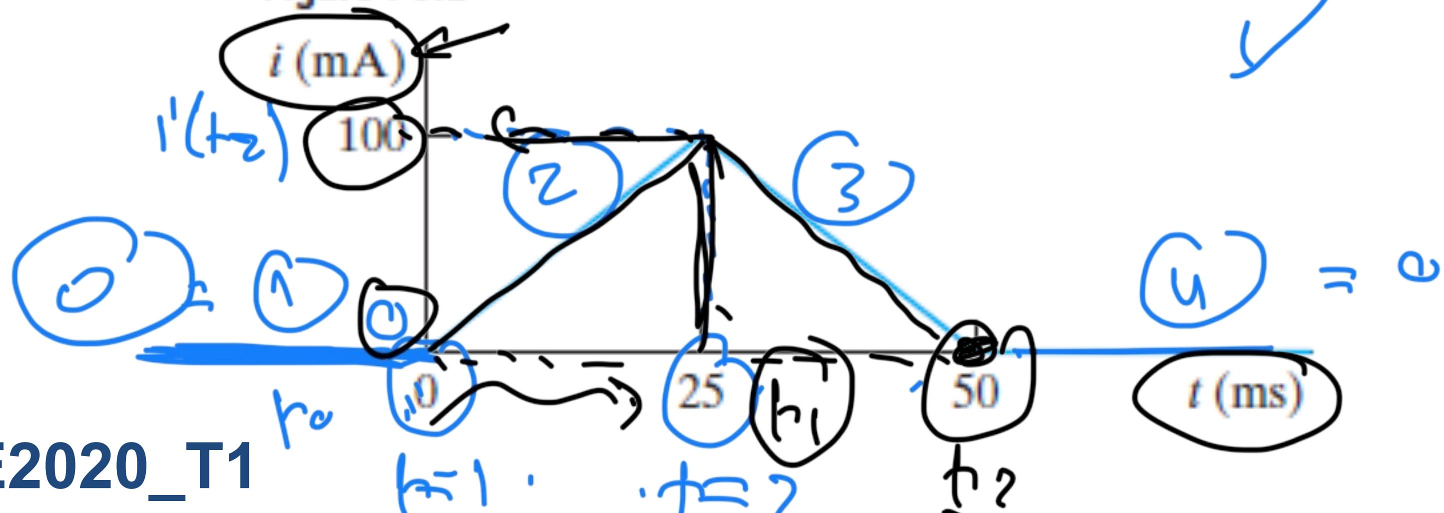
Figure P6.2



$$i(t) = m(t - t_0) + i(t_0)$$

$i(t_1)$

$t = 1$

$\boxed{2}$

$$m = \frac{i(t_2) - i(t_1)}{t_2 - t_1} = \frac{100 - 0}{25 - 0}$$

$$\boxed{m = 4}$$

$$i(t) = 4t + 0 = \boxed{i(t) = 4t}$$

A

$\boxed{3}$ $m = \dfrac{i(t_2) - i(t_1)}{t_2 - t_1} = \dfrac{0 - 100}{50 - 25}$

$$m = \frac{-100}{25} = -4$$

$$i(t) = -4(t - 25\,ms) + 100\,mA$$

$$-4t + 100\,ms + 100\,ms$$

$$i(t) = \underline{-4t + 200\,ms}$$

$$
i(t) = \begin{cases} 0\ \text{A}, & 0 < t \\ 4t\ \text{A}, & 0 < t < 25\text{ms} \\ -4t + \cancel{200\text{ms}}\ \text{A}, & 25\text{ms} < t < 50\text{ms} \\ 0\ \text{A}, & t > 50\text{m} \end{cases}
$$

0,2

$$200 \times 10^{-3} = 0{,}2\ \text{s}$$

$$V = L\,\frac{\partial i}{\partial t}$$

$L = 500\,mH$

$0{,}5\,H$

$\frac{1}{2}$

$$\frac{\partial i}{\partial t} = \begin{cases} 0 & t < 0 \\ M & 0 < t < 25\,ms \\ -M & 25\,ms < t < 50\,ms \\ 0 & t > 50\,ms \end{cases}$$

$$v = \begin{cases} 0\ \text{V} & t < 0 \\ 2\ \text{V} & 0 < t < 25\,\text{ms} \\ -2\ \text{V} & 25\,\text{ms} < t < 50\,\text{ms} \\ 0\ \text{V} & t > 50\,\text{ms} \end{cases}$$

$$P = v\,i$$

$$-2(-4t + 0{,}2)$$

$$P = \begin{cases} 0 \quad w & t<0 \\ 8t \quad w & 0<t<25ms \\ 8t-0.4 \quad w & 25<t<50ms \\ 0 \quad w & t>50ms \end{cases}$$

$w = \frac{1}{2} L i^2 \quad \times \quad \to \varepsilon j$

$$W = \int P(t)$$

$$W = \begin{cases} 0\ \text{J} & t<0 \\ 4t^2\ \text{J} & 0<t<25\text{ms} \\ 4t^2-0{,}4t+0{,}1\ \text{J} & 25\text{ms}<t<50\text{ms} \\ 0{,}09\ \text{J} & t>50\text{ms} \end{cases}$$

$$w = \int_{0}^{r} 8x \, dx + w(r=0) = 0$$

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$$= 4\cancel{8}\frac{x^2}{\cancel{2}}\Big]_0^r = 4(r^2 - 0) = 4r^2$$

$$w = \int_{25\times10^{-3}}^{r} 8x - 0.4 \, dx + w(r = 0.025)$$

25×10^{-3}

0.025

$4(0.025)^2$

$= 2.5\times10^{-3}$

$\underline{\underline{0.0025}}$

$$w = \left[4x^2 - 0{,}4x\right]_{0{,}025}^{t} + 0{,}0025$$

$$\left(4t^2 - 0{,}4t\right) - \left[4(0{,}025)^2 - 0{,}4(0{,}025)\right] +$$

$$- \left(0{,}0025 - 0{,}1\right) + 0{,}0025$$

$$+ \left(+0{,}0975\right)$$

$$w = 4t^2 - 0{,}4t + 0{,}1$$

$$w = \int_{0}^{t} \cancel{0\,dt} + \underline{w(t = 50ms)}$$

$50ms$

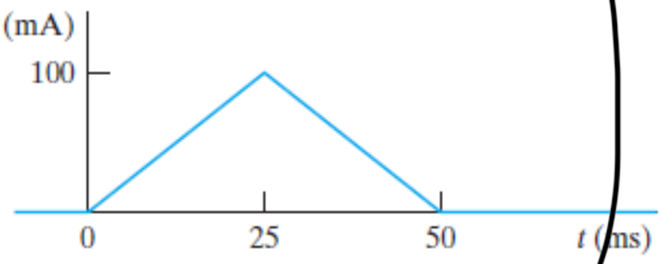
$0{,}05$

$$w(t = 50ms) = 4(50\times10^{-3})^2 - 0{,}4(50\times10^{-3}) + 0{,}1$$

$$= 0{,}09$$

Solution for Problem #1

[a]
$$\begin{aligned} i &= 0 & t &< 0 \\ i &= 4t \text{ A} & 0 \le t &\le 25 \text{ ms} \\ i &= 0.2 - 4t \text{ A} & 25 \le t &\le 50 \text{ ms} \\ i &= 0 & 50 \text{ ms} &< t \end{aligned}$$



[b] $v = L\dfrac{di}{dt} = 500 \times 10^{-3}(4) = 2\text{ V} \qquad 0 \le t \le 25 \text{ ms}$

$v = 500 \times 10^{-3}(-4) = -2\text{ V} \qquad 25 \le t \le 50 \text{ ms}$

$$\begin{aligned} v &= 0 & t &< 0 \\ v &= 2\text{ V} & 0 < t &< 25 \text{ ms} \\ v &= -2\text{ V} & 25 < t &< 50 \text{ ms} \\ v &= 0 & 50 \text{ ms} &< t \end{aligned}$$

Cont. Solution for Problem #1

$p = vi$

$$
\begin{array}{lcll}
p & = & 0 & t < 0 \\
p & = & (4t)(2) = 8t\,\text{W} & 0 < t < 25\,\text{ms} \\
p & = & (0.2 - 4t)(-2) = 8t - 0.4\,\text{W} & 25 < t < 50\,\text{ms} \\
p & = & 0 & 50\,\text{ms} < t \\
w & = & 0 & t < 0 \\
w & = & \int_0^t (8x)\,dx = 8\frac{x^2}{2}\Big|_0^t = 4t^2\,\text{J} & 0 < t < 25\,\text{ms} \\
w & = & \int_{0.025}^t (8x - 0.4)\,dx + 2.5\times 10^{-3} & \\
 & = & 4x^2 - 0.4x\Big|_{0.025}^t + 2.5\times 10^{-3} & \\
 & = & 4t^2 - 0.4t + 10\times 10^{-3}\,\text{J} & 25 < t < 50\,\text{ms} \\
w & = & 0 & 10\,\text{ms} < t
\end{array}
$$

0.109 J

Problem #2

6.7 The voltage at the terminals of the 750 μH inductor in Fig. P6.7(a) is shown in Fig. P6.7(b). The inductor current i is known to be zero for $t \le 0$.

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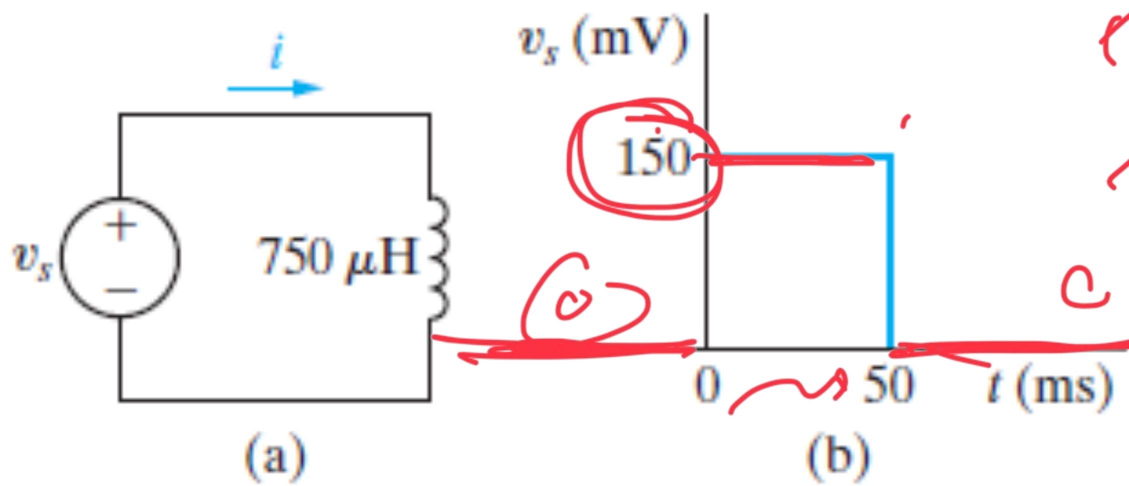
a) Derive the expressions for i for $t \ge 0$.

b) Sketch i versus t for $0 \le t \le \infty$.

Figure P6.7



$$i(t) = \begin{cases} 0 \ A & t \leq 0 \\ 200t \ A & 0 < t < 50ms \\ 10 \ A & t > 50ms \end{cases}$$

$$v(t) = \begin{cases} 0\,V, & t < 0 \\ 150mV, & 0 < t < 50ms \\ 0\,V, & t > 50ms \end{cases}$$

$$i_L(t) = \frac{1}{L}\int_{t_0}^{t} v_L(t) + i_L(t_0)$$

$t_0 = 0$

$$i_L(t) = \frac{1}{750 \times 10^{-6}} \int_0^t 150 \times 10^{-3}\, dx + 0$$

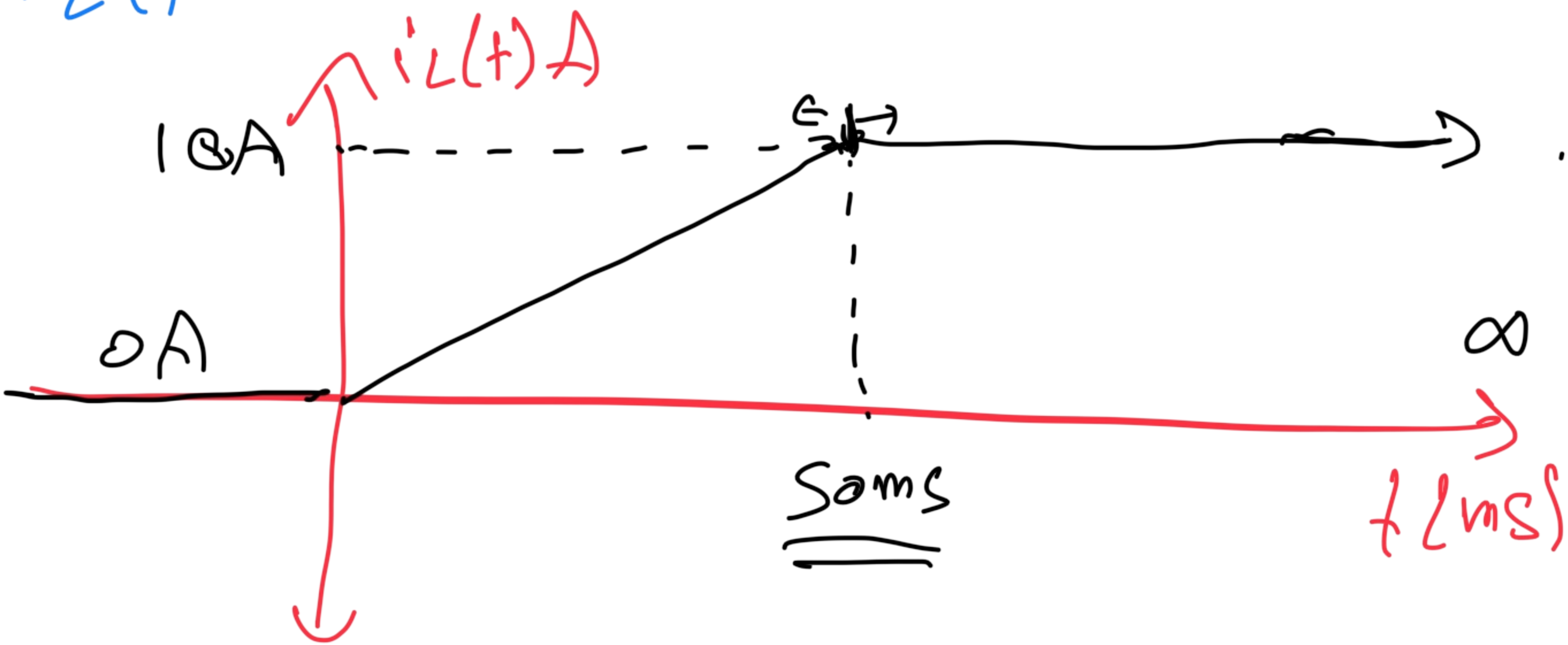
$$\frac{150 \times 10^{-3}}{750 \times 10^{-6}} \int_0^t 1\, dx = x \Big]_0^t = t$$

$$i_L(t) = 200t$$

$$i_L(t) = \cancel{\frac{1}{50\times 10^{-6}}\int_{50ms}^{t} 0\, dt} + i_L(t=50ms)$$

0

$$i_L(t=50ms) = 200\times 50\times 10^{-3} = 10A$$



Solution for Problem #2

[a] $0 \le t \le 50\,\text{ms}:$

$$i = \frac{1}{L}\int_0^t v_s\,dx + i(0) = \frac{10^6}{750}\int_0^t 0.15\,dx + 0$$

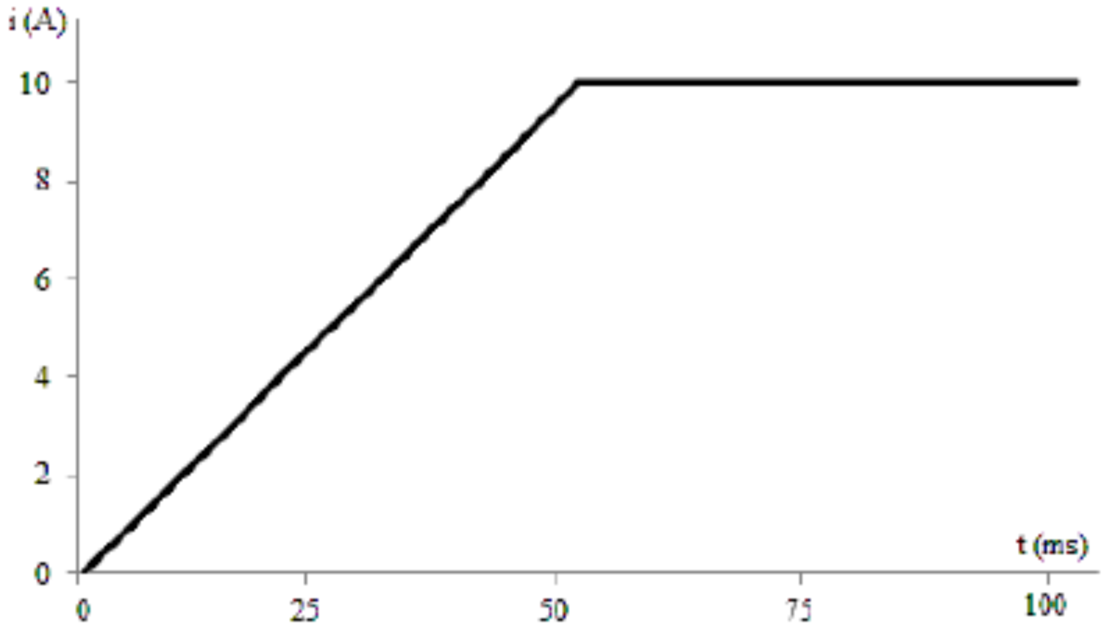
$$= 200x \Big|_0^t = 200t\,\text{A}$$

$$i(0.05) = 200(0.05) = 10\,\text{A}$$

$t \ge 50\,\text{ms}:$ $$i = \frac{10^6}{750}\int_{50\times10^{-3}}^t (0)\,dx + 10 = 10\,\text{A}$$

Cont. Solution for Problem #2

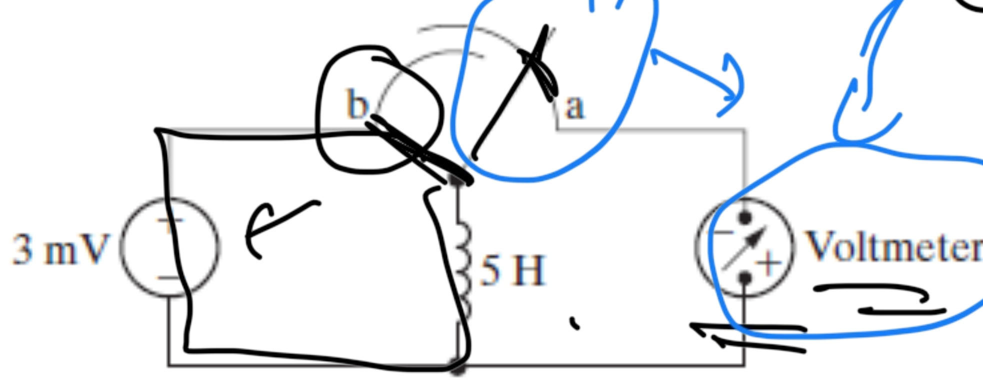
[b] $i = 200t\,\text{A}, \quad 0 \le t \le 50\,\text{ms}; \qquad i = 10\,\text{A}, \quad t \ge 50\,\text{ms}$



Problem #3

6.12 Initially there was no energy stored in the 5 H inductor in the circuit in Fig. P6.12 when it was placed across the terminals of the voltmeter. At $t = 0$ the inductor was switched instantaneously to position b where it remained for 1.6 s before returning instantaneously to position a. The d'Arsonval voltmeter has a full-scale reading of 20 V and a sensitivity of 1000 Ω/V. What will the reading of the voltmeter be at the instant the switch returns to position a if the inertia of the d'Arsonval movement is negligible?

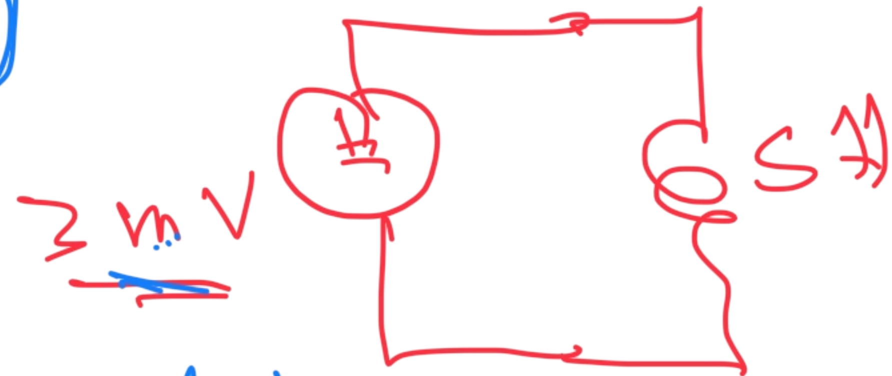
Figure P6.12



$V = 0$

$20\,V$

$0 \le t \le 1,6\,s$



$$i_L(t) = \frac{1}{L}\int_0^t v\,dx \qquad i_c(0) = 0$$

$$i = \frac{1}{5}\int_0^t 3\times10^{-3}\,dx =$$

$$\frac{3\times10^{-3}}{5}\int_0^t 1 = x\Big]_0^t = t$$

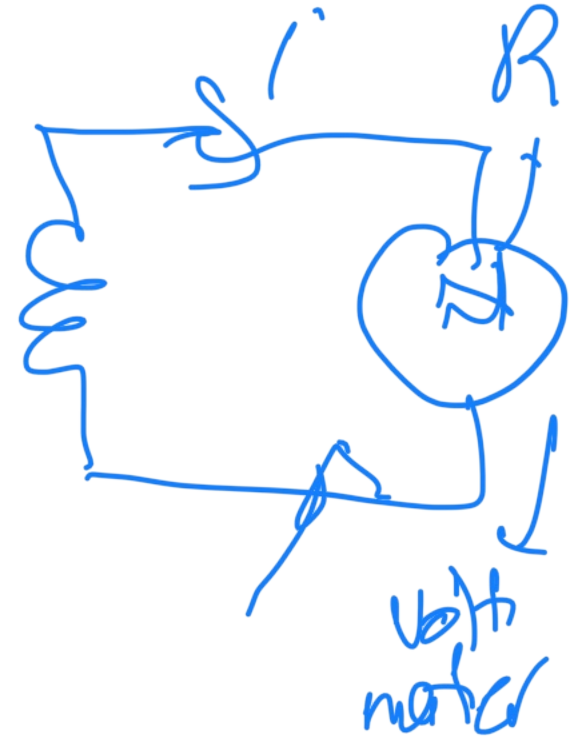
$$\Rightarrow 0,6t \text{ mA}$$

$$\boxed{i_L(t) = 0{,}6\,t \text{ mA}}$$

$$t \geq 1{,}6 \text{ ms}$$

$$\cancel{i_L(t) = \frac{1}{L}\int_0^{1,6} v\,dt + i_L(1{,}6)}$$

$$i_L(t) = 0{,}6 \times 1{,}6 \times 10^{-3} \times 10^{-3} = \underline{\underline{0{,}96 \text{ mA}}}$$

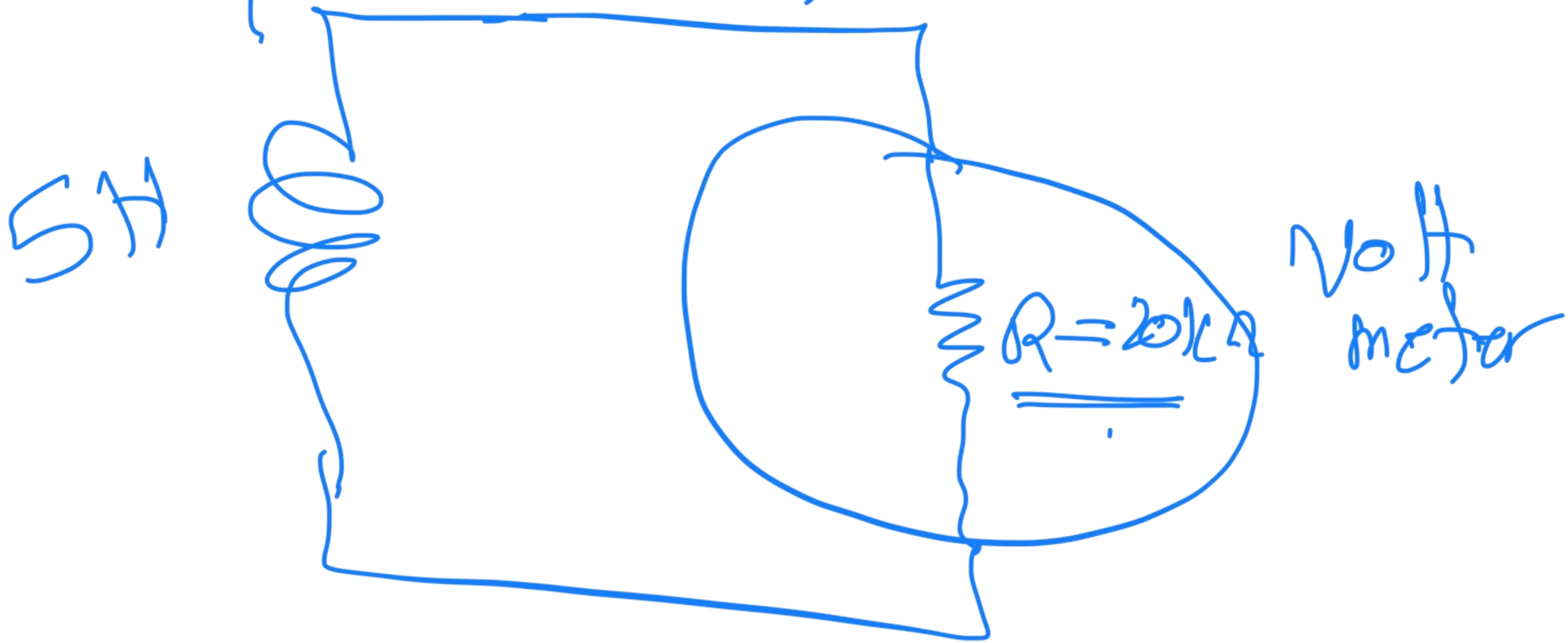


$$i_L(t) = \begin{cases} 0\,A & t < 0 \\ 0{,}6\,mA & 0 < t < 1{,}6\,ms \\ \underline{\underline{0{,}96\,mA}} & t > 1{,}6\,ms \end{cases}$$

$\underline{1000\,\Omega / V}$ x $V = 20V$

$R = 1.000\,\frac{\Omega}{\cancel{V}} \times 20\cancel{V} = \boxed{20k\,\Omega}$

$i_c(1{,}6ms) = 0{,}96mA$



$$V_m(t = 1{,}6ms) = IR$$

$$0{,}96\,\cancel{m}A \times 20\,\cancel{k}\Omega \Rightarrow \boxed{19.2V}$$

Solution for Problem #3

For $0 \le t \le 1.6\,\mathrm{s}$:

$$i_L = \frac{1}{5}\int_0^t 3 \times 10^{-3}\,dx + 0 = 0.6 \times 10^{-3} t$$

$$i_L(1.6\,\mathrm{s}) = (0.6 \times 10^{-3})(1.6) = 0.96\,\mathrm{mA}$$

$$R_m = (20)(1000) = 20\,\mathrm{k}\Omega$$

$$v_m(1.6\,\mathrm{s}) = (0.96 \times 10^{-3})(20 \times 10^3) = 19.2\,\mathrm{V}$$

Problem #4

The voltage pulse applied to the 100 mH inductor shown in Fig. 6.5 is 0 for $t < 0$ and is given by the expression

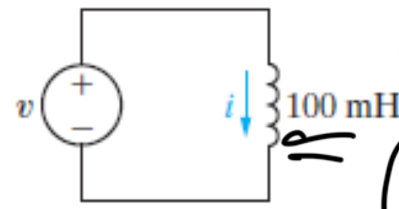
$$v(t) = 20te^{-10t}\ \text{V}$$

for $t > 0$. Also assume $i = 0$ for $t \leq 0$.

a) Sketch the voltage as a function of time.

b) Find the inductor current as a function of time.

c) Sketch the current as a function of time.

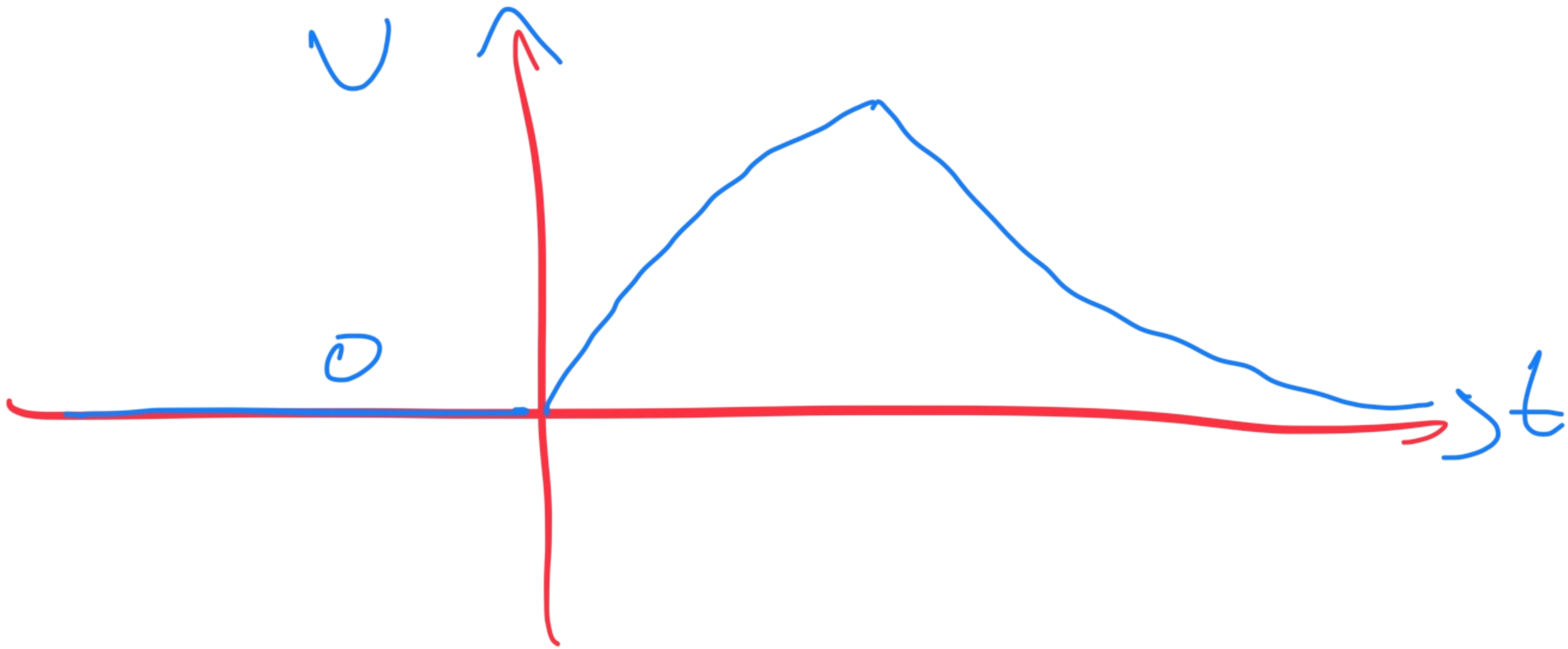


$v = 0, \quad t < 0$

$v = 20te^{-10t}\ \text{V}, \quad t > 0$

Figure 6.5 ▲ The circuit for Example 6.2

a) $v = \begin{cases} 0 & t < 0 \\ 20t\,e^{-10t} & t > 0 \end{cases}$



b) $i = \frac{1}{L}\int_0^t v_L(t) + \cancel{i(0)} \to 0$

$$i(t) = \frac{2e^{-0}}{100 \times 10^{-3}} \int_0^t x e^{-10x}\,dx + 0$$

$\int x e^{-10x} \to$ by parts

	du	dv
+	x	e^{-10x}
−	1	$e^{-10x}/-10$
+	0	$e^{-10x}/100$

$$200\left[-\frac{x e^{-10x}}{10}-\frac{e^{-10x}}{100}\right]_0^t \quad e^0=1$$

$$200\left[\left(-\frac{t e^{-10t}}{10}-\frac{e^{-10t}}{100}\right)-\left(0-\frac{1}{100}\right)\right]$$

$$200\left[-\frac{t e^{-10t}}{10}-\frac{e^{-10t}}{100}+\frac{1}{100}\right]$$

$$\boxed{i_1(t)=2-20t e^{-10t}-2e^{-10t}\ \text{A}}$$

$$i_2(t) = \begin{cases} 0 & , t<0 \\ 2 - 20te^{-10t} - 2e^{-10t} & , t>0 \end{cases}$$

Solution for Problem #4

a) The voltage as a function of time is shown in Fig. 6.6.

$$v(t) = 20te^{-10t}\ \text{V}$$

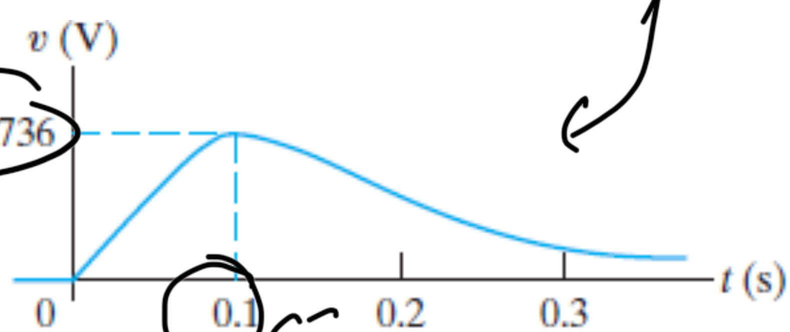


Figure 6.6 ▲ The voltage waveform for Example 6.2.

$20te^{-10t}$

$$\frac{dv}{dt} = 0 \qquad 20\left[1 \cdot e^{-10t} + t \cdot e^{-10t} \cdot (-10)\right] = 0$$

$$20\, e^{-10t}\left[1 - 10t\right] = 0$$

$$0 \neq 20 \qquad 0 \neq e^{-10t} \qquad 1 - 10t = 0$$

$$\frac{1}{10} = \frac{\cancel{10}t}{\cancel{10}} \rightarrow t = \frac{1}{10} = 0{,}1s$$

$$v(0{,}1) = 20(0{,}1)\, e^{-10(0{,}1)} = 0{,}736$$

Cont. Solution for Problem #4

b) The current in the inductor is 0 at $t = 0$. Therefore, the current for $t > 0$ is

$$i = \frac{1}{0.1}\int_0^t 20\tau e^{-10\tau}\,d\tau + 0$$

Use integration by parts: $\int_0^t u\,dv = uv|_0^t - \int_0^t v\,du$

$$= 2(1 - 10te^{-10t} - e^{-10t})\ \text{A}, \qquad t > 0.$$

$t(s)$	0	0.1	0.2	0.3	0.4	0.5
$v(t)$	0	0.53	1.2	1.6	1.8	1.9

c) Figure 6.7 shows the current as a function of time.

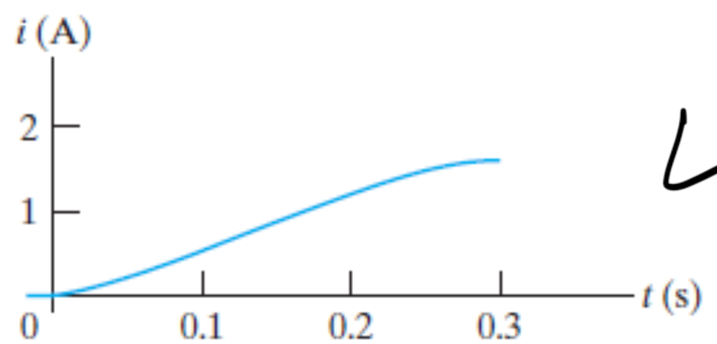


Figure 6.7 ▲ The current waveform for Example 6.2.

Homework

6.1 The current in a 150 μH inductor is known to be

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$$i_L = 25te^{-500t}\text{ A} \quad \text{for } t \geq 0.$$

a) Find the voltage across the inductor for $t > 0$. (Assume the passive sign convention.)

b) Find the power (in microwatts) at the terminals of the inductor when $t = 5$ ms.

c) Is the inductor absorbing or delivering power at 5 ms?

d) Find the energy (in microjoules) stored in the inductor at 5 ms.

e) Find the maximum energy (in microjoules) stored in the inductor and the time (in milliseconds) when it occurs.