EE2020-Electrical Circuits Analysis

Inductors

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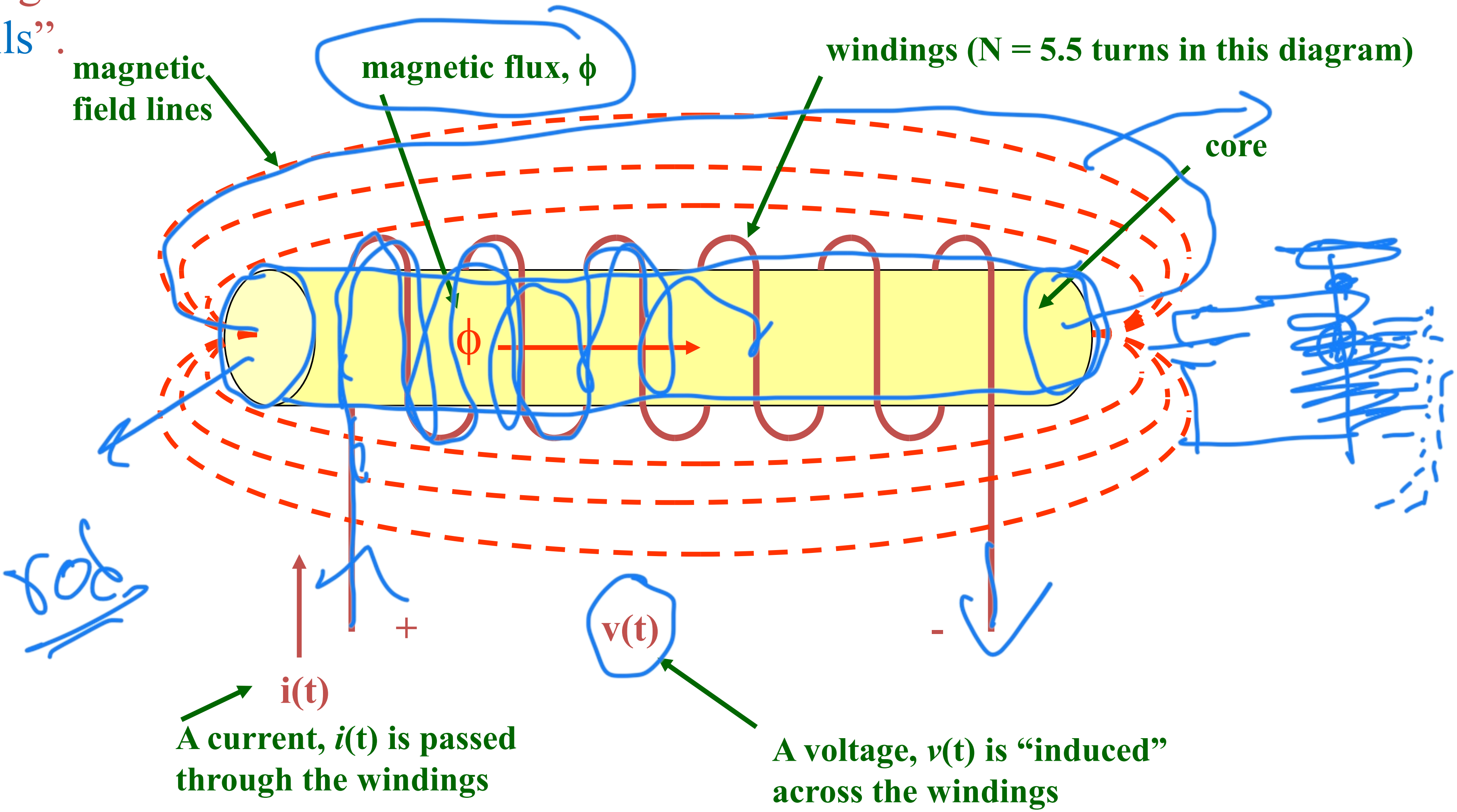
Outline

- The Inductor
- Series--Parallel Combinations of Inductance

Inductors

An inductor is a passive device created by wrapping wire around a core. When time-varying current passes through the coil a magnetic field is created and a voltage is "induced" across the coil. Inductors are also called "chokes" or "coils".



Magnetic flux

In the previous diagram it was shown that a magnetic flux, ϕ, flowed through the core.

Magnetic flux is measured in units of Webers, Wb. It is somewhat like a magnetic current flowing through the core. The direction of the magnetic flux is determined using the "***right-hand rule***".

> Right-hand rule: Using your right hand, curl your fingers in the direction that the current flows through the coil and your thumb will indicate the direction of the magnetic flux.

Inductance

N = number of windings around the core

λ = flux linkage = $N\phi$

λ is also proportional to the current or λ = (constant)i

This constant is referred to as inductance, L

$$\lambda = N\phi = Li$$

$$\text{so } L = \text{inductance} = \frac{\lambda}{i} \quad \left(\text{in } \frac{\text{Webers}}{\text{Ampere}} = \text{Henries, H}\right)$$

Note: More detailed information on magnetic fields is covered in a later course in electromagnetics.

$\lambda = Li$

The voltage induced across the coil is equal to the derivative of the flux linkage, so

$$v = \frac{d\lambda}{dt} = \frac{d(Li)}{dt}$$

$v_L = L\frac{di}{dt}$

So a key relationship for inductors is: $\boxed{v = L\frac{di}{dt}}$

Notes:

1) **This equation is sort of like "Ohm's Law" for an inductor.**
2) **Be sure to use passive sign convention**

$v = iR$

3) **Note that the inductor symbol looks like a coil of wire.**

Physical Characteristics

The value of L can also be determined from the physical properties of the inductor using

$$L = \frac{N^2 \mu A}{l_c}$$

$\mu = \mu_0 \mu_r$

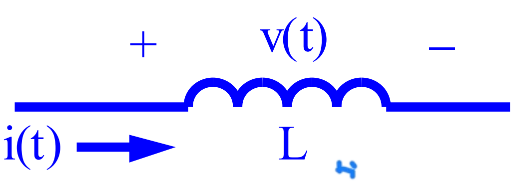
Where N = number of turns

A = cross-sectional area of the core (in m^2)

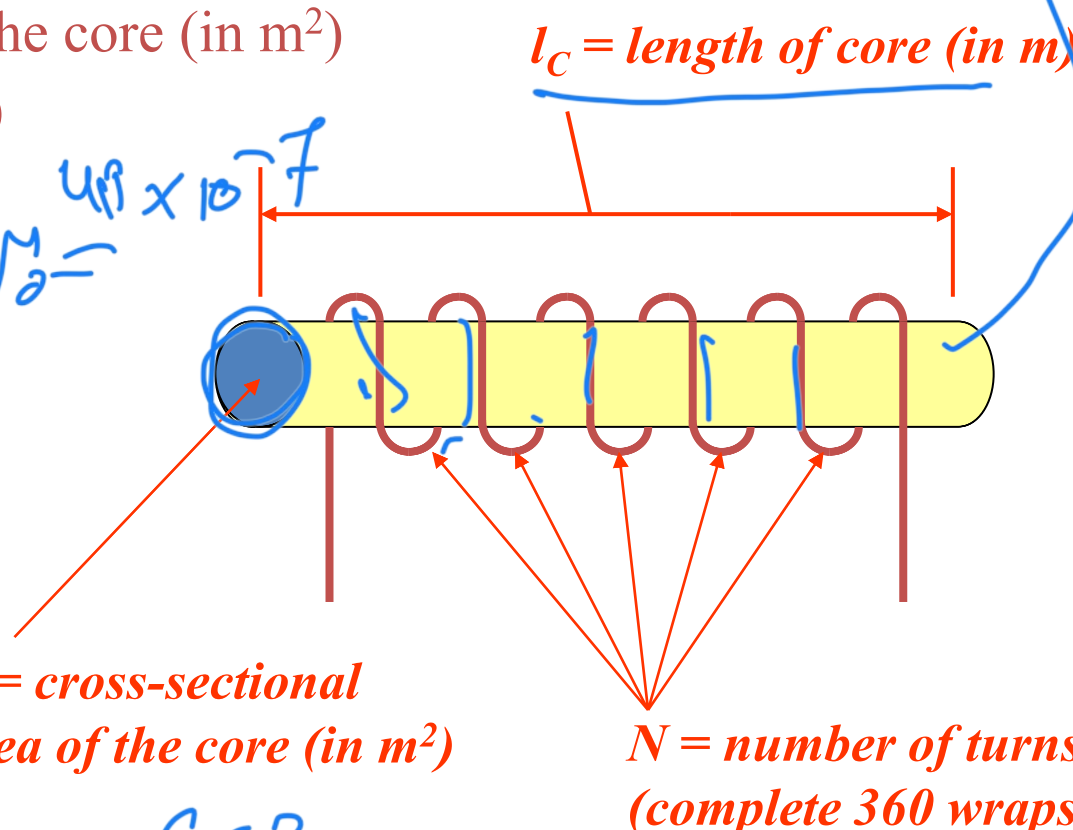
l_c = length of the core (in m)

μ = permeability of the core

$\mu_0 = 4\pi \times 10^{-7}$



Inductor symbol



$= \pi r^2$

Permeability of the core

Permeability can be thought of as a measure of how well a type of material can sustain a magnetic field.

μ = permeability of the core. This is typically expressed as:

$$\boxed{\mu = \mu_R \mu_o}$$

where $\mu_o = 4\pi \times 10^{-7}$ Wb/A·m and μ_R = relative permeability

There are only basically two values for μ_R :

- $\mu_R = 1$ for non-ferrous materials
- $\mu_R \approx 200$ for ferrous materials

So the value of L is increased by a factor of 200 simply by using an iron core!

The equation for inductance can now be written as:

$$\boxed{L = \frac{N^2 \mu_R \mu_o A}{l_c}}$$

L : Henry (H)

Typical values

Inductors are sometimes classified in two broad categories:

1) iron-core inductors - typical values in the H range
2) non-iron core inductors - typical values in the μH or mH range

General inductor symbol

Iron-core inductor symbol

3 mH

1 μH

$m = 10^{-3}$

$\mu = 10^{-6}$

5 A

3 H

Examples of inductors (www.allelectronics.com)

220uH drum choke

Variable choke with adjustable ferrite

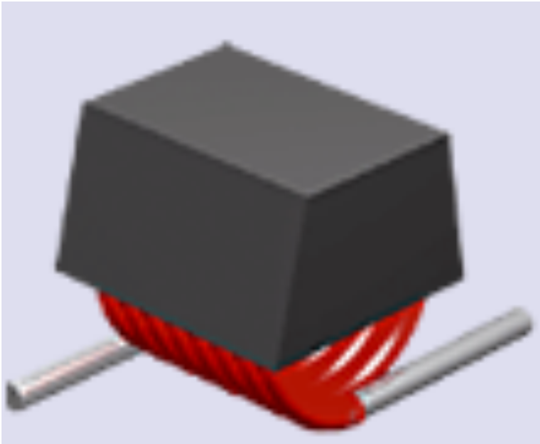
3.5mH bobbin choke

390uH choke coil

4mH high-current choke

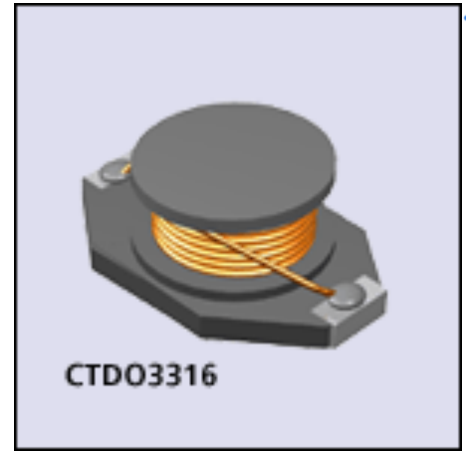
346uH inductor (toroid)

Examples of inductors (www.ctparts.com)

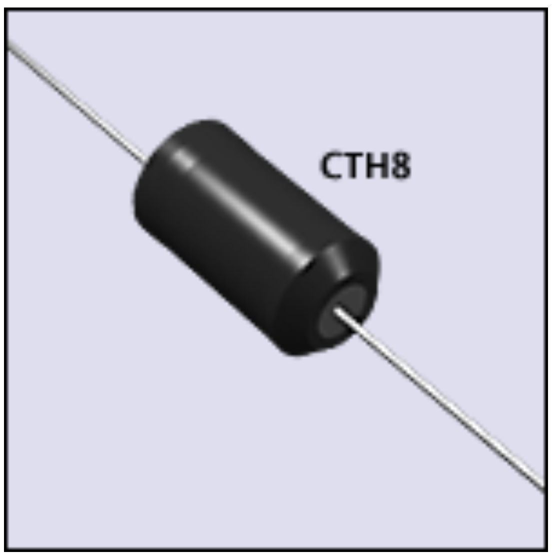


Air-core inductor

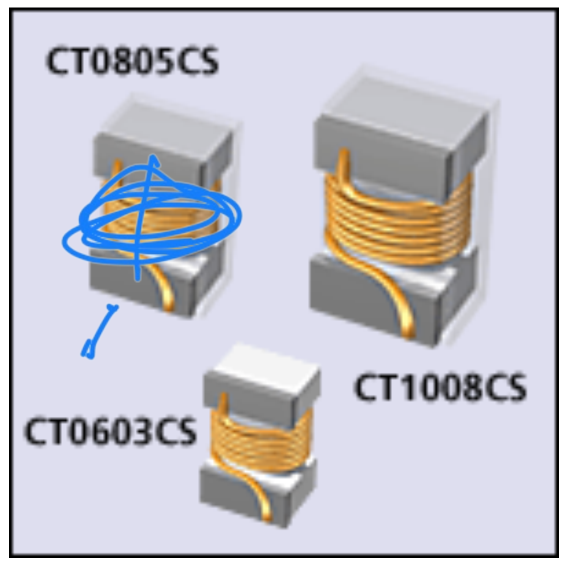
Peaking coils



Power inductor



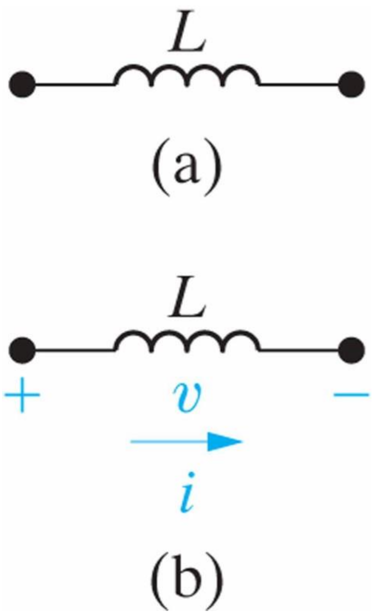
Power line choke



Wire-wound inductors

Toroidal power chokes (www.coilcraft.com)

The Inductor



Inductance is the circuit parameter used to describe an inductor. Inductance is *symbolized by the letter L*, is *measured in henrys (H)*, and is *represented graphically as a coiled wire*—a reminder that inductance is a consequence of a conductor linking a magnetic field.

Assigning the reference direction of the current in *the direction of the voltage drop across the terminals of the inductor*, as shown in (b), yields

$$v = L\frac{di}{dt}$$

Two Important Observations

a) If the current is constant (***steady-state***), the voltage across the ideal inductor is zero. Thus ***the inductor behaves as a short circuit in the presence of a constant or dc, current***.

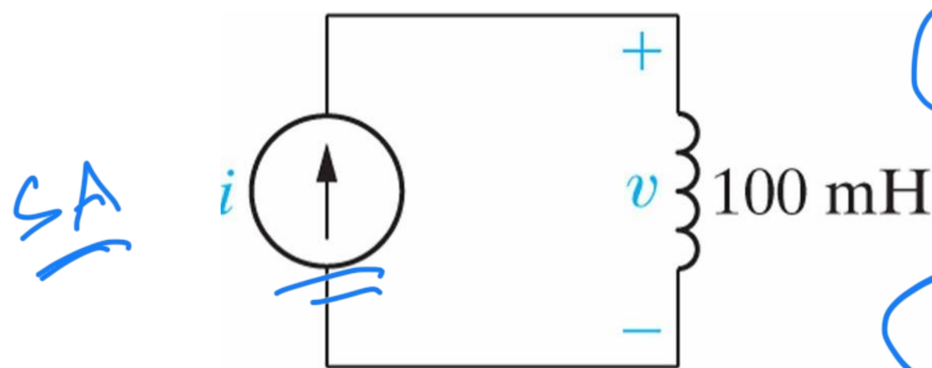
b) Current cannot change instantaneously in an inductor; that is, ***the current cannot change by a finite amount in zero time.***

- This is sometimes expressed as $i_L(0^+) = i_L(0^-)$

- When someone opens the switch on an inductive circuit in an actual system, the current initially continues to flow in the air across the switch, a phenomenon called *arcing*.
- Arcing must be controlled to prevent equipment damage!

Example #1

- The independent current source in the circuit generates zero current for $t < 0$ and a pulse $10te^{-5t}$A, for $t > 0$.



$$i = 0, \quad t < 0$$

$$i = 10te^{-5t}\text{A}, \quad t > 0$$

a) Sketch the current waveform.

b) At what instant of time is the current maximum? → calculus

c) Express the voltage across the terminals of the 100 mH inductor as a function of time.

dep SA

$t = 0{,}25$

d) Sketch the voltage waveform.

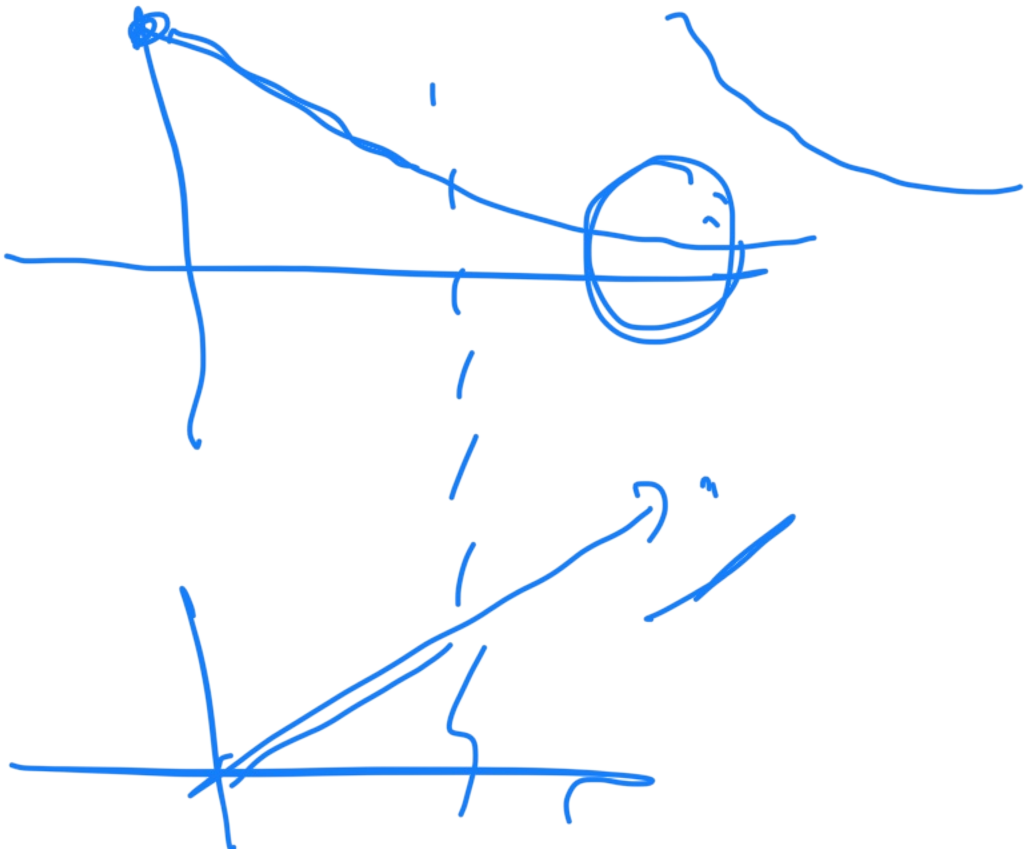
e) Are the voltage and the current at a maximum at the same time?

f) At what instant of time does the voltage change polarity?

g) Is there ever an instantaneous change in voltage across the inductor? If so, at what time?

$10t\,e^{-5t}$

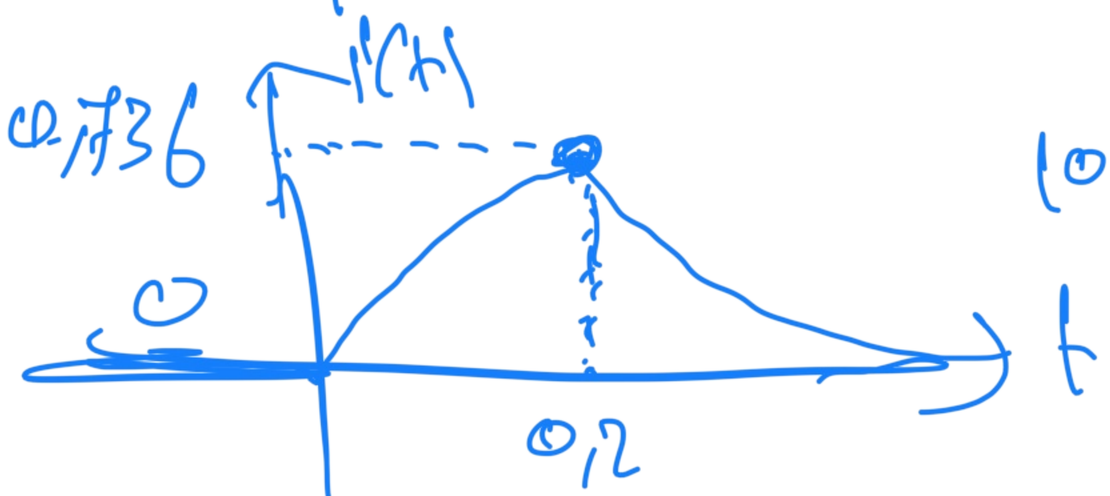
a)



e^{-at}

$+$

t



$10te^{-5t}$

b) $i'(t) = 10\left[t \cdot e^{-st}\right]$ ← max → $\frac{\partial i'}{\partial t} = 0$

$$\frac{\partial i'}{\partial t} = 10\left[(1)e^{-st} + t\,e^{-st} \cdot -s\right] = 0$$

$$\boxed{\frac{\partial i'}{\partial t} = 10\, e^{-st}\left[1 - st\right] = 0}$$

$\neq 0$ $\quad \neq 0$ $\qquad 1 - st = 0$

$$\frac{1}{s} = \frac{st}{s} \rightarrow t = \frac{1}{s} = 0{,}2\ s$$

$$i'(0{,}2) = 10\,(0{,}2)\, e^{-5 \times 0{,}2} = 0{,}73\ GA$$

c) $v = L \dfrac{di}{dt}$ $\qquad L = 100mH$

$v_L(t) = \underbrace{100 \times 10^{-3} \times 10}\, e^{-st}[1-st]$

$$\boxed{v_L(t) = e^{-st}[1-st] \quad V} \quad t>0$$

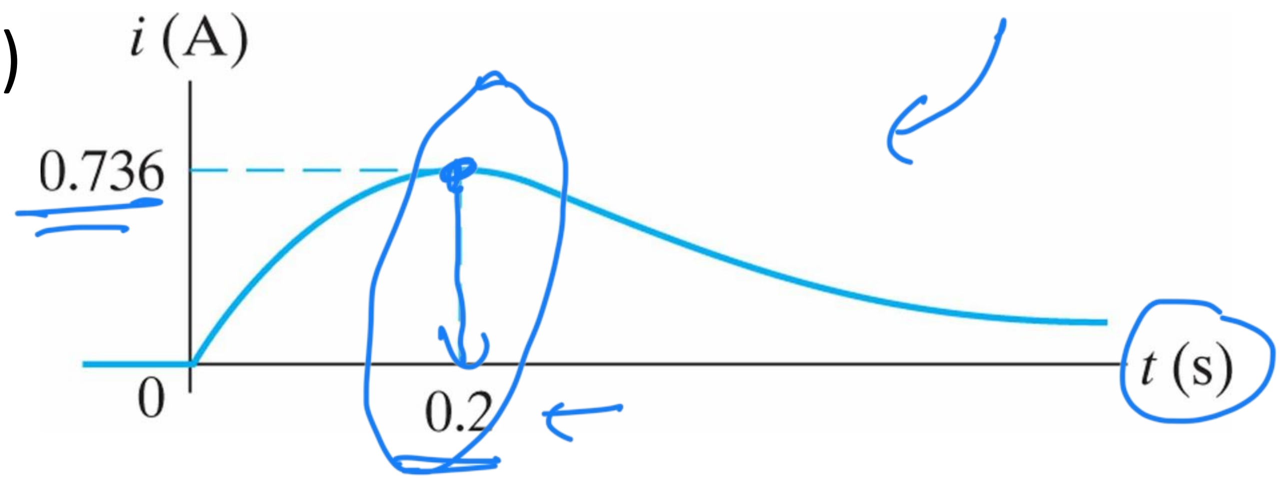
$v_L(t) = 0 \quad t<0$

$e^0 = 1$

$$V_L(t=0^-) \quad t<0 \quad = 0\,V$$
$$V_L(t=0^+) \quad t>0 \quad = 1\,V$$

Solution for Example #1

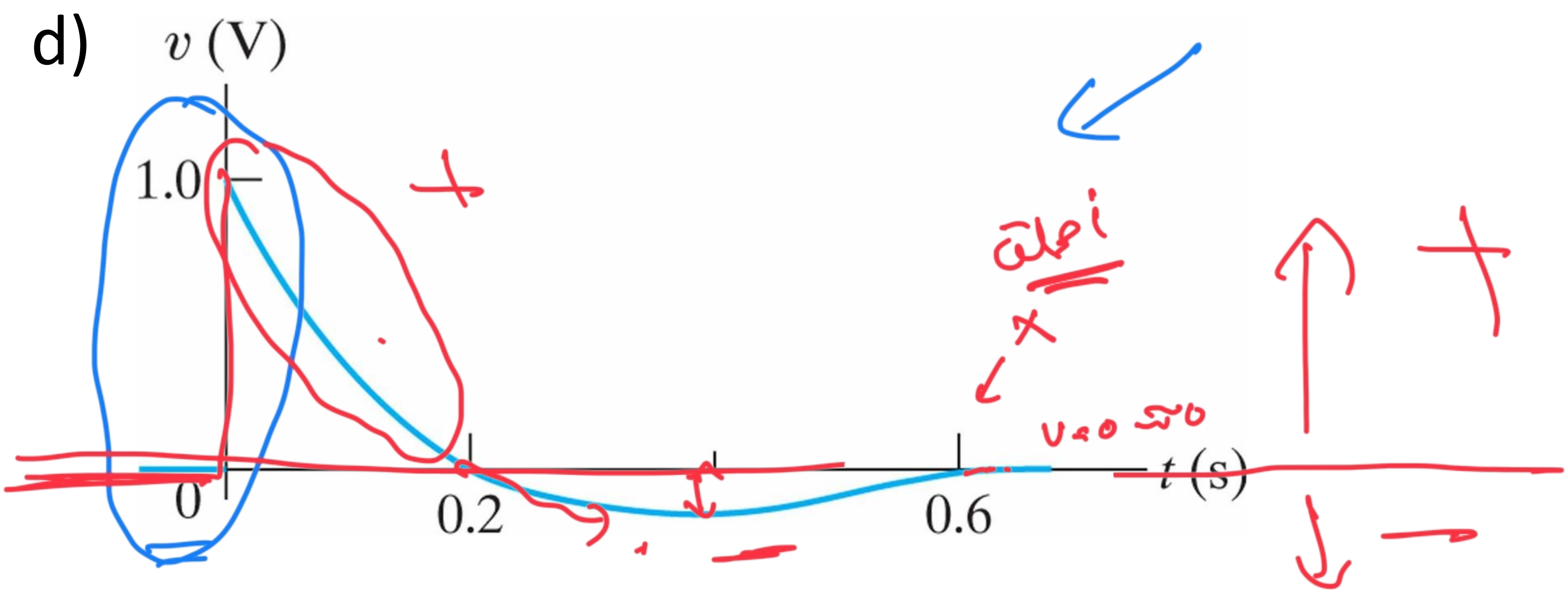
a)



b) $di/dt = 10(-5te^{-5t} + e^{-5t}) = 10e^{-5t}$ $(1 - 5t)$ A/s; $di/dt = 0$ when $t = \frac{1}{5}$ s. $= 0{,}2\ s$

c) $v = L\,di/dt = (0.1)10e^{-5t}(1 - 5t) = e^{-5t}$ $(1 - 5t)$ V, $t > 0$; $v = 0$, $t < 0$.

d)



e) No; the voltage is proportional to di/dt, not i.

f) At 0.2 s, which corresponds to the moment when di/dt is passing through zero and changing sign.

g) Yes, at t - 0. Note that the voltage can change instantaneously across the terminals of an inductor.

Current in an Inductor in Terms of the Voltage Across the Inductor

$$v = L\frac{di}{dt} \;\Longrightarrow\; v\,dt = L\,di \;\xrightarrow{\text{Integrate}}\; L\int_{i(t_0)}^{i(t)} dx = \int_{t_0}^{t} v\,d\tau$$

$$\Longrightarrow\; i(t) = \frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0)$$

$$\xrightarrow{t_0 = 0}\; \boxed{i(t) = \frac{1}{L}\int_{0}^{t} v\,d\tau + i(0)}$$

أهم سطر

$$V_L(t) = L\,\frac{\partial i}{\partial t}$$

$$i_L(t) = \frac{1}{L}\int_{t_0}^{t} v_L(t) + i_L(t_0)$$

t_0 ← نقطة البداية

بداية الفترة

Power and Energy in the Inductor

$$p = vi \;\Rightarrow\; p = Li\frac{di}{dt} \;\Rightarrow\; p = v\left[\frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0)\right]$$

$$p = \frac{dw}{dt} = Li\frac{di}{dt} \;\Rightarrow\; dw = Li\,di$$

$$\int_0^w dx = L\int_0^i y\,dy \;\Rightarrow\; w = \frac{1}{2}Li^2$$

$W = \int p(t)\,dt$

Key inductor relationships:

$$v = L\frac{di}{dt} \qquad \text{(V)}$$

$$i = \frac{1}{L}\int_{t_0}^{t} v\, d\tau + i(t_0) \qquad \text{(A)}$$

$$p = vi = Li\frac{di}{dt} \qquad \text{(W)}$$

$$w = \frac{1}{2}Li^2 \qquad \text{(J)}$$

Example #2

a) For Example #1, Plot i, v, p, and w versus time. Line up the plots vertically to allow easy assessment of each variable's behavior.

b) In what time interval is energy being stored in the inductor?

c) In what time interval is energy being extracted from the inductor?

d) What is the maximum energy stored in the inductor?

e) Evaluate the integrals and comment on their significance.

$$\int_0^{0.2} p\,dt \quad \text{and} \quad \int_{0.2}^{\infty} p\,dt$$

$$v_L(t) = 1 \cdot e^{-5t}[1-5t]\ \text{V} \qquad t>0$$

$$v_L(t) = \underline{\underline{0}}\ \text{V} \qquad t<0$$

$$i_L(t) = 10t\, e^{-5t}\ \text{A} \qquad t>0$$

$$i_L(t) = \underline{\underline{0}}\ \text{A} \qquad t<0$$

$$P = v\,i = \begin{cases} \boxed{10t\, e^{-10t}[1-5t]\ \text{W}} & t>0 \\ \underline{\underline{0\ \text{W}}} & \underline{\underline{t<0}} \end{cases}$$

$$w = \int_0^t p(t)\, dt = \int_0^t 10x\, e^{-10x}[1-5x]\, dx$$

$$\boxed{w = \underbrace{5t^2 e^{-10t}}\ J}$$

$t > 0$

$$w = \frac{1}{2} L i^2$$

$i\,(t = \sin\theta)$

$(t = 1)$

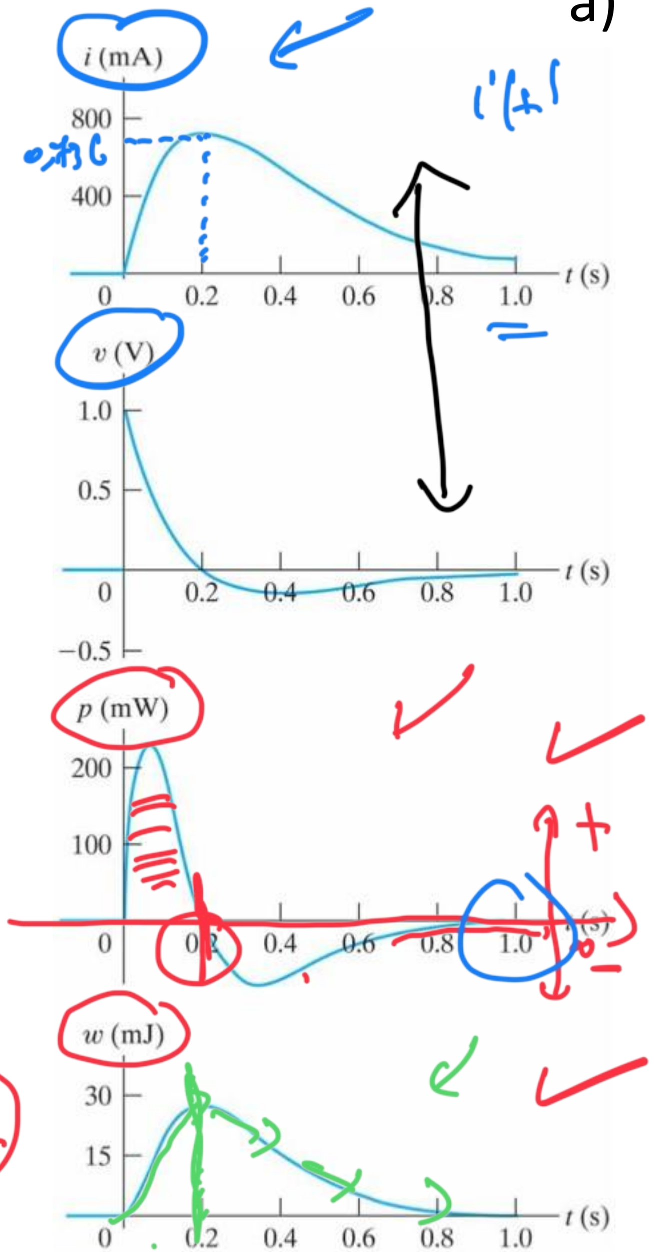
$e^{?}$

Solution for Example #2

b) An increasing energy curve indicates that energy is being stored. Thus energy is being stored in the time interval 0 to 0.2 s. *Note that this corresponds to the interval when $p > 0$.*

c) A decreasing energy curve indicates that energy is being extracted. Thus energy is being extracted in the time interval 0.2 s to ∞. *Note that this corresponds to the interval when $p < 0$.*

a)



تخزينه Energy stored $0 \to 0,2$ $P+$

تفريغ $0,2 \to \infty$ Energy extracted $P-$

$0 \to 0,2$ stored

$0,2 \rightarrow \infty \rightarrow$ ضل يشتغل

$\rightarrow$ Extracted

max energy = max current

*

$t = 0,2s \rightarrow i = 0,736A$

$$W = 5t^2 e^{-10t} = 5(0,2)^2 e^{-10(0,2)}$$

$W_{max} = 27,07\ mJ$

$$w = \int_0^{0,2} p(t) = \int_0^{0,2} 10t\,e^{-10t}[1-5t]\,dt$$

$$w = +27,07 \text{ mJ}$$

stored

+

$$w = \int_{0,2}^{\infty} 10t\,e^{-10t}[1-5t]\,dt$$

$$w = -27,07 \text{ mJ}$$

Extracted

d) *Energy is at a maximum when current is at a maximum*; glancing at the graphs confirms this. From Example #1, maximum current = 0.736 A. Therefore, $w_{max} = 27.07$ mJ.

e) From Example #1, we have $i(t) = 10te^{-5t}$ A and $L = 0.1$ H, therefore,

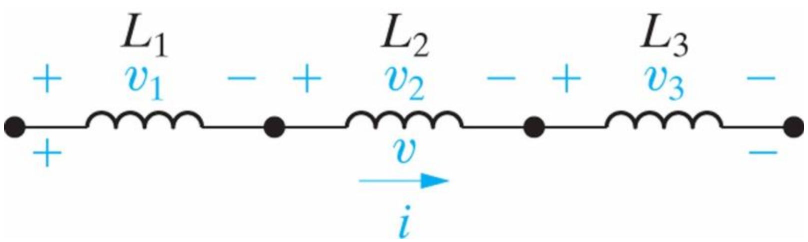
$$\int_0^{0.2} p\,dt = w(0.2) - w(0) = L/2 \times [i(0.2)^2 - i(0)^2] = 0.2e^{-2} = 27.07 \text{ mJ}$$

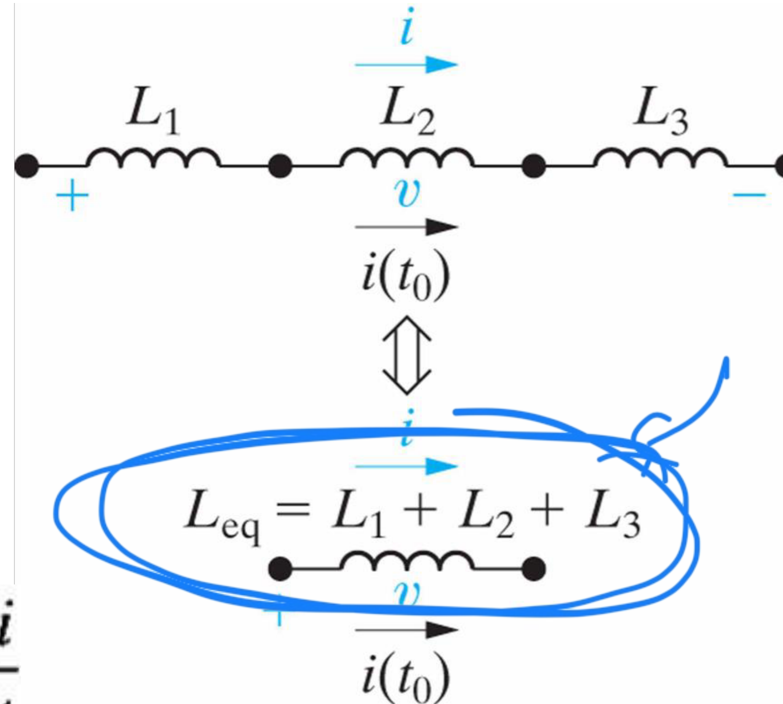
$$\int_{0.2}^{\infty} p\,dt = w(\infty) - w(0.2) = L/2 \times [i(\infty)^2 - i(0.2)^2] = -0.2e^{-2} = -27.07 \text{ mJ}$$

Based on the definition of p, the area under the plot of p versus t represents the energy expended over the interval of integration. Hence the integration of the power between 0 and 0.2 s represents the energy stored in the inductor during this time interval. The integral of p over the interval 0.2 s $-\infty$ is the energy extracted. *Note that in this time interval, all the energy originally stored is removed; that is, after the current peak has passed, no energy is stored in the inductor.*

Resistor

Series Combination of Inductance





$$v_1 = L_1 \frac{di}{dt} \quad v_2 = L_2 \frac{di}{dt} \quad v_3 = L_3 \frac{di}{dt}$$

$$v = v_1 + v_2 + v_3 = (L_1 + L_2 + L_3) \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_n$$

Parallel Combination of Inductance

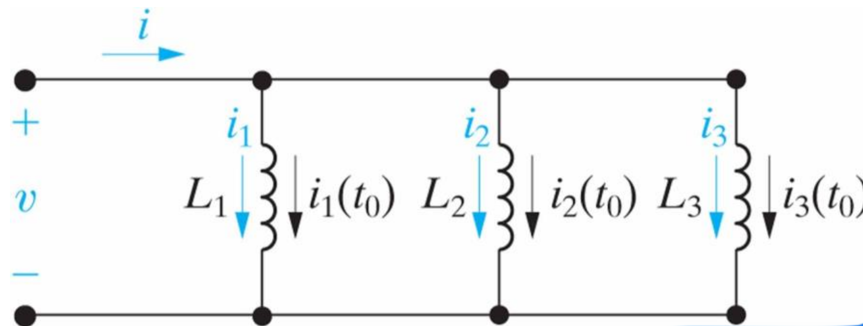
$$i_1 = \frac{1}{L_1}\int_{t_0}^{t} v\,d\tau + i_1(t_0)$$

$$i_2 = \frac{1}{L_2}\int_{t_0}^{t} v\,d\tau + i_2(t_0)$$

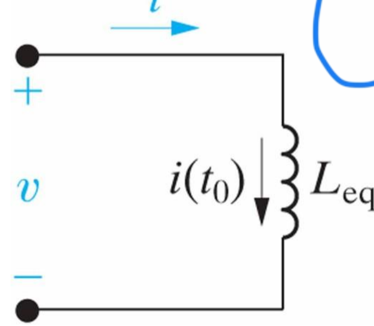
$$i_3 = \frac{1}{L_3}\int_{t_0}^{t} v\,d\tau + i_3(t_0)$$

$$i = i_1 + i_2 + i_3$$

$$i = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)\int_{t_0}^{t} v\,d\tau + i_1(t_0) + i_2(t_0) + i_3(t_0)$$



$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$$



$$i(t_0) = i_1(t_0) + i_2(t_0) + i_3(t_0)$$

$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \cdots}$$

Key inductor relationships:

Primary v-i equation	$v(t) = L\dfrac{di(t)}{dt}$
Alternate v-i equation	$i(t) = \dfrac{1}{L}\displaystyle\int_{t_0}^{t} v(\tau)\,d\tau + i(t_0)$
Initial condition	$i(t_0)$
Behavior with a constant source	If $i(t) = I$, $v(t) = 0$ and the inductor behaves like a short circuit
Continuity requirement	$i(t)$ is continuous for all time so $v(t)$ is finite
Power equation	$p(t) = v(t)i(t) = Li(t)\dfrac{di(t)}{dt}$
Energy equation	$w(t) = \dfrac{1}{2}Li(t)^2$
Series-connected equivalent	$L_{eq} = \displaystyle\sum_{j=1}^{n} L_j$ $i_{eq}(t_0) = i_j(t_0)$ for all j
Parallel-connected equivalent	$\dfrac{1}{L_{eq}} = \displaystyle\sum_{j=1}^{n} \dfrac{1}{L_j}$ $i_{eq}(t_0) = \displaystyle\sum_{j=1}^{n} i_j(t_0)$

Summary

- Inductance: *definitions*, *characteristics*, and *equations*
- Series--Parallel combinations of Inductance