(1 0 ∠ 5 0 °) × (2 ∠ 2 0 °)

▶ Polar ENTER 20.00E0 ∠ 70.00E0

FIG. 14.62
Performing the operation (10 ∠50°)(2 ∠20°).

form even though both quantities in the calculation are in polar form. In the rest of the examples, the scrolling required to obtain mathematical functions is not included to minimize the length of the sequence.

For the product of mixed complex numbers, the sequence of Fig. 14.63 results. Again, the polar form was selected for the solution.

(5 ∠ 5 3 . 1 °) × (2 + 2 *i*)

▶ Polar ENTER ENTER 14.14E0 ∠ 98.10E0

FIG. 14.63
Performing the operation (5 ∠53.1°)(2 + j 2).

Finally, Example 14.26(c) is entered as shown by the sequence in Fig. 14.64. Note that the results exactly match those obtained earlier.

(2 ∠ 2 0 °) ^ 2 × (3 + 4 *i*)

÷ (8 − 6 *i*) ▶ Polar ENTER ENTER 2.00E0 ∠ 130.0E0

FIG. 14.64
Verifying the results of Example 14.26(c).

14.11 PHASORS

As noted earlier in this chapter, the addition of sinusoidal voltages and currents is frequently required in the analysis of ac circuits. One lengthy but valid method of performing this operation is to place both sinusoidal waveforms on the same set of axes and add algebraically the magnitudes of each at every point along the abscissa, as shown for $c = a + b$ in Fig. 14.65. This, however, can be a long and tedious process with limited

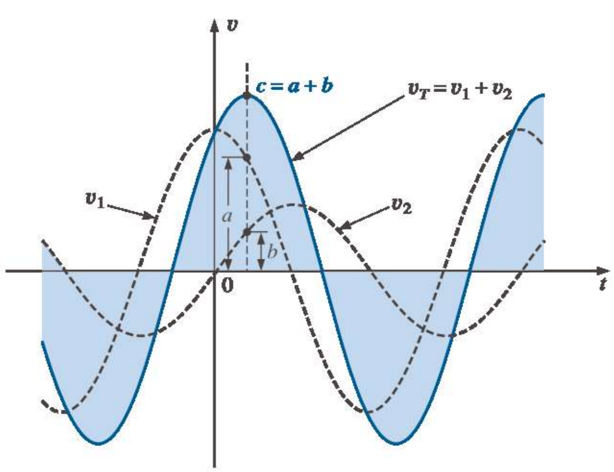

FIG. 14.65
Adding two sinusoidal waveforms on a point-by-point basis.

accuracy. A shorter method uses the rotating radius vector first appearing in Fig. 13.16. This *radius vector,* having a *constant magnitude* (length) with *one end fixed at the origin,* is called a **phasor** when applied to electric circuits.

Because of the importance of the discussion to follow and the benefits it will provide in your future analysis, it is strongly suggested that you return to Section 13.4 and carefully review how the rotating vector of fixed magnitude can generate a sinusoidal waveform at a frequency determined by the speed of rotation of the vector. If the two sinusoidal voltages to be added are in phase, as shown in Fig. 14.66(a), the radius vectors representing each appear on the positive axis at zero degrees because the vertical projection of each at that instant is zero, as shown in Fig. 14.66(b). Note also that the length of each phasor representation is the same as the peak value in Fig. 14.66(a). It should be clear from Fig. 14.66(a) that when the sinusoidal voltages are in phase the sum is simply the sum of the peak values of each as verified in Fig. 14.66(b). In general, therefore,

the addition (or subtraction) of two sinusoidal voltages of the same frequency and phase angle is simply the sum (or difference) of the peak values of each with the sum (or difference) having the same phase angle.

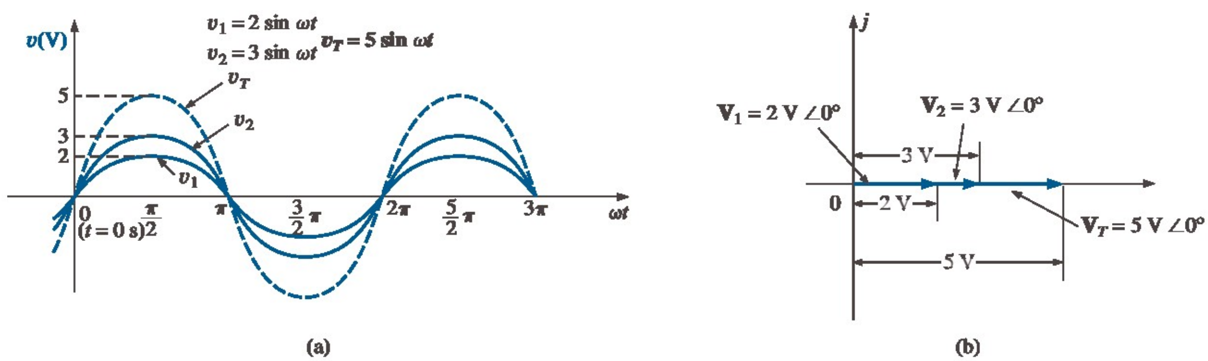

FIG. 14.66
Finding the sum of two sinusoidal waveforms with the same frequency and phase angle.

If the waveforms do not have the same phase angle, a summation of waveforms must be performed as indicated in Fig. 14.65 or using the approach to be described in this section.

Consider the addition of the two sinusoidal voltages of Fig. 14.67(a) out of phase by 90°. The peak value of one is 2 V and the other is 4 V, as shown in Fig. 14.67(a) and in the phasor representation of Fig. 14.67(b). At $t = 0$ s ($\theta = 0°$) the rotating vector of one is passing through the horizontal axis at zero degrees while the other is at its peak value due to the 90° phase shift. If we add the two waveforms of Fig. 14.67(a) on a point-to-point basis, the dashed blue sinusoidal waveform shown in the same figure would result. Note at $\theta = 0°(t = 0\text{ s})$ that $v_T = v_1 = 4$ V since $v_1 = 0$ V and at $\theta = \pi/2$ that $v_T = v_1 = 2$ V since $v_2 = 0$ V. The peak value will turn out to be close to 4.1 V at a phase angle of about 76°. It is difficult when adding waveforms to obtain a high level of accuracy unless the graphs are quite large and very carefully drawn. Now, if we look at the **phasor diagram** and simply find the hypotenuse

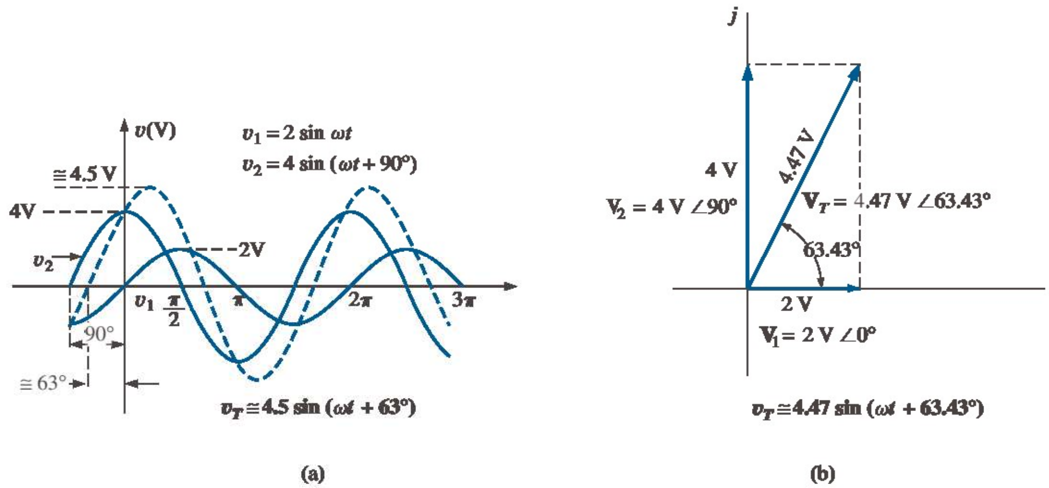

FIG. 14.67
Finding the sum of two sinusoidal waveforms that are out of phase.

of the triangle formed by the two vectors, we find that the magnitude of the projection is also 4.12 V—wonderful. A solution has been found for finding the sum of two sinusoidal waveforms that are not in phase. Simply draw a snapshot of the rotating vectors at $\theta = 0°(t = 0\text{ s})$ and find the sum of the two vectors. A closer examination of Fig. 14.67(b) also reveals that the phase angle associated with the resultant waveform leads the voltage by 63.43°. In other words, using the phasor diagram we can calculate both the magnitude and phase angle of the sinusoidal waveform representing the sum of the two waveforms. In addition, note the high level of accuracy obtained with a vector addition compared to the artistic approach.

If we now return to Fig. 14.67(b), the phasors representing each sinusoidal waveform can be written as

$$\mathbf{V}_1 = 2\text{ V} \angle 0° \qquad \text{and} \qquad \mathbf{V}_2 = 4\text{ V} \angle 90°$$

Their vector sum then becomes the following using the vector algebra introduced in the previous section. That is,

$$\begin{aligned} \mathbf{V}_T = \mathbf{V}_1 + \mathbf{V}_2 &= 2\text{ V} \angle 0° + 4\text{ V} \angle 90° \\ &= 2\text{ V} + j\,4\text{ V} \\ &= 4.47\text{ V} \angle 63.43° \end{aligned}$$

The result can then be written in the sinusoidal time domain format:

$$v_T = 4.47 \sin(\omega t + 63.43°)$$

If the sinusoidal voltages to be added have different peaks and phase angles, the required calculations are a bit more complex but not extensively so. The next few examples will demonstrate the power of the conclusions just introduced.

EXAMPLE 14.27 Find the sum of the following sinusoidal functions

$$i_1 = 5 \sin(\omega t + 30°)$$
$$i_2 = 6 \sin(\omega t + 60°)$$

a. Using a graphical approach
b. Using a phasor approach

Ex 27

$$i_1 = 5\cancel{\sin(\omega t} + 30)$$

$$+\ i_2 = \underline{6}\ \cancel{\sin(\omega t} + 60)$$

$$\rightarrow\ \cancel{\boxed{5\sin(\omega t + 30) + 6\sin(\omega t + 60)}}$$

عشان نجمع ⟸ شروط 3

1 إنه يكون علاها موجب ✓

2 إنه يكون كلاها sin cos ✓

3 حول الـ phasor ⟵ sin(ωt) اشطب

$$I_1 = 5\angle 30$$

$$+$$

$$I_2 = 6\angle 60$$

$(5 \angle 30) + (6 \angle 60) =$

$7.33 + j7.696$ ← مابعرف اجيبها

polar →

$10,628 \angle 46,395$

$$\boxed{i_1 + i_2 = 10,628 \sin(\omega t + 46,395)}$$

Solutions:

a. The two waveforms and the resultant sum appear in Fig. 14.68. It was obviously a tedious process to add the two waveforms with this approach. Take note that the position of each vector generating the waveforms shown is a snapshot of their position at $\theta = 0°(t = 0\text{ s})$. The sum of the two waveforms is obviously a vector addition of the two waveforms as shown to the left of Fig. 14.68.

b. In phasor form:

$$i_1 = 5\sin(\omega t + 30°) \Rightarrow 5\text{ A}\angle 30°$$
$$i_2 = 6\sin(\omega t + 60°) \Rightarrow 6\text{ A}\angle 60°$$

$$\begin{aligned}\mathbf{I}_T &= \mathbf{I}_1 + \mathbf{I}_2\\ &= 5\text{ A}\angle 30° + 6\text{ A}\angle 60°\\ &= (4.33\text{ A} + j\,2.5\text{ A}) + (3\text{ A} + j\,5.2\text{ A})\\ &= 7.33\text{ A} + j\,7.7\text{ A}\\ &= 10.63\text{ A}\angle 46.41°\end{aligned}$$

and $\boldsymbol{i_T = 10.63\sin(\omega t + 46.41°)}$ as obtained graphically.

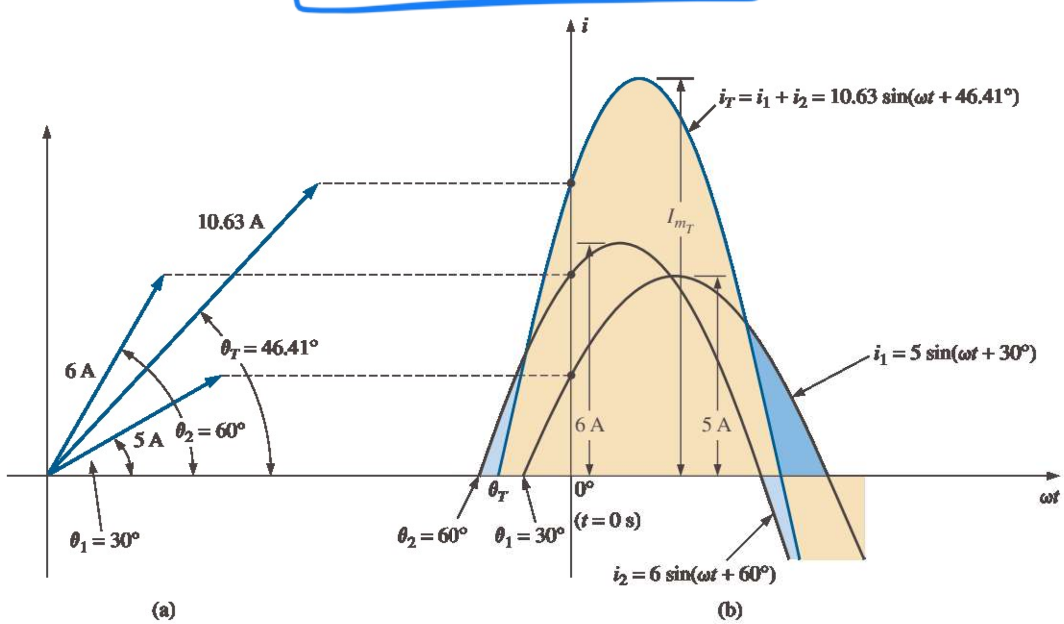

FIG. 14.68
Example 14.27

Since the rms, rather than the peak, values are used almost exclusively in the analysis of ac circuits, the phasor will now be redefined for the purposes of practicality and uniformity as having a magnitude equal to the *rms value* of the sine wave it represents. The angle associated with the phasor will remain as previously described—the phase angle.

In general, for all of the analyses to follow, the phasor form of a sinusoidal voltage or current will be

$$\mathbf{V} = V\angle\theta \quad \text{and} \quad \mathbf{I} = I\angle\theta$$

where V and I are rms values and θ is the phase angle. It should be pointed out that in phasor notation, the sine wave is always the reference, and the frequency is not represented.

Phasor algebra for sinusoidal quantities is applicable only for waveforms having the same frequency.

The use of phasor notation in the analysis of ac networks was first introduced by Charles Proteus Steinmetz in 1897 (Fig. 14.69).

FIG. 14.69
Charles Proteus Steinmetz.
Bain News Service/George Grantham Bain Collection/Library of Congress

German-American (Breslau, Germany; Yonkers and Schenectady, NY, USA)
(1865–1923)
Mathematician, Scientist, Engineer, Inventor, Professor of Electrical Engineering and Electrophysics, Union College
Department Head, General Electric Co.

Although the holder of some 200 patents and recognized worldwide for his contributions to the study of hysteresis losses and electrical transients, Charles Proteus Steinmetz is best recognized for his contribution to the study of ac networks. His "Symbolic Method of Alternating-current Calculations" provided an approach to the analysis of ac networks that removed a great deal of the confusion and frustration experienced by engineers of that day as they made the transition from dc to ac systems. His approach (on which the phasor notation of this text is premised) permitted a direct analysis of ac systems using many of the theorems and methods of analysis developed for dc systems. In 1897 he authored the epic work *Theory and Calculation of Alternating Current Phenomena,* which became the authoritative guide for practicing engineers. Dr. Steinmetz was fondly referred to as "The Doctor" at General Electric Company where he worked for some 30 years in a number of important capacities. His recognition as a multigifted genius is supported by the fact that he maintained active friendships with such individuals as Albert Einstein, Guglielmo Marconi, and Thomas A. Edison, to name just a few. He was President of the American Institute of Electrical Engineers (AIEE) and the National Association of Corporation Schools and actively supported his local community (Schenectady) as president of the Board of Education and the Commission on Parks and City Planning.

EXAMPLE 14.28 Convert the following from the time to the phasor domain:

Time Domain	Phasor Domain
a. $\sqrt{2}(50)\sin\omega t$	$\mathbf{50\ \angle 0^\circ}$
b. $69.6\sin(\omega t + 72^\circ)$	$(0.707)(69.6)\ \angle 72^\circ = \mathbf{49.21\ \angle 72^\circ}$
c. $45\cos\omega t$	$(0.707)(45)\ \angle 90^\circ = \mathbf{31.82\ \angle 90^\circ}$

EXAMPLE 14.29 Write the sinusoidal expression for the following phasors if the frequency is 60 Hz:

Phasor Domain	Time Domain
a. $\mathbf{I} = 10\ \angle 30^\circ$	$i = \sqrt{2}(10)\sin(2\pi 60t + 30^\circ)$ and $i = \mathbf{14.14\sin(377t + 30^\circ)}$
b. $\mathbf{V} = 115\ \angle -70^\circ$	$v = \sqrt{2}(115)\sin(377t - 70^\circ)$ and $v = \mathbf{162.6\sin(377t - 70^\circ)}$

EXAMPLE 14.30 Find the input voltage of the circuit in Fig. 14.70 if

$$\left.\begin{aligned} v_a &= 50\sin(377t + 30^\circ) \\ v_b &= 30\sin(377t + 60^\circ) \end{aligned}\right\} f = 60\text{ Hz}$$

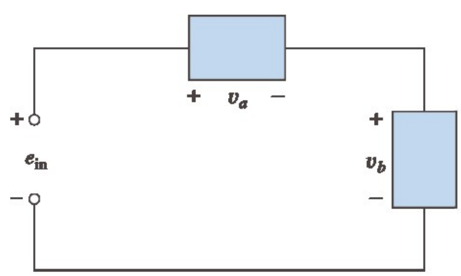

FIG. 14.70
Example 14.30.

Solution: Applying Kirchhoff's voltage law, we have

$$e_{in} = v_a + v_b$$

Converting from the time to the phasor domain yields

$$v_a = 50\sin(377t + 30^\circ) \Rightarrow \mathbf{V}_a = 35.35\text{ V}\ \angle 30^\circ$$
$$v_b = 30\sin(377t + 60^\circ) \Rightarrow \mathbf{V}_b = 21.21\text{ V}\ \angle 60^\circ$$

Converting from polar to rectangular form for addition yields

$$\mathbf{V}_a = 35.35\text{ V}\ \angle 30^\circ = 30.61\text{ V} + j\,17.68\text{ V}$$
$$\mathbf{V}_b = 21.21\text{ V}\ \angle 60^\circ = 10.61\text{ V} + j\,18.37\text{ V}$$

Ex 14.28

ص80

Time Domain $\longrightarrow$ Phaser Domain

sin (wt شطبها)

$\boxed{\sqrt{2}(50)}\ \cancel{\underline{\sin}}\ \cancel{(wt}+0)$ $\longrightarrow$ $50\angle 0$

peak

max

نقسم على √ $\longleftarrow$ { rms, eff }

$69.6\ \cancel{\sin}\ \cancel{(wt+}72)$ $\longrightarrow$ $\frac{69.6}{\sqrt{2}}\angle 72$

$49.21\angle 72$

$45 \cos wt \longrightarrow$ sin ال دوم

$45\ \cancel{\sin}(w\cancel{t}+90) \longrightarrow \boxed{\frac{45}{\sqrt{2}}\angle 90}$

ص80

Ex 14.20

80

من rms لل peak اضرب $\sqrt{2}$

من peak لل rms اقسم $\sqrt{2}$

Phasor Domain → Time Domain

$$I = 10 \angle 30 \rightarrow 10\sqrt{2}\sin(\omega t + 30)$$

rms
eff

peak
max

$$V = 115 \angle -70 \rightarrow 115\sqrt{2}\sin(\omega t - 70)$$

$$f = 60\,Hz \rightarrow \omega = 2\pi f = 2\times\pi\times 60$$

$$= 377$$

$$\boxed{V = 162.6 \sin(377t - 70)}$$

Then

$$\mathbf{E}_{in} = \mathbf{V}_a + \mathbf{V}_b = (30.61\text{ V} + j\,17.68\text{ V}) + (10.61\text{ V} + j\,18.37\text{ V})$$
$$= 41.22\text{ V} + j\,36.05\text{ V}$$

Converting from rectangular to polar form, we have

$$\mathbf{E}_{in} = 41.22\text{ V} + j\,36.05\text{ V} = 54.76\text{ V}\angle 41.17^\circ$$

Converting from the phasor to the time domain, we obtain

$$\mathbf{E}_{in} = 54.76\text{ V}\angle 41.17^\circ \Rightarrow e_{in} = \sqrt{2}(54.76)\sin(377t + 41.17^\circ)$$

and $$e_{in} = \mathbf{77.43\sin(377}\boldsymbol{t} \mathbf{+ 41.17^\circ)}$$

A plot of the three waveforms is shown in Fig. 14.71. Note that at each instant of time, the sum of the two waveforms does in fact add up to e_{in}. At $t = 0(\omega t = 0)$, e_{in} is the sum of the two positive values, while at a value of ωt, almost midway between $\pi/2$ and π, the sum of the positive value of v_a and the negative value of v_b results in $e_{in} = 0$.

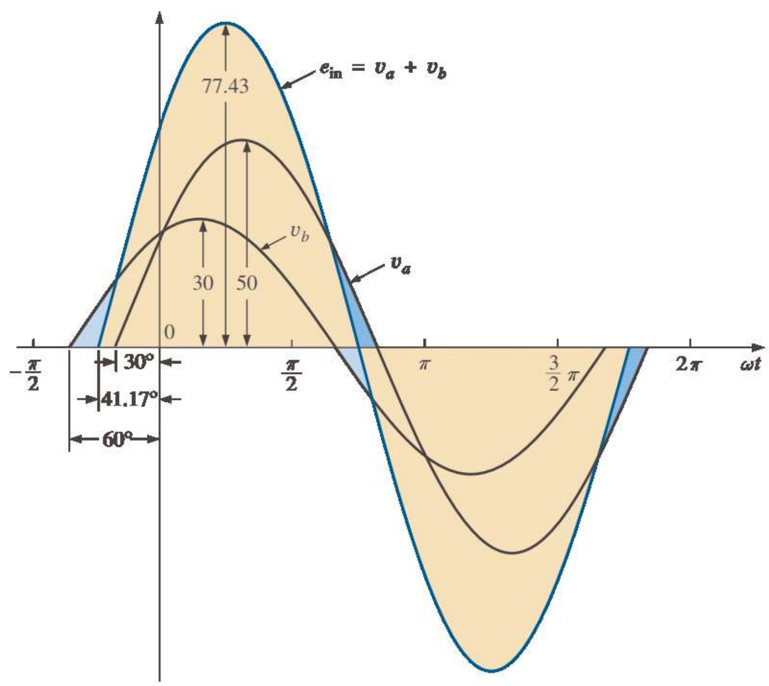

FIG. 14.71
Solution to Example 14.30.

EXAMPLE 14.31 Determine the current i_2 for the network in Fig. 14.72.

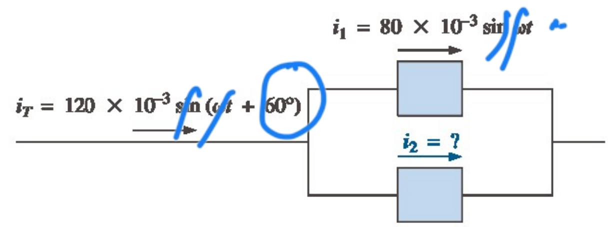

FIG. 14.72
Example 14.31.

$$I_T = i_1 + i_2$$

$$i_2 = I_T - I_1$$

$$I_T = 120\times10^{-3}\angle 60 = 0{,}12\angle 60$$

$$I_1 = 80\times10^{-3}\angle 0 = 0{,}08\angle 0$$

$$i_2 = (0{,}12\angle 60) - (0{,}08\angle 0)$$

$$i_2 = 0{,}1058\angle 100{,}89$$

$$\boxed{i_2(t) = 0{,}1058\sin(\omega t + 100{,}89)}$$

Time Domain $\rightarrow$ $\dfrac{\text{Phasor}}{\sqrt{2}}$

Solution: Applying Kirchhoff's current law, we obtain

$$i_T = i_1 + i_2 \qquad \text{or} \qquad i_2 = i_T - i_1$$

Converting from the time to the phasor domain yields

$$i_T = 120 \times 10^{-3} \sin(\omega t + 60°) \Rightarrow 84.84 \text{ mA} \angle 60°$$
$$i_1 = 80 \times 10^{-3} \sin \omega t \Rightarrow 56.56 \text{ mA} \angle 0°$$

Converting from polar to rectangular form for subtraction yields

$$\mathbf{I}_T = 84.84 \text{ mA} \angle 60° = 42.42 \text{ mA} + j\,73.47 \text{ mA}$$
$$\mathbf{I}_1 = 56.56 \text{ mA} \angle 0° = 56.56 \text{ mA} + j\,0$$

Then

$$\mathbf{I}_2 = \mathbf{I}_T - \mathbf{I}_1$$
$$= (42.42 \text{ mA} + j\,73.47 \text{ mA}) - (56.56 \text{ mA} + j\,0)$$

and $$\mathbf{I}_2 = -14.14 \text{ mA} + j\,73.47 \text{ mA}$$

Converting from rectangular to polar form, we have

$$\boxed{\mathbf{I}_2 = 74.82 \text{ mA} \angle 100.89°}$$

Converting from the phasor to the time domain, we have

$$\mathbf{I}_2 = 74.82 \text{ mA} \angle 100.89° \Rightarrow$$
$$i_2 = \sqrt{2}(74.82 \times 10^{-3}) \sin(\omega t + 100.89°)$$

and $$\mathbf{i_2 = 105.8 \times 10^{-3} \sin(\omega t + 100.89°)}$$

A plot of the three waveforms appears in Fig. 14.73. The waveforms clearly indicate that $i_T = i_1 + i_2$.

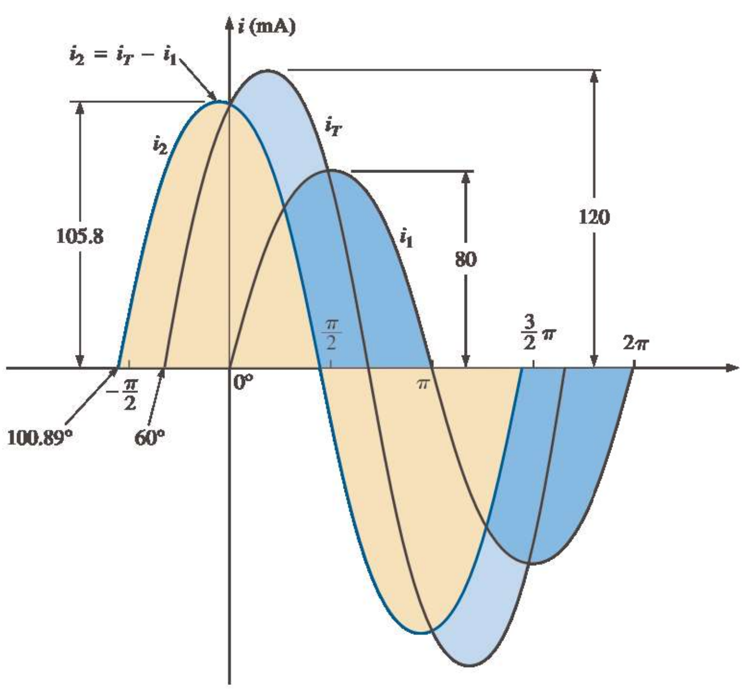

FIG. 14.73
Solution to Example 14.31.

14.12 COMPUTER ANALYSIS

PSpice

Capacitors and the ac Response The simplest of ac capacitive circuits is now analyzed to introduce the process of setting up an ac source and running an ac transient simulation. The ac source in Fig. 14.74 is obtained through **Place part** key-**SOURCE-VSIN-OK.** Change the name or value of any parameter by double-clicking on the parameter on the display. The peak value (**VAMPL**) of the source voltage is 5 V, the frequency 1 kHz, and the phase angle zero degrees.

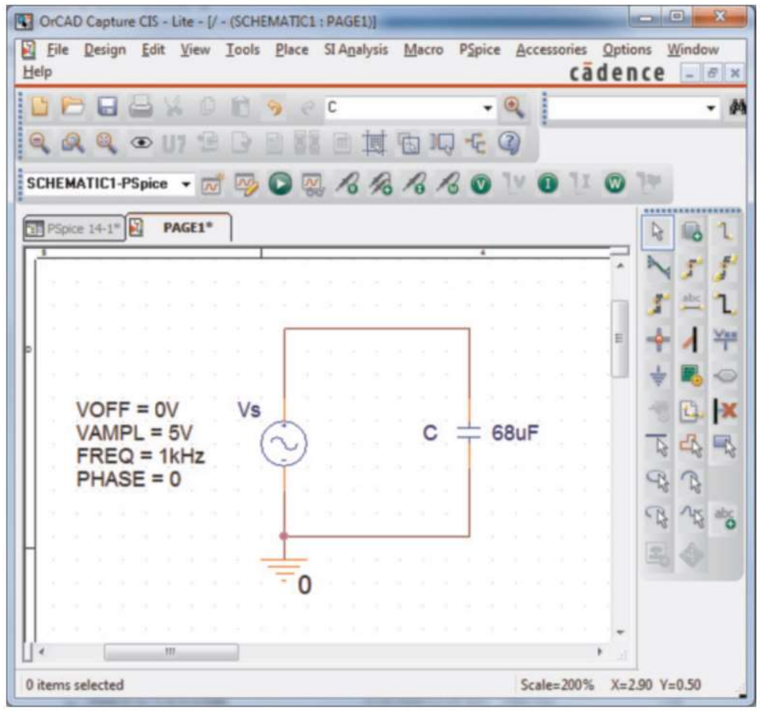

FIG. 14.74
Using PSpice to analyze the response of a capacitor to a sinusoidal ac signal.

The simulation process is initiated by selecting the **New Simulation Profile.** Under **New Simulation,** enter **PSpice 14-1** for the **Name** followed by **Create.** The result will be a blinking **Simulation Setting-PSpice 14-1** dialog box at the bottom of the window that can be deposited on the screen by simply clicking on the dialog box. In the **Simulation Settings** dialog box, select **Analysis** and choose **Time Domain(Transient)** under **Analysis type.** Set the **Run to time** at 3 ms to permit a display of three cycles of the sinusoidal waveforms ($T = 1/f = 1/1000\text{ Hz} = 1\text{ ms}$). Leave the **Start saving data after** at 0 s, and set the **Maximum step size** at $3\text{ ms}/1000 = 3\ \mu s$. Clicking **OK** and then selecting the **Run PSpice** icon results in a **SCHEMATIC1-PSpice 14-1** dialog box at the bottom of the window that can be deposited on the screen by simply clicking on the dialog box. The resulting plot has a horizontal axis that extends from 0 to 3ms.

Now you must tell the computer which waveforms you are interested in. First, take a look at the applied ac source by selecting **Trace-Add Trace-V(Vs:+)** followed by **OK.** The result is the sweeping ac voltage in the bottom region of the screen in Fig. 14.75. Note that it has a peak

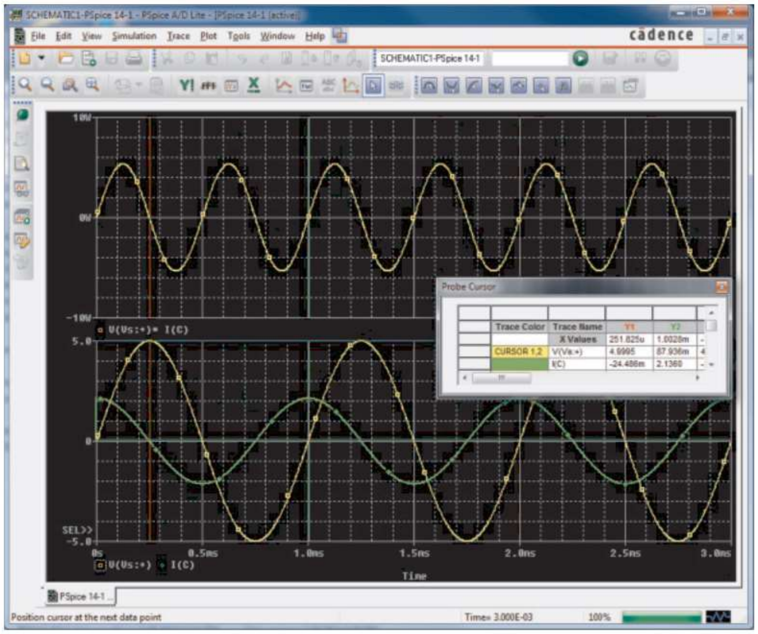

FIG. 14.75
A plot of the voltage, current, and power for the capacitor in Fig. 14.74.

value of 5 V, and three cycles appear in the 3 ms time frame. The current for the capacitor can be added by selecting **Trace-Add Trace** and choosing **I(C)** followed by **OK.** The resulting waveform for **I(C)** appears at a 90° phase shift from the applied voltage, with the current leading the voltage (the current has already peaked as the voltage crosses the 0 V axis). Since the peak value of each plot is in the same magnitude range, the 5 appearing on the vertical scale can be used for both. A theoretical analysis results in $X_C = 2.34\ \Omega$, and the peak value of $I_C = E/X_C = 5\text{ V}/2.34 = 2.136\text{ A}$, as shown in Fig. 14.75.

For practice, let us obtain the curve for the power delivered to the capacitor over the same time period. First select **Plot-Add Plot to Window-Trace-Add Trace** to obtain the **Add Traces** dialog box. Select **W(C)** followed by **OK** and the top plot of Fig. 14.75 will appear showing that over time the net power delivered is zero (the average value). The power to the capacitor can also be found by first choosing **V(Vs: +)** followed by * from the **Function** listing on the right side of the **Add Traces** dialog box and then **I(C).** The result is the expression **V(Vs: +)*I(C)** of the power format: $p = vi$. Click **OK,** and the power plot at the top of Fig. 14.75 appears. Note that over the full three cycles, the area above the axis equals the area below—there is no net transfer of power over the 3 ms period. Note also that the power curve is sinusoidal (which is quite interesting) with a frequency twice that of the applied signal. Using the cursor control, we can determine that the maximum power (peak value of the sinusoidal waveform) is 5.34 W. The cursors, in fact, have been added to the lower curves to show the peak value of the applied sinusoid and the resulting current.

After selecting the **Toggle cursor** icon, left-click to surround the symbol to the left of **V(Vs: +)** at the bottom of the plot with a dashed line to establish that the cursor is providing the levels of that quantity. Then a left-click on the plot will establish the cursor option. When placed at ¼ of the total period (250 μs), the peak value is approximately

5 V (Y_1) as shown in the **Probe Cursor** dialog box. Placing the cursor over the symbol next to **I(C)** at the bottom of the plot and right-clicking assigns the right cursor to the current. Placing it at exactly 1 ms (**Y2**) results in a peak value of 2.136 A to match the solution above. To further distinguish between the voltage and current waveforms, the color and the width of the lines of the traces were changed. With the **Toggle cursor** key disabled, place the cursor right on the plot line and right-click. The **Properties** option appears. When **Properties** is selected, a **Trace Properties** dialog box appears in which the yellow color can be selected and the width widened to improve the visibility on the black background. Note that yellow was chosen for **Vs** and green for **I(C).** Note also that the axis and the grid have been changed to a more visible color using the same procedure.

Multisim

Since PSpice reviewed the response of a capacitive element to an ac voltage, Multisim repeats the analysis for an inductive element. The ac voltage source was derived from the **Place Source** parts bin as described in Chapter 13 with the values appearing in Fig. 14.76 set in the **AC-Voltage** dialog box.

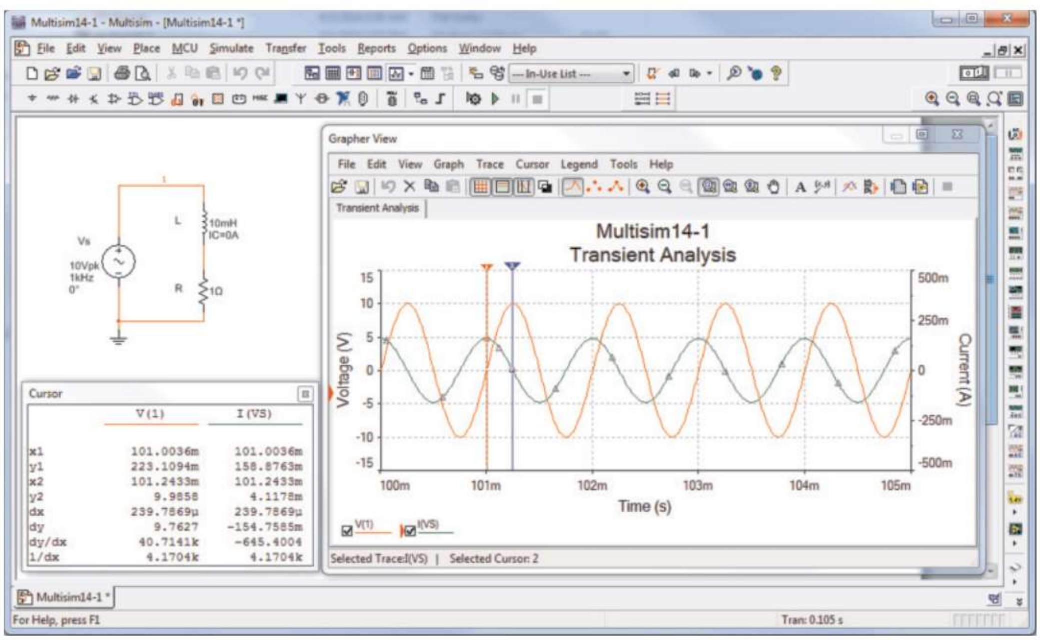

FIG. 14.76
Using Multisim to review the response of an inductive element to a sinusoidal ac signal.

Once the circuit has been constructed, the sequence **Simulate-Analyses-Transient Analysis** results in a **Transient Analysis** dialog box. Select **Analysis** parameters and set **Start Time** to 0 s and **End Time** to 105 ms using 0.105 s or 105E-3 s. Then select **Analysis options** and set maximum number of points to 10,000 to ensure a good display for the rapidly changing waveform. The 105 ms was set as the **End Time** to give the network 100 ms to settle down in its steady-state mode and 5 ms for five cycles in the output display.

Next the **Output** heading was chosen within the dialog box, and the source voltage **V(1)** and source current **I(VS)** were moved from the **Variables in Circuit** to **Selected variables for analysis** using the **Add** option. Choosing **Simulate** results in a waveform that extends from 0 s to 105 ms. Even though we plan to save only the response that occurs after 100 ms, the computer is unaware of our interest, and it plots the response for the entire period. This is corrected by selecting **Trace-Trace Properties** to obtain the **Graph Properties** dialog box. Selecting **Bottom Axis** permits setting the **Range** from a **Minimum of 0.100 s = 100 ms** to a **Maximum of 0.105 s = 105 ms.** Click **OK,** and the time period of Fig. 14.76 is displayed. The grid structure is added by selecting the **Show Grid** keypad, and the color associated with each curve is displayed if we choose **Legend-Show Legend.**

It is clear from the plot that the scale for the source current has to be improved for us to be able to clearly read its peak and negative values. This is done by first clicking on the **I(VS)** curve to set the **Selected Trace** at the bottom of the graph as **I(VS).** A right click, and one can choose the **Properties** option to obtain the **Graph Properties** dialog box. Under **Traces,** select **Right axis** under **Y-vertical axis.** Then select **Right Axis** to establish the right axis as the scale to be used for the source current. Insert the **Label: Current(A),** select **Enabled** under the **Axis** heading, and finally choose **Pen Size** as 1. The **Scale** is **Linear** and of range -0.5 to 0.5 (-500 mA to 500 mA), with **Total Ticks** of 8 and **Minor Ticks** of 2. The result is the plot of Fig. 14.76. The right axis can now be improved by selecting **Graph Properties** again, followed by **Left Axis,** whereby the **Current(A)** can be deleted. We can now see that the source current has a peak value of about 160 mA. For more detail on the waveforms, select **Cursor-Show Cursors** to obtain the **Transient Analysis** dialog box with box **V(1)** and **I(VS)** listed with the same color headings as used on the graph. Clicking on one of the cursors and moving it horizontally to the maximum value of the current will result in **x1** = 101.0 ms with **y1** at 158.88 mA. Actually, the **max y** appears below at 159.07 mA, which could have been obtained if we had increased the number of data points. Moving the other cursor to find the minimum value of current will result in **x2** = 101.24 ms with **y2** at 4.1 mA (the closest to the level of 0 mA obtainable with this data level setting). The maximum value of **V(1)** appears below as 9.986 V $\cong$ 10 V (at **x1** = 101 ms), which it should be, and the distance between the maximum value of **I(VS)** and the its minimum value is **dx** = 239.79 μs, which is very close to 0.25 ms, or one fourth of the period of the applied signal.

PROBLEMS

SECTION 14.1 Introduction

1. Plot the following waveform versus time showing one clear, complete cycle. Then determine the derivative of the waveform using Eq. (14.1), and sketch one complete cycle of the derivative directly under the original waveform. Compare the magnitude of the derivative at various points versus the slope of the original sinusoidal function.

$$v = 4 \sin 62.8t$$

2. Repeat Problem 1 for the following sinusoidal function, and compare results. In particular, determine the frequency of the waveforms of Problems 1 and 2, and compare the magnitude of the derivative.

$$v = 10 \sin 377t$$

3. What is the derivative of each of the following sinusoidal expressions?
   - **a.** $10 \sin 377t$
   - **b.** $20 \sin(400t + 60°)$
   - **c.** $\sqrt{2}\,20 \sin(157t - 20°)$
   - **d.** $-200 \sin(t + 180°)$

SECTION 14.2 Response of Basic *R*, *L*, and *C* Elements to a Sinusoidal Voltage or Current

4. The voltage across a 20 Ω resistor is as indicated. Find the sinusoidal expression for the current. In addition, sketch the v and i sinusoidal waveforms on the same axis.
   a. $160 \sin 100t$
   b. $60 \sin(2000t + 45°)$
   c. $6 \cos(\omega t + 10°)$
   d. $-12 \sin(\omega t + 40°)$

   V

5. The current through a 7.8 kΩ resistor is as indicated. Find the sinusoidal expression for the voltage. In addition, sketch the v and i sinusoidal waveforms on the same axis.
   a. $0.2 \sin 500t$
   b. $5 \times 10^{-3} \sin(600t - 120°)$
6. Determine the inductive reactance (in ohms) of a 3 mH coil for
   a. dc

   and for the following frequencies:
   b. 60 Hz
   c. 8 kHz
   d. 1.4 MHz
7. Determine the closest standard value inductance that has a reactance of
   a. 2.5 kΩ at $f = 12.47$ kHz.
   b. 45 kΩ at $f = 5.8$ kHz.
8. Determine the frequency at which a 47 mH inductance has the following inductive reactances:
   a. 10 Ω
   b. 4 kΩ
   c. 12 kΩ
9. The current through a 20 Ω inductive reactance is given. What is the sinusoidal expression for the voltage? Sketch the v and i sinusoidal waveforms on the same axis.
   a. $i = 25 \times 10^{-3} \sin 200t$
   b. $i = 40 \times 10^{-3} \sin(\omega t + 60°)$
   c. $i = -6 \sin(\omega t - 30°)$
10. The current through a 0.15 H coil is given. What is the sinusoidal expression for the voltage?
    a. $15 \sin 150t$
    b. $6 \times 10^{-6} \sin(400t + 20°)$
11. The voltage across a 40 Ω inductive reactance is given. What is the sinusoidal expression for the current? Sketch the v and i sinusoidal waveforms on the same set of axes.
    a. $120 \sin \omega t$
    b. $30 \sin(\omega t + 20°)$
12. The voltage across a 0.25 H coil is given. What is the sinusoidal expression for the current?
    a. $2.5 \sin 90t$
    b. $16 \times 10^{-3} \sin(20t + 5°)$

    X_C

13. Determine the capacitive reactance (in ohms) of a 0.4 μF capacitor for
    a. dc $\omega = 0 \rightarrow X_C = \frac{1}{0} = \infty$ open

    and for the following frequencies:
    b. 80 Hz
    c. 2.5 kHz 10^3
    d. 2.5 MHz $\times 10^6$
14. Determine the closest standard value capacitance that has a reactance of
    a. 75 Ω at $f = 250$ Hz.
    b. 2.2 kΩ at 36 kHz.
15. Determine the frequency at which a 3.9 μF capacitor has the following capacitive reactances:
    a. 10 Ω
    b. 60 kΩ
    c. 0.1 Ω
    d. 2000 Ω
16. The voltage across a 2.5 Ω capacitive reactance is given. What is the sinusoidal expression for the current? Sketch the v and i sinusoidal waveforms on the same set of axes.
    a. $120 \sin \omega t$
    b. $4 \times 10^{-3} \sin(\omega t + 40°)$
17. The voltage across a 1 μF capacitor is given. What is the sinusoidal expression for the current?
    a. $30 \sin 250t$
    b. $90 \times 10^{-3} \sin 377t$
18. The current through a 2 kΩ capacitive reactance is given. Write the sinusoidal expression for the voltage. Sketch the v and i sinusoidal waveforms on the same set of axes.
    a. $i = 50 \times 10^{-3} \sin \omega t$
    b. $i = 2 \times 10^{-6} \sin(\omega t + 60°)$
19. The current through a 0.50 μF capacitor is given. What is the sinusoidal expression for the voltage?
    a. $0.20 \sin 500t$
    b. $5 \times 10^{-3} \sin(377t - 45°)$

*20. For the following pairs of voltages and currents, indicate whether the element involved is a capacitor, an inductor, or a resistor, and find the value of C, L, or R if sufficient data are given:
    L ← a. $v = 550 \sin(377t + 50°)$
       $i = 11 \sin(377t - 40°)$
    b. $v = 36 \sin(754t - 80°)$
       $i = 4 \sin(754t - 170°)$
    R ← c. $v = 10.5 \sin(\omega t - 13°)$
       $i = 1.5 \sin(\omega t - 13°)$

    $\theta_v > \theta_i \rightarrow$ lag $\rightarrow$ L
    $\theta_v < \theta_i \rightarrow$ lead $\rightarrow$ C
    $\theta_v = \theta_i \rightarrow$ unity $\rightarrow$ R

*21. Repeat Problem 20 for the following pairs of voltages and currents with $\omega = 157$ rad/s.
    a. $v = 2000 \sin \omega t$
       $i = 5 \cos \omega t$
    b. $v = 80 \sin(157t + 150°)$
       $i = 2 \sin(157t + 60°)$
    c. $v = 35 \sin(\omega t - 20°)$
       $i = 7 \cos(\omega t - 110°)$

SECTION 14.3 Frequency Response of the Basic Elements

22. Plot X_L versus frequency for a 3 mH coil using a frequency range of zero to 100 kHz on a linear scale.
23. Plot X_C versus frequency for a 1 μF capacitor using a frequency range of zero to 10 kHz on a linear scale.
24. At what frequency will the reactance of a 1.5 μF capacitor equal the resistance of a 2 kΩ resistor?
25. The reactance of a coil equals the resistance of a 10 kΩ resistor at a frequency of 5 kHz. Determine the inductance of the coil.

Q 4 $\quad R = 20\,\Omega$

b) $V = 60 \sin(2000t + 45)$

$V = IR \rightarrow I = \frac{V}{R}$

$i = \frac{60}{20} = 3 \sin(2000t + 45)$

$$\boxed{i(t) = 3 \sin(2000t + 45)}$$

R

$V = IR \qquad I = \frac{V}{R}$

Q7 find L ← $\omega = 2\pi f$

a) $2.5\,k\Omega$ at $f = 12.47\,kHz$

$X_L = 2.5\,k\Omega = \omega L$

~~$2\pi \times 12.47 \times 10^3$~~

~~$2\pi\ 12.47 \times 10^3$~~

$L = \dfrac{2.5 \times 10^3}{2\pi \times 12.47 \times 10^3}$

$$\boxed{L = 32\,mH}$$

Q_{10} $L = 0{,}15\ H$

b) $\underline{i} = 6\times10^{-6}\ \cancel{\sin}(\boxed{\cancel{400}}\,t + 20)$ $\quad \omega$

phasor Impedance Z

$V = \underline{I}\,Z \qquad Z = jX_L$

$X_L = \omega L \Rightarrow Z_L = j\underline{\omega}L$

$Z_L = j\cdot \underline{2\pi f}\cdot \underline{L}$

$Z_L = j\ 400\times0{,}15 = \underline{j60\ \Omega}$

$I = \dfrac{6\times10^{-6}}{\sqrt{2}} \angle 20\ A$

$V = \hat{I} Z$

$0{,}000006$

$$V = \left(\frac{(6\times10^{-6}\angle 20)}{\sqrt{2}}\right)(60\,\underline{\underline{j}})$$

$$V = 2.5456\times10^{-4}\angle 110$$

$$V = \sqrt{2}\,(2.5456\times10^{-4})\sin(400t + 110)$$

$$V = \underline{\underline{3.6\times10^{-4}}}\sin(400t + 110)$$

$$360 \quad \sin(400t + 110)\ \mu V$$

$$10^{-6}$$

Q15 find F

$C = 3.9 \mu F \rightarrow 10^{-6}$

$X_C = 10 \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = X_C$$

$$f = \frac{1}{2\pi X_C C} = \frac{1}{2\pi \times 10 \times 3.9 \times 10^{-6}}$$

$$\boxed{f = 4080 \text{ Hz}}$$

Q18

b) $\boxed{X_C = 2k\,\Omega} \rightarrow$ cap

$i = 2\times10^{-6}\,\cancel{\sin}(\cancel{\omega t}+60)$

$\underline{\underline{\text{find}\quad V}}$

$V = \underline{\underline{I}}\,Z \qquad Z_C = -jX_C$

$$\boxed{Z_C = -2000j\ \Omega}$$

$$I = \frac{2\times10^{-6}}{\sqrt{2}}\angle 60$$

$$V = \left(\frac{2\times10^{-6}\angle 60}{\sqrt{2}}\right)(-2000j)$$

$$V = 2.828\times10^{-3}\angle -30$$

$$v(t) = \underbrace{\sqrt{2}\,(2.828\times10^{-3})}\,\sin(\omega t - 30)$$

$$\boxed{v(t) = 4\sin(\omega t - 30)\ \text{mV}}$$

26. Determine the frequency at which a 2 μF capacitor and an 80 mH inductor will have the same reactance.

27. Determine the capacitance required to establish a capacitive reactance that will match that of a 2 mH coil at a frequency of 60 kHz.

SECTION 14.4 Average Power and Power Factor

***28.** Find the average power loss and power factor for each of the circuits whose input current and voltage are as follows:

a. $v = 60 \sin(\omega t + 30°)$
$i = 15 \sin(\omega t + 60°)$

b. $v = -50 \sin(\omega t - 20°)$
$i = -2 \sin(\omega t - 20°)$

c. $v = 50 \sin(\omega t + 80°)$
$i = 3 \cos(\omega t - 20°)$

d. $v = 75 \sin(\omega t - 5°)$
$i = 0.08 \sin(\omega t + 35°)$

29. If the current through and voltage across an element are $i = 8 \sin(\omega t + 40°)$ and $v = 56 \sin(\omega t + 50°)$, respectively, compute the power by I^2R, $(V_mI_m/2)\cos\theta$, and $VI\cos\theta$, and compare answers.

30. A circuit dissipates 150 W (average power) at 200 V (effective input voltage) and 2.5 A (effective input current). What is the power factor? Repeat if the power is 0 W; 500 W.

***31.** The power factor of a circuit is 0.5 lagging. The power delivered in watts is 600. If the input voltage is $60 \sin(\omega t + 20°)$, find the sinusoidal expression for the input current.

32. In Fig. 14.77, $e = 120 \sin(2\pi 60t + 20°)$.

a. What is the sinusoidal expression for the current?
b. Find the power loss in the circuit.
c. How long (in seconds) does it take the current to complete six cycles?

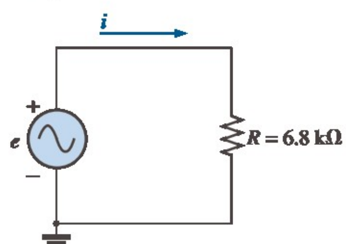

FIG. 14.77
Problem 32.

33. In Fig. 14.78, $e = 240 \sin(1500t + 45°)$.

a. Find the sinusoidal expression for i.
b. Find the average power loss by the inductor.

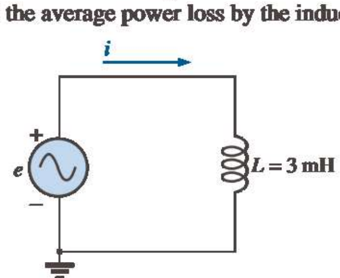

FIG. 14.78
Problem 33.

34. In Fig. 14.79, $i = 20 \times 10^{-3} \sin(2\pi 600t - 30°)$.

a. Find the sinusoidal expression for e.
b. Find the average power loss in the capacitor.

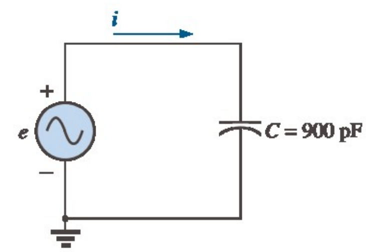

FIG. 14.79
Problem 34.

***35.** For the network in Fig. 14.80 and the applied signal:

a. Determine the sinusoidal expressions for i_1 and i_2.
b. Find the sinusoidal expression for i_s by combining the two parallel capacitors.

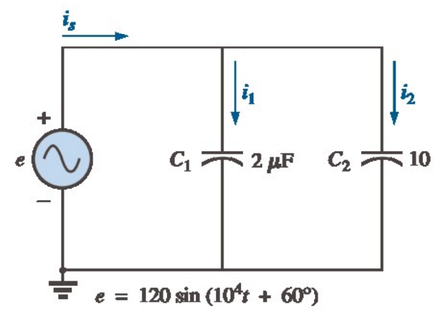

FIG. 14.80
Problem 35.

***36.** For the network in Fig. 14.81 and the applied source:

a. Determine the sinusoidal expression for the source voltage v_s.
b. Find the sinusoidal expression for the currents i_1 and i_2.

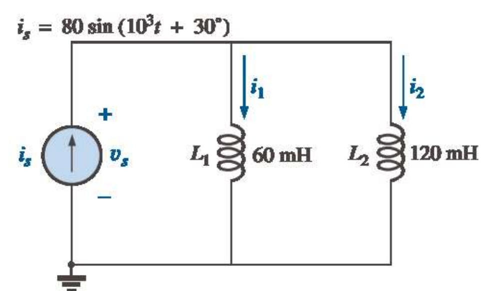

FIG. 14.81
Problem 36.

SECTION 14.8 Conversion between Forms

37. Convert the following from rectangular to polar form:
a. $4 + j6$ b. $3 + j3$
c. $5 + j15$ d. $500 + j50$
e. $-1000 + j2000$ f. $-0.2 + j0.4$

***38.** Convert the following from rectangular to polar form:
a. $-8 - j16$
b. $+8 - j4$
c. $0.02 - j0.003$
d. $-6 \times 10^{-3} - j6 \times 10^{-3}$
e. $200 + j0.02$
f. $-1000 + j20$

39. Convert the following from polar to rectangular form:
a. $6 \angle 40°$ b. $12 \angle 120°$
c. $2000 \angle -90°$ d. $0.0064 \angle +200°$
e. $48 \angle 2°$ f. $5 \times 10^{-4} \angle -20°$

40. Convert the following from polar to rectangular form:
a. $42 \angle 0.15°$
b. $2002 \angle -60°$
c. $0.006 \angle -120°$
d. $8 \times 10^{-3} \angle -220°$
e. $15 \angle +180°$
f. $1.2 \angle -89.9°$

SECTION 14.9 Mathematical Operations with Complex Numbers

41. Perform the following additions in rectangular form:
a. $(4.8 + j7.8) + (4.6 + j0.6)$
b. $(242 + j7) + (3.8 + j44) + (0.4 + j0.7)$
c. $(5 \times 10^{-6} + j75) + (7.4 \times 10^{-7} - j9)$

42. Perform the following subtractions in rectangular form:
a. $(8.8 + j6.2) - (5.6 + j5.6)$
b. $(197 + j243) - (-42.3 - j58)$
c. $(-36.0 + j70) - (-5 - j6) + (10.5 - j72)$

43. Perform the following operations with polar numbers, and leave the answer in polar form:
a. $6 \angle 20° + 8 \angle 80°$
b. $42 \angle 45° + 62 \angle 60° - 70 \angle 120°$
c. $20 \angle -120° - 10 \angle -150° + 8 \angle -210° + 8 \angle +240°$

44. Perform the following multiplications in rectangular form:
a. $(2 + j3)(6 + j8)$
b. $(7.8 + j1)(4 + j2)(7 + j6)$
c. $(400 - j200)(-0.01 - j0.5)(-1 + j3)$

45. Perform the following multiplications in polar form:
a. $(2 \angle 60°)(4 \angle -40°)$
b. $(6.9 \angle 8°)(7.2 \angle -72°)$
c. $(0.002 \angle 120°)(0.5 \angle 200°)(40 \angle +80°)$

46. Perform the following divisions in polar form:
a. $(42 \angle 10°)/(7 \angle 60°)$
b. $(0.006 \angle 120°)/(30 \angle +60°)$
c. $(4360 \angle -20°)/(40 \angle -210°)$

47. Perform the following divisions, and leave the answer in rectangular form:
a. $(8 + j8)/(2 + j2)$
b. $(8 + j42)/(-6 - j4)$
c. $(-4.5 - j6)/(0.1 - j0.8)$

***48.** Perform the following operations, and express your answer in rectangular form:
a. $\dfrac{(4 + j3) + (6 - j8)}{(3 + j3) - (2 + j3)}$
b. $\dfrac{8 \angle 60°}{(2 \angle 0°) + (100 + j400)}$
c. $\dfrac{(6 \angle 20°)(120 \angle -40°)(3 + j8)}{2 \angle -30°}$

***49.** Perform the following operations, and express your answer in polar form:
a. $\dfrac{(0.4 \angle 60°)^2(300 \angle 40°)}{3 + j9}$
b. $\left(\dfrac{1}{(0.02 \angle 10°)^2}\right)\left(\dfrac{2}{j}\right)^3\left(\dfrac{1}{6^2 - j\sqrt{900}}\right)$

***50.** a. Determine a solution for x and y if
$$(x + j5) + (3x + jy) - j6 = 16 \angle 0°$$
b. Determine x if
$$(18 \angle 20°)(x \angle -60°) = 38.64 - j25.72$$

***51.** a. Determine a solution for x and y if
$$(5x + j10)(2 - jy) = 90 - j70$$
b. Determine θ if
$$\frac{80 \angle 0°}{20 \angle \theta} = 3.464 - j2$$

SECTION 14.11 Phasors

52. Express the following in phasor form:
a. $\sqrt{2}(180)\sin(\omega t + 40°)$
b. $\sqrt{2}(25 \times 10^{-3})\sin(157t - 60°)$
c. $300 \sin(\omega t - 120°)$

***53.** Express the following in phasor form:
a. $30 \sin(377t - 180°)$
b. $6 \times 10^{-6} \cos \omega t$
c. $5.6 \times 10^{-6} \cos(754t - 40°)$

54. Express the following phasor currents and voltages as sine waves if the frequency is 60 Hz:
a. $\mathbf{I} = 40 \text{ A} \angle 20°$
b. $\mathbf{V} = 120 \text{ V} \angle 10°$
c. $\mathbf{I} = 8 \times 10^{-3} \text{A} \angle -110°$
d. $V = \dfrac{6000}{\sqrt{2}} \text{V} \angle -180°$

55. For the system in Fig. 14.82, find the sinusoidal expression for the unknown voltage v_a if
$$e_{in} = 60 \sin(377t + 90°)$$
$$v_b = 20 \sin(377t - 45°)$$

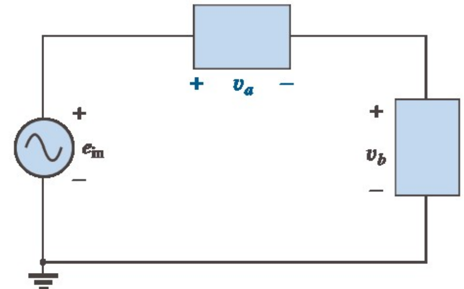

FIG. 14.82
Problem 55.

56. For the system in Fig. 14.83, find the sinusoidal expression for the unknown current i_1 if

$$i_s = 30 \times 10^{-6} \sin(\omega t + 80°)$$
$$i_2 = 4 \times 10^{-6} \sin(\omega t - 50°)$$

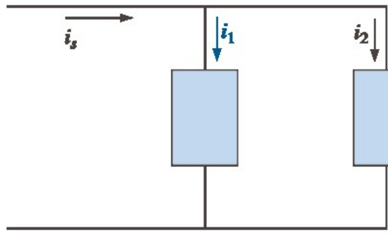

FIG. 14.83
Problem 56.

57. Find the sinusoidal expression for the voltage v_a for the system in Fig. 14.84 if

$$e_{\text{in}} = 120 \sin(\omega t + 30°)$$
$$v_b = 30 \sin(\omega t + 60°)$$
$$v_c = 40 \sin(\omega t - 90°)$$

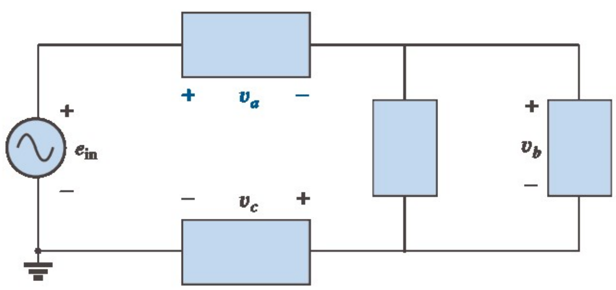

FIG. 14.84
Problem 57.

*58. Find the sinusoidal expression for the current i_1 for the system in Fig. 14.85 if

$$i_s = 18 \times 10^{-3} \sin(377t + 180°)$$
$$i_2 = 8 \times 10^{-3} \sin(377t + 90°)$$
$$i_3 = 2i_2$$

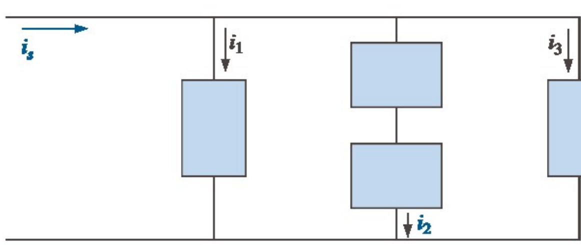

FIG. 14.85
Problem 58.

SECTION 14.12 Computer Analysis

PSpice or Multisim

59. Plot i_c and v_c versus time for the network in Fig. 14.74 for two cycles if the frequency is 0.2 kHz.

60. Plot the magnitude and phase angle of the current i_c versus frequency (100 Hz to 100 kHz) for the network in Fig. 14.74.

*61. Plot the total impedance of the configuration in Fig. 14.24(a) versus frequency (100 kHz to 100 MHz) for the following parameter values: $C = 0.1\ \mu\text{F}$, $L_s = 0.2\ \mu\text{H}$, $R_s = 2\text{M}\Omega$, and $R_p = 100\ \text{M}\Omega$. For what frequency range is the capacitor "capacitive"?

GLOSSARY

Average or real power The power delivered to and dissipated by the load over a full cycle.

Complex conjugate A complex number defined by simply changing the sign of an imaginary component of a complex number in the rectangular form.

Complex number A number that represents a point in a two-dimensional plane located with reference to two distinct axes. It defines a vector drawn from the origin to that point.

Derivative The instantaneous rate of change of a function with respect to time or another variable.

Leading and lagging power factors An indication of whether a network is primarily capacitive or inductive in nature. Leading power factors are associated with capacitive networks and lagging power factors with inductive networks.

Phasor A radius vector that has a constant magnitude at a fixed angle from the positive real axis and that represents a sinusoidal voltage or current in the vector domain.

Phasor diagram A "snapshot" of the phasors that represent a number of sinusoidal waveforms at $t = 0$.

Polar form A method of defining a point in a complex plane that includes a single magnitude to represent the distance from the origin and an angle to reflect the counterclockwise distance from the positive real axis.

Power factor (F_p) An indication of how reactive or resistive an electrical system is. The higher the power factor, the greater is the resistive component.

Reactance The opposition of an inductor or a capacitor to the flow of charge that results in the continual exchange of energy between the circuit and magnetic field of an inductor or the electric field of a capacitor.

Reciprocal A format defined by 1 divided by the complex number.

Rectangular form A method of defining a point in a complex plane that includes the magnitude of the real component and the magnitude of the imaginary component, the latter component being defined by an associated letter j.