

6.7 DERIVATIVES AND INTEGRALS INVOLVING INVERSE TRIGONOMETRIC FUNCTIONS

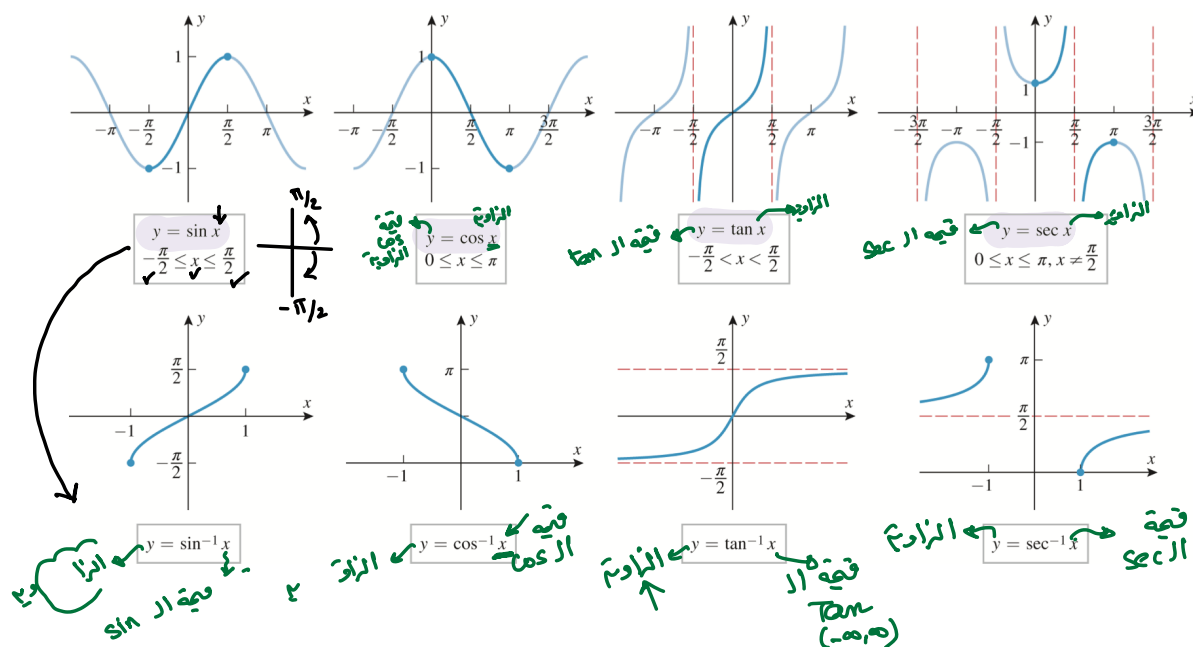


Table 6.7.1 summarizes the basic properties of the inverse trigonometric functions we have considered. You should confirm that the domains and ranges listed in this table are consistent with the graphs shown in Figure 6.7.1.

Table 6.7.1
PROPERTIES OF INVERSE TRIGONOMETRIC FUNCTIONS

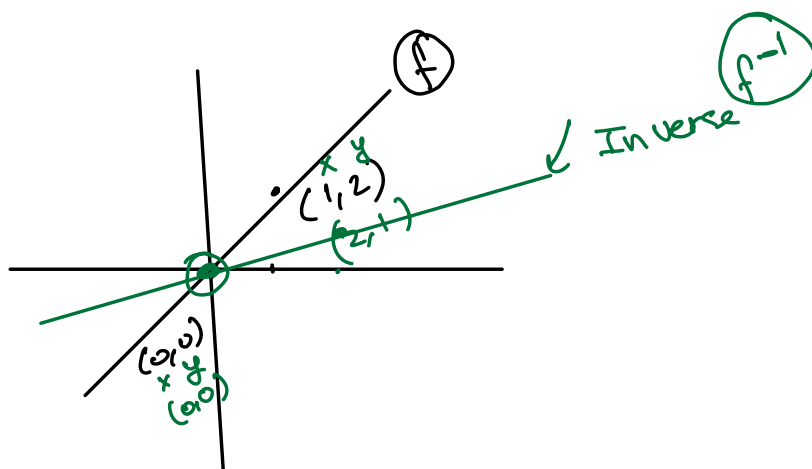
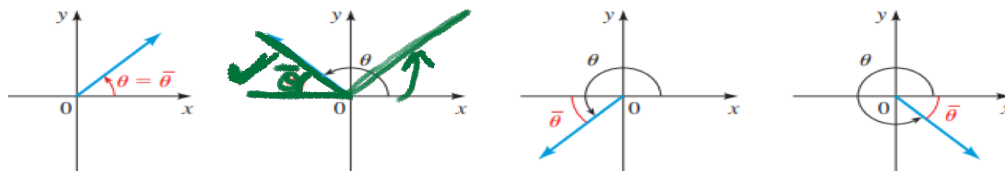
FUNCTION	DOMAIN	RANGE	BASIC RELATIONSHIPS
$\sin^{-1} x$	$[-1, 1]$	$y \in [-\pi/2, \pi/2]$	$\sin^{-1}(\sin x) = x$ if $-\pi/2 \leq x \leq \pi/2$ $\sin(\sin^{-1} x) = x$ if $-1 \leq x \leq 1$
$\cos^{-1} x$	$[-1, 1]$	$y \in [0, \pi]$	$\cos^{-1}(\cos x) = x$ if $0 \leq x \leq \pi$ $\cos(\cos^{-1} x) = x$ if $-1 \leq x \leq 1$
$\tan^{-1} x$	$(-\infty, +\infty)$	$(-\pi/2, \pi/2)$	$\tan^{-1}(\tan x) = x$ if $-\pi/2 < x < \pi/2$ $\tan(\tan^{-1} x) = x$ if $-\infty < x < +\infty$
$\sec^{-1} x$	$(-\infty, -1] \cup [1, +\infty)$	$[0, \pi/2) \cup (\pi/2, \pi]$	$\sec^{-1}(\sec x) = x$ if $0 \leq x \leq \pi, x \neq \pi/2$ $\sec(\sec^{-1} x) = x$ if $ x \geq 1$

Revision
g.4
Math 2

REFERENCE ANGLE

Let θ be an angle in standard position. The **reference angle** $\bar{\theta}$ associated with θ is the acute angle formed by the terminal side of θ and the x -axis.

Figure 6 shows that to find a reference angle $\bar{\theta}$, it's useful to know the quadrant in which the terminal side of the angle θ lies.



15-16 Use a calculating utility to approximate the solution of each equation. Where radians are used, express your answer to four decimal places, and where degrees are used, express it to the nearest tenth of a degree. [Note: In each part, the solution is not in the range of the relevant inverse trigonometric function.] Math21

15. (a) $\sin x = 0.37$, $\pi/2 < x < \pi$ → radian
 (b) $\sin \theta = -0.61$, $180^\circ < \theta < 270^\circ$ → Degree

15. (a) $\sin x = 0.37$, $\pi/2 < x < \pi$

$$x = \sin^{-1}(0.37)$$

Calculator

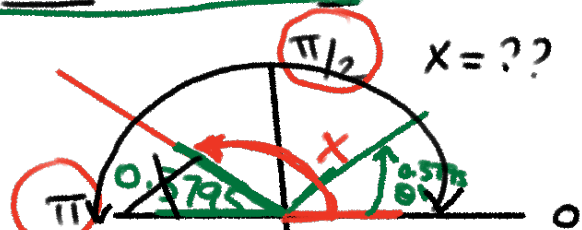
Shift Sin (0.37) → rad

المزاوية $x = 0.3795$ radian
 reference angle.

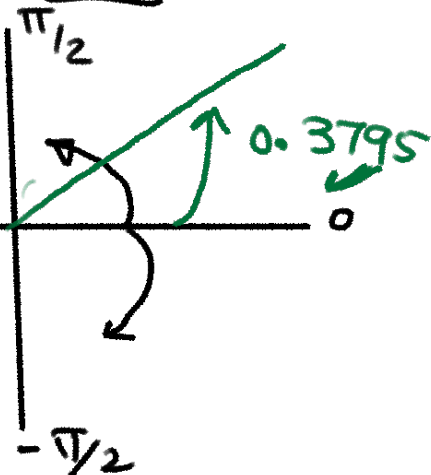
$$x = \pi - 0.3795$$

$$= \frac{22}{7} - 0.3795$$

$x \approx 2.7621$ radian



Mode $\frac{3\pi}{2}$
 rad ✓



(b) $\sin \theta = -0.61$, $180^\circ < \theta < 270^\circ$

$$\theta' = \sin^{-1}(-0.61)$$

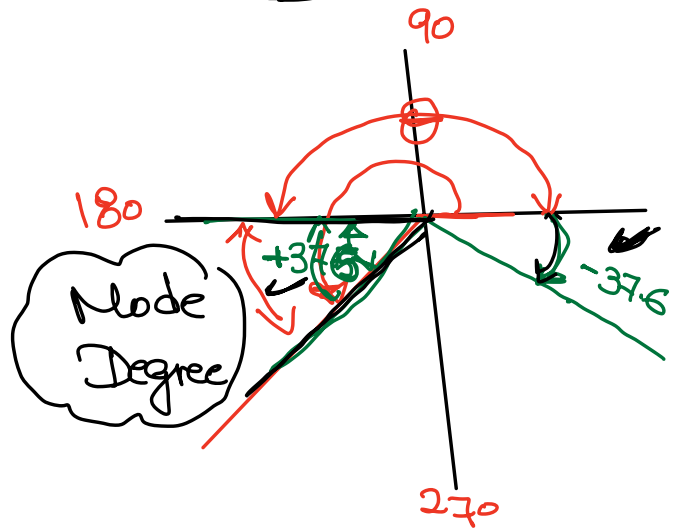
Calculator

Shift $\sin(-0.61)$

$$\theta' = -37.6^\circ$$

reference angle

$$\begin{aligned}\theta &= 180 + 37.6 \\ &= 217.6^\circ\end{aligned}$$



16. (a) $\cos x = -0.85$, $\pi < x < 3\pi/2$ radian

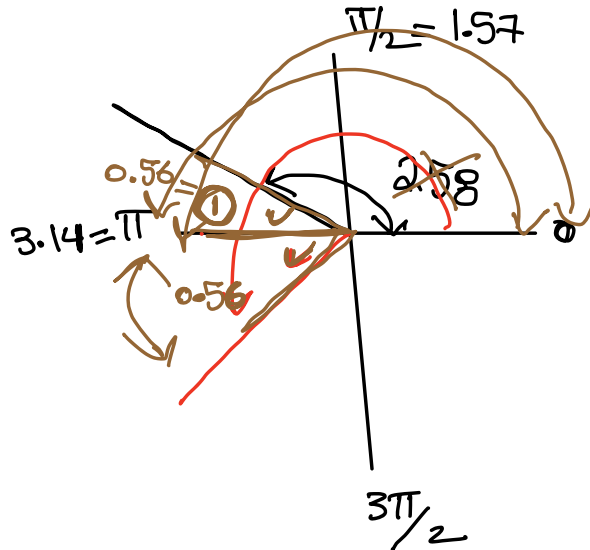
$x = \cos^{-1}(-0.85)$

Mode rad

$x = 2.58 \dots \text{rad}$

Shift $\cos(-0.85)$

$x' = \pi - 2.58$
 $= 0.56$



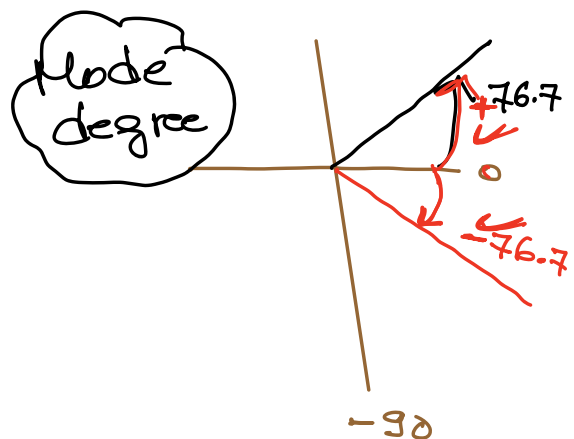
الحل $x = \pi + 0.56$

$x \approx 3.7 \text{ radian}$

(b) $\cos \theta = 0.23$, $-90^\circ < \theta < 0^\circ$

$\theta = \cos^{-1}(0.23)$
 $= 76.7^\circ$ ✓

$\theta' = -76.7^\circ$ ✓



19-20 Find the limits. ■

19. $\lim_{x \rightarrow +\infty} \sin^{-1} \left(\frac{x}{1-2x} \right) \rightarrow \lim_{x \rightarrow +\infty} \left(\frac{x}{1-2x} \right)$

$\frac{\infty}{1-2\infty} = \frac{\infty}{-\infty}$

Intermediate
forte

نیم فتحہ کا اردو فی لے، (فقا)
عی اکبر ای فی (فقا)

$$\lim_{x \rightarrow +\infty} \left(\frac{\cancel{x}/\cancel{x}}{\cancel{x} - 2\cancel{x}} \right) = \lim_{x \rightarrow \infty} \left(\frac{1}{\frac{1}{x} - 2} \right)$$

$\boxed{x = \infty}$

$$= \frac{1}{\frac{1}{\infty} - 2} = \frac{1}{0 - 2} = -\frac{1}{2}$$

$$\lim_{x \rightarrow +\infty} \sin^{-1} \left(-\frac{1}{2} \right) = -30^\circ$$

↑ calculator shift $\sin(-1/2) = -30^\circ$

20. $\lim_{x \rightarrow +\infty} \cos(2 \tan^{-1} x) = \lim_{x \rightarrow +\infty} \cos 2\alpha$

Let $\alpha = \tan^{-1}(x)$

$\alpha = \tan^{-1} \infty$

$\tan \alpha = \infty$

$\infty = \tan$ زاوية التانجنت التي لا نهاية لها

$\frac{\infty}{0} = \infty$ undefined.

$\alpha = 90^\circ$

$\lim_{x \rightarrow +\infty} \cos(2 \tan^{-1} x) = \lim_{x \rightarrow +\infty} \cos 2\alpha = \lim_{x \rightarrow \infty} \cos(2 \cdot 90)$

$= \lim_{x \rightarrow \infty} \cos 180 = \cos 180^\circ = \underline{\underline{-1}}$

Math 2,

θ in degrees	θ in radians	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\cot \theta$
0	0	0	1	0	—	1	—
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	1	0	—	1	—	0