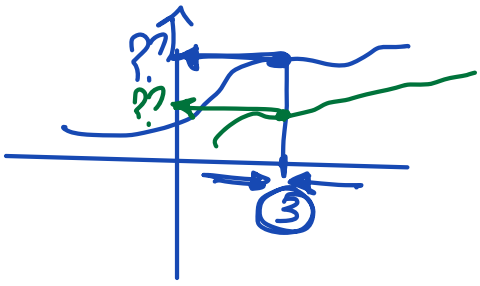
Kingdom of Saudi Arabia
Council of Universities' Affairs
Al-Fayha College

المملكة العربية السعودية
مجلس شؤون الجامعات
كلية الفيحاء

Work Sheet 1 for Math 101



PART I:

1. If $\lim_{x \to 3} f(x) = 1$, $\lim_{x \to 3} g(x) = -1$, then

$$\lim_{x \to 3} [3f(x) - 2g(x)] =$$

$3 \cdot 1 - 2 \cdot (-1)$

$3 + 2 = 5$

a. -4

b. 4

c. 0

d. -5

✓ e. 5

2. The slope of the tangent line to the graph of the function $f(x) = x^2$ at $x = 2$, is

معدل الخط المماس للجراف

$f(x) = x^2 \rightarrow x = 2 \rightarrow m_{tan} = ??$

$m_{tan} = f'(x) = 2x^{2-1} = 2x$ ← derivative مشتقة

$m_{tan} = f'(x)|_{x=2} = 2 \cdot 2 = 4$

a. 2

b. -4

c. 3

d. -2

✓ e. 4

$-2, 1$

3. The vertical asymptote(s) of f(x) = $(x^2 - x - 2)/(x^2 - 4)$ is (are)

(a) x = 2

(b) x = 2, x = -2

(c) x = -2

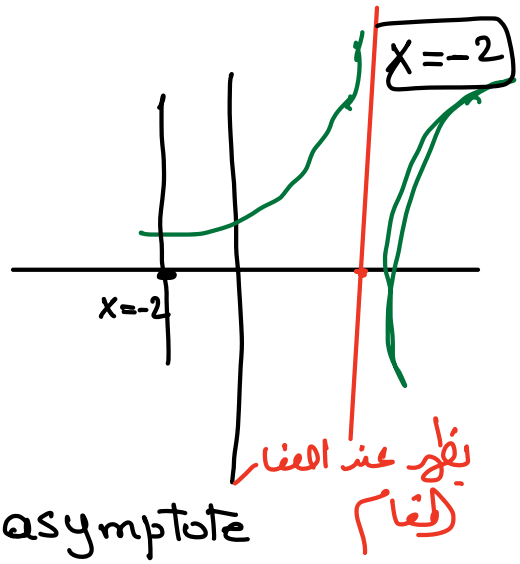
(d) y = 2

بسط مقام

$f(x) = \frac{(x+1)(x-2)}{(x+2)(x-2)}$

$x + 2 = 0$

$x = -2$ ← v. asymptote



يظهر عند الصفر المقام

معادلة المماس

$m_{tan} = f'(x)\big|_{x=1}$

$f(x) = x^4 - 3x^3 + 2x^2$

$f'(x) = 4x^3 - 3.3x^2 + 2.2x^1$

$f'(x) = 4x^3 - 9x^2 + 4x \longrightarrow m_{tan} = f'(x)\big|_{x=1} = 4(1)^3 - 9(1)^2 + 4(1) = \checkmark$

$f(x) = 2x^3 + 3x^2$

$f'(x) = 2.3x^2 + 3.2x^1 = 6x^2 + 6x$

$m_{tan} = f'(x)\big|_{x=1} = 6.1^2 + 6.1 = 12$

$f(x) = 5x^2 + 1$

اى ثوابت مشتقه = صفر

$f'(x) = 5.2x^1 + 0$

$f'(x) = 10x$

$f(x) = 6x^3 - x^1 + 5$

$f'(x) = 6.3x^2 - 1x^0 + 0$

$1 \quad 1$

$f(x) = x^{(1)}$

$f'(x) = 1$

$f(x) = 3x$

$f'(x) = 3(1) = 3$

$f(x) = \frac{x^2 - x - 2}{x^2 - 9} = \frac{(x-2)(x+1)}{(x+3)(x-3)}$

V. A → $(x+3)(x-3) = 0$

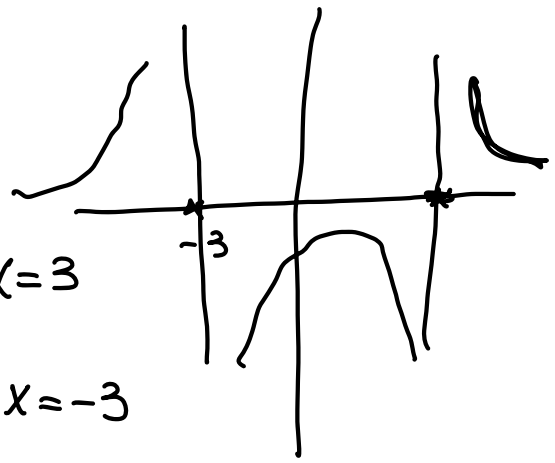
$x + 3 = 0$

$x = -3$

$x - 3 = 0$

$x = 3$

2. V. A → $x = 3$, $x = -3$



from math ① → slope $= \frac{y_2 - y_1}{x_2 - x_1}$

(x_1, y_1) (x_2, y_2)

from calculus 1 → $m_{tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

4. The slope of the tangent line to the curve $y = e^x + x$ at the point $(0,1)$ is given by

(a) $\lim_{x \to 0} \frac{e^x + x - 1}{x - 1}$

(b) $\lim_{x \to 0} \frac{e^x + x}{x - 1}$

(c) $\lim_{x \to 0} \frac{e^x + x - 1}{x}$

(d) $\lim_{x \to 0} \frac{e^x + x + 1}{x}$

(e) $\lim_{x \to 0} \frac{e^x + x}{x}$

$m_{tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \lim_{x \to 0} \frac{y - 1}{x - 0}$

$m_{tan} = \lim_{x \to 0} \frac{e^x + x - 1}{x}$

5. **The slope of the tangent line to the curve**

$y = 505\,x^2 + 1$

at the point Q (2,2021)

(a) 2010

(b) 2020

(c) 2030

(d) 2000

(e) 2017

$m_{tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$= \lim_{x \to 2} \frac{y - 2021}{x - 2} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \lim_{x \to 2} \frac{505x^2 + 1 - 2021}{x - 2}$

$= \lim_{x \to 2} \frac{505x^2 - 2020}{x - 2}$

$= \lim_{x \to 2} \frac{505[x^2 - 4]}{x - 2}$ $= \frac{(2)^2}{2-2}$ 0

$= \lim_{x \to 2} \frac{505(x-2)(x+2)}{x-2}$ $= \frac{0}{0}$ → Intermediate form

$= \lim_{x \to 2} 505(x+2) = 505(2+2) = 2020$

6. Evaluate

$\lim_{x \to 2} \frac{x^2 - 1x - 2}{x - 2} = \frac{2^2 - 2 - 2}{2-2} = \frac{0}{0}$ (Intermediate form)

(-2 × +1)

a) 3 ✓

b) − 4

c) − 3

d) 0

e) 4

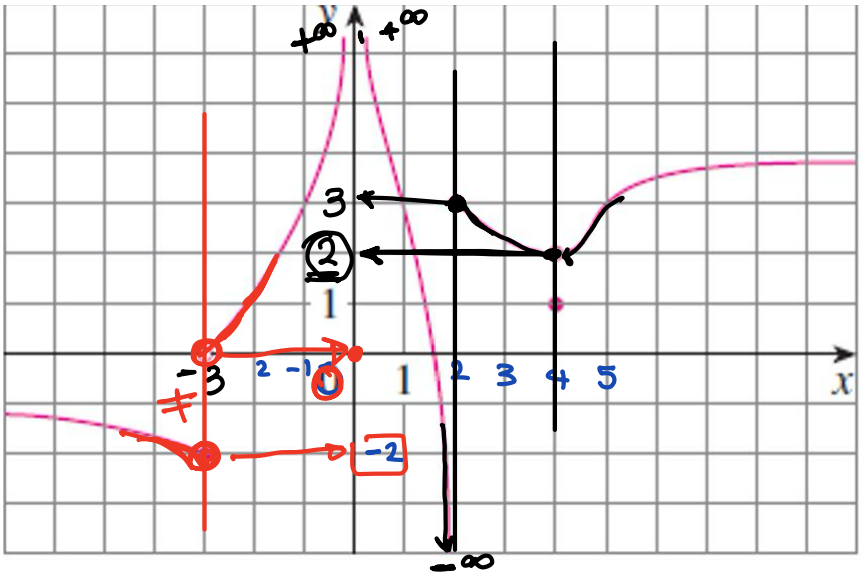
$\lim_{x \to 2} \frac{(x-2)(x+1)}{x-2}$

$= \lim_{x \to 2} x+1 = 2+1 = 3$

PART II:

7. Problem #1

For the graph of the function f below, find.



$\lim_{x \to -3} f(x)$ D.N.E

$\lim f(x)$ $x \to -3^+$ $= 0$ $\neq$ $\lim f(x)$ $x \to -3^-$ $= -2$

$\lim f(x) = +\infty$ $(x \to 0)$

$\lim f(x)$ $x \to 0^+$ $= +\infty$ $\lim f(x)$ $x \to 0^-$ $= +\infty$

$\lim f(x)$ D.N.E $(x \to 2)$

$\lim f(x)$ $x \to 2^+ = 3$ $\neq$ $\lim f(x)$ $x \to 2^- = -\infty$

$\lim f(x) = 2$ $(x \to 4)$

$\lim f(x)$ $x \to 4^+ = 2$ $\lim f(x)$ $x \to 4^- = 2$

8. Problem #2 Apply the operations (add-subtract-multiplication-division)

جمع طرح ضرب قسمة

Let's take:

$f(x) = x + 1$, $g(x) = x^2$

We want the limits as $x \to 1$.

Solution

$\lim_{x \to 1}$ $f+g$

$f-g$

$f.g$

f/g

Step 2: Apply operations

1. **Addition**

$$\lim_{x\to 1} f+g = \lim_{x\to 1} (x+1)+x^2 = 1+1+1^2 = 3$$

2. **Subtraction**

$$\lim_{x\to 1} f-g = \lim_{x\to 1} x+1-x^2 = 1+\cancel{1}-\cancel{1^2} = 1$$

3. **Multiplication**

$$\lim_{x\to 1} f\cdot g = \lim_{x\to 1} (x+1)(x^2) = (1+1)(1)^2 = 2$$

4. **Division**

$$\lim_{x\to 1} f/g = \lim_{x\to 1} \frac{(x+1)}{x^2} = \frac{1+1}{1^2} = \frac{2}{1} = 2$$

5. **Power** 2// $\rightarrow f(x)^2$

$$\lim_{x\to 1} (x+1)^2 = (1+1)^2 = 2^2 = 4$$

6. **Root** $\sqrt{g(x)}$

$$\lim_{x\to 1} \sqrt{x^2} = \sqrt{1^2} = \sqrt{1} = 1$$

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9. **Problem #3**

Estimate the value of $\lim_{t\to 0} \dfrac{\sqrt{t^2+9}-3}{t^2}$. $= \dfrac{\sqrt{0^2+9}-3}{0^2} = \dfrac{0}{0}$

Intermediate form

في مسائل الحدود والكسور
يتم البدء في كل مسألة بالتعويض والقيام بالحل

$$\lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2} \cdot * \frac{\sqrt{t^2+9}+3}{\sqrt{t^2+9}+3}$$

$$= \lim_{t\to 0} \frac{(\sqrt{t^2+9})^2-(3)^2}{t^2(\sqrt{t^2+9}+3)} = \lim_{t\to 0} \frac{t^2+9-9}{t^2(\sqrt{t^2+9}+3)}$$

$$= \lim_{t\to 0} \frac{t^2/t^2}{t^2/t^2(\sqrt{t^2+9}+3)} = \lim_{t\to 0} \frac{1}{\sqrt{t^2+9}+3}$$

$$= \frac{1}{\sqrt{0^2+9}+3} = \frac{1}{3+3} = \frac{1}{6}$$