Function الدالة

(input) ⟶ (output)

Ex : $f(x) = 2x + 3$

If $x = 3$ (input)

$f(3) = 2(3) + 3$

$= 6 + 3$

$= (9) \longrightarrow$ output

Function

every input has only one output

Input → domain → x

output → Range → y

EX 1 $x^2 + y = 1$ Is this a function?

If $x = 2$ → $(2)^2 + y = 1$

$4 + y = 1$

$y = 1 - 4$

$y = -3$

there is only one value of y

This equation is a function

Ex 2 @ $y^2 + x^2 = 10$

let $x = 2$

$$y^2 + (2)^2 = 10$$
$$y^2 + 4 = 10$$
$$y^2 = 10 - 4$$
$$y^2 = 6$$
$$y = \pm\sqrt{6} \quad \begin{cases} y = \sqrt{6} \\ y = -\sqrt{6} \end{cases}$$

2 solutions (not a function)

(b) $3x + y = -4$

let $x = 3$

$$3(3) + y = -4$$
$$9 + y = -4$$
$$y = -4 - 9$$
$$= -13$$

(a function)

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Ex:3 @ { (1,2), (-2,5), (8,0) } (x, y)

domain Range

1 → 2

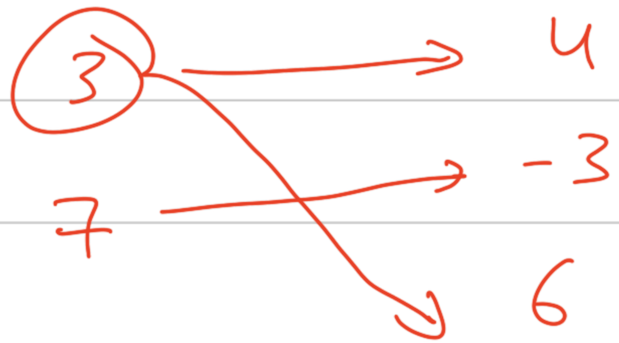
-2 → 5

8 → 0

(function)

(b) { (3,4), (7,-3), (3,6) }

domain Range



not a function

Ex-4 $g(x) = -x^2 + 3x + 17$

Find * $g(0) = -(0)^2 + 3(0) + 17 = 17$

* $g(-2) = -(-2)^2 + 3(-2) + 17$

$= -(4) - 6 + 7 = -10 + 7 = 7$

Rule

$(a+b)^2 = a^2 + 2ab + b^2$

* $g(x+h) = -(x+h)^2 + 3(x+h) + 17$

$= -(x^2 + 2xh + h^2) + 3x + 3h + 17$

$= -x^2 - 2xh - h^2 + 3x + 3h + 17$ ✓

Ex-5 Evaluate the piece-wise function.

$$f(x) = \begin{cases} x^2 + 3x + 8 & x < 0 \\ 2x^2 - 7 & x \geq 0 \end{cases}$$

when $x = -2, 0, 3$

Sol * $f(-2) = (-2)^2 + 3(-2) + 8 = 4 - 6 + 8 = 6$

* $f(0) = 2(0)^2 - 7 = -7$

* $f(3) = 2(3)^2 - 7 = 2(9) - 7 = 11$

Ex 6 Find the domain

(A) $\{(-2,0),(-1,4),(0,2),(2,2),(4,-1)\}$

Domain → $\{-2,-1,0,2,4\}$ → function.

(B) $\{(5,-2),(4,-10),(5,9)\}$ → not a function

Domain → $\{5,4\}$

(C) $f(x) = \frac{1}{x+5}$

domain is all real numbers except $x=-5$

$x+5 \neq 0$

$x \neq -5$

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Ex-7 Find the domain.

(i) $f(x) = \frac{1}{x^2-9}$

$x^2 - 9 \neq 0$

$x^2 \neq 9$

$x \neq \pm\sqrt{9}$, $x \neq \pm 3$

Domain is all real numbers except $\{-3, 3\}$

or $\longrightarrow$ $R - \{-3, 3\}$

(ii) $f(x) = \frac{1}{x^2+25}$

$x^2 + 25 \neq 0$

$x^2 \neq -25$ → There is no number for $x = \sqrt{-25}$

* Domain is all real numbers

Ex-8 Find the domain

(f) $f(x) = \sqrt{x-3}$ $\longrightarrow$ $x-3 \geq 0$

$x \geq 3$

Domain is $[3, \infty)$

(g) $F(x) = \dfrac{1}{\sqrt{x-8}}$ $\longrightarrow$ $x-8 > 0$

$x > 8$

Domain is $(8, \infty)$

Ex 9 find the domain.

(H) $f(x) = x^2 - 7x + 10$ (function is polynomial)

Domain is all real numbers $\rightarrow (-\infty, \infty)$

(i) $f(x) = |x - 5| \rightarrow$ V-shape

Domain is all real numbers $\rightarrow (-\infty, \infty)$.

* Summary

  * function is a polynomial or Absolute
    * Domain is all real numbers → $(-\infty, \infty)$

  * fraction → denominator $\neq 0$

  * square root → $\sqrt{M}$ → $M \geq 0$

* **Different Quotient** $\rightarrow \dfrac{f(x+h)-f(x)}{h} \rightarrow h \neq 0$

Ex-10 Find the Different quotient

$$f(x) = x^2 - 3$$

$(a+b)^2 = a^2 + 2ab + b^2$

$-(x^2-3)$

$-x^2+3$

* $f(x+h) = (x+h)^2 - 3 = x^2 + 2xh + h^2 - 3$

* $\dfrac{f(x+h)-f(x)}{h} = \dfrac{x^2+2xh+h^2-3-x^2+3}{h} = \dfrac{2xh+h^2}{h} =$

$$= \frac{h(2x+h)}{h} = 2x+h$$

Ex-11 Find the Quotient $f(x) = x^2 - 2x + 5$

Quotient = $\dfrac{f(x+h) - f(x)}{h}$

* $f(x+h) = (x+h)^2 - 2(x+h) + 5 = x^2 + 2xh + h^2 - 2x - 2h + 5$

* $\dfrac{f(x+h) - f(x)}{h} = \dfrac{x^2 + 2xh + h^2 - 2x - 2h + 5 - x^2 + 2x - 5}{h}$

$= \dfrac{2xh + h^2 - 2h}{h} = \dfrac{h(2x + h - 2)}{h} = \boxed{2x + h - 2}$