CHAPTER

3

Simple Resistive Circuits

CHAPTER CONTENTS

✓CHAPTER OBJECTIVES

1. Be able to recognize resistors connected in series and in parallel and use the rules for combining series-connected resistors and parallel-connected resistors to yield equivalent resistance.
2. Know how to design simple voltage-divider and current-divider circuits.
3. Be able to use voltage division and current division appropriately to solve simple circuits.
4. Be able to determine the reading of an ammeter when added to a circuit to measure current; be able to determine the reading of a voltmeter when added to a circuit to measure voltage.
5. Understand how a Wheatstone bridge is used to measure resistance.
6. Know when and how to use delta-to-wye equivalent circuits to solve simple circuits.

Our analytical toolbox now contains Ohm's law and Kirchhoff's laws. In Chapter 2 we used these tools in solving simple circuits. In this chapter we continue applying these tools, but on more-complex circuits. The greater complexity lies in a greater number of elements with more complicated interconnections. This chapter focuses on reducing such circuits into simpler, equivalent circuits. We continue to focus on relatively simple circuits for two reasons: (1) It gives us a chance to acquaint ourselves thoroughly with the laws underlying more sophisticated methods, and (2) it allows us to be introduced to some circuits that have important engineering applications.

The sources in the circuits discussed in this chapter are limited to voltage and current sources that generate either constant voltages or currents; that is, voltages and currents that are invariant with time. Constant sources are often called **dc sources**. The *dc* stands for *direct current*, a description that has a historical basis but can seem misleading now. Historically, a direct current was defined as a current produced by a constant voltage. Therefore, a constant voltage became known as a direct current, or dc, voltage. The use of *dc* for *constant* stuck, and the terms *dc current* and *dc voltage* are now universally accepted in science and engineering to mean constant current and constant voltage.

Practical Perspective

Resistive Touch Screens

Some mobile phones and tablet computers use resistive touch screens, created by applying a transparent resistive material to the glass or acrylic screens. Two screens are typically used, separated by a transparent insulating layer. The resulting touch screen can be modeled by a grid of resistors in the *x*-direction and a grid of resistors in the *y*-direction, as shown in the figure on the right.

A separate electronic circuit applies a voltage drop across the grid in the *x*-direction, between the points *a* and *b* in the circuit, then removes that voltage and applies a voltage drop across the grid in the *y*-direction (between points *c* and *d*), and continues to repeat this process. When the screen is touched, the two resistive layers are pressed together, creating a voltage that is sensed in the *x*-grid and another voltage that is sensed in the *y*-grid. These two voltages precisely locate the point where the screen was touched.

How is the voltage created by touching the screen related to the position where the screen was touched? How are the properties of the grids used to calculate the touch position? We will answer these questions in the Practical Perspective at the end of this chapter. The circuit analysis required to answer these questions uses some circuit analysis tools developed next.

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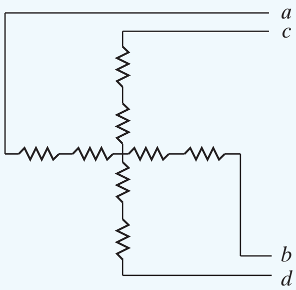

$V_s = V_{R_1} + V_{R_2} + \cdots$

$i_s = i_1 = i_2 \cdots$

$R_1 \quad R_2$

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3.1 Resistors in Series

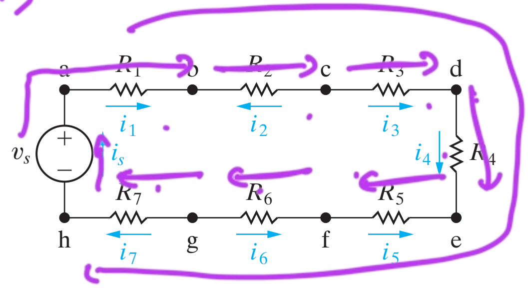


Figure 3.1 ▲ Resistors connected in series.

In Chapter 2, we said that when just two elements connect at a single node, they are said to be in series. **Series-connected circuit elements** carry the same current. The resistors in the circuit shown in Fig. 3.1 are connected in series. We can show that these resistors carry the same current by applying Kirchhoff's current law to each node in the circuit. The series interconnection in Fig. 3.1 requires that

$$i_s = i_1 = -i_2 = i_3 = i_4 = -i_5 = -i_6 = i_7, \tag{3.1}$$

which states that if we know any one of the seven currents, we know them all. Thus we can redraw Fig. 3.1 as shown in Fig. 3.2, retaining the identity of the single current i_s.

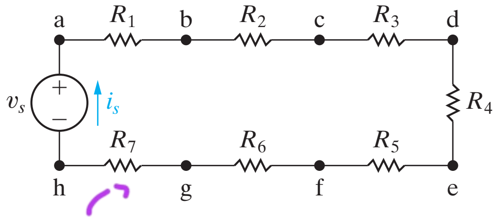


Figure 3.2 ▲ Series resistors with a single unknown current i_s.

To find i_s, we apply Kirchhoff's voltage law around the single closed loop. Defining the voltage across each resistor as a drop in the direction of i_s gives

$$-v_s + i_sR_1 + i_sR_2 + i_sR_3 + i_sR_4 + i_sR_5 + i_sR_6 + i_sR_7 = 0, \tag{3.2}$$

or

$$v_s = i_s(R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7). \tag{3.3}$$

equavalent

The significance of Eq. 3.3 for calculating i_s is that the seven resistors can be replaced by a single resistor whose numerical value is the sum of the individual resistors, that is,

$$\boxed{R_{eq} = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7} \tag{3.4}$$

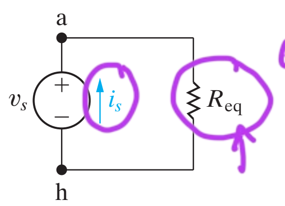


Figure 3.3 ▲ A simplified version of the circuit shown in Fig. 3.2.

and

$$\boxed{v_s = i_sR_{eq}.} \tag{3.5}$$

Thus we can redraw Fig. 3.2 as shown in Fig. 3.3.

In general, if k resistors are connected in series, the equivalent single resistor has a resistance equal to the sum of the k resistances, or

Combining resistors in series ▶

$$R_{eq} = \sum_{i=1}^{k} R_i = R_1 + R_2 + \cdots + R_k. \tag{3.6}$$

Note that the resistance of the equivalent resistor is always larger than that of the largest resistor in the series connection.

Another way to think about this concept of an equivalent resistance is to visualize the string of resistors as being inside a black box. (An electrical engineer uses the term **black box** to imply an opaque container; that is, the contents are hidden from view. The engineer is then challenged to model the contents of the box by studying the relationship between the voltage and current at its terminals.) Determining whether the box contains k resistors or a single equivalent resistor is impossible. Figure 3.4 illustrates this method of studying the circuit shown in Fig. 3.2.

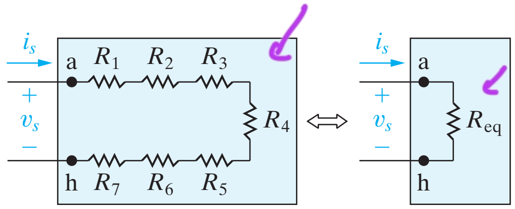


Figure 3.4 ▲ The black box equivalent of the circuit shown in Fig. 3.2.

3.2 Resistors in Parallel

$N_s = N_1 = V_2 = \dots$

When two elements connect at a single node pair, they are said to be in parallel. **Parallel-connected circuit elements** have the same voltage across their terminals. The circuit shown in Fig. 3.5 illustrates resistors connected in parallel. Don't make the mistake of assuming that two elements are parallel connected merely because they are lined up in parallel in a circuit diagram. The defining characteristic of parallel-connected elements is that they have the same voltage across their terminals. In Fig. 3.6, you can see that R_1 and R_3 are not parallel connected because, between their respective terminals, another resistor dissipates some of the voltage.

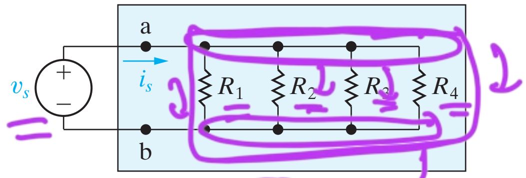


Figure 3.5 ▲ Resistors in parallel.

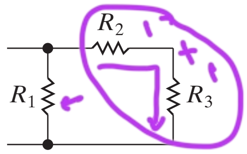


Figure 3.6 ▲ Nonparallel resistors.

Resistors in parallel can be reduced to a single equivalent resistor using Kirchhoff's current law and Ohm's law, as we now demonstrate. In the circuit shown in Fig. 3.5, we let the currents i_1, i_2, i_3, and i_4 be the currents in the resistors R_1 through R_4, respectively. We also let the positive reference direction for each resistor current be down through the resistor, that is, from node a to node b. From Kirchhoff's current law,

$$\boxed{i_s = i_1 + i_2 + i_3 + i_4.} \tag{3.7}$$

The parallel connection of the resistors means that the voltage across each resistor must be the same. Hence, from Ohm's law,

$$\boxed{i_1R_1 = i_2R_2 = i_3R_3 = i_4R_4 = v_s.} \tag{3.8}$$

$V_1 = V_2 = V_3 = V_4$

Parallel

Therefore,

$$i_1 = \frac{v_s}{R_1},$$

$$i_2 = \frac{v_s}{R_2},$$

$$i_3 = \frac{v_s}{R_3}, \quad \text{and}$$

$$i_4 = \frac{v_s}{R_4}. \tag{3.9}$$

Substituting Eq. 3.9 into Eq. 3.7 yields

$$i_s = v_s\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right), \tag{3.10}$$

from which

$$\frac{i_s}{v_s} = \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}. \tag{3.11}$$

Equation 3.11 is what we set out to show: that the four resistors in the circuit shown in Fig. 3.5 can be replaced by a single equivalent resistor. The circuit shown in Fig. 3.7 illustrates the substitution. For k resistors connected in parallel, Eq. 3.11 becomes

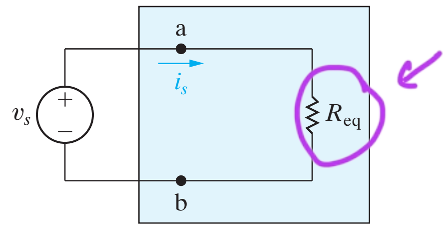


Figure 3.7 ▲ Replacing the four parallel resistors shown in Fig. 3.5 with a single equivalent resistor.

◀ **Combining resistors in parallel**

$$\frac{1}{R_{\text{eq}}} = \sum_{i=1}^{k} \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_k}. \tag{3.12}$$

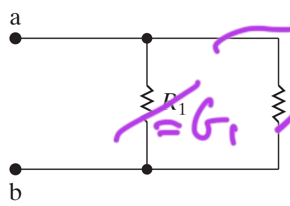
$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots}$$

Note that the resistance of the equivalent resistor is always smaller than the resistance of the smallest resistor in the parallel connection. Sometimes,

$R = \frac{1}{G}$ $G = \frac{1}{R}$

R : restance

G : Conductance



$= G_1$ $= G_2$

Figure 3.8 ▲ Two resistors connected in parallel.

using conductance when dealing with resistors connected in parallel is more convenient. In that case, Eq. 3.12 becomes

$$G_{eq} = \sum_{i=1}^{k} G_i = G_1 + G_2 + \cdots + G_k. \tag{3.13}$$

Many times only two resistors are connected in parallel. Figure 3.8 illustrates this special case. We calculate the equivalent resistance from Eq. 3.12:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2}, \tag{3.14}$$

or

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}.} \tag{3.15}$$

ضربهم

جمعهم

Thus for just two resistors in parallel the equivalent resistance equals the product of the resistances divided by the sum of the resistances. Remember that you can only use this result in the special case of just two resistors in parallel. Example 3.1 illustrates the usefulness of these results.

Example 3.1 Applying Series-Parallel Simplification

Find i_s, i_1, and i_2 in the circuit shown in Fig. 3.9.

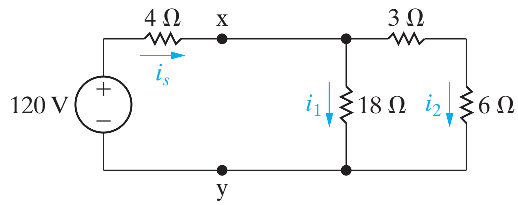


Figure 3.9 ▲ The circuit for Example 3.1.

Solution

We begin by noting that the 3 Ω resistor is in series with the 6 Ω resistor. We therefore replace this series combination with a 9 Ω resistor, reducing the circuit to the one shown in Fig. 3.10(a). We now can replace the parallel combination of the 9 Ω and 18 Ω resistors with a single resistance of (18 × 9)/(18 + 9), or 6 Ω. Figure 3.10(b) shows this further reduction of the circuit. The nodes x and y marked on all diagrams facilitate tracing through the reduction of the circuit.

From Fig. 3.10(b) you can verify that i_s equals 120/10, or 12 A. Figure 3.11 shows the result at this point in the analysis. We added the voltage v_1 to help clarify the subsequent discussion. Using Ohm's law we compute the value of v_1:

$$\boxed{v_1 = (12)(6) = 72 \text{ V}.} \tag{3.16}$$

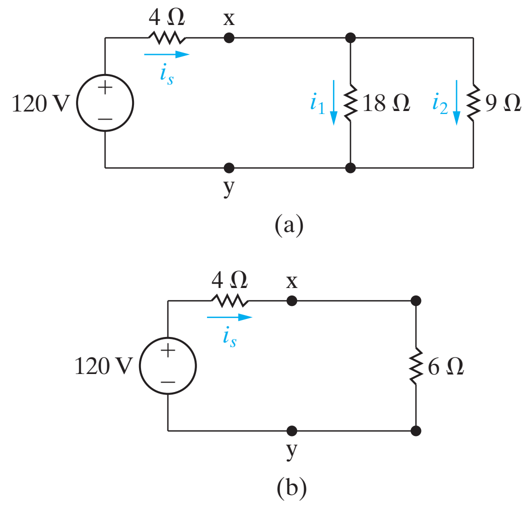


Figure 3.10 ▲ A simplification of the circuit shown in Fig. 3.9.

But v_1 is the voltage drop from node x to node y, so we can return to the circuit shown in Fig. 3.10(a) and again use Ohm's law to calculate i_1 and i_2. Thus,

$$i_1 = \frac{v_1}{18} = \frac{72}{18} = 4 \text{ A}, \tag{3.17}$$

$$i_2 = \frac{v_1}{9} = \frac{72}{9} = 8 \text{ A}. \tag{3.18}$$

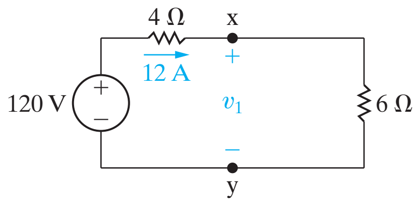


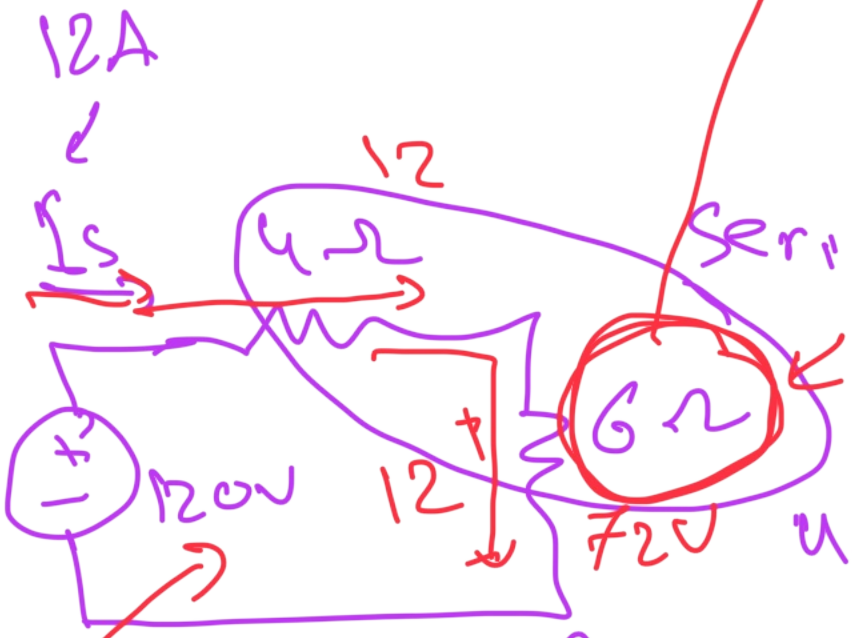
Figure 3.11 ▲ The circuit of Fig. 3.10(b) showing the numerical value of i_s.

We have found the three specified currents by using series-parallel reductions in combination with Ohm's law.

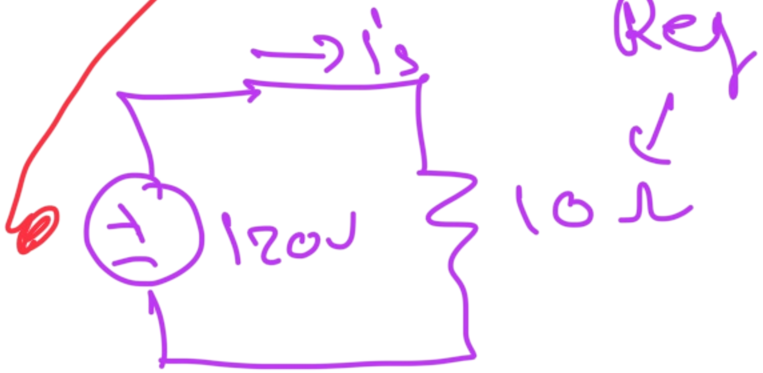
Series
9Ω

Parallel

$$R_{eq} = \frac{1}{\frac{1}{18}+\frac{1}{9}} = 6\Omega$$



$4+6 = 10\Omega$



Req

$$I_s = \frac{V_s}{R_{eq}} = \frac{120}{10}$$

$$= 12A$$

series $I_s = I_1 = I_2$

$V_s = V_1 + V_2$

---

parallel $V_s = V_1 = V_2$

$I_s = I_1 + I_2$

$V_{6\Omega} = IR = 12 \times 6 = 72V$

$I_1 = \frac{V_{18\Omega}}{R_{18\Omega}} = \frac{72}{18} = 4A$

$\Rightarrow I_2 = I_s - I_1 = 12 - 4 = \boxed{8A}$

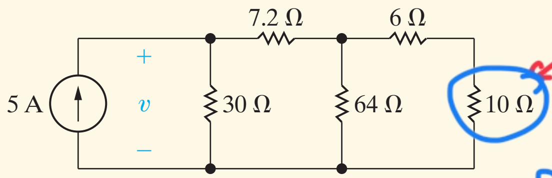
$I_2 = \frac{V_{9\Omega}}{R_{9\Omega}} = \frac{72}{9} = \boxed{8A}$

Before leaving Example 3.1, we suggest that you take the time to show that the solution satisfies Kirchhoff's current law at every node and Kirchhoff's voltage law around every closed path. (Note that there are three closed paths that can be tested.) Showing that the power delivered by the voltage source equals the total power dissipated in the resistors also is informative. (See Problems 3.1 and 3.2.)

✓ASSESSMENT PROBLEM

Objective 1—Be able to recognize resistors connected in series and in parallel

3.1 For the circuit shown, find (a) the voltage v, (b) the power delivered to the circuit by the current source, and (c) the power dissipated in the 10 Ω resistor.



Answer: (a) 60 V;
(b) 300 W;
(c) 57.6 W.

يعطي للدائرة
(-)

يستهلك
(+)
absorbed

المقاومة $P = I^2R$
$= V^2/R$

NOTE: Also try Chapter Problems 3.3–3.6.

3.3 The Voltage-Divider and Current-Divider Circuits

At times—especially in electronic circuits—developing more than one voltage level from a single voltage supply is necessary. One way of doing this is by using a **voltage-divider circuit**, such as the one in Fig. 3.12.

We analyze this circuit by directly applying Ohm's law and Kirchhoff's laws. To aid the analysis, we introduce the current i as shown in Fig. 3.12(b). From Kirchhoff's current law, R_1 and R_2 carry the same current. Applying Kirchhoff's voltage law around the closed loop yields

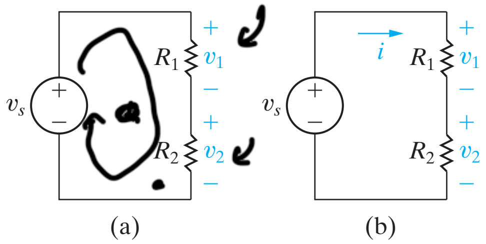


Figure 3.12 ▲ (a) A voltage-divider circuit and (b) the voltage-divider circuit with current i indicated.

series

$I_s = I_1 = I_2$

$V_s = V_1 + V_2$

Voltage divider

$$\boxed{V_{R_2} = \frac{R_2}{R_1+R_2} \times V_s}$$

$$v_s = iR_1 + iR_2, \tag{3.19}$$

or

$$i = \frac{v_s}{R_1 + R_2}. \quad \rightarrow ① \tag{3.20}$$

Now we can use Ohm's law to calculate v_1 and v_2:

$$v_1 = iR_1 = v_s\frac{R_1}{R_1 + R_2}, \quad \rightarrow ② \tag{3.21}$$

$$\boxed{v_2 = iR_2 = v_s\frac{R_2}{R_1 + R_2}.} \tag{3.22}$$

Equations 3.21 and 3.22 show that v_1 and v_2 are fractions of v_s. Each fraction is the ratio of the resistance across which the divided voltage is defined to the sum of the two resistances. Because this ratio is always less than 1.0, the divided voltages v_1 and v_2 are always less than the source voltage v_s.