The Velocity Problem

Let's consider the *velocity problem:* Find the instantaneous velocity of an object moving along a straight path at a specific time if the position of the object at any time is known. In the next example, we investigate the velocity of a falling ball. Through experiments carried out four centuries ago, Galileo discovered that the distance fallen by any freely falling body is proportional to the square of the time it has been falling. (This model for free fall neglects air resistance.) If the distance fallen after t seconds is denoted by $s(t)$ and measured in meters, then (at the earth's surface) Galileo's observation is expressed by the equation

$$s(t) = 4.9t^2$$

EXAMPLE 3 Suppose that a ball is dropped from the upper observation deck of the CN Tower in Toronto, 450 m above the ground. Find the velocity of the ball after 5 seconds.

Find velocity

$t_1 = 5 \rightarrow s(t_1)$

$t_2 = 5.1 \rightarrow s(t_2)$

$$\text{average velocity} = \frac{\text{change in position}}{\text{time elapsed}}$$

$$= \frac{s_{t_1} - s_{t_2}}{5 - 5.1}$$

$$= \frac{4.9(5)^2 - 4.9(5.1)^2}{5 - 5.1}$$

$$= 49.49 \text{ m/s}$$

$s(t) = 4.9t^2$

distance

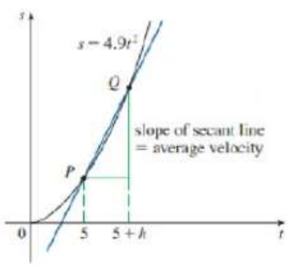
he following table shows the results of similar calculations of the average velocity over successively smaller time periods.

Time interval	Average velocity (m/s)
$5 \leq t \leq 5.1$	49.49
$5 \leq t \leq 5.05$	49.245
$5 \leq t \leq 5.01$	49.049
$5 \leq t \leq 5.001$	49.0049

$$\frac{4.9(5)^2 - 4.9(5.05)^2}{5 - 5.05}$$

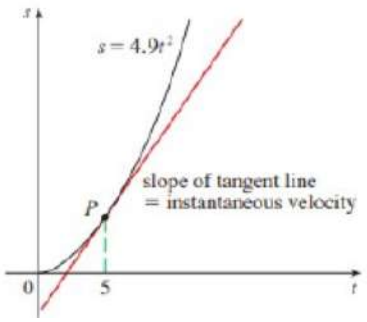
$$\frac{4.9(5)^2 - 4.9(5.01)^2}{5 - 5.01} =$$

$$\frac{4.9(5)^2 - 4.9(5.001)^2}{5.001}$$



It appears that as we shorten the time period, the average velocity is becoming closer to 49 m/s. The **instantaneous velocity** when $t = 5$ is defined to be the *limiting value* of these average velocities over shorter and shorter time periods that start at $t = 5$. Thus it appears that the (instantaneous) velocity after 5 seconds is 49 m/s. ■

متوسط سرعة الجسم عند
اللحظة الخامسة
يقترب من 49m/s



7. The table shows the position of a motorcyclist after accelerating from rest.

الزمن t (seconds)	0	1	2	3	4	5	6
المسافة s (meters)	0	1.5	6.3	14.2	24.1	38.0	53.9

(a) Find the average velocity for each time period:

(i) [2, 4] (ii) [3, 4] (iii) [4, 5] (iv) [4, 6]

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[2,4] average velocity = $\frac{\text{Change in position}}{\text{time elapsed}}$

$= \frac{6.3 - 24.1}{2 - 4} = 8.9$ m/s

[3,4] → average velocity $= \frac{14.2 - 24.1}{3 - 4} = 9.9$ m/s

[4,5] → average velocity $= \frac{24.1 - 38}{4 - 5} = 13.9$ m/s

[4,6] → average velocity $= \frac{24.1 - 53.9}{4 - 6} = 14.9$ m/s

(b) Use the graph of s as a function of t to estimate the instantaneous velocity when $t = 3$.

Velocity [2,3] $= \frac{6.3 - 14.2}{2 - 3} = 7.9$ m/s

Velocity [3,4] $= \frac{14.2 - 24.1}{3 - 4} = 9.9$ m/s

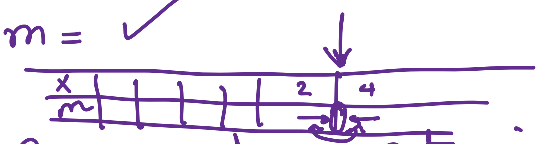
average velocity when $t = 3$ → $\frac{7.9 + 9.9}{2} = 8.9$ m/s

Today's Goal

1. $m = \text{slope} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Given 2 points $P(\checkmark_{x_1}, \checkmark_{y_1})$ $Q.(x_{x_2}, y_{y_2})$

[illegible] $m = \checkmark$



2. Guess slope at indicated point

3. Equation of tangent line   Point P Given

$$y - y_1 = m(x - x_1)$$

Velocity

$$\text{Distance} \propto t^2$$

ازاحة $S(t) = 4.9\, t^2$

$$\text{average Velocity} = \frac{\text{Change in Position}}{\text{Time elapsed}}.$$