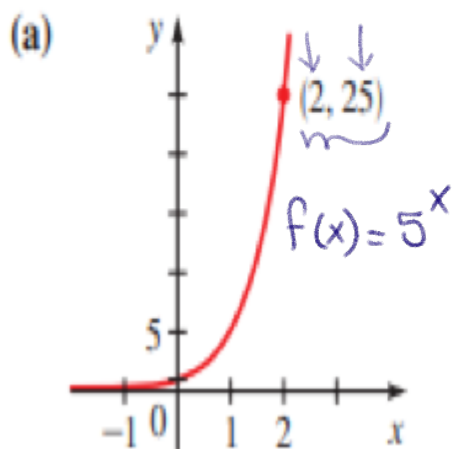


EXAMPLE 3 ■ Identifying Graphs of Exponential Functions

Find the exponential function $f(x) = a^x$ whose graph is given.



$$(-2, \frac{1}{a^2}) (-1, \frac{1}{a}) (0, 1) (1, a) (2, a^2)$$

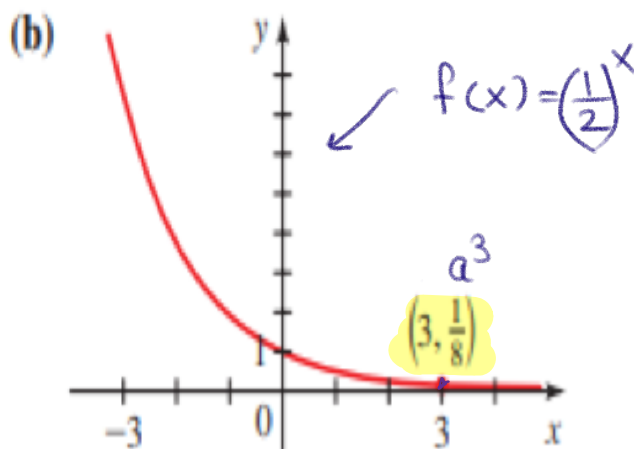
at $x=2 \rightarrow f(x)=a^2$

$$a^2 = 25$$

$$a^2 = 5^2$$

$$a = 5$$

$$f(x) = a^x = 5^x$$



$$(-2, \frac{1}{a^2}) (-1, \frac{1}{a}) (0, 1) (1, a) (2, a^2) (3, a^3)$$

at $x=3 \rightarrow f(x)=a^3$

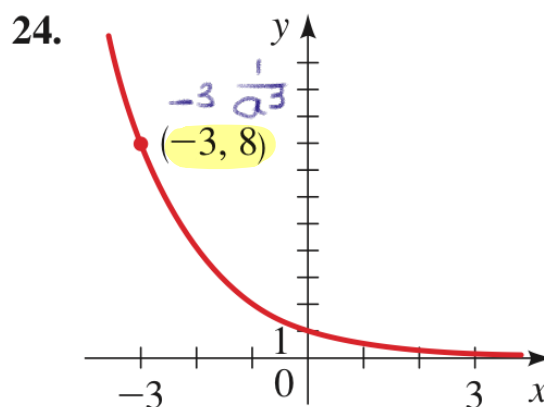
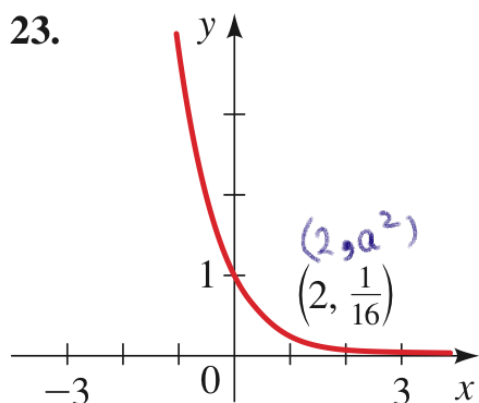
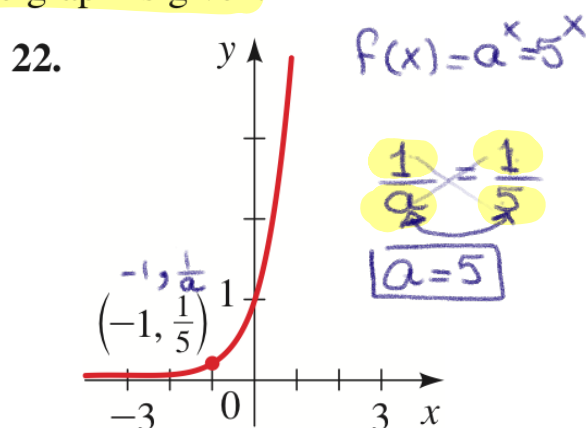
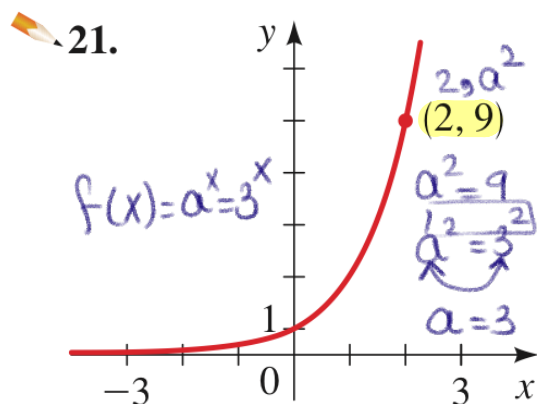
$$a^3 = \frac{1^3}{8}$$

$$= \frac{1^3}{2^3} = \left(\frac{1}{2}\right)^3$$

$$a = \frac{1}{2}$$

$$f(x) = a^x = \left(\frac{1}{2}\right)^x$$

21–24 ■ Exponential Functions from a Graph Find the exponential function $f(x) = a^x$ whose graph is given.



$$a^2 = \frac{1}{16}$$

$$a^2 = \frac{1^2}{4^2} = \left(\frac{1}{4}\right)^2$$

$$a = \frac{1}{4}$$

$$f(x) = a^x = \left(\frac{1}{4}\right)^x$$

$$\frac{1}{a^3} = 8$$

$$\frac{1}{8a^3} = \frac{1}{8}$$

$$a^3 = \frac{1}{8} = \frac{1^3}{2^3} = \left(\frac{1}{2}\right)^3$$

$$a = \frac{1}{2} \rightarrow f(x) = \left(\frac{1}{2}\right)^x$$

Translation انزياح

$$f(x) = a^x$$

Horizontal

Vertical

Left

right

up ↑

↓ down

$$F(x) = f(x \pm c)$$

بعد افرة ← x

$$F(x) = a^{x \pm c}$$

← حركة افقية

$$+ \rightarrow \text{Left}$$

$$- \rightarrow \text{right}$$

$$F(x) = a^{x+c} \rightarrow \text{Left}$$

$$F(x) = a^{x-c} \rightarrow \text{right}$$

$$F(x) = f(x) \pm c$$

← الارتفاع

$$F(x) = a^x \pm c$$

$$+ \uparrow \text{ up}$$

$$- \downarrow \text{ down}$$

$$F(x) = a^x + c \rightarrow \text{up}$$

$$F(x) = a^x - c \rightarrow \text{down}$$

$f(x) = 5^x \rightarrow$ 2 units up, down
right, left

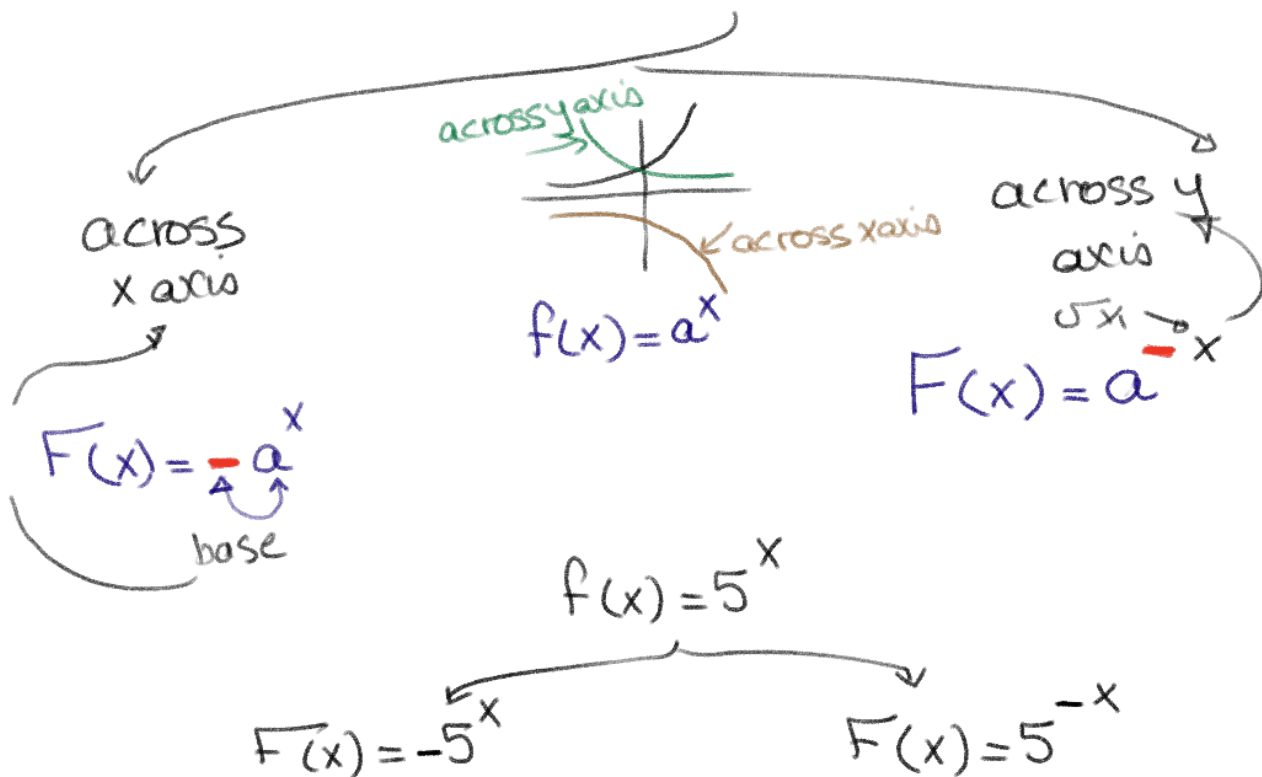
① up by 2 units $\rightarrow F(x) = 5^{x+2}$

② down by 2 units $\rightarrow F(x) = 5^{x-2}$

③ Right by 2 units $\rightarrow F(x) = 5^{x-2}$

④ Left by 2 units $\rightarrow F(x) = 5^{x+2}$

Reflection



Let $f(x) = b^x$, $b \neq 1$

1. Vertical Translation

If $F(x) = b^x + c = f(x) + c$, then graph of F is vertical translation of f c units upward.

If $F(x) = b^x - c = f(x) - c$, then graph of F is vertical translation of f c units downward.

2. Horizontal Translation

If $G(x) = b^{x+c} = f(x+c)$, then graph of G is horizontal translation of f c units to the left.

If $G(x) = b^{x-c} = f(x-c)$, then graph of G is horizontal translation of f c units to the right.

3. Reflection

If $H(x) = b^{-x} = f(-x)$, then graph of H is the graph of f reflected across the y -axis.

If $H(x) = -b^x = -f(x)$, then the graph of H is the graph of f reflected across the x -axis

EXAMPLE 4 ■ Transformations of Exponential Functions

Use the graph of $f(x) = 2^x$ to sketch the graph of each function. State the domain, range, and asymptote.

(a) $g(x) = 1 + 2^x \rightarrow 2^x + 1 \rightarrow$

Graph of $f(x) = 2^x$
 $a=2$

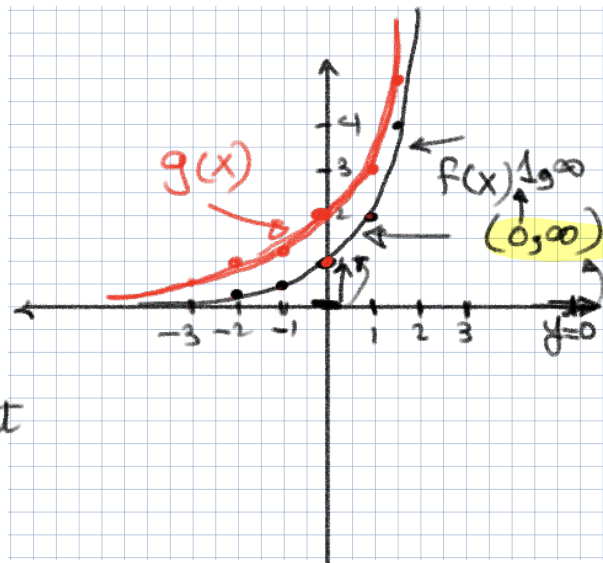
$(-2, \frac{1}{a^2}) (-1, \frac{1}{a}) (0, 1) (1, a) (2, a^2)$

$(-2, \frac{1}{4}) (-1, \frac{1}{2}) (0, 1) (1, 2) (2, 4)$

The graph of 2^x shift up by 1 unit
 Domain $\mathbb{R} \rightarrow (-\infty, \infty)$

Range $\rightarrow (1, \infty)$

H. Asymptote $\rightarrow y=1$



(b) $h(x) = -2^x$

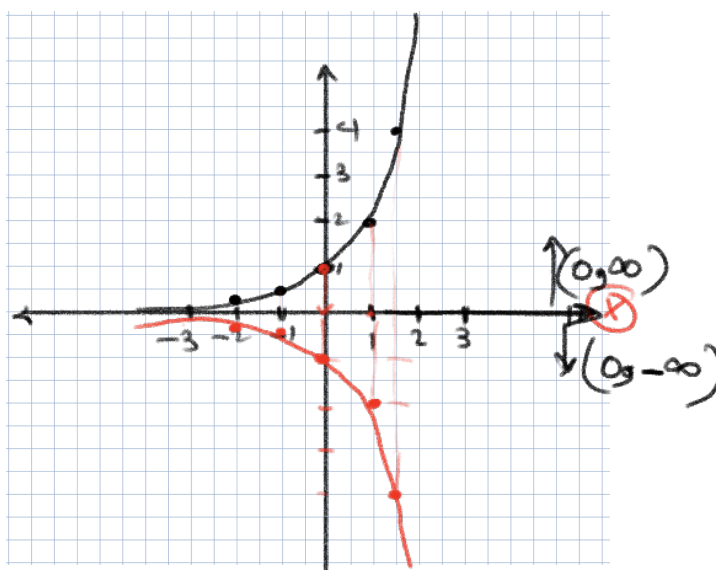
Downward

Graph of 2^x reflect across
 x axis

Domain $\rightarrow (-\infty, \infty)$

Range $\rightarrow (0, -\infty)$

H. Asymptote $y=0$





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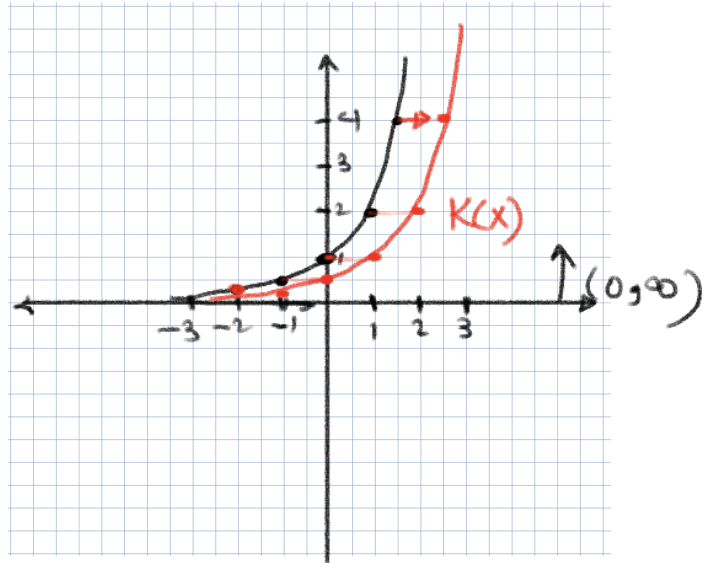
← 2^x ← 2^{x-1}
(c) $k(x) = 2^{x-1}$

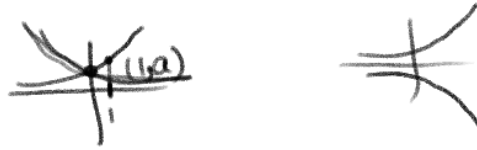
The graph of 2^x shift
to the right by 1 unit

Domain $(-\infty, \infty)$

Range $(0, \infty)$

H. Asymptote $y=0$





2. Match the exponential function with one of the graphs labeled I, II, III, or IV, shown below.

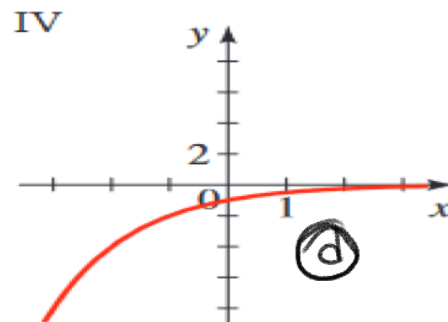
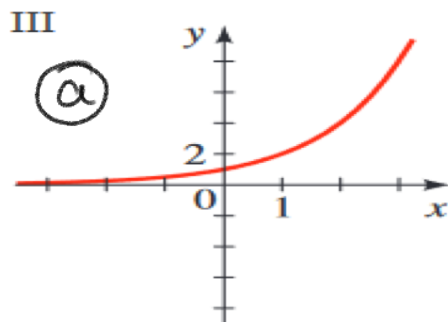
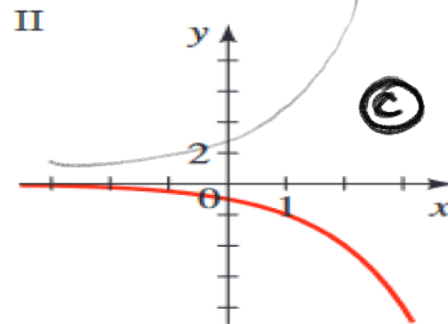
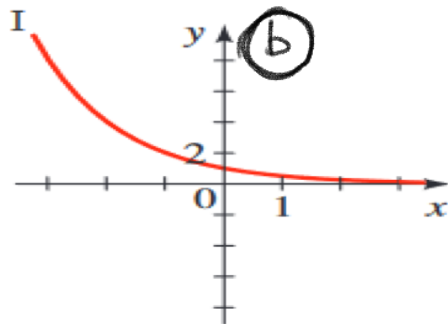
(a) $f(x) = 2^x$ (III)

(b) $f(x) = 2^{-x}$

(c) $f(x) = -2^x$

(d) $f(x) = -2^{-x}$

ایکس سے حوالہ

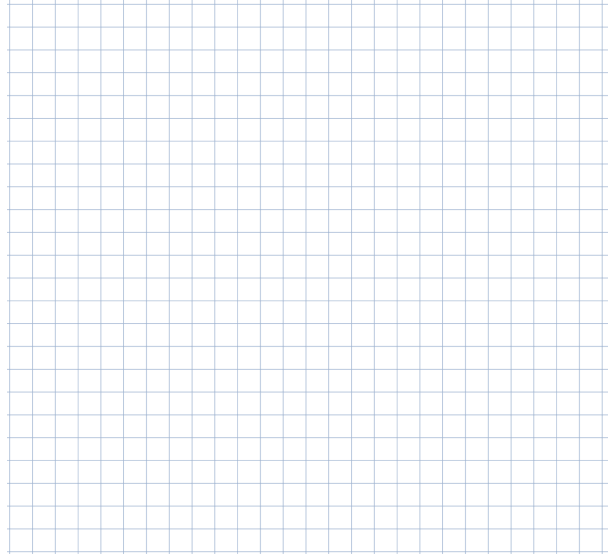


17–20 ■ Graphing Exponential Functions Graph both functions on one set of axes.

H.W

18. $f(x) = 3^{-x}$ and $g(x) = \left(\frac{1}{3}\right)^x$

الطاس الجراف
حول محور y



$$f(x) = 3^x$$

27–40 ■ Graphing Exponential Functions Graph the function, not by plotting points, but by starting from the graphs in Figure 2. State the domain, range, and asymptote.

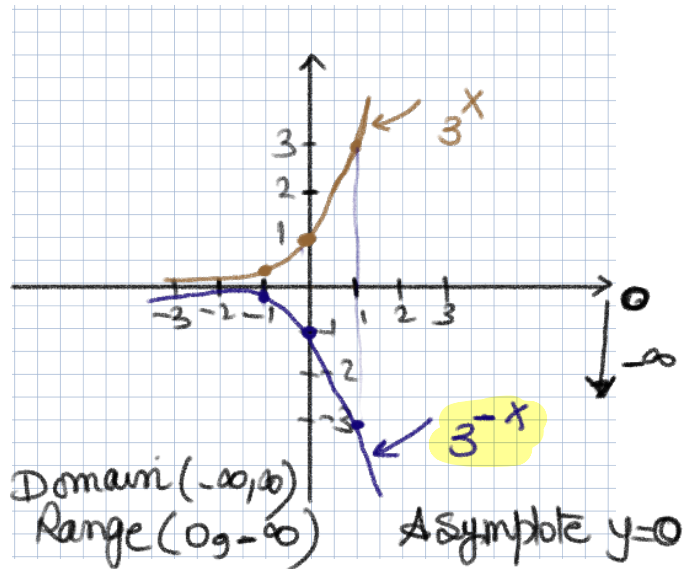
29. $f(x) = -3^x$

الكامس حو 3
(a=3)

$$(-1, \frac{1}{a}) (0, 1) (1, a)$$

$$(-1, \frac{1}{3}) (0, 1) (1, 3)$$

The graph reflect across
x axis



31. $f(x) = 10^{x+3}$
 $10^x \rightarrow a=10$

$$(-1, \frac{1}{a}) (0, 1) (1, a)$$

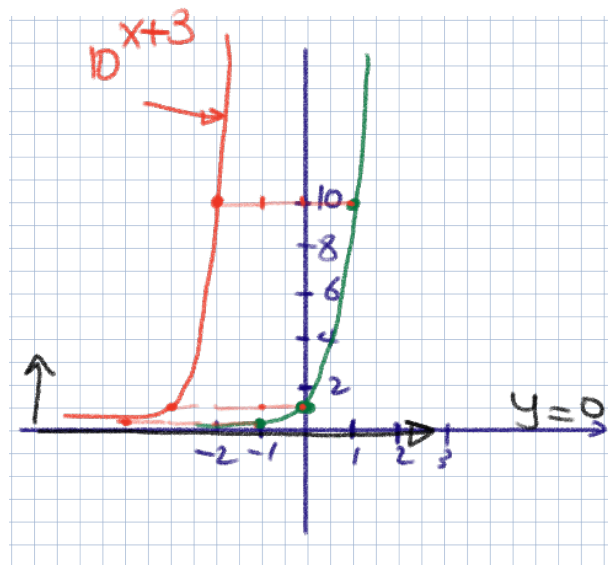
$$(-1, \frac{1}{10}) (0, 1) (1, 10)$$

The graph is shifted to the
Left by 3 units

$$\text{Domain } (-\infty, \infty)$$

$$\text{Range } (0, \infty)$$

$$\text{Asymptote } \rightarrow y=0$$



32. $g(x) = 2^{x-3}$

$2^x \rightarrow a=2$

$(-1, \frac{1}{a}) (0, 1) (1, a)$

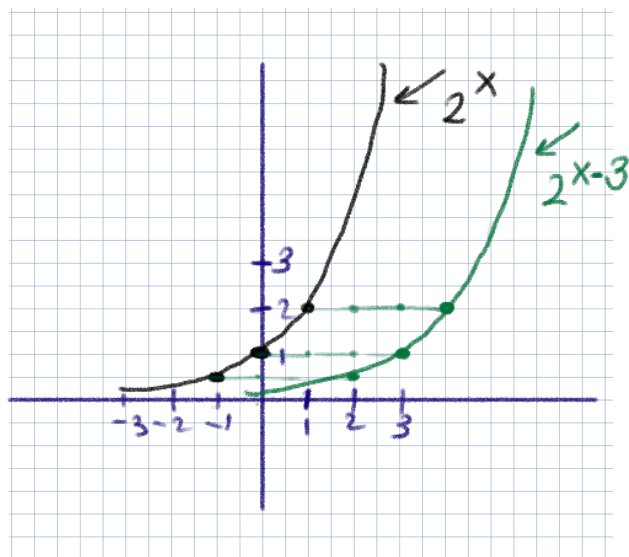
$(-1, \frac{1}{2}) (0, 1) (1, 2)$

Domain $(-\infty, \infty)$

Range $(0, \infty)$

H. Asymptote $y=0$

النظام عددي في x



35. $y = +2 - (\frac{1}{3})^x$

مراجعة ايفية
لايف

$(\frac{1}{3})^x \rightarrow a=\frac{1}{3}$

$(-1, \frac{1}{a}) (0, 1) (1, a)$

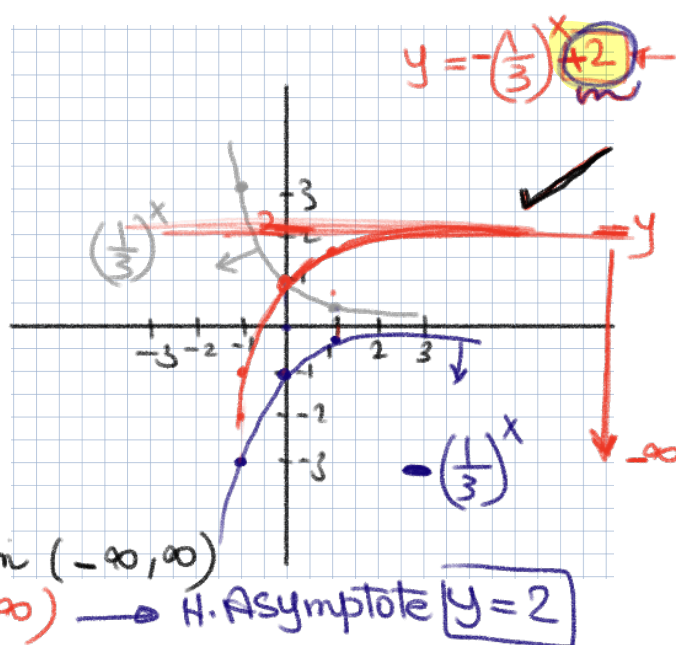
$(-1, 3) (0, 1) (1, \frac{1}{3})$

$y = -(\frac{1}{3})^x + 2$

The graph of $(\frac{1}{3})^x$ reflect
across x axis then shifted

up by 2 units $\rightarrow$ Domain $(-\infty, \infty)$

Range $\rightarrow (2, \infty)$ $\rightarrow$ H. Asymptote $y=2$



36. $y = 5^{-x-3}$

reflect across y axis
 shift down by 3 units
 $5^x \rightarrow a=5$

$(-1, \frac{1}{a}) (0, 1) (1, a)$

$(-1, \frac{1}{5}) (0, 1) (1, 5)$

The graph of $y = 5^x$

Reflect across y axis
 then shifted down

by 3 units

Domain $(-\infty, \infty)$

Range $(-3, \infty)$

H. Asymptote $y = -3$

