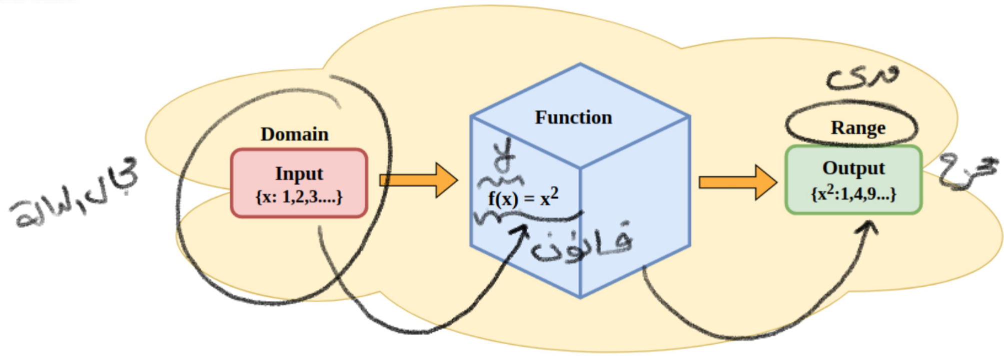
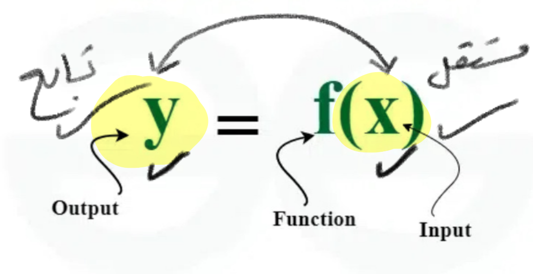
FUNCTIONS



FUNCTION NOTATION



Function Notation

$f(x) = 2x + 1$

$f(3)$

$f(3) = 2(3) + 1 = 7$

Input	Output
مدخل	مخرج
domain	Range

Input	→	Function	→	Output
3		$2x + 1$		7
		$2(3) + 1$		
Domain				Range.

Polynomial

$f(x) = x^b$

Exponential

$f(x) = b^x$ base $= a^x$ اكثر

beIN Class
ONLINE CLASSES

4.1 # Exponential Functions

- A polynomial function has the basic form: $f(x) = x^3$ (base, Exponent)
- An exponential function has the basic form: $f(x) = 3^x$ (base, Exponent)
- An exponential function has the variable in the exponent, not in the base.
- General Form of an Exponential Function:

EXPONENTIAL FUNCTIONS

The **exponential function with base a** is defined for all real numbers x by

$$f(x) = a^x$$

base. ← → Exponent is a variable.

موجب

where $\boxed{a > 0}$ and $\boxed{a \neq 1}$.

We assume that $a \neq 1$ because the function $f(x) = 1^x = 1$ is just a constant function. Here are some examples of exponential functions:

$$f(x) = 2^x \qquad g(x) = 3^x \qquad h(x) = 10^x$$

Base 2 — Base 3 — Base 10

EXAMPLE 1 ■ Evaluating Exponential Functions

Let $f(x) = 3^x$, and evaluate the following:

(a) $f(5)$ **(b)** $f(-\frac{2}{3})$

(c) $f(\pi)$ **(d)** $f(\sqrt{2})$

SOLUTION We use a calculator to obtain the values of f.

$f(5) \to x = 5 \to f(x) = 3^x = 3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 9 \cdot 9 \cdot 3 = 81 \cdot 3 = 243$

$f(-\frac{2}{3}) \to x = -\frac{2}{3} \to f(-\frac{2}{3}) = \frac{3^{-2/3}}{1} = \frac{1}{3^{2/3}} = \checkmark$

$f(\pi) \to x = \pi \to f(\pi) = 3^{\pi} = \checkmark$

$f(\sqrt{2}) \to x = \sqrt{2} \to f(\sqrt{2}) = 3^{\sqrt{2}} = \checkmark$

Given $f(x) = 2^{x+3} + 2$

$$f(1) = 2^{1+3} + 2 = 2^4 + 2 = \underbrace{\underbrace{2 \cdot 2}_{4} \cdot \underbrace{2 \cdot 2}_{4}} + 2$$

$$= 16 + 2 = 18$$

Graph

Plotting point

$x = \checkmark$

$y = \checkmark$

$f(x) = a^x$

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	

Keypoints

$f(x) = a^x \rightarrow \boxed{a = \checkmark}$

$\left(-3, \frac{1}{a^3}\right)\left(-2, \frac{1}{a^2}\right)\left(-1, \frac{1}{a}\right)(0, 1)\ (1, a)\left(2, a^2\right)\left(3, a^3\right)$

GRAPHS OF EXPONENTIAL FUNCTIONS

The exponential function

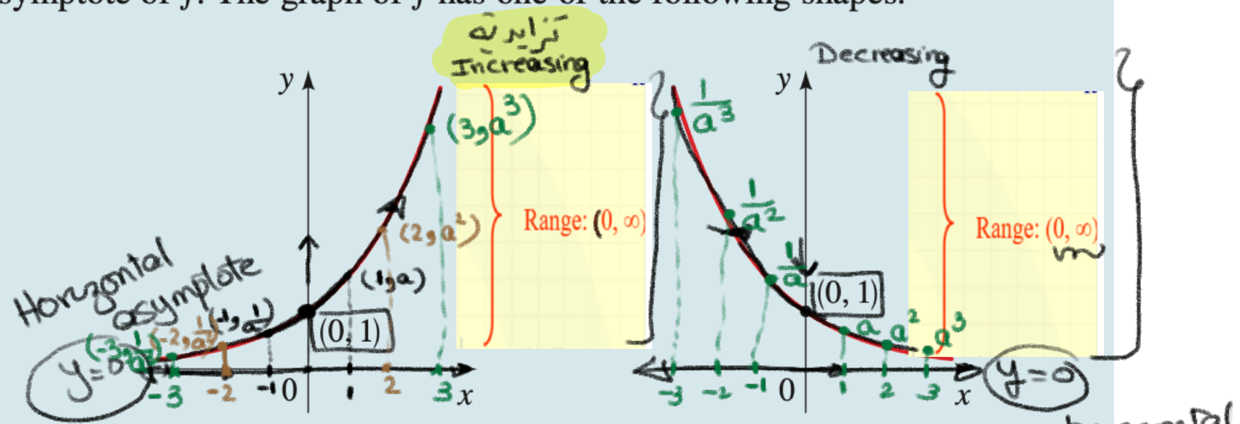
$$f(x) = a^x \qquad a > 0,\ a \neq 1$$

Domain

y

$0 < a < 1$

has domain $\mathbb{R}$ and range $(0, \infty)$. The line $y = 0$ (the x-axis) is a horizontal asymptote of f. The graph of f has one of the following shapes.



$f(x) = a^x$ for $a > 1$ $\qquad$ $f(x) = a^x$ for $0 < a < 1$

Domain: $(-\infty, \infty)$ $\qquad$ Domain: $(-\infty, \infty)$

- Exponential functions with positive bases <u>greater</u> than 1 have graphs that are <u>increasing</u>.

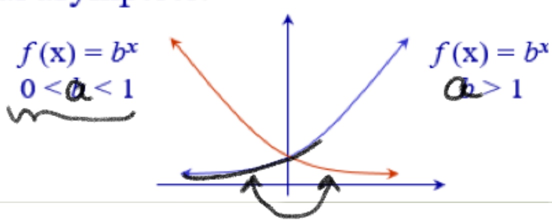
beIN Class

ONLINE CLASSES

Characteristics of Exponential Functions

$(-\infty, \infty)$

- The domain of $f(x) = a^x$ consists of all real numbers. The range of $f(x) = b^x$ consists of all positive real numbers. $(0, \infty)$
- The graphs of all exponential functions pass through the point (0, 1) because $f(0) = a^0 = 1$. $\boxed{a \neq 0}$
- If $a > 1$, $f(x) = a^x$ has a graph that goes up to the right and is an increasing function.
- If $0 < a < 1$, $f(x) = b^x$ has a graph that goes down to the right and is a decreasing function.
- $f(x) = b^x$ is a one-to-one function and has an inverse that is a function.
- The graph of $f(x) = b^x$ approaches but does not cross the x-axis. The x-axis is a horizontal asymptote.



- Domain: $D : \{x \in \mathbb{R}\}$
- Range: $R : \{y \in \mathbb{R} \mid y > 0\}$
- Horizontal asymptote: $y = 0$
- y-intercept: $(0, 1)$

f^{-1}

is inverse one to another

EXAMPLE 2 ■ Graphing Exponential Functions by Plotting Points

Draw the graph of each function.

(a) $f(x) = 3^x$

x	$f(x) = 3^x$
-3	$\frac{1}{27}$
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9
3	27

$3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

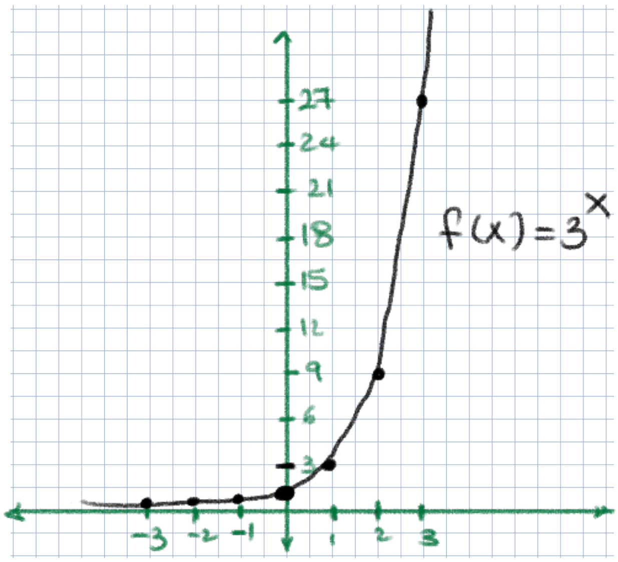
$3^{-1} = \frac{1}{3^1}$

$3^0 = 1$

$3^1 = 3$

$3^2 = 9$

$3^3 = 27$

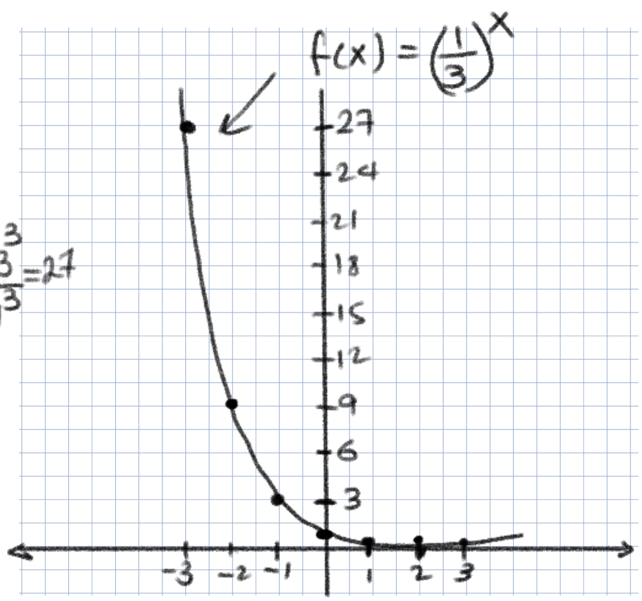


(b) $g(x) = \left(\frac{1}{3}\right)^x$

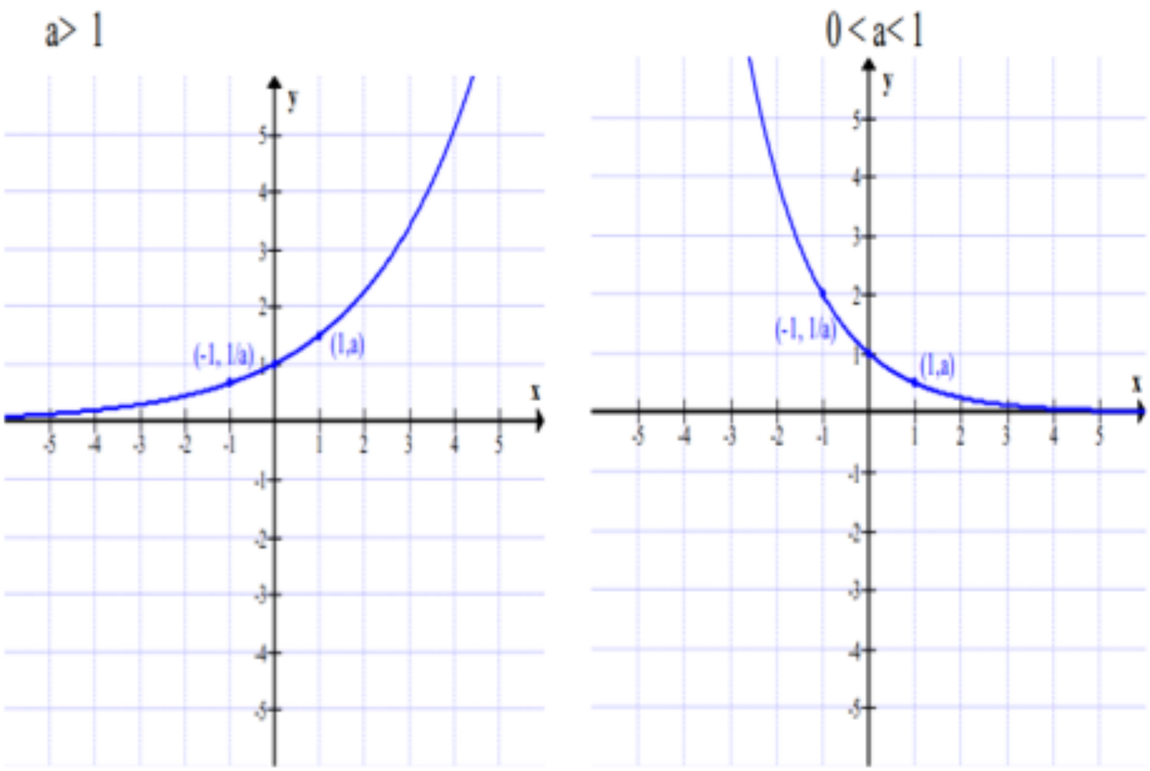
x	$g(x) = \left(\frac{1}{3}\right)^x$
-3	27
-2	9
-1	3
0	1
1	$\frac{1}{3}$
2	$\frac{1}{9}$
3	$\frac{1}{27}$

$\left(\frac{1}{3^1}\right)^{-3} = \frac{1^{-3}}{3^{-3}} = \frac{3^3}{1^3} = 27$

$\left(\frac{1}{3}\right)^{-2} = \frac{1^{-2}}{3^{-2}} = \frac{3^2}{1^2} = 9$



Points on the graph method



Points on the graph: (-1, 1/a), (0,1), (1, a)

17–20 ■ Graphing Exponential Functions Graph both functions on one set of axes.

19. $f(x) = 4^x$ and $g(x) = 7^x$

$\boxed{a = 4}$

Keypoints

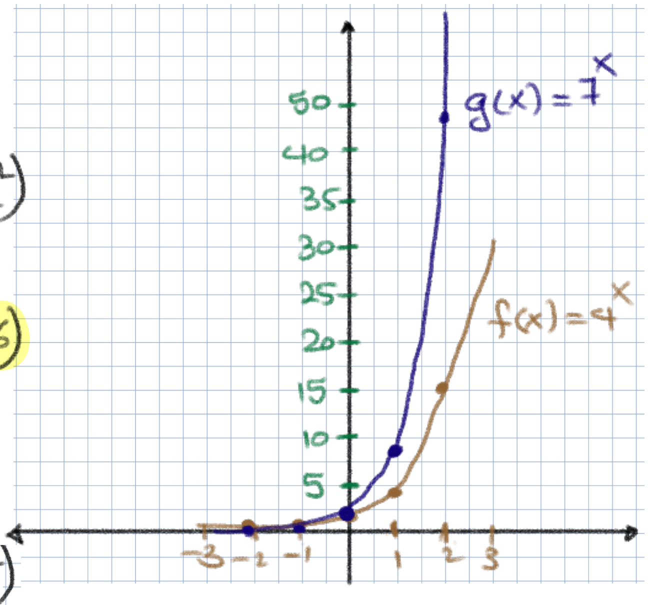
$\left(-2, \frac{1}{a^2}\right)\left(-1, \frac{1}{a}\right)(0, 1)(1, a)(2, a^2)$

$\left(-2, \frac{1}{16}\right)\left(-1, \frac{1}{4}\right)(0, 1)(1, 4)(2, 16)$

$\boxed{a = 7}$

$\left(-2, \frac{1}{a^2}\right)\left(-1, \frac{1}{a}\right)(0, 1)(1, a)(2, a^2)$

$\left(-2, \frac{1}{49}\right)\left(-1, \frac{1}{7}\right)(0, 1)(1, 7)(2, 4$



$0 < \frac{3}{4} < 1$

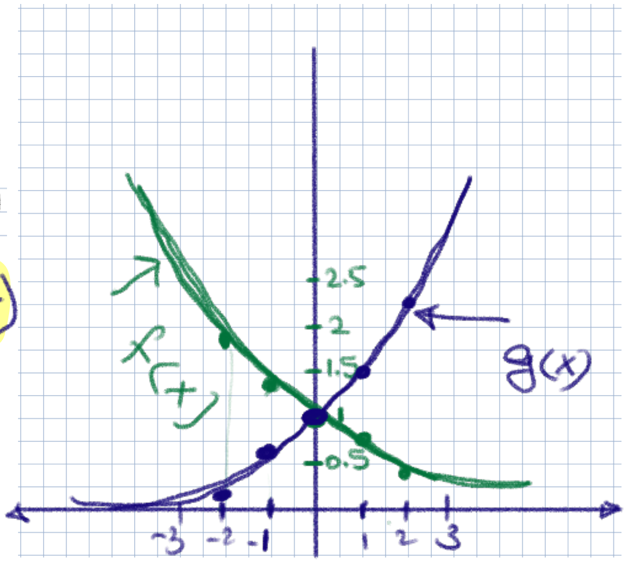
20. $f(x) = \left(\frac{3}{4}\right)^x$ and $g(x) = 1.5^x$

$a = \frac{3}{4} \longrightarrow a^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

$(-2, \frac{1}{a^2})(-1, \frac{1}{a})(0, 1)(1, a)(2, a^2)$

$(-2, \frac{16}{9})(-1, \frac{4}{3})(0, 1)(1, \frac{3}{4})(2, \frac{9}{16})$

1.8



$a = \left(\frac{3}{2}\right) \longrightarrow \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$(-2, \frac{1}{a^2})(-1, \frac{1}{a})(0, 1)(1, a)(2, a^2)$

$(-2, \frac{4}{9})(-1, \frac{2}{3})(0, 1)(1, \frac{3}{2})(2, \frac{9}{4})$

2.25