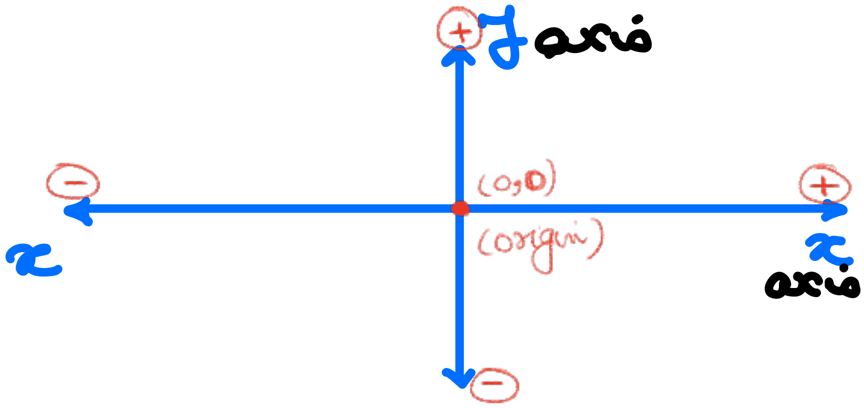
1.1 The Coordinate Plane

Objectives

- The Coordinate Plane
- The Distance Formula
- The Midpoint Formula

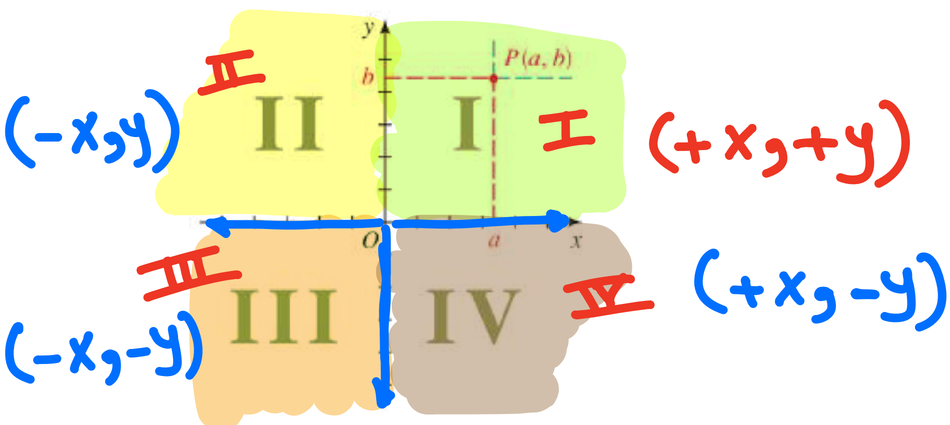


Just as points on a line can be identified with real numbers to form the coordinate line, points in a plane can be identified with ordered pairs of numbers to form the **coordinate plane** or **Cartesian plane**.

To do this, we draw two perpendicular real lines that intersect at 0 on each line.

Usually, one line is horizontal with positive direction to the right and is called the ***x*-axis**; the other line is vertical with positive direction upward and is called the ***y*-axis**.

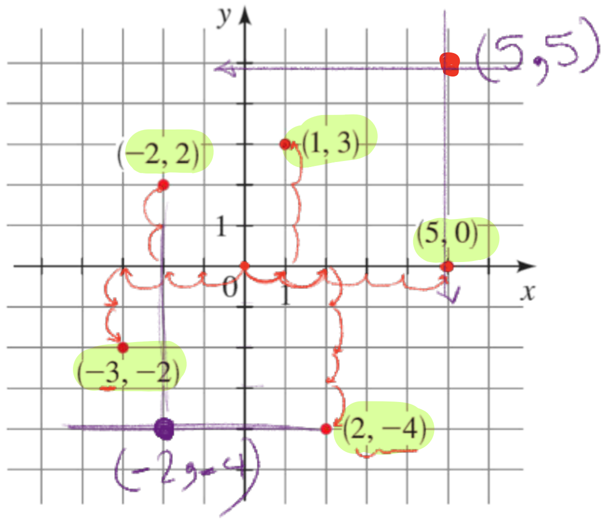
The point of intersection of the *x*-axis and the *y*-axis is the **origin *O***, and the two axes divide the plane into four **quadrants**, labeled I, II, III, and IV in Figure 1. (The points *on* the coordinate axes are not assigned to any quadrant.)



Any point *P* in the coordinate plane can be located by a unique **ordered pair** of numbers (*a*, *b*), as shown in Figure 1.

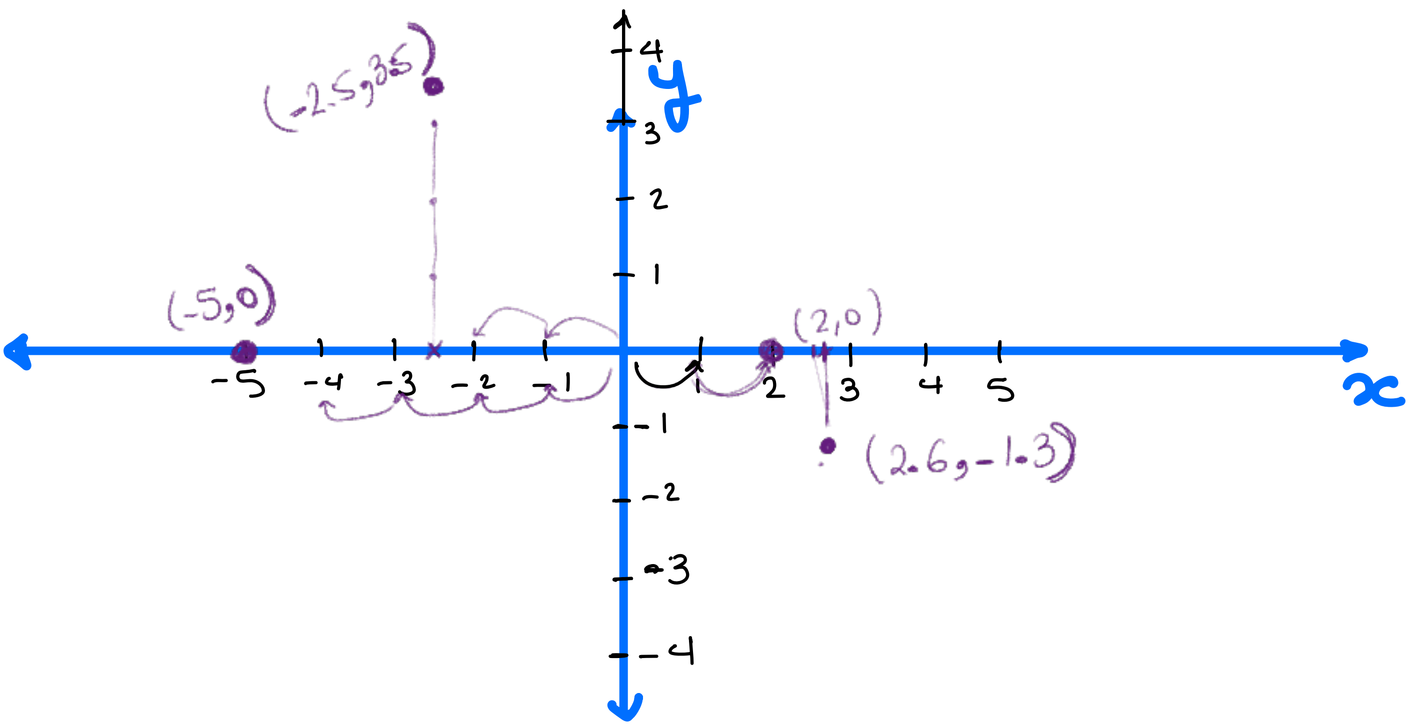
The first number *a* is called the ***x*-coordinate** of *P*; the second number *b* is called the ***y*-coordinate** of *P*.

We can think of the coordinates of *P* as its "address," because they specify its location in the plane. Several points are labeled with their coordinates in Figure 2.



7–8 ■ Points in a Coordinate Plane Plot the given points in a coordinate plane.

8. $(-5, 0)$, $(2, 0)$, $(2.6, -1.3)$, $(-2.5, 3.5)$

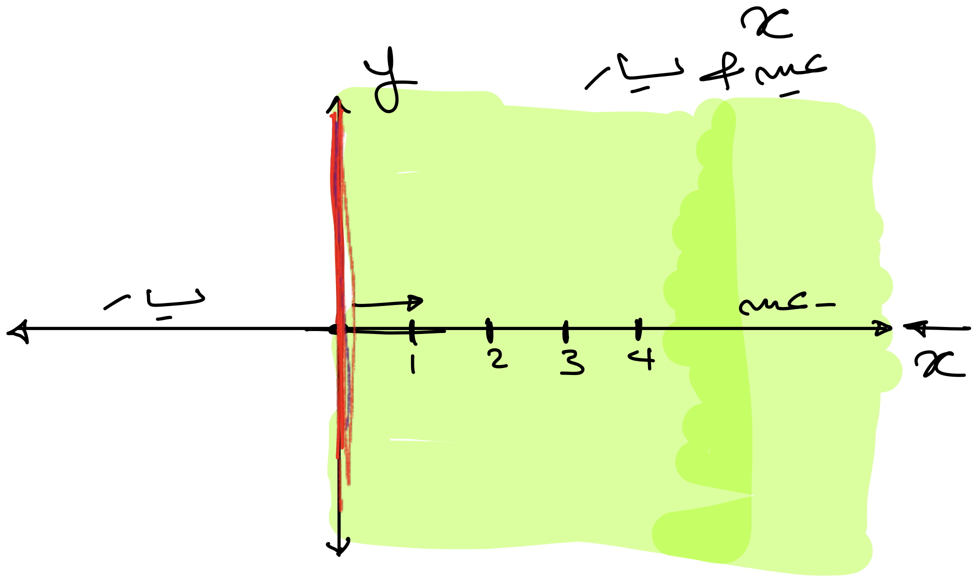


Describe and sketch the regions given by each set.

(a) $\{(x, y) \mid x \geq 0\}$

الخط بيكون رفيع

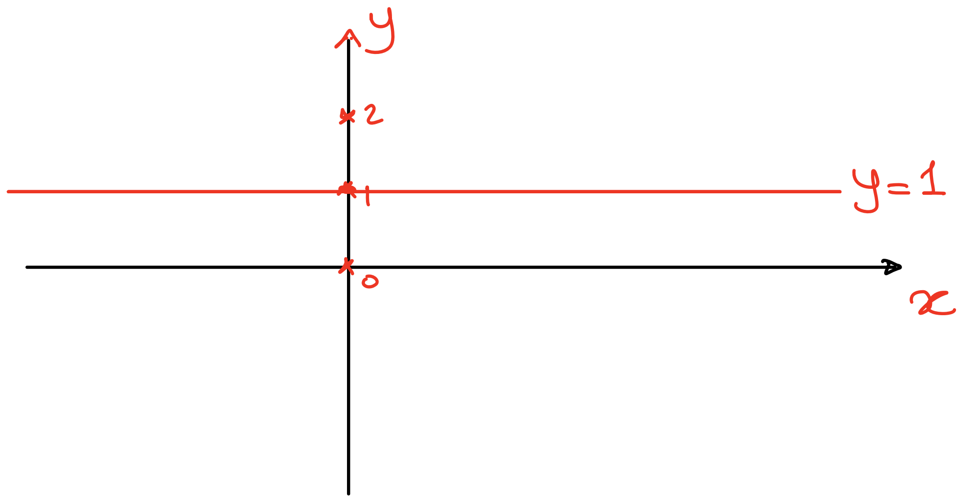
فيه علامة يساوي
الخط بيكون ثقيل
bold



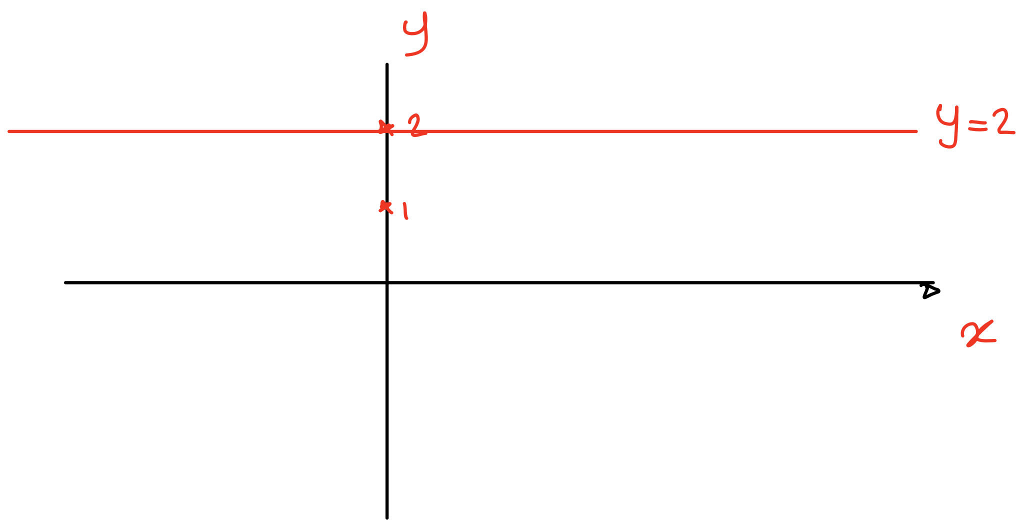
خط مستقيم

(b) $\{(x, y) \mid y = 1\}$

الخط بيكون افقي
horizontal
افقي



10. $\{(x, y) \mid y = 2\}$

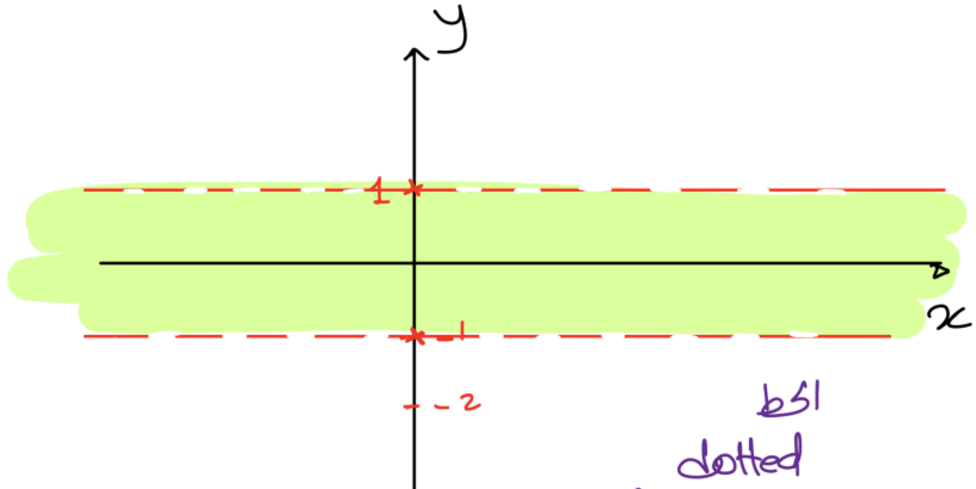


(c) $\{(x, y) \mid -1 < y < 1\}$

لايوجد علامة يساوي

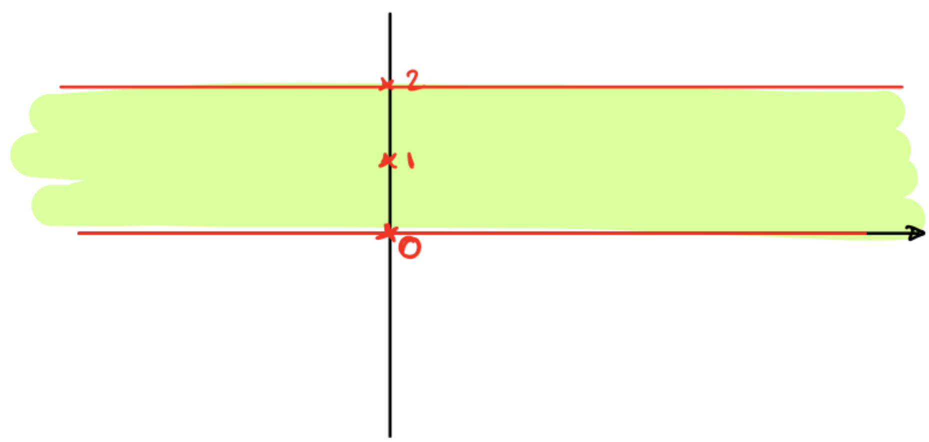
الخط سيكون منقط

dotted



وذلك لعدم وجود علامة اليساوي في المتباينة

14. $\{(x, y) \mid 0 \leq y \leq 2\}$



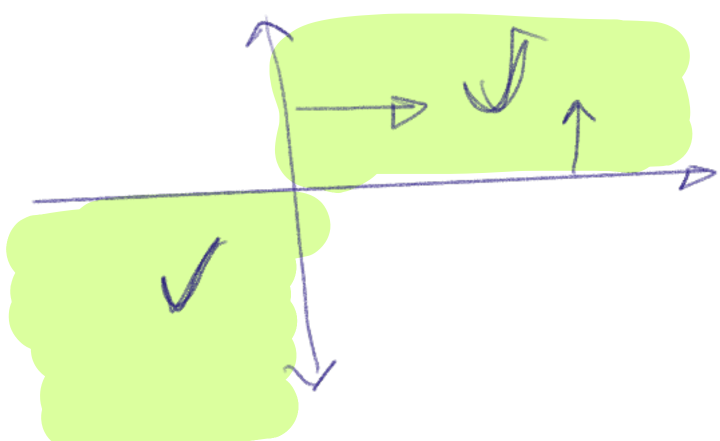
16. $\{(x, y) \mid xy > 0\}$

$x > 0 \quad y > 0$

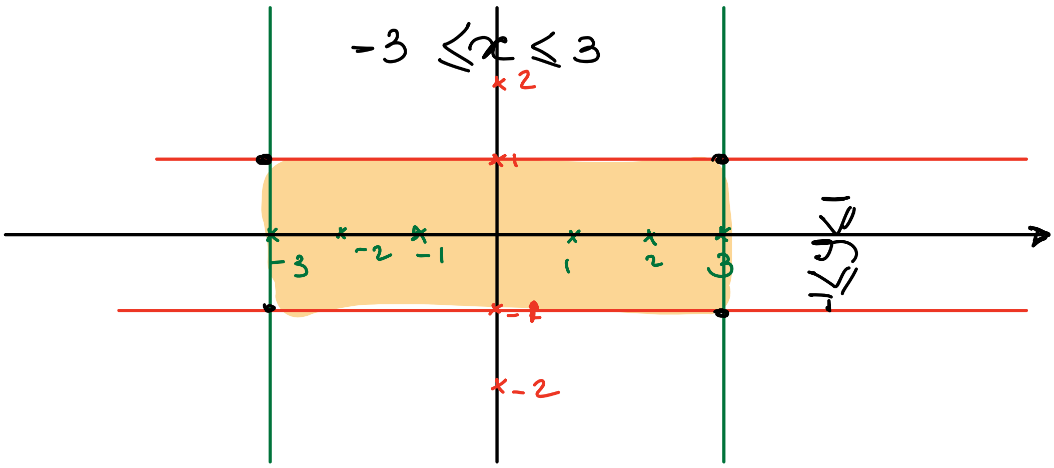
$x * y$

$\oplus \quad \oplus = \oplus$

$\ominus \quad \ominus = \oplus$



20. $\{(x, y) \mid -3 \le x \le 3 \text{ and } -1 \le y \le 1\}$



The Distance Formula

We now find a formula for the distance $d(A, B)$ between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the plane.

We know that the distance between points a and b on a number line is $d(a, b) = |b - a|$.

So from Figure 4 we see that the distance between the points $A(x_1, y_1)$ and $C(x_2, y_1)$ on a horizontal line must be $|x_2 - x_1|$, and the distance between $B(x_2, y_2)$ and $C(x_2, y_1)$ on a vertical line must be $|y_2 - y_1|$.

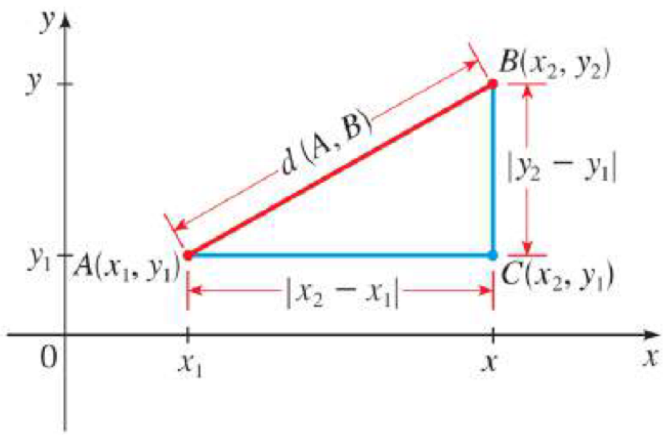


Figure 4

Since triangle ABC is a right triangle, the Pythagorean Theorem gives

$$d(A, B) = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance Formula

The distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the plane is

$$d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

فرق x تربيع   فرق ال y

Find the distance between the points $A(\overset{x_1}{2}, \overset{y_1}{5})$ and $B(\overset{x_2}{4}, \overset{y_2}{-1})$.

$$d(A,B) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= \sqrt{(4-2)^2 + (-1-5)^2} = \sqrt{2^2 + (-6)^2}$$

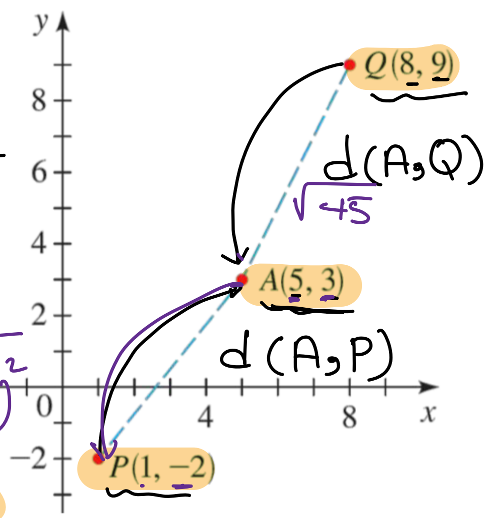
$$= \sqrt{4+36} = \sqrt{40} = \sqrt{4 \times 10} = 2\sqrt{10}$$

Which of the points $P(1, -2)$ or $Q(8, 9)$ is closer to the point $A(5, 3)$?

$$d(A,Q) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= \sqrt{(5-8)^2 + (3-9)^2} = \sqrt{(-3)^2 + (-6)^2}$$

$$= \sqrt{9+36} = \sqrt{45}$$



$$d(A,P) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(5-1)^2 + (3--2)^2}$$

$$= \sqrt{4^2 + 5^2} = \sqrt{16+25} = \sqrt{41}$$

point P is closer from A

36. Which of the points $C(-6, 3)$ or $D(3, 0)$ is closer to the point $E(-2, 1)$?

$$d(C,E) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(-6--2)^2 + (3-1)^2} = \sqrt{(-4)^2 + (2)^2}$$

$$= \sqrt{20}$$

$$d(D,E) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(3--2)^2 + (0-1)^2} = \sqrt{5^2 + (-1)^2} = \sqrt{25+1}$$

$$= \sqrt{26}$$

Point C is closer to point E

The Midpoint Formula

نقطة المنتصف

Now let's find the coordinates (x, y) of the midpoint M of the line segment that joins the point $A(x_1, y_1)$ to the point $B(x_2, y_2)$.

In Figure 7, notice that triangles APM and MQB are congruent because $d(A, M) = d(M, B)$ and the corresponding angles are equal.

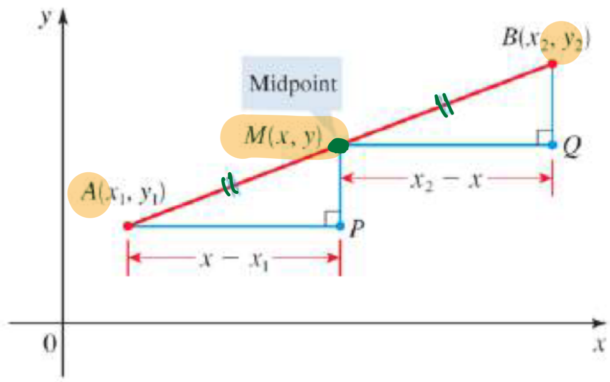


Figure 7

Midpoint Formula

+ /2

The midpoint of the line segment from $A(x_1, y_1)$ to $B(x_2, y_2)$ is

+ /2

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

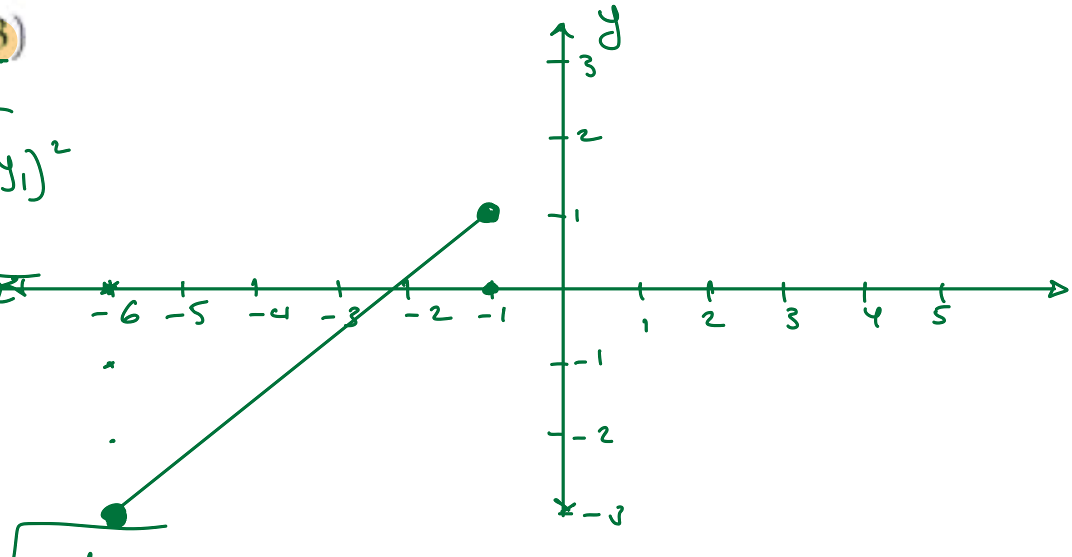
Find the midpoint of the line segment that joins $(-2, 1)$ and $(4, 5)$.

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-2+4}{2}, \frac{1+5}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{6}{2}\right) = (1, 3)$$

25–30 ■ Distance and Midpoint A pair of points is given. (a) Plot the points in a coordinate plane. (b) Find the distance between them. (c) Find the midpoint of the segment that joins them.

28. $(-1, 1), (-6, -3)$



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-6 - -1)^2 + (-3 - 1)^2}$$

$$= \sqrt{(-5)^2 + (-4)^2}$$

$$= \sqrt{25 + 16} = \sqrt{41}$$

$$\text{Midpoint} = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right) = \left(\frac{-6-1}{2}, \frac{-3+1}{2}\right) = \left(\frac{-7}{2}, \frac{-2}{2}\right)$$

$$= \left(-\frac{7}{2}, -1\right)$$

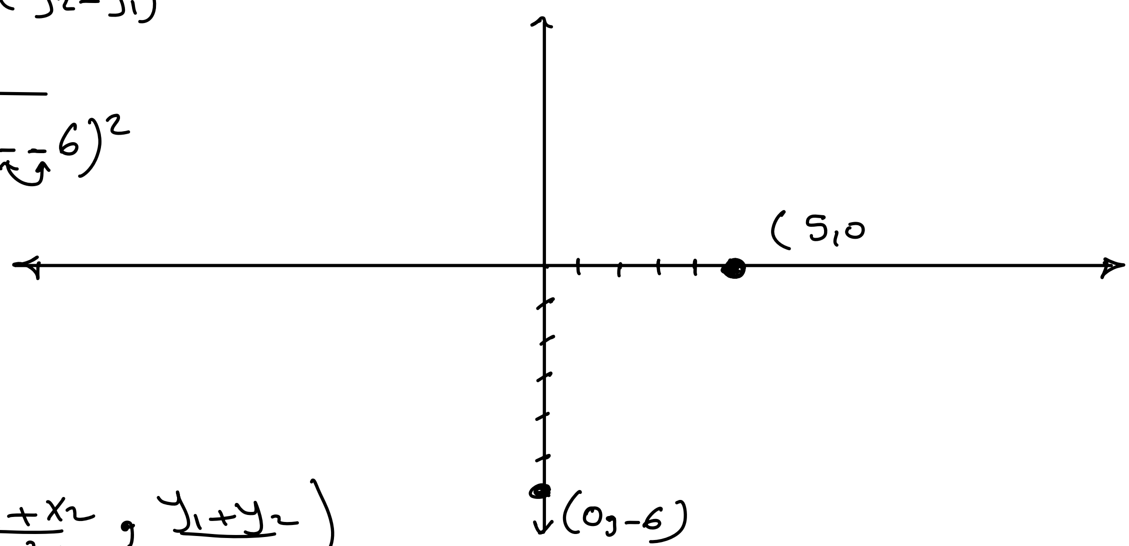
30. $(0, -6), (5, 0)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(5 - 0)^2 + (0 - -6)^2}$$

$$= \sqrt{25 + 36}$$

$$= \sqrt{61}$$

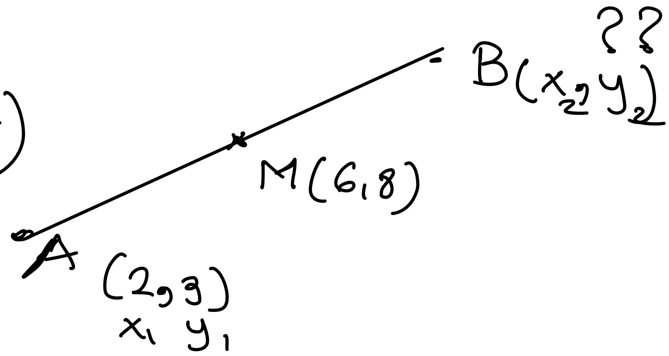


$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{0+5}{2}, \frac{-6+0}{2}\right) = \left(\frac{5}{2}, -3\right)$$

48. If $M(6, 8)$ is the midpoint of the line segment AB and if A has coordinates $(2, 3)$, find the coordinates of B.

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$



$$(6, 8) = \left(\frac{2+x_2}{2}, \frac{3+y_2}{2}\right)$$

$$\frac{6}{1} = \frac{2+x_2}{2} \quad \& \quad \frac{8}{1} = \frac{3+y_2}{2}$$

$6 * 2 = (2+x_2) * 1$

$12 = (2+x_2)$

$12 - 2 = x_2$

$\boxed{10 = x_2}$

$8 * 2 = (3+y_2)1$

$16 = 3+y_2$

$16 - 3 = y_2$

$\boxed{13 = y_2}$

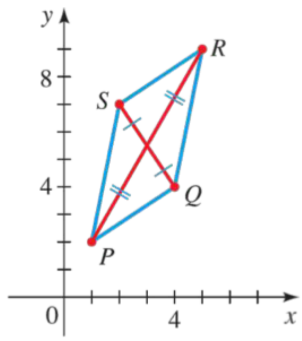
Point $B(x_2, y_2) = (10, 13)$

EXAMPLE 5 ■ Applying the Midpoint Formula

Show that the quadrilateral with vertices $P(1, 2)$, $Q(4, 4)$, $R(5, 9)$, and $S(2, 7)$ is a parallelogram by proving that its two diagonals bisect each other.

متوازي الاضلاع

Midpoint of $\boxed{SQ}$ $\quad \overset{x_1 \; y_1}{(2, 7)} \quad \overset{x_2 \; y_2}{(4, 4)}$



$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{2+4}{2}, \frac{7+4}{2}\right) = \left(3, \frac{11}{2}\right)$$

Midpoint of $\boxed{PR}$ $\quad (1, 2) \quad (5, 9)$

$$\text{Mid point} \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{1+5}{2}, \frac{2+9}{2}\right) = \left(3, \frac{11}{2}\right)$$

2 diagonals have same midpoint they bisect each other
PQRS is parallelogram

43. Show that the points $A(-2, 9)$, $B(4, 6)$, $C(1, 0)$, and $D(-5, 3)$ are the vertices of a square.

مربع

$d(A, C)$

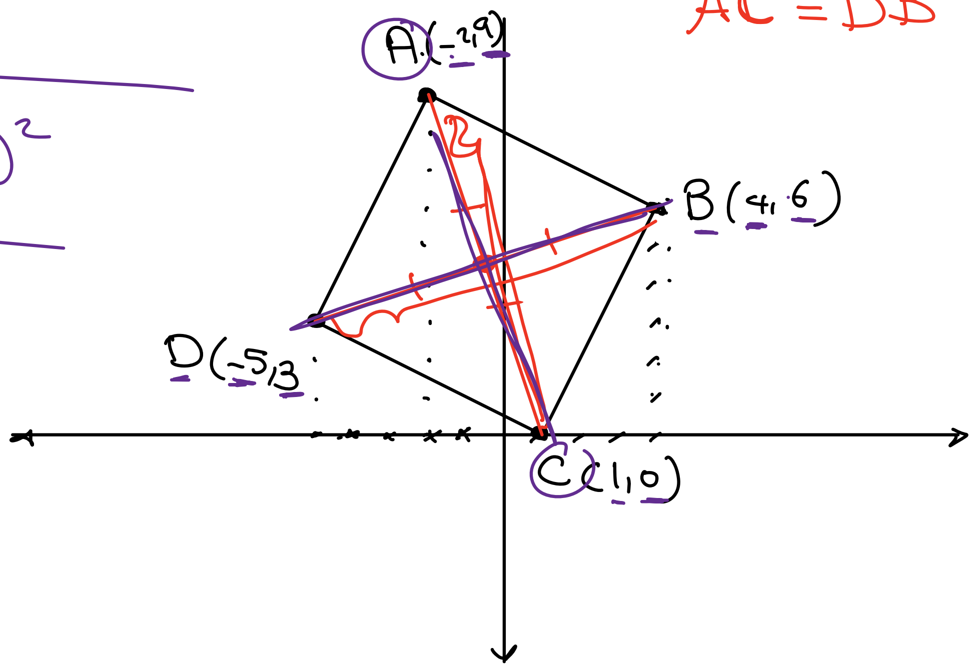
$$= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= \sqrt{(-2-1)^2 + (9-0)^2}$$

$$= \sqrt{9+81}$$

$$= \sqrt{90}$$

AC = DB



$$d(B, D) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4 - -5)^2 + (6-3)^2}$$

$$= \sqrt{9^2 + 3^2}$$

$$= \sqrt{81+9} = \sqrt{90}$$

Midpoint between A & C

$$= \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} = \left(\frac{-2+1}{2}, \frac{9+0}{2}\right) = \left(-\frac{1}{2}, \frac{9}{2}\right)$$

Midpoint between B & D

$$= \left(\frac{-5+4}{2}, \frac{6+3}{2}\right) = \left(\frac{-1}{2}, \frac{9}{2}\right)$$

line AC & BD they have same length & same mid point → they bisect each other

. then ABCD is a square