6.1 / 5.3 MATRICES ← مصفوفات

Matrix مصفوفة

$$A = \begin{bmatrix} 10 & 12 & 16 \\ 5 & 9 & 7 \end{bmatrix} \begin{matrix} \leftarrow R \\ \leftarrow R \end{matrix}$$

The horizontal rows of a matrix are numbered consecutively from top to bottom, and the vertical columns are numbered from left to right. For the foregoing matrix A, we have

$$\begin{array}{r|ccc|l} & \text{column } 1 & \text{column } 2 & \text{column } 3 & \\ \hline \text{row } 1 & 10 & 12 & A_{13}\ 16 & \\ \text{row } 2 & A_{21}\ 5 & 9 & 7 & = A \end{array}$$

Since A has two rows and three columns, we say that A has *size* 2×3 (read "2 by 3")

The numbers in a matrix are called its **entries.** To denote the entries in a matrix A of size 2×3, say, we use the name of the matrix, with *double subscripts* to indicate *position*, consistent with the conventions above:

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{bmatrix}$$

For the entry A_{12} (read "A sub one-two" or just "A one-two"), the first subscript, 1, specifies the row and the second subscript, 2, the column in which the entry appears.

$$A_{13} = 16 \quad , \quad A_{21} = 5$$

Definition

A rectangular array of numbers A consisting of m horizontal rows and n vertical columns,

$$\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ A_{m1} & A_{m2} & \cdots & A_{mn} \end{bmatrix}$$

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is called an $m \times n$ ***matrix*** and $m \times n$ is the ***size*** of A. For the entry A_{ij}, the row subscript is i and the column subscript is j.

A matrix that has exactly one row, such as the 1×4 matrix

- $$A = [1 \quad 7 \quad 12 \quad 3]$$ ← row vector

is called a **row vector.**

- A matrix consisting of a single column is called a **column vector**.

$$\begin{bmatrix} 1 \\ -2 \\ 15 \\ 9 \\ 16 \end{bmatrix}$$

Single row is called a row vector

$[1 \quad -2 \quad 15 \quad 9 \quad 16]$

EXAMPLE 1 **Size of a Matrix**

a. The matrix $[1 \quad 2 \quad 0]$ has size 1×3.

b. The matrix $\begin{bmatrix} 1 & -6 \\ 5 & 1 \\ 9 & 4 \end{bmatrix}$ has size 3×2.

c. The matrix $[7]$ has size 1×1.

d. The matrix $\begin{bmatrix} 1 & 3 & 7 & -2 & 4 \\ 9 & 11 & 5 & 6 & 8 \\ 6 & -2 & -1 & 1 & 1 \end{bmatrix}$ has size 3×5 and $(3)(5) = 15$ entries.

Special Matrices

- An $m \times n$ matrix whose entries are all 0 is called the $m \times n$ **zero matrix** and is denoted by $0_{m \times n}$ or, more simply, by 0 if its size is understood. Thus, the 2×3 zero matrix is

$$0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- A matrix having the same number of columns as rows—for example, n rows and n columns—is called a **square matrix** of order n. That is, an $m \times n$ matrix is square if and only if $m = n$. For example, matrices

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$$\begin{bmatrix} 2 & 7 & 4 \\ 6 & 2 & 0 \\ 4 & 6 & 1 \end{bmatrix}_{3*3} \quad \text{and} \quad [3]$$

order 3 // 3*3

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A square matrix A is called a **diagonal matrix** if all the entries that are off the main diagonal are zero—that is, if $A_{ij} = 0$ for $i \neq j$. Examples of diagonal matrices are

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 3 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

Zeros

- A square matrix A is said to be an **upper triangular matrix** if all entries *below* the main diagonal are zero—that is, if $A_{ij} = 0$ for $i > j$.

$$\begin{bmatrix} 5 & 1 & 1 \\ 0 & -3 & 7 \\ 0 & 0 & 4 \end{bmatrix}$$

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upper

- Similarly, a matrix A is said to be a **lower triangular matrix** if all entries *above* the main diagonal are zero—that is, if $A_{ij} = 0$ for $i < j$. When a matrix is either upper triangular or lower triangular, it is called a **triangular matrix.** Thus, the matrices

$$\begin{bmatrix} 7 & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ 6 & 5 & -4 & 0 \\ 1 & 6 & 0 & 1 \end{bmatrix}$$

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Lower

1. Let

$$A = \begin{bmatrix} 1 & -6 & 2 \\ -4 & 2 & 1 \end{bmatrix}_{2*3} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3*3} \quad C = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{bmatrix}_{3*2}$$

$$D = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}_{2*2} \quad E = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 6 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 6 & 1 \end{bmatrix}_{4*4} \quad F = \begin{bmatrix} 6 & 2 \end{bmatrix}_{1*2}$$

$$G = \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix}_{3*1} \quad H = \begin{bmatrix} 1 & 6 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3*3} \quad J = [4]_{1*1}$$

(a) State the size of each matrix.

(b) Which matrices are square?

B E H J D

(c) Which matrices are upper triangular? lower triangular?

upper Triangular → all entries below the main diagonal are zeros

H

Lower Triangular   all entries above the main diagonal are zero.

D

(d) Which are row vectors? مصفوفة الصف الواحد

F و J

(e) Which are column vectors? مصفوفة العمود الواحد

G و J

let $A = [A_{ij}] = \begin{bmatrix} 7 & -2 & 14 & 6 \\ 6 & 2 & 3 & -2 \\ 5 & 4 & 1 & 0 \\ 8 & 0 & 2 & 0 \end{bmatrix}$ size 4×4

2. What is the order of A? order 4 //

Find the following entries.

3. A_{21} $= 6$ **4.** A_{42} $= 0$ **5.** A_{32} $= 4$ **6.** A_{34} $= 0$ **7.** A_{44} $= 0$ **8.** A_{55} Not defined.

Equality of Matrices

Definition

Matrices A and B are ***equal*** if and only if they have the same size and $A_{ij} = B_{ij}$ for each i and j (that is, corresponding entries are equal).

Thus,

$$\begin{bmatrix} 1+1 & \frac{2}{2} \\ 2 \cdot 3 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 6 & 0 \end{bmatrix} \longleftarrow \text{same size}$$

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but

$$\underset{1*2}{[1 \quad 1]} \neq \underset{2*1}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}} \quad \text{and} \quad \underset{1*2}{[1 \quad 1]} \neq \underset{1*3}{[1 \quad 1 \quad 1]} \qquad \text{different sizes}$$

They are not equal.

A matrix equation can define a system of equations. For example, suppose that

$$\begin{bmatrix} x & y+1 \\ 2z & 5w \end{bmatrix} = \begin{bmatrix} 2 & 7 \\ 4 & 2 \end{bmatrix}$$

By equating corresponding entries, we must have

$$\begin{cases} x = 2 \\ y+1 = 7 \longrightarrow y = 7-1 = 6 \\ 2z = 4 \longrightarrow \frac{2z}{2} = \frac{4}{2} \longrightarrow z = 2 \\ 5w = 2 \longrightarrow \frac{5w}{5} = \frac{2}{5} \longrightarrow w = \frac{2}{5} \end{cases}$$

Solving gives $x = 2, y = 6, z = 2$, and $w = \frac{2}{5}$.

solve the matrix equation.

24. $\begin{bmatrix} 3x & 2y-1 \\ z & 5w \end{bmatrix} = \begin{bmatrix} 9 & 6 \\ 7 & 15 \end{bmatrix}$

$2*2$ $\quad$ $2*2$

$\frac{3x}{3} = \frac{9}{3} \longrightarrow \boxed{x = 3}$

$2y - 1 = 6 \longrightarrow 2y = 6+1 \longrightarrow \frac{2y}{2} = \frac{7}{2} \longrightarrow \boxed{y = \frac{7}{2}}$

$z = 7$

$\frac{5w}{5} = \frac{15}{5} \longrightarrow \boxed{w = 3}$

25. $\begin{bmatrix} 6 & 3 \\ x & 7 \\ 3y & 2z \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 6 & 7 \\ 2 & 7 \end{bmatrix}$

$x = 6$

$\frac{3y}{3} = \frac{2}{3}$

$y = \frac{2}{3}$

$\frac{2z}{2} = \frac{7}{2}$

$z = \frac{7}{2}$

Transpose of a Matrix

If A is a matrix, the matrix formed from A by interchanging its rows with its columns is called the *transpose* of A.

Definition

The ***transpose*** of an $m \times n$ matrix A, denoted A^T, is the $n \times m$ matrix whose ith row is the ith column of A.

EXAMPLE 3 **Transpose of a Matrix**

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, find A^T.

Solution: Matrix A is 2×3, so A^T is 3×2. Column 1 of A becomes row 1 of A^T, column 2 becomes row 2, and column 3 becomes row 3. Thus,

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \longrightarrow (A^T)^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Now Work Problem 19 ◁

Observe that the columns of A^T are the rows of A. Also, if we take the transpose of our answer, the original matrix A is obtained. That is, the transpose operation has the property that

$$(A^T)^T = A$$

In Problems 17–20, find A^T.

17. $A = \begin{bmatrix} 6 & -3 \\ 2 & 4 \end{bmatrix} \longrightarrow A^T = \begin{bmatrix} 6 & 2 \\ -3 & 4 \end{bmatrix}$

18. $A = [2 \quad 4 \quad 6 \quad 8] \longrightarrow A^T = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 8 \end{bmatrix}$