$$A = \begin{bmatrix} 10 & 12 & 16 \\ 5 & 9 & 7 \end{bmatrix}$$
 همونه ممنونه $A = \begin{bmatrix} 10 & 12 & 16 \\ 5 & 9 & 7 \end{bmatrix}$

The horizontal rows of a matrix are numbered consecutively from top to bottom, and the vertical columns are numbered from left to right. For the foregoing matrix A, we have

column 1 column 2 column 3

row 1
$$\begin{bmatrix} 10 & 12 & A_{3} & 16 \\ 10 & 12 & 9 & 7 \end{bmatrix} = A$$

Since A has two rows and three columns, we say that A has size 2×3 (read "2 by 3")

The numbers in a matrix are called its **entries.** To denote the entries in a matrix A of size 2×3 , say, we use the name of the matrix, with *double subscripts* to indicate *position*, consistent with the conventions above:

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{bmatrix}$$

For the entry A_{12} (read "A sub one-two" or just "A one-two"), the first subscript, 1, specifies the row and the second subscript, 2, the column in which the entry appears.

$$A_{13} = 16$$
 , $A_{21} = 5$

Definition

A rectangular array of numbers A consisting of m horizontal rows and n vertical columns,

is called an $m \times n$ matrix and $m \times n$ is the size of A. For the entry A_{ij} , the row subscript is i and the column subscript is j.

A matrix that has exactly one row, such as the 1×4 matrix

$$A = \begin{bmatrix} 1 & 7 & 12 & 3 \end{bmatrix}$$
 TOW VECTOR

A matrix consisting of a single column; is called a column vector.

EXAMPLE 1 Size of a Matrix

- **a.** The matrix $[1 \ 2 \ 0]$ has size 1×3 .
- **b.** The matrix $\begin{bmatrix} 1 & -6 \\ 5 & 1 \\ 9 & 4 \end{bmatrix}$ has size 3×2 .
- **c.** The matrix [7] has size 1×1 .
- **d.** The matrix $\begin{bmatrix} 1 & 3 & 7 & -2 & 4 \\ 9 & 11 & 5 & 6 & 8 \\ 6 & -2 & -1 & 1 & 1 \end{bmatrix}$ has size 3×5 and (3)(5) = 15 entries.

Special Matrices

An $m \times n$ matrix whose entries are all 0 is called the $m \times n$ zero matrix and is denoted by $0_{m \times n}$ or, more simply, by 0 if its size is understood. Thus, the 2 × 3 zero matrix is

$$0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

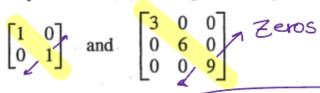
A matrix having the same number of columns as rows—for example, n rows and n columns—is called a **square matrix** of order n. That is, an $m \times n$ matrix is square if and only if m = n. For example, matrices

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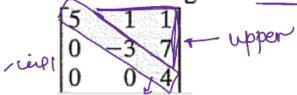
$$\begin{bmatrix} 2 & 7 & 4 \\ 6 & 2 & 0 \\ 4 & 6 & 1 \end{bmatrix}$$
 and [3] 3×3

08 der 3/1

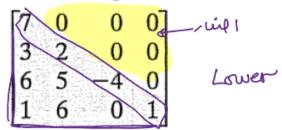
A square matrix A is called a **diagonal matrix** if all the entries that are off the main diagonal are zero—that is, if $A_{ij} = 0$ for $i \neq j$. Examples of diagonal matrices are



A square matrix A is said to be an upper triangular matrix if all entries below the main diagonal are zero—that is, if $A_{ij} = 0$ for i > j.



• Similarly, a matrix A is said to be a lower triangular matrix if all entries above the main diagonal are zero—that is, if $A_{ij} = 0$ for i < j. When a matrix is either upper triangular or lower triangular, it is called a **triangular matrix**. Thus, the matrices



$$A = \begin{bmatrix} 1 & -6 & 2 \\ -4 & 2 & 1 \end{bmatrix}_{2 \times 3} \underbrace{B} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3 \times 3} C = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{bmatrix}_{3 \times 2}$$

$$D = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}_{2*2} E = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 6 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 6 & 1 \end{bmatrix}_{2*2} E = \begin{bmatrix} 6 & 2 \end{bmatrix}_{1*2}$$

$$G = \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix}_{3*1} \quad H = \begin{bmatrix} 1 & 6 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3*3} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}_{1*1}$$

(a) State the size of each matrix.

(b) Which matrices are square?

(c) Which matrices are upper triangular? lower triangular?

upper Triangular

allentries below the mount diagonal are Zeros

Lower Triangular

all entries above the main D diagonal are zero.

(d) Which are row vectors?

(e) Which are column vectors? إوا العود إوا العام الع

let
$$A = [A_{ij}] = \begin{bmatrix} 7 & -2 & 14 & 6 \\ 6 & 2 & 3 & -2 \\ 5 & 4 & 1 & 0 \\ 8 & 0 & 2 & 0 \end{bmatrix}$$

2. What is the order of A? Order 4//

Find the following entries.

3.
$$A_{21}$$
 4. A_{42} 5. A_{32} 6. A_{34} 7. A_{44} 8. A_{55} = 0 = 0 Not defined.

Equality of Matrices

Definition

Matrices A and B are equal if and only if they have the same size and $A_{ij} = B_{ij}$ for each i and j (that is, corresponding entries are equal).

A matrix equation can define a system of equations. For example, suppose that

$$\begin{bmatrix} x & y+1 \\ 2z & 5w \end{bmatrix} = \begin{bmatrix} 2 & 7 \\ 4 & 2 \end{bmatrix}$$

By equating corresponding entries, we must have

$$\begin{cases} x = 2 \\ y + 1 = 7 \\ 2z = 4 \\ 5w = 2 \end{cases} \xrightarrow{2Z} = \frac{4}{2} \xrightarrow{2} Z = 2$$

$$5w = 2 \xrightarrow{5} = \frac{2}{5} \xrightarrow{5} \omega_{=} \frac{2}{5}$$

Solving gives x = 2, y = 6, z = 2, and $w = \frac{2}{5}$.

solve the matrix equation.

24.
$$\begin{bmatrix} 3x & 2y - 1 \\ 5w \end{bmatrix} = \begin{bmatrix} 9 & 6 \\ 7 & 15 \end{bmatrix}$$

$$2 \times 2$$

$$\frac{3x}{3} = \frac{9}{3} \longrightarrow 2 \times = 3$$

$$2y - 1 = 6 \longrightarrow 2y = 6 + 1 \longrightarrow 2y = \frac{17}{2} \longrightarrow y = \frac{17}{2}$$

$$= 7$$

$$= 7$$

$$5w = 15$$

$$= 15$$

$$= 15$$

$$= 15$$

25.
$$\begin{bmatrix} 6 & 3 \\ x & 7 \\ 3y & 2z \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 6 & 7 \\ 2 & 7 \end{bmatrix}$$

$$\mathcal{K} = 6$$

$$\frac{3y = 2}{3}$$

$$y = \frac{2}{3}$$

$$y = \frac{4}{2}$$

$$y = \frac{4}{2}$$

Transpose of a Matrix

If A is a matrix, the matrix formed from A by interchanging its rows with its columns is called the *transpose* of A.

Definition

The transpose of an $m \times n$ matrix A, denoted A^{T} , is the $n \times m$ matrix whose ith row is the ith column of A.

EXAMPLE 3 Transpose of a Matrix

If
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$
, find A^{T} .

Solution: Matrix A is 2×3 , so A^{T} is 3×2 . Column 1 of A becomes row 1 of A^{T} , column 2 becomes row 2, and column 3 becomes row 3. Thus,

$$A^{T} = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ \hline 3 & 6 \end{bmatrix} \longrightarrow \begin{bmatrix} A^{T} \neq \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \end{bmatrix}$$

Now Work Problem 19 ⊲

Observe that the columns of A^{T} are the rows of A. Also, if we take the transpose of our answer, the original matrix A is obtained. That is, the transpose operation has the property that

$$(A^{\mathrm{T}})^{\mathrm{T}} = A$$



In Problems 17–20, find A^T.

17.
$$A = \begin{bmatrix} 6 & -3 \\ \underline{2} & 4 \end{bmatrix}$$
 \longrightarrow $A^T = \begin{bmatrix} 6 & 2 \\ -3 & 4 \end{bmatrix}$

18.
$$A = \begin{bmatrix} 2 & 4 & 6 & 8 \end{bmatrix}$$
 \longrightarrow $A^{\top} = \begin{bmatrix} 2 & 4 & 6 & 8 \end{bmatrix}$