13.1 Relative Extrema

Objective

To find when a function is increasing or decreasing, to find critical values, to locate relative maxima and relative minima, and to state the first-derivative test. Also, to sketch the graph of a function by using the information obtained from the first derivative.

➢ Increasing or Decreasing Nature of a Function

- Examining the graphical behavior of functions is a basic part of mathematics and has applications to many areas of study.

- When we sketch a curve, just plotting points may not give enough information about its shape.

Example. The points $(-1, 0)$, $(0, -1)$ and $(1, 0)$ satisfy the equation given by

$$y = (x+1)^3(x-1)$$

On the basis of these points, we might hastily conclude that the graph should appear as in Figure 13.1(a), but in fact the true shape is given in Figure 13.1(b).

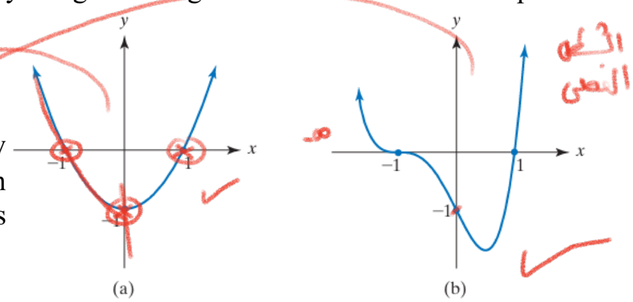


FIGURE 13.1 Curves passing through $(-1, 0)$, $(0, -1)$, and $(1, 0)$.

> **Definition**
>
> A function f is said to be **increasing** on an interval I when, for any two numbers x_1, x_2 in I, if $x_1 < x_2$, then $f(x_1) < f(x_2)$. A function f is **decreasing** on an interval I when, for any two numbers x_1, x_2 in I, if $x_1 < x_2$, then $f(x_1) > f(x_2)$.

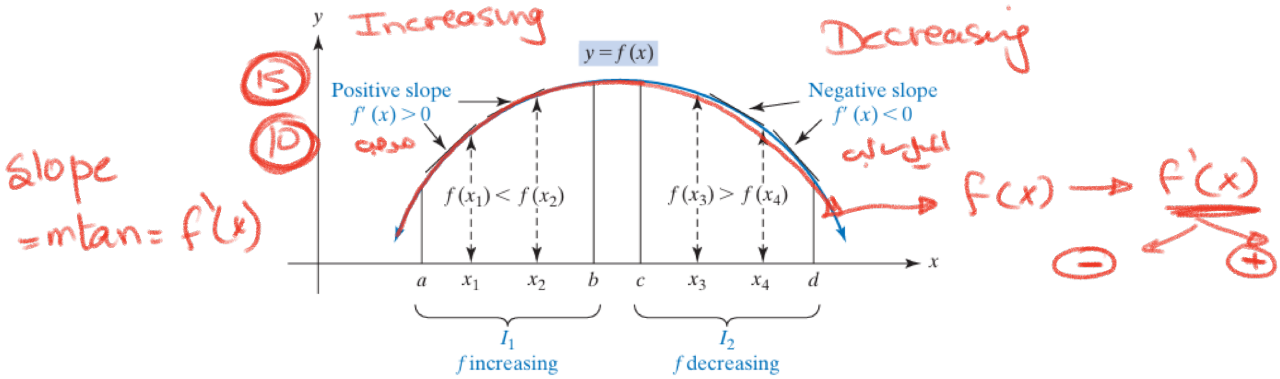


FIGURE 13.2 Increasing or decreasing nature of function.

The curve is falling where the derivative is negative. We, thus, have the following rule, which allows us to use the derivative to determine when a function is increasing or decreasing:

> **Rule 1 Criteria for Increasing or Decreasing Function**
>
> Let f be differentiable on the interval (a, b). If $f'(x) > 0$ (موجب) for all x in (a, b), then f is increasing on (a, b). If $f'(x) < 0$ (سالب) for all x in (a, b), then f is decreasing on (a, b).

- **Extrema**

> **Definition**
>
> A function f has a **relative maximum** at a if there is an open interval containing a on which $f(a) \geq f(x)$ for all x in the interval. The relative maximum value is $f(a)$. A function f has a **relative minimum** at a if there is an open interval containing a on which $f(a) \leq f(x)$ for all x in the interval. The relative minimum value is $f(a)$.

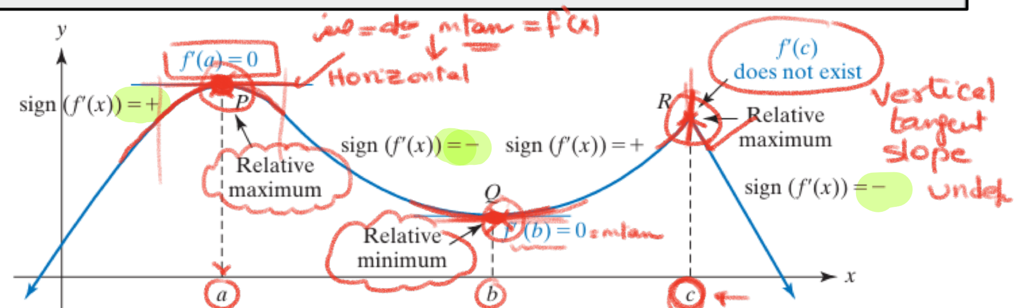


FIGURE 13.3 Relative Maxima and Relative Minima.

Note that there is a difference between relative extreme values and where they occur.

Increasing ⊕ → decreasing ⊖ → relative maximum
decreasing ⊖ → Increasing ⊕ → relative minimum

Rule 2 A Necessary Condition for Relative Extrema

If f has a relative extremum at a, then $f'(a) = 0$ or $f'(a)$ does not exist.

Max Min Horizontal tangent Vertical Tangent Exist

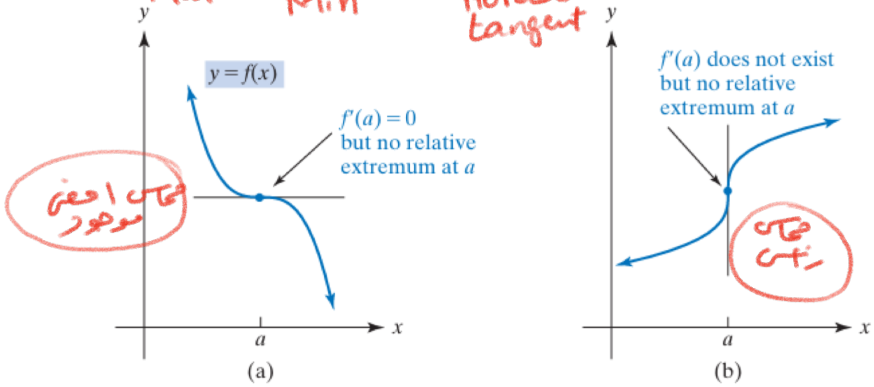


FIGURE 13.4 No Relative Extremum at a.

Definition

For a in the domain of f, if either $f'(a) = 0$ or $f'(a)$ does not exist, then a is called a **critical value** for f. If a is a critical value, then the point $(a, f(a))$ is called a **critical point** for f.

Rule 3 Criteria for Relative Extrema

Suppose f is continuous on an open interval I that contains the critical value a and f is differentiable on I, except possibly at a.

1. If $f'(x)$ changes from positive to negative as x increases through a, then f has a relative maximum at a.
2. If $f'(x)$ changes from negative to positive as x increases through a, then f has a relative minimum at a.

First-Derivative Test for Relative Extrema

Step 1. Find $f'(x)$.

Step 2. Determine all critical values of f (those a where $f'(a) = 0$ or $f'(a)$ does not exist) and any a that are not in the domain of f but that are near values in the domain of f, and construct a sign chart that shows for each of the intervals determined by these values whether f is increasing ($f'(x) > 0$) or decreasing ($f'(x) < 0$).

Step 3. For each critical value a at which f is continuous, determine whether $f'(x)$ changes sign as x increases through a. There is a relative maximum at a if $f'(x)$ changes from $+$ to $-$ going from left to right and a relative minimum if $f'(x)$ changes from $-$ to $+$ going from left to right. If $f'(x)$ does not change sign, there is no relative extremum at a.

Step 4. For critical values a at which f is not continuous, analyze the situation by using the definitions of extrema directly.

$\boxed{1}$ Given $f(x) = \checkmark$ → a) find the interval in which $f(x)$ increasing / Decreasing

b) find Maximum or Minimum point

Relative maxima or relative minima

Extremeum

c) find c.v

$\boxed{1}$ $f'(x)$

$\boxed{2}$ $f'(x) = 0$ $\boxed{c.v}$ وبعد ايجاد ← الكل

$\boxed{3}$ بعد وضع c.v على خط الاعداد وبعد اختبار الاشارات → Increasing / Decreasing

$\boxed{4}$ $+\ -$ → relative Max

$-\ +$ → relative Min

$+\ +$ / $-\ -$ NO Extremeum

determine where the function is **(a)** *increasing;* **b)** *decreasing; and* **(c)** *determine where relative extrema occur.*

$$y = 18x - \frac{2}{3}x^3$$

$\boxed{1}$ $y' = 18(1) - \frac{2}{\cancel{3}}(\cancel{3}x^2) = 18 - 2x^2$

$\boxed{2}$ $f'(x) = 0$

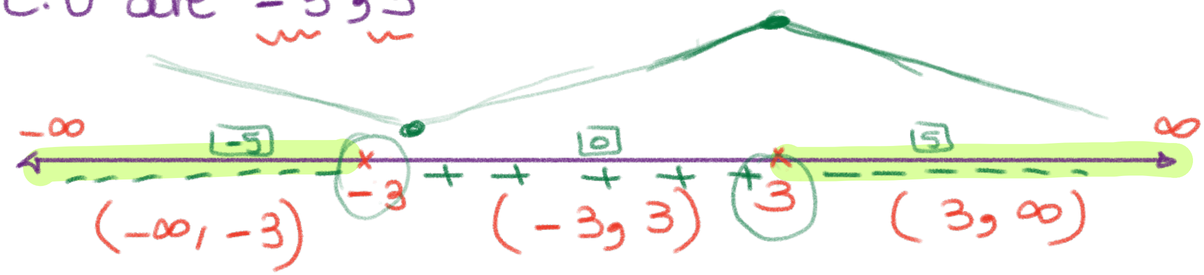
$18 - 2x^2 = 0$

$2[\underbrace{9}_{3^2} - x^2] = 0 \longrightarrow 2(3+x)(3-x) = 0$

$3 + x = 0$ or $3 - x = 0$

$x = -3$ $\qquad$ $x = 3$

C.V are $-3, 3$



	$(-\infty, -3)$	$(-3, 3)$	$(3, \infty)$
$+2$	$+$	$+$	$+$
$(3+x)$	$-$	$+$	$+$
$(3-x)$	$+$	$+$	$-$

✓ $(-\infty, -3) \longrightarrow$ decreasing $\ominus$

✓ $(-3, +3) \longrightarrow$ Increasing $\oplus$

✓ $(3, \infty) \longrightarrow$ decreasing $\ominus$

There is relative minimum at $x = -3$

There is relative Maximum at $x = 3$