P.2 REAL NUMBERS

الاعداد الحقيقية

- Real Numbers
- Properties of Real Numbers
- Addition and Subtraction
- Multiplication and Division
- The Real Line
- Sets and Intervals.
- Absolute Value and Distance

الحية NATURAL NUMBERS الطبيعية

The numbers that we use to count things, such as the number of books in a library .

Natural Numbers =N $= \{1, 2, 3, 4, 5, 6, 7, \dots\dots\dots.\}$

Not prime
Not composite

Natural Numbers greater than one

2,3,4,5,6,7,8,9…………

الاولية التي لاتقبل القسمة الا على نفسها

Prime

Divisible evenly only by 1 and itself

for example:

2,3,5,7,11,13,17, 19, 23, 29, ...

المركبة تقبل القسمة على رقم آخر

Composite

(not a prime number)

For example:

4,6,8,9,10,12

2.2, 3.2, 4.2, 3.3, 5.2, 6.2, 4.3

WHOLE NUMBERS

The Whole numbers include zero and the natural numbers.

$$W = \{0, 1, 2, 3, 4, 5, 6, \dots \dots \dots .\}$$

الاعداد الصحيحة INTEGERS

-ve   0   +ve

We also need numbers to measure temperature below zero or , in accounting, when a company incurs a loss.

Integers included negative integers, zero and positive integers(natural numbers).

an INTEGERS is rational number

$$Integers = Z = \{\ldots\ldots\ldots\ldots,\ -4,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ \ldots\ldots\ldots\ldots\}$$

صفر

Why we call it Z ?

Zahlen

Natural
whole
Integers تضم كل من $+$ 0 $-$

- + ← RATIONAL NUMBERS الاعداد النسبية

Rational Numbers $= \left\{\frac{p}{q}, where\ p\ and\ q\ are\ integers\ and\ q \neq 0\right\}$

$\neq 0$

اي عدد ليته وصعه على صورة كسر $\frac{p}{q}$ او رقم عشري منتهي 0.45

Examples:

$\frac{3}{4}, -\frac{5}{9}, \frac{8}{1}\ and\ \frac{2}{7}$

او رقم عشري غير منتهي لكنه مكرر
0.66666---

A rational number written as a fraction can be written as a decimal by dividing the numerator by the denominator. The result is either a terminating decimal such as $0.45 = \frac{9}{20}$ or a repeating decimal such as $\frac{12}{55} = 0.218181818\ldots = 0.2\overline{18}$

غير منتهي لكن مكررة

الاعداد الغير نسبية

IRRATIONAL NUMBERS

اعداد عشرية غير منتهية وغير مكررة

Numbers that are not rational numbers are called irrational numbers. In decimal form, an irrational numbers has a decimal representation that never terminates nor repeats.

Examples:

$$\pi = 3.145926 \ldots \ldots$$

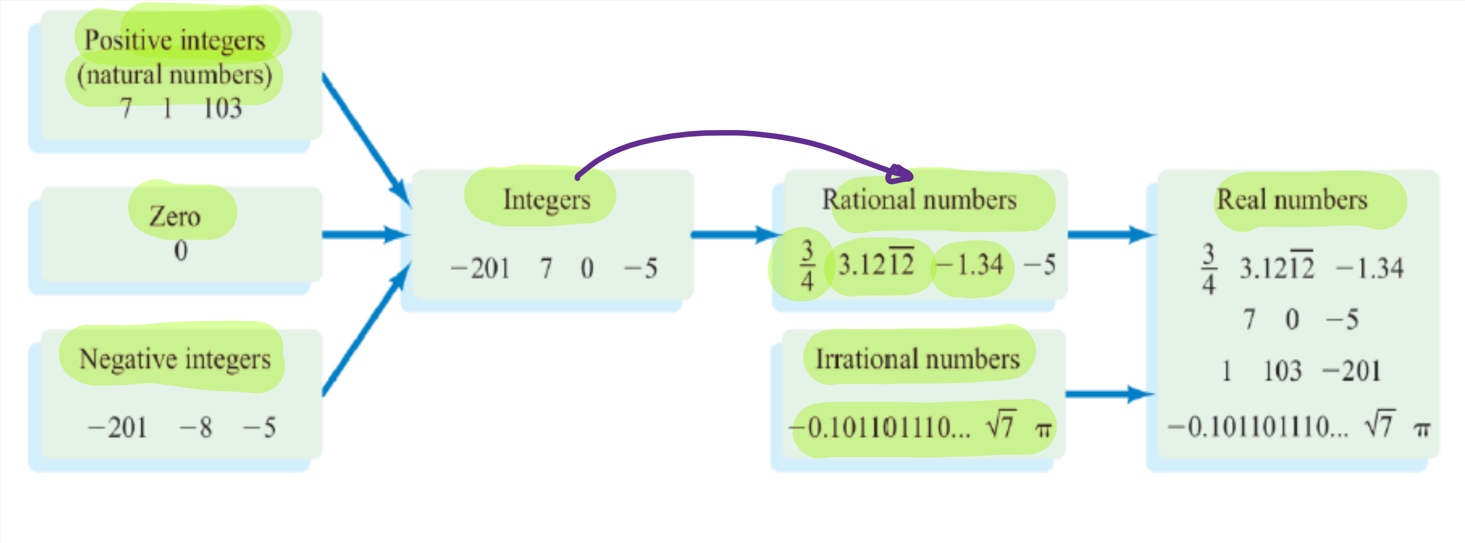
اي جذر ليس له نتيجة صحيحة
فهو غير نسبي

$$\sqrt{11} = 3.316 \ldots\ldots$$

غير منتهي وغير مكرر

$\sqrt{7}$ $\sqrt{5}$ $\sqrt{13}$

$$2.13113111311113 \ldots \ldots$$



Positive integers
(natural numbers)
7 1 103
Zero
0
Negative integers
−201 −8 −5
Integers
−201 7 0 −5
Rational numbers
3/4 3.12$\overline{12}$ −1.34 −5
Irrational numbers
−0.101101110... √7 π
Real numbers
3/4 3.12$\overline{12}$ −1.34
7 0 −5
1 103 −201
−0.101101110... √7 π

EXERCISE

Determine whether each number is an integer, a rational number, an irrational number, a prime numbers, or a real number.

$3*17$

$\frac{5}{\sqrt{7}}$	$\frac{5}{7}$	$-2\frac{1}{2}$	31	4.235653907493	51	π	0.888…..
				غير منتهي وغير مكرر Irrational			

ليس لصيه

Inlegers 31, 51

Rational 31, 51, $\frac{5}{7}$, $-2\frac{1}{5}$, $0.\dot{8}88$,

Irratiotial $\frac{5}{\sqrt{7}}$, 4.235653907493, π

Prime 31

EXERCISE

Determine whether each number is an integer, a rational number, an irrational number, a prime numbers, or a real number.

كل الاعداد
Real number

$\frac{-1}{5}$ R	Integer $\mathbf{0}$ rational	$\mathbf{44}$ Integer, rational	π Irrational
3.14 Rational	5.0500500050000 5 … … Irrational غير منتهي غير متكرر	$\sqrt{81} = 9$ Integer, rational	$\sqrt{3}$ Irrational
$\sqrt{-3}$. Imaginary	52 Integer	$\frac{-27}{6}$ Rational	$e \simeq 2.718281828$ Irrational
$2.\overline{76}$ rational	-1 Integer, rational	97 Integer, Rat	$-0.88888 \ldots \ldots$ Rational
3.9745618 … … . . Irrational.	-44 Integer rational	2 Integer Prime rational	$\frac{3}{4}$ Rational

EXERCISES

Which of the following numbers are prime numbers?

i. 39 → 13*3 xx
ii. 53 → prime
iii. 102 → 51*2 xx
iv. 97 → prime

PROPERTIES OF REAL NUMBERS

PROPERTIES OF REAL NUMBERS

Property	Example	Description
Commutative Properties (التبديل)		
$a + b = b + a$	$7 + 3 = 3 + 7$	When we add two numbers, order doesn't matter.
$ab = ba$	$3 \cdot 5 = 5 \cdot 3$	When we multiply two numbers, order doesn't matter.
Associative Properties (التجميع)		
$(a + b) + c = a + (b + c)$	$\underbrace{(2 + 4)}_{6} + 7 = 2 + \overbrace{(4 + 7)}^{11}$ $6 + 7 = 13$; $2 + 11 = 13$	When we add three numbers, it doesn't matter which two we add first.
$(ab)c = a(bc)$	$(3 \cdot 7) \cdot 5 = 3 \cdot (7 \cdot 5)$ $21 \times 5 = 105$; $3 \times 35 = 105$	When we multiply three numbers, it doesn't matter which two we multiply first.
Distributive Property (التوزيع)		
$a(b + c) = ab + ac$ $(b + c)a = ab + ac$	$2 \cdot (3 + 5) = 2 \cdot 3 + 2 \cdot 5$ $(3 + 5) \cdot 2 = 2 \cdot 3 + 2 \cdot 5$	When we multiply a number by a sum of two numbers, we get the same result as we get if we multiply the number by each of the terms and then add the results.

EXAMPLE 1 ■ Using the Distributive Property

(a) $2(x+3) = \underline{2} \cdot x + \underline{2} \cdot 3$ Distributive Property وزعت

$= \underline{2x} + \underline{6}$ Simplify سبط

(b) $(a + b)(x + y)$	$= (a + b)x + (a + b)y$	Distributive Property
	$= (ax + bx) + (ay + by)$	Distributive Property
	$= ax + bx + ay + by$	Associative Property of Addition

$(a+b)(x+y)$

$a(x+y)+b(x+y)$

$ax+ay+bx+by$

ADDITION AND SUBTRACTION

The number 0 is special for addition; it is called the **additive identity** because $a + 0 = a$ for any real number a. Every real number a has a **negative**, $-a$, that satisfies $a + (-a) = 0$. **Subtraction** is the operation that undoes addition; to subtract a number from another, we simply add the negative of that number. By definition

$$a - b = a + (-b)$$

To combine real numbers involving negatives, we use the following properties.

الضرب والقسمة

$- \;\; -$

$+$

PROPERTIES OF NEGATIVES

Property	Example
1. $(-1)a = -a$	$(-1)5 = -5$
2. $-(-a) = a$	$-(-5) = 5$
3. $(-a)b = a(-b) = -(ab)$	$\overset{-35}{(-5)7} = \overset{-35}{5(-7)} = \overset{-35}{-(5 \cdot 7)}$
4. $(-a)(-b) = ab$	$(-4)(-3) = 4 \cdot 3$
5. $-(a + b) = -a - b$	$-(3 + 5) = -3 - 5$
6. $-(a - b) = b - a$	$-(5 - 8) = 8 - 5$

$-a+b \longrightarrow b-a$

EXAMPLE 2 ■ Using Properties of Negatives

Let x, y, and z be real numbers.

(a) $-(x + 2) = -x - 2$ Property 5: $-(a + b) = -a - b$

$-x-2$

(b) $-(x + y - z) = -x - y - (-z)$ Property 5: $-(a + b) = -a - b$

$-x-y+z$

$= -x - y + z$ Property 2: $-(-a) = a$

Multiplication and Division

محايد ضربي

The number 1 is special for multiplication; it is called the **multiplicative identity** = 1 because $a \cdot 1 = a$ for any real number a. Every nonzero real number a has an **inverse**, $1/a$, that satisfies $a \cdot (1/a) = 1$. **Division** is the operation that undoes multiplication;

اي رقم

المعكوس الضربي

$\frac{2}{1} * \frac{1}{2} = 1 \qquad \frac{2}{3} * \frac{3}{2} = 1$

to divide by a number, we multiply by the inverse of that number. If $b \neq 0$, then, by definition,

$$a \div b = a \cdot \frac{1}{b}$$

$$a \div b \quad * \; 1/b$$

We write $a \cdot (1/b)$ as simply a/b. We refer to a/b as the **quotient** of a and b or as the **fraction** a over b; a is the **numerator** and b is the **denominator** (or **divisor**). To combine real numbers using the operation of division, we use the following properties.

$$3 \div \frac{2}{5} = 3 * \frac{5}{2}$$

$$4 \div \frac{3}{1} = 4 * \frac{1}{3}$$

PROPERTIES OF FRACTIONS

Property	Example	Description
1. $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$	$\frac{2}{3} \cdot \frac{5}{7} = \frac{2 \cdot 5}{3 \cdot 7} = \frac{10}{21}$	When **multiplying fractions**, multiply numerators and denominators.
2. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$	$\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \cdot \frac{7}{5} = \frac{14}{15}$	When **dividing fractions**, invert the divisor and multiply.
3. $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$	$\frac{2}{5} + \frac{7}{5} = \frac{2+7}{5} = \frac{9}{5}$	When **adding fractions** with the **same denominator**, add the numerators.
4. $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$	$\frac{2}{5} + \frac{3}{7} = \frac{\overset{14}{2 \cdot 7} + \overset{15}{3 \cdot 5}}{35} = \frac{29}{35}$	When **adding fractions** with **different denominators**, find a common denominator. Then add the numerators.
5. $\frac{ac}{bc} = \frac{a}{b}$	$\frac{2 \cdot \cancel{5}}{3 \cdot \cancel{5}} = \frac{2}{3}$	**Cancel** numbers that are **common factors** in numerator and denominator.
6. If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$ $\boxed{ad = bc}$	$\frac{2}{3} = \frac{6}{9}$, so $2 \cdot 9 = 3 \cdot 6$	**Cross-multiply**.

$2 * 9 = 3 * 6$
$18 = 18$

EXAMPLE 3 ■ Using the LCD to Add Fractions

Evaluate: $\dfrac{5}{36} + \dfrac{7}{120}$

$$\begin{array}{r|l} 2 & 120 \\ 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$

SOLUTION Factoring each denominator into prime factors gives

$$36 = 2^2 \cdot 3^2 \quad \text{and} \quad 120 = 2^3 \cdot 3 \cdot 5$$

We find the least common denominator (LCD) by forming the product of all the prime factors that occur in these factorizations, using the highest power of each prime factor. Thus the LCD is $2^3 \cdot 3^2 \cdot 5 = 360$. So

$2^3 = 8$, $3^2 = 9$, 72, 5

$$\frac{5}{36} + \frac{7}{120} = \frac{5 \cdot 10}{36 \cdot 10} + \frac{7 \cdot 3}{120 \cdot 3} \qquad \text{Use common denominator}$$

$$= \frac{50}{360} + \frac{21}{360} = \frac{71}{360} \qquad \text{Property 3: Adding fractions with the same denominator}$$

$\frac{360}{36} = 10$

$\frac{360}{120} = 3$

$$\frac{5 * 10}{36 * 10} + \frac{7 * 3}{120 * 3}$$

EXERCISES

2. Complete each statement and name the property of real numbers you have used.

(a) $ab = $ ba ; ________ Property Commutative

(b) $a + (b + c) = $ (a+b)+c ________ Property Associative property

(c) $a(b + c) = $ ab+ac ; ________ Property Distributive

9–10 ■ Real Numbers List the elements of the given set that are

(a) natural numbers → $\{1, 2, 3, 4, ---\}$

(b) integers $\{---\ -2, -1, 0, 1, 2, ----\}$

(c) rational numbers a/b, 0.45, $0.888---$

(d) irrational numbers $\sqrt{7}$, $\sqrt{5}$, π

9. $\{-1.5, \underline{0}, \frac{5}{2}, \sqrt{7}, 2.71, -\pi, 3.1\overline{4}, \underline{100}, -8\}$

10. $\{1.3, 1.3333\ldots, \sqrt{5}, 5.34, -500, 1\frac{2}{3}, \sqrt{16}, \frac{246}{579}, -\frac{20}{5}\}$

9) Natural number 100

Integer $0, 100, -8$

rational $-1.5, 0, \frac{5}{2}, 2.71, 3.1\overline{4}, 100, -8$

Irrational $\sqrt{7}, -\pi$

19–22 ■ Properties of Real Numbers Rewrite the expression using the given property of real numbers.

19. Commutative Property of Addition, $x + 3 =$ $3+x$

20. Associative Property of Multiplication, $7(3x) =$ $(7*3)x$

21. Distributive Property, $4(A + B) =$ $4A+4B$

22. Distributive Property, $5x + 5y =$ $5(x+y)$

EXERCISE

Perform the indicated operations

31) $\frac{2}{3}\left(6-\frac{3}{2}\right) = \frac{2}{\cancel{3}_1} * \overset{2}{\cancel{6}} + \frac{2}{\cancel{3}} * \frac{-\cancel{3}}{\cancel{2}}$

$= 2*2 - 1$

$= 4 - 1$

$= 3$

$\frac{2}{3} * \frac{3}{2} = 1$

32) $\frac{2}{\frac{2}{3}} - \frac{\frac{2}{3}}{3} = \overbrace{2 \div \frac{2}{3}} - \overbrace{\frac{2}{3} \div 3}$

$= \cancel{2} * \frac{3}{\cancel{2}} - \frac{2}{3} * \frac{1}{3}$

$= \frac{3}{1} - \frac{2}{9} = \frac{3.9 - 2.1}{9} = \frac{27-2}{9} = \frac{25}{9}$