Chapter 9 : Introduction to Hypothesis Testing

Learning Outcomes

9-1

جرب الكتاب

Outcome 1. Formulate null and alternative hypotheses for applications involving a single population mean or proportion. ← 9.2

Outcome 2. Know what Type I and Type II errors are.

Outcome 3. Correctly formulate a decision rule for testing a hypothesis.

Outcome 4. Know how to use the test statistic, critical value, and p-value approaches to test a hypothesis.

9-2 ✓

Outcome 5. Formulate null and alternative hypothesis for applications involving a single population proportion.
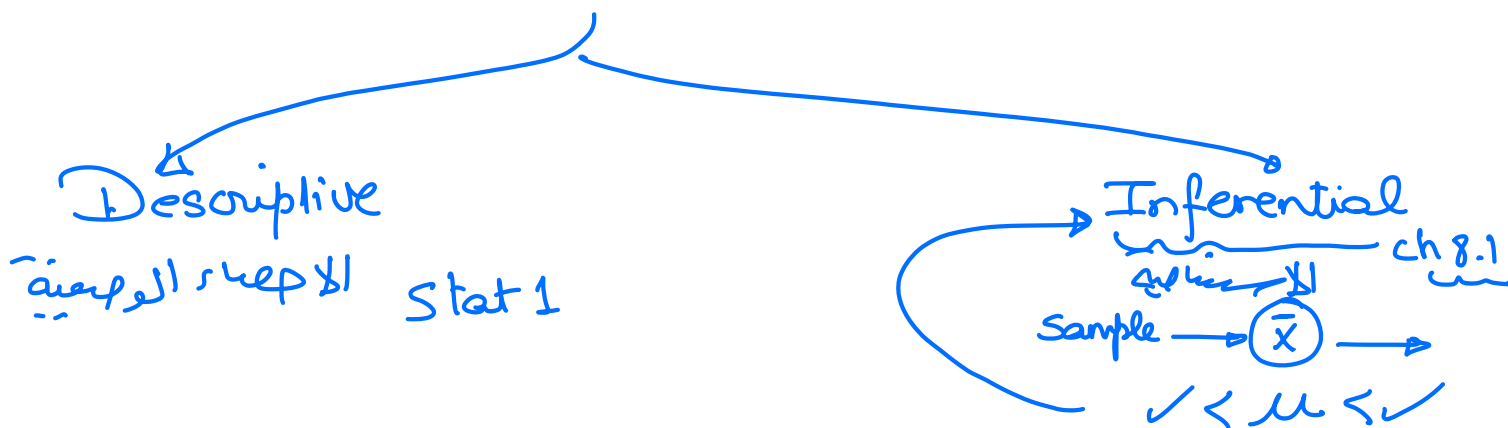
9-3 #

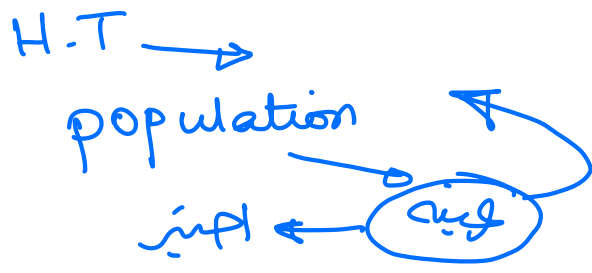**Outcome 6.** Compute the probability of a Type II error. ← Q_1 Midterm

WHY YOU NEED TO KNOW

Estimating a population parameter based on a sample statistic is one area of business statistics called *statistical inference*. The basic tools for estimation were introduced in Chapter 8. Another important application of statistical inference is *hypothesis testing*. In hypothesis testing, we make a hypothesis (or statement) concerning a population parameter. We then use sample data to either deny or confirm the validity of the proposed hypothesis.

Descriptive
الاحصاء الوصفي   Stat 1

Inferential — ch 8.1
الاحصاء الاستدلالي
Sample → $\bar{X}$ →
√ ≤ μ ≤ √

H.T →

population ← عينة

عينة → مجتمع

One pop
9

11.1

(قسم الطالبات)

< 50%

Two pop
ch 10 & 11.2

متغير الطالبات   مهم الطبيب
↓   ↓

more than
2 pop

12.1

A & R
B
MIS

H.T

✓ mean
✓ proportion
✓ S.D

Stat ①
Ho & Hα
parameter   Statistic
↓   ↓
pop   Sample
mean   (x̄)
(μ)   X X X

✓ Statistics

Business
MIS
HR

CS

الطب العلوم
الصحية
Biostatistics

هندسة

Engineering
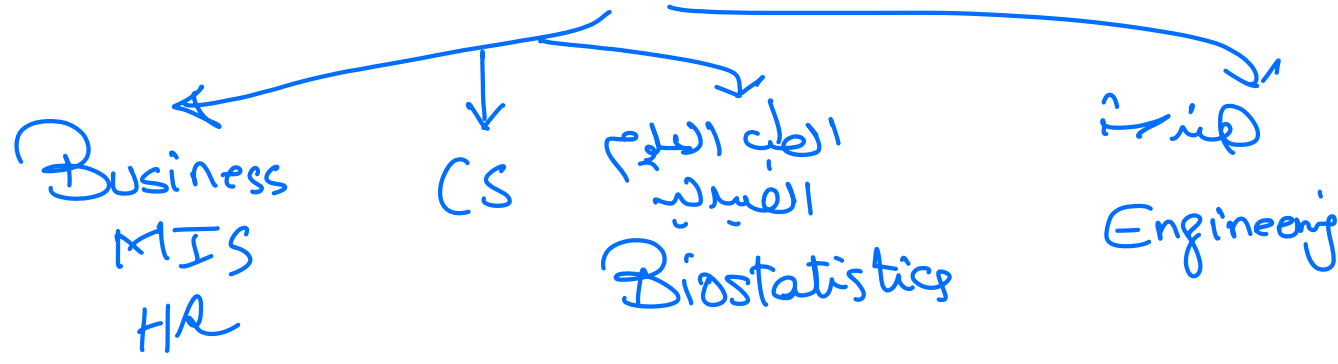
one population

Hypothesis test for population mean

الإحصاء الاستدلالي

\# Hypothesis testing is an application of statistical inference.

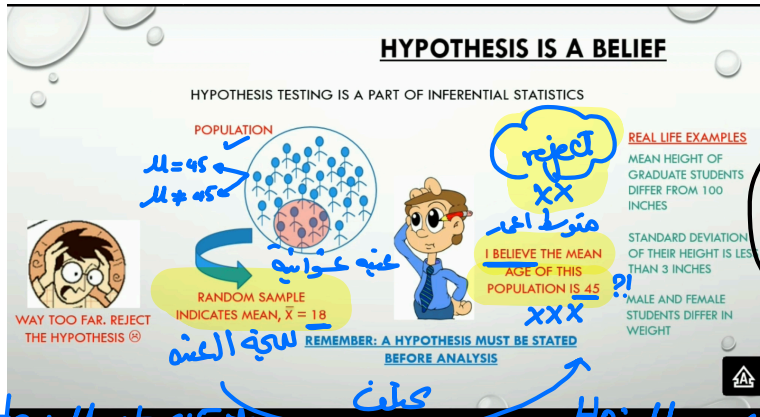\# Is performed in many industries.
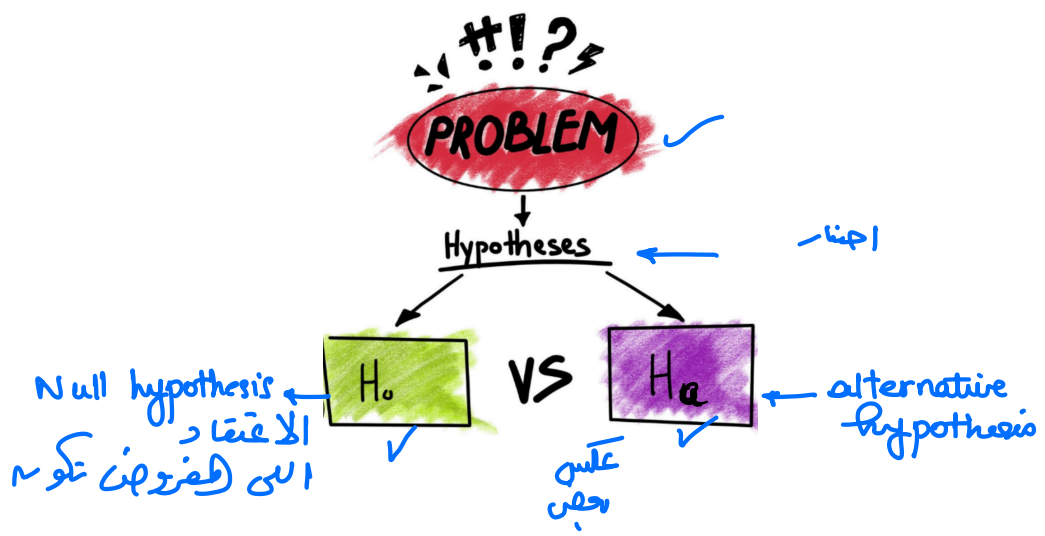
\# Is a major part of business statistics.

\# Provide managers with a structured analytical method for making decisions about population means, proportions, as well as comparing different populations

هي عبارة عن اختبارات مرضية يتم اجرائها في حالة الشك أو التكوى (claim)
وهي عبارة عن تطبيقه احصائي يتم استخدامه في الجالات الصناعية والمؤسسية والطبية
..... الخ و بناءً عليه يتم اتخاذ قرارات معينة من المدراء و حسنن القرار

HYPOTHESIS IS A BELIEF

HYPOTHESIS TESTING IS A PART OF INFERENTIAL STATISTICS

POPULATION
$\mu = 45$
$\mu \neq 45$
reject XX متوسط اعمار
عينة عشوائية
I BELIEVE THE MEAN AGE OF THIS POPULATION IS 45
XXX
RANDOM SAMPLE INDICATES MEAN, $\bar{x} = 18$
سنة العينة
WAY TOO FAR. REJECT THE HYPOTHESIS ☹
REMEMBER: A HYPOTHESIS MUST BE STATED BEFORE ANALYSIS

REAL LIFE EXAMPLES
MEAN HEIGHT OF GRADUATE STUDENTS DIFFER FROM 100 INCHES
STANDARD DEVIATION OF THEIR HEIGHT IS LESS THAN 3 INCHES
MALE AND FEMALE STUDENTS DIFFER IN WEIGHT

رفض reject Ho
$H_0: \mu = 45 \; xxxxx$
$H_a: \mu \neq 45$ accept Ha
true
قبول
دليلنا $\bar{x} = \boxed{18}$

$H_a: \mu \neq 45$ years كلف مع الاعتبار
$H_0: \mu = 45$ years

?!#?
PROBLEM

↓

Hypotheses ← احنا

↓ ↓

Null hypothesis → H_0 VS H_a ← alternative hypothesis
الاعتقاد الى الفروض تكون
عكس نحن

based on population that is normally distributed

Hypothesis

[H0&Ha combine to cover all possible values of pop]

Null hypothesis

H_0

Blood Pressure is 120/80

The GPA of any student could be more than or equal 2 — $GPA \geqslant 2$

The age of freshman student is less than or equal 21 — $Age \leqslant 21$

alternative hypothesis Ha

Blood Pressure is NOT 120/80

The GPA of any student could be less than 2 — $GPA < 2$

The age of freshman student is more than 21 — $Age > 21$

(Note)

Ho علامة المساوى دائماً عند ال Ho

Equal sign is always at (Ho)

Ho:
Null hypothesis:. The statement about the population parameter that will be assumed to be true during the conduct of the hypothesis test. The null hypothesis will be rejected If the sample data provide evidence.

Ha:
Alternative hypothesis: includes all population values not included in the null hypothesis. it will be selected if there is strong evidence to support it. Ha is deemed to be true if Ho is rejected

Ex8. If the sample mean is inconsistent (نتعارض)
with Ho about the population mean → Reject Ho
and accept (don't reject) Ha
null hypothesis الـ مع نتعارض وكانت عينة أخذ حالة في

Ex8 ^{XX} Ho: $\mu = 45$ → sample $\bar{X} = 18$ → Reject Ho
Ha: $\mu \neq 45$ and accept Ha

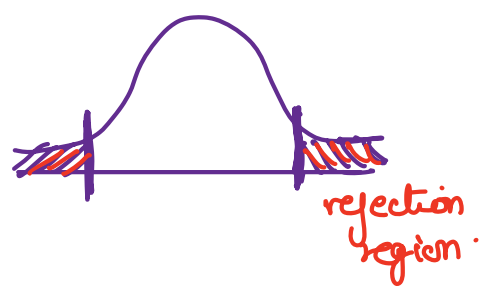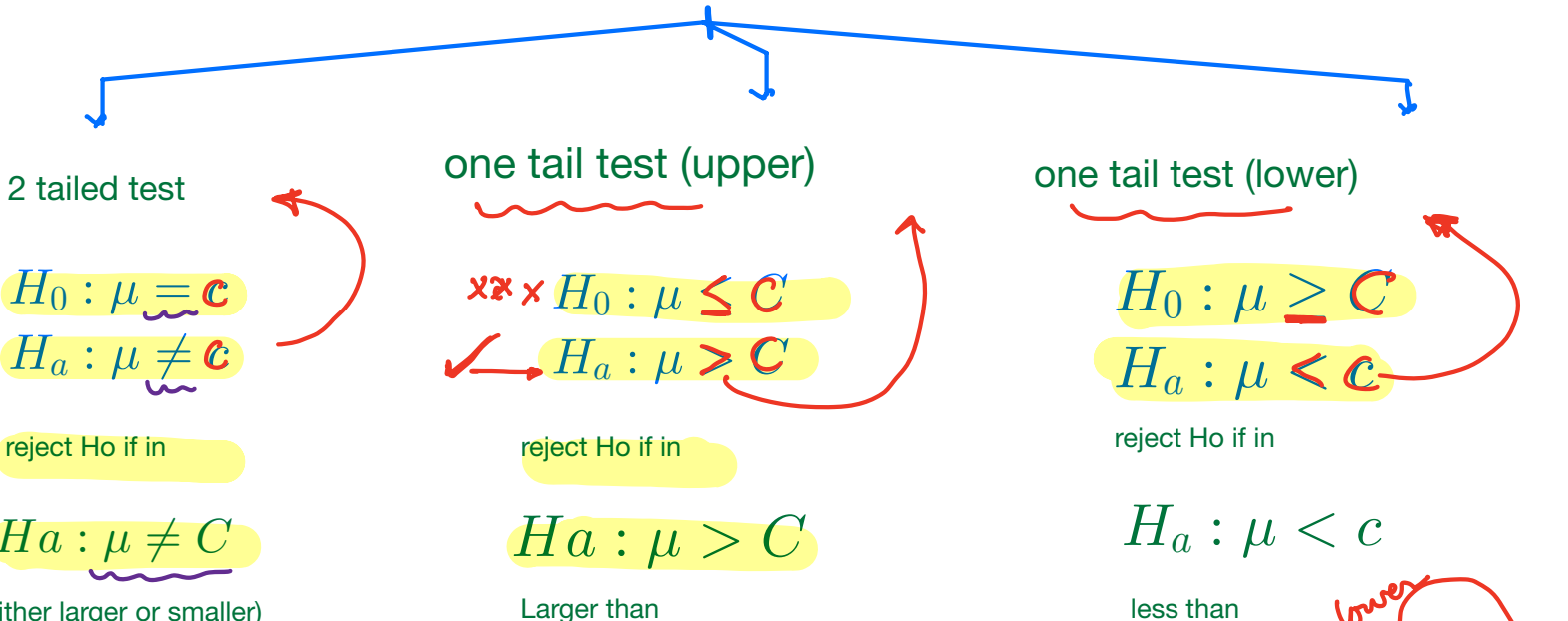هذا التعارض يوضح أن العينة مختلفة عن ال pop. كما ينبه الى وجود خطأ

By inconsistent, we mean that $\bar{X}$ is enough different
from μ, the difference cannot be attributed to
sampling error

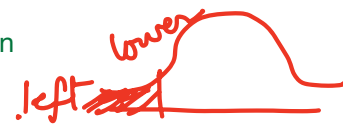# How to formulate Ho & Ha for the population mean?

It depends on hypothesis type

تحقق من نوع/منشأ

μ

2 tailed test

$$H_0 : \mu = c$$
$$H_a : \mu \neq c$$

reject Ho if in

$$Ha : \mu \neq C$$

(either larger or smaller)

rejection region

one tail test (upper)

xx x $H_0 : \mu \leq C$

✓ $H_a : \mu > C$

reject Ho if in

$$Ha : \mu > C$$

Larger than

upper right

1 Kg

$\mu = 1Kg$
$\mu \neq 1Kg$

عنبه ← وزنها ← متوسط الوزن

$\mu = 900 g$

one tail test (lower)

$$H_0 : \mu \geq C$$
$$H_a : \mu < c$$

reject Ho if in

$$Ha : \mu < c$$

less than

lower left

xxx reject Ho

$\mu = 1Kg$ xx
$\mu \neq 1Kg$ xx

Important notes

=	$\leq$	$\geq$
$\neq$	$>$	$<$

• علامة الساوى داعياً عند Ho

• الاشارات داعياً عكس بعض

• الرقم الثابت واحد داعيا في الـ Ho & HA

hypothesis test always based on population not sample

Ho : μ ✓   Ho : $\bar{x}$
Ha : μ ✓   Ha : $\bar{x}$ x xxx

Quiz ① (y)

Examples: Formulate the Hypothesis

$\mu = 16$

1) <u>Testing the Status Quo</u> : The box of Kellogg's cereal has a mean fill of 16 ounces.

$$Ho: \mu = 16 \text{ ounces}$$
$$Ha: \mu \neq 16 \text{ ounces}$$

2) <u>Testing a Research Hypothesis:</u> Goodyear's tire will last longer than 60,000 miles on average.

$\mu > 60,000 \text{ miles}$
↑ ha
الحوجة بالكلام سياسي

$$Ho: \mu \leq 60,000 \text{ miles}$$
$$(\text{claim}) \; Ha: \mu > 60,000 \text{ miles}$$

3) <u>Testing a claim</u> about a population: A radio commercial stated the average waiting time at a medical clinic is less than 15 minutes.

< 15

$$Ho: \mu \geq 15 \text{ mins}$$
$$Ha: \mu < 15 \text{ mins}$$

① A city mayor claims that the mean time spent by people in dow town parking spaces exceeds 33 mins

μ

$>$

exceeds $>$

$$Ho: \mu \leq 33 \text{ mins}$$
$$Ha: \mu > 33 \text{ mins}$$

② The department of agriculture has stated that average annual rainfall in a particular county is at least 23 inches

μ — على

at least $\geq$
على الاقل

عدد الاقل $\geq$

$$Ho: \mu \geq 23 \text{ inches}$$
$$Ha: \mu < 23 \text{ inches}$$

③ The company that bottles a popular soft drink has set the filling machine so that the mean contents per bottle is equal to 12 ounces

$$H_0: \mu = 12 \text{ ounces}$$
$$H_a: \mu \neq 12 \text{ ounces}$$

④ A customer who buys pipe fitting from a supplier claims that the mean diameter of 2nd pipe is not equal to 2"

$$H_0: \mu = 2''$$
$$H_a: \mu \neq 2''$$

⑤ A pharmaceutical company has produced a new drug to treat hurt burn. The company beleives that the drug will stop hurt burn in less than 15 mins

$$H_0: \mu \geq 15 \text{ mins}$$
$$H_a: \mu < 15 \text{ mins}$$

Note:
The claim always at Ha [upper & lower tail]
The statement always at Ho [upper & lower tail
The status quo always at Ho [two tailed test]

Exercises:

Determine whether each of the following pairs of hypotheses is valid for a hypothesis test. Explain reasons for any pair that is indicated to be invalid

Valid ✓ صالح

Invalid ✗

a) $H_0: \mu = 15$, $H_a: \mu > 15$ Invalid

≠

b) $H_0: \mu = 20$, $H_a: \mu \neq 20$ valid

$H_0: \mu = 45$ ✓
$H_a: \mu \neq 40$ Invalid
قيم

c) $H_0: \mu \leq 30$, $H_a: \mu > 30$ Invalid

أحد يساوى عند H_0

7

d) $H_0: \mu \leq 40$, $H_a: \mu \geq 40$ Invalid

✗✗✗ مكررة

e) $H_0: \bar{x} \leq 45$, $H_a: \bar{x} > 45$ Invalid

μ μ

Today's Goal

✔ Understand the difference between HO&Ha

✔ How to formulate HO&Ha

Quiz ✓