FUNCTIONS

الدوال

- A **function** is a relation in which each element of the domain is paired with *exactly one* element of the range. Another way of saying it is that there is one and only one output (y) with each input (x).

FUNCTION NOTATION

$$y = f(x)$$

Output

Name of Function

Input

$x = س$

علاقة

الأس

Exponent

(base)

الاساس (القاعدة) → $a > 0$

$\neq 1$

4.1 # Exponential Functions

الدوال الاسية

- A polynomial function has the basic form: $f(x) = x^3$
- An exponential function has the basic form: $f(x) = 3^x$

Exponent Variable

$a^x \quad b^x$ → base

- An exponential function has the variable in the exponent, not in the base.
- General Form of an Exponential Function:

EXPONENTIAL FUNCTIONS

The **exponential function with base *a*** is defined for all real numbers x by

$$f(x) = a^x = b^x$$

where $a > 0$ and $a \neq 1$.

base

We assume that $a \neq 1$ because the function $f(x) = 1^x = 1$ is just a constant function. Here are some examples of exponential functions:

$$f(x) = 2^x \qquad g(x) = 3^x \qquad h(x) = 10^x$$

Base 2   Base 3   Base 10

① Evaluate

② Graph

Properties of Exponents

خصائص الا

- Simplify: $\left(\frac{2}{3}\right)^{-3} = \frac{2^{-3}}{3^{-3}} = \frac{3^3}{2^3} = \frac{27}{8}$

$$\frac{3^7}{3^9} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$7-9$

$$\left(2^{1/2}\right)\left(8^{1/2}\right) = (2 \cdot 8)^{1/2} = 16^{1/2} = \sqrt{16} = 4$$

$$\# \; 3^2 \cdot 3^{-5} = 3^{2+-5} = 3^{2-5} = \frac{3^{-3}}{1} = \frac{1}{3^3} = \frac{1}{27}$$

EXAMPLE 1 ■ Evaluating Exponential Functions

Let $f(x) = 3^x$, and evaluate the following:

(a) $f(5)$ **(b)** $f(-\frac{2}{3})$

(c) $f(\pi)$ **(d)** $f(\sqrt{2})$

SOLUTION We use a calculator to obtain the values of f.

$f(x) = 3^x$

a] $f(5) = 3^5 = 3.3.3.3.3 = 243$ $\quad 81 \times 3 = 243$

b] $f(-\frac{2}{3}) = 3^{-2/3} = \frac{1}{3^{2/3}} \simeq 0....$

c) $f(\pi) = 3^{\pi}$

d) $f(\sqrt{2}) = 3^{\sqrt{2}}$

7–10 ■ Evaluating Exponential Functions Use a calculator to evaluate the function at the indicated values. Round your answers to three decimals.

7. $f(x) = 4^x$; $f(\frac{1}{2}), f(\sqrt{5}), f(-2), f(0.3)$ ✓

~~**8.**~~ $f(x) = 3^{x-1}$; $f(\frac{1}{2}), f(2.5), f(-1), f(\frac{1}{4})$

9. $g(x) = (\frac{1}{3})^{x+1}$; $g(\frac{1}{2}), g(\sqrt{2}), g(-3.5), g(-1.4)$ ✓

7] $f(1/2) = 4^{1/2} = \sqrt{4} = 2$

$f(\sqrt{5}) = 4^{\sqrt{5}}$

$f(-2) =$

$f(0.3) =$

8) $f(x) = 3^{x-1}$

a) $f(\frac{1}{2}) = 3^{\frac{1}{2}-1} = 3^{-\frac{1}{2}} = \frac{1}{3^{\frac{1}{2}}} = \frac{1}{\sqrt{3}} * \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{3}}{3}$

b) $f(2.5) = 3^{2.5-1} = 3^{1.5}$

✓c) $f(-1) = 3^{-1-1} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

d) $f(\frac{1}{4}) = 3^{\frac{1}{4}-1} = 3^{-3/4} = \frac{1}{3^{3/4}}$

Exponent

1. The function $f(x) = 5^x$ is an exponential function with base

base

$\underline{5}$; $f(-2) = \underline{5^{-2} = \frac{1}{5^2} = \frac{1}{25}}$, $f(0) = \underline{5^0 = 1}$,

$f(2) = \underline{5^2 = 25}$, and $f(6) = \underline{5^6 = 5.5.5.5.5.5}$

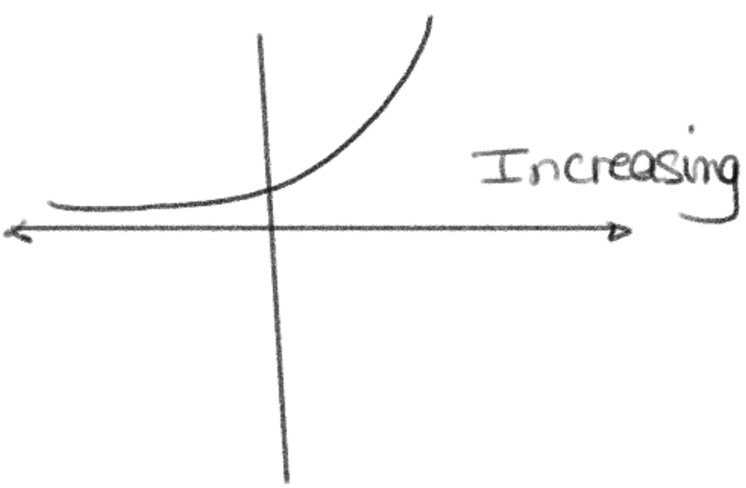
$x = 2$

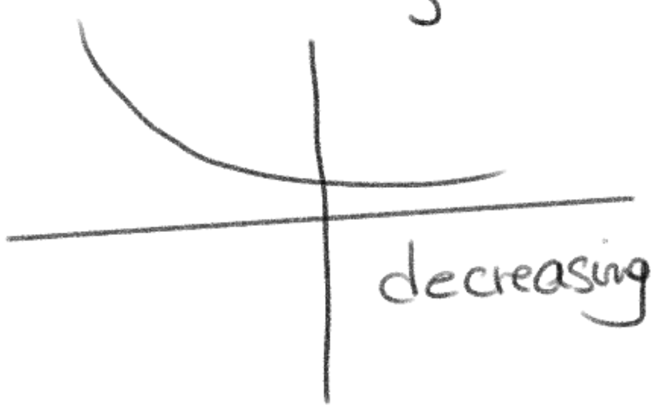
$\boxed{125 \times 125} =$ ✓

$\# \quad f(x) = a^x \qquad a > 0 \quad a \neq 1$

Graph

base $\rightarrow a > 1$	$0 < a < 1$
Ex $f(x) = 2^x$	$f(x) = \frac{1}{2}^x$
$= 3^x$	$= \frac{2}{3}^x$
$= 10^x$	$= \frac{1}{5}^x$
$= 2.5^x$	



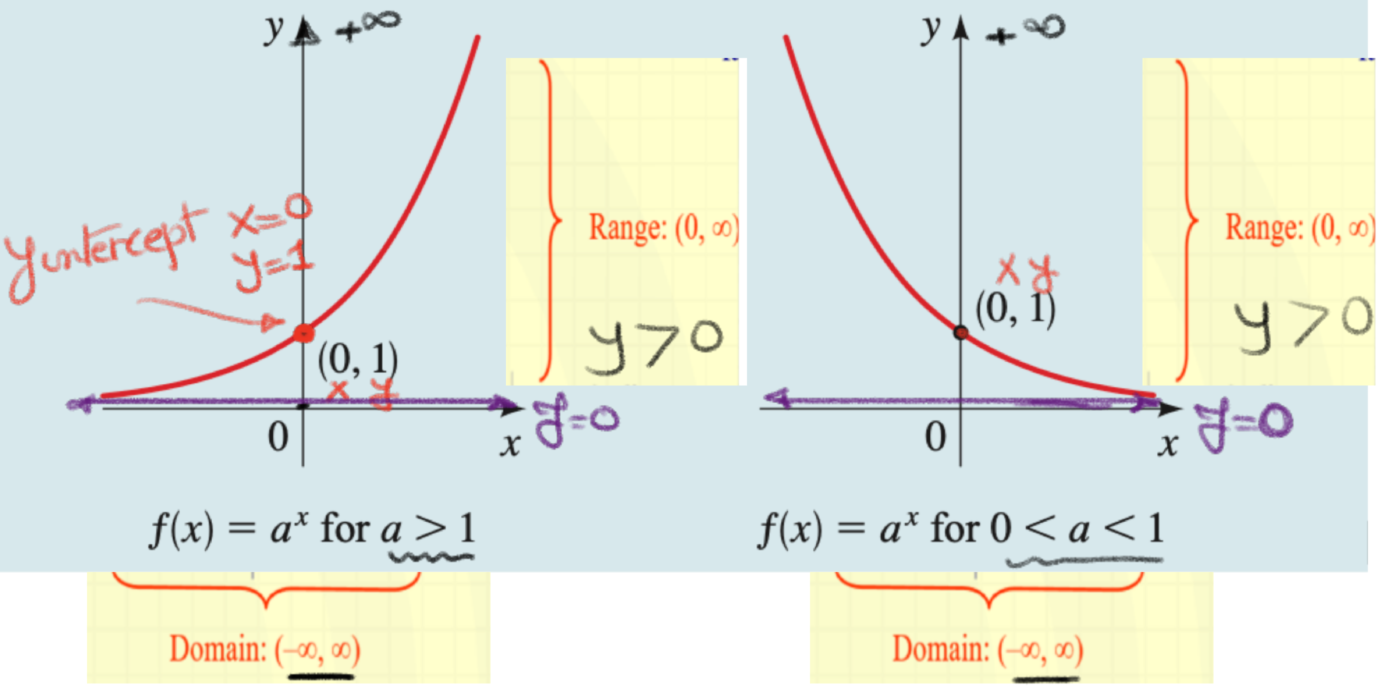


GRAPHS OF EXPONENTIAL FUNCTIONS

The exponential function

$$f(x) = a^x \qquad a > 0, a \neq 1$$

has domain $\mathbb{R}$ and range $(0, \infty)$. The line $y = 0$ (the x-axis) is a horizontal asymptote of f. The graph of f has one of the following shapes.



$f(x) = a^x$ for $a > 1$

$f(x) = a^x$ for $0 < a < 1$

- Exponential functions with positive bases greater than 1 have graphs that are increasing.

Exponential functions with positive base $0 < a < 1 \rightarrow$ have graphs that are decreasing

Horizontal Asymptote $\longrightarrow$ at $y=0$

Graph

① Substitution

$x = \checkmark \longrightarrow y = \checkmark$

-3
-2
-1
0
1
2
3

② Key points Graph

EXAMPLE 2 ■ Graphing Exponential Functions by Plotting Points

Draw the graph of each function.

$3 > 1 \longrightarrow$

(a) $f(x) = 3^x$

x	$f(x) = 3^x$
-3	$\frac{1}{27}$
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9
3	27

$3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

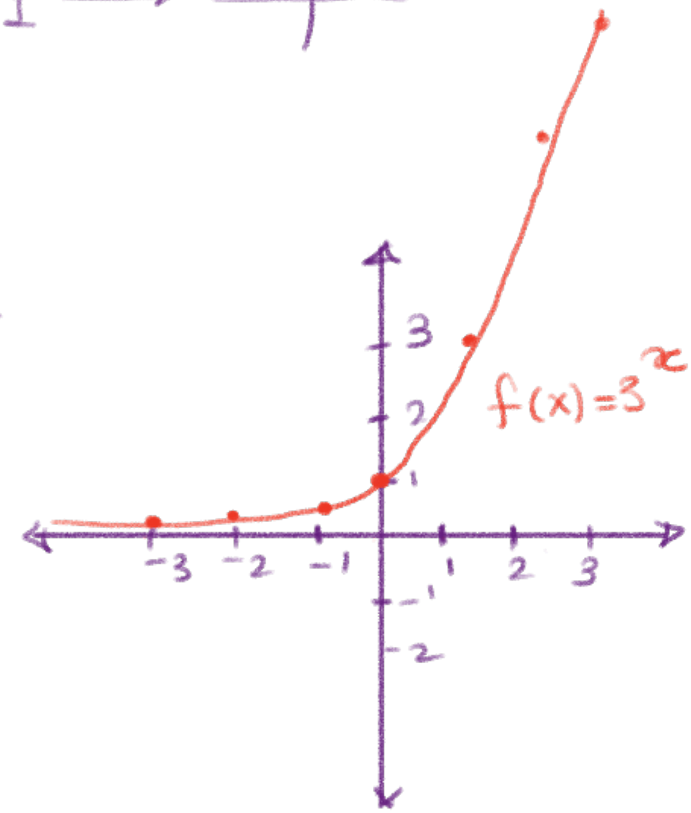
$3^{-1} = \frac{1}{3^1} = \frac{1}{3}$

$3^0 = 1$

$3^1 = 3$

$3^2 = 9$

$3^3 = 27$



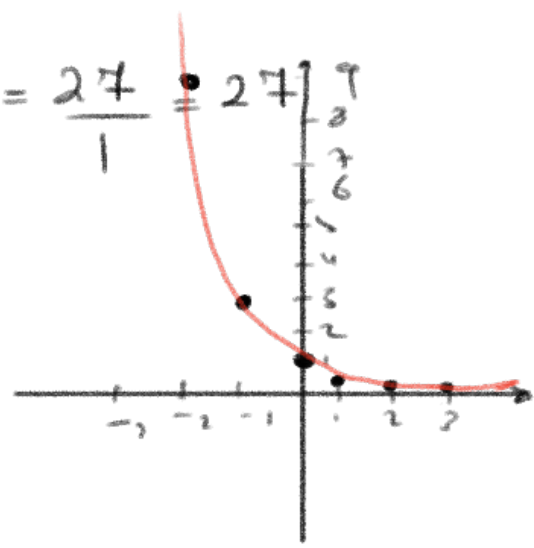
(b) $g(x) = \left(\frac{1}{3}\right)^x \longrightarrow 0 < \frac{1}{3} < 1$

x	$g(x) = \left(\frac{1}{3}\right)^x$
-3	27
-2	9
-1	3
0	1
1	$\frac{1}{3}$
2	$\frac{1}{9}$
3	$\frac{1}{27}$

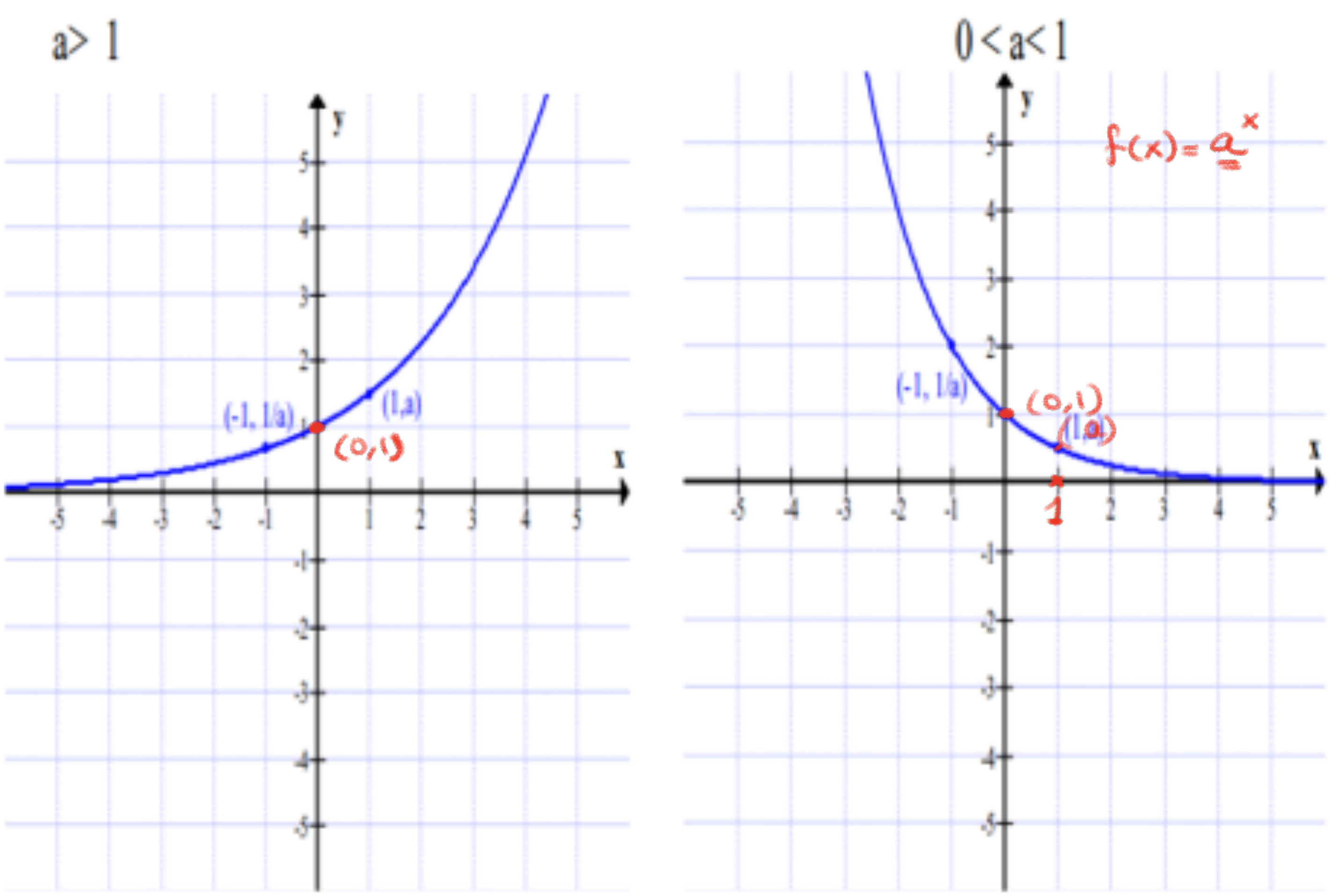
$\left(\frac{1}{3}\right)^{-3} = \left(\frac{1}{3^{-3}}\right) = \frac{3^3}{1^3} = \frac{27}{1} = 27$

$\left(\frac{1}{3}\right)^0 = 1$

$\left(\frac{1}{3}\right)^2 = \frac{1^2}{3^2} = \frac{1}{9}$



Points on the graph method



Points on the graph: (-1, 1/a), (0,1), (1,a)

Keypoints to plot Exponential function

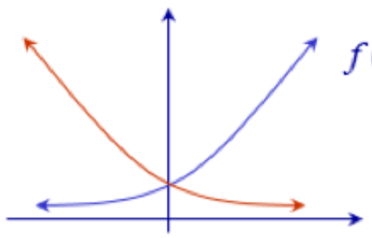
$\left(-3, \frac{1}{a^3}\right) \left(-2, \frac{1}{a^2}\right) \left(-1, \frac{1}{a}\right) (0, 1) (1, a) (2, a^2) (3, a^3)$

$f(x) = b^x$

Characteristics of Exponential Functions

- The domain of $f(x) = b^x$ consists of all real numbers. $(-\infty, \infty)$ The range of $f(x) = b^x$ consists of all positive real numbers. → $y > 0$ → $(0, \infty)$
- The graphs of all exponential functions pass through the point $(0, 1)$ because $f(0) = b^0 = 1$.
- If $b > 1$, $f(x) = b^x$ has a graph that goes up to the right and is an increasing function.
- If $0 < b < 1$, $f(x) = b^x$ has a graph that goes down to the right and is a decreasing function.
- $f(x) = b^x$ is a one-to-one function and has an inverse that is a function.
- The graph of $f(x) = b^x$ approaches but does not cross the x-axis. The x-axis is a horizontal asymptote.

$f(x) = b^x$
$0 < b < 1$



$f(x) = b^x$
$b > 1$

- Domain: $D : \{x \in \mathbb{R}\}$
- Range: $R : \{y \in \mathbb{R} \mid y > 0\}$
- Horizontal asymptote: $y = 0$
- y-intercept: $(0, 1)$

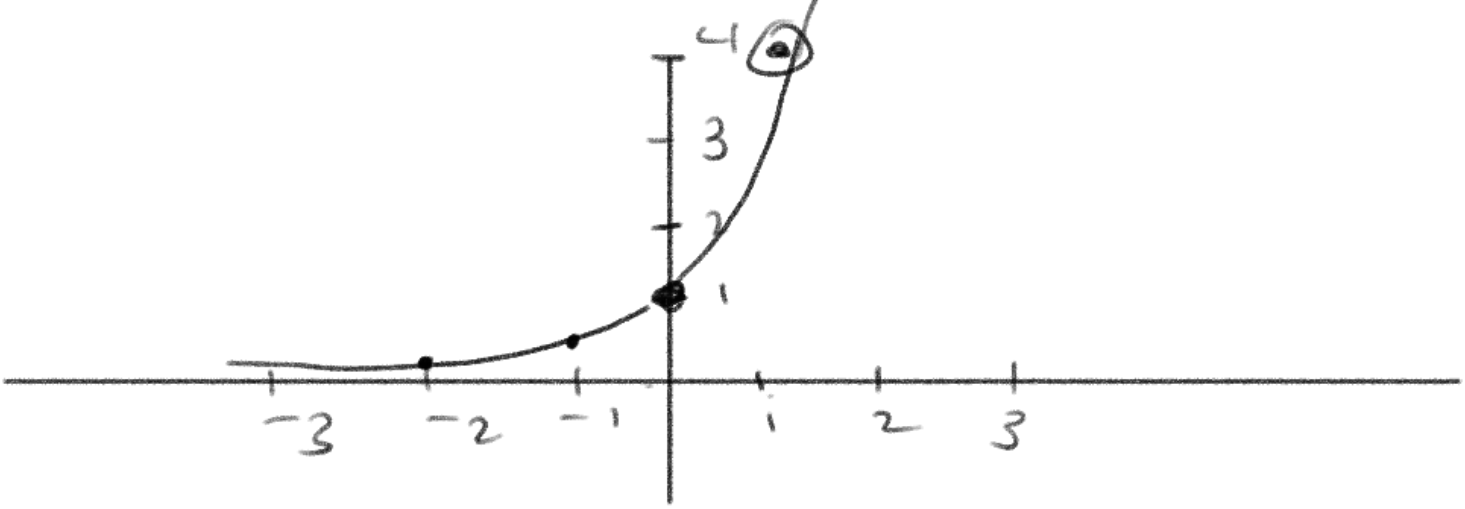
- Graph both functions on one set of axes.

19. $f(x) = 4^x$ and $g(x) = 7^x$ H.W

Keypoints $a=4$

$(-2, \frac{1}{a^2})(-1, \frac{1}{a})(0, 1)(1, a)(2, a^2)$

$(-2, \frac{1}{16})(-1, \frac{1}{4})(0, 1)(1, 4)(2, 16)$



$g(x) = 7^x$

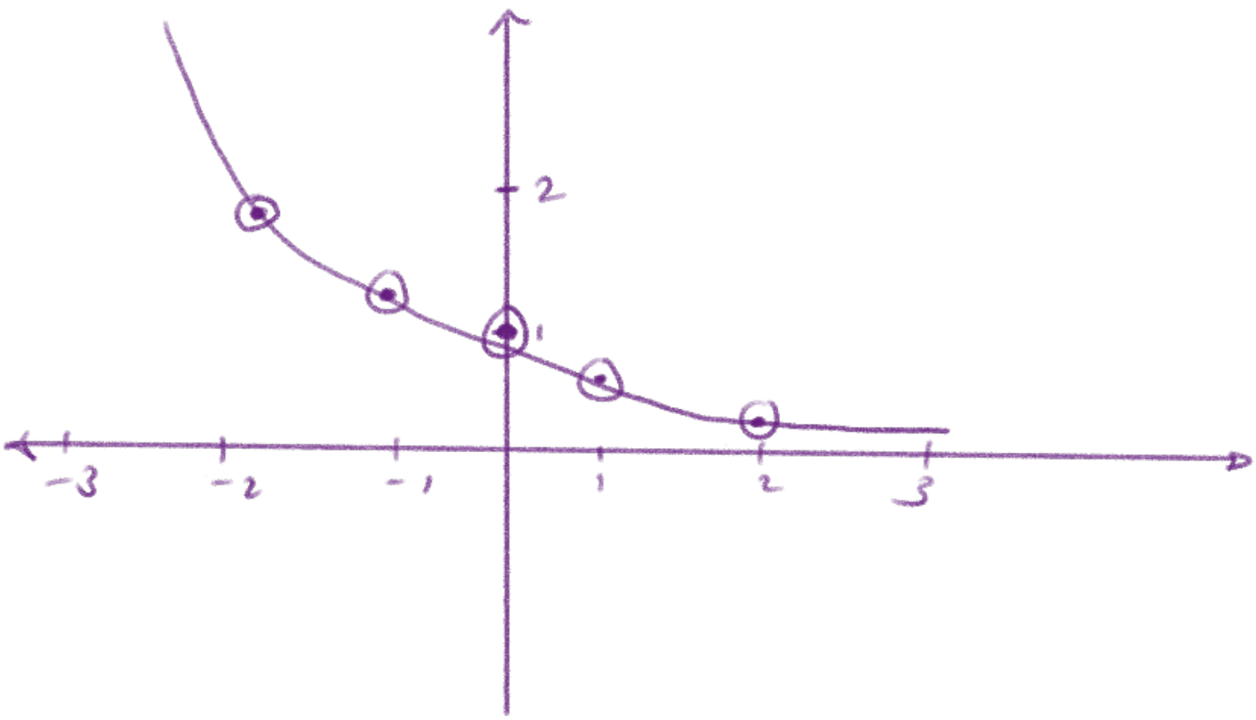
20. $f(x) = \left(\frac{3}{4}\right)^x$ and $g(x) = 1.5^x$

$a = \frac{3}{4}$

Keypoints

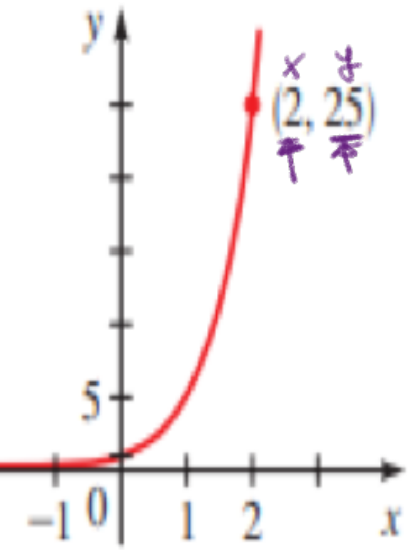
$(-2, \frac{1}{a^2})\ (-1, \frac{1}{a})\ (0, 1)\ (1, a)\ (2, a^2)$

$(-2, \frac{16}{9})\ (-1, \frac{4}{3})\ (0, 1)\ (1, \frac{3}{4})\ (2, \frac{9}{16})$

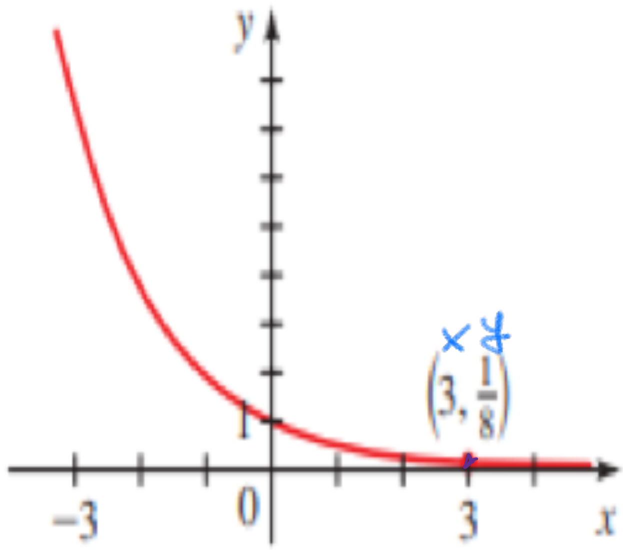


MPLE 3 ■ Identifying Graphs of Exponential Functions

the exponential function $f(x) = a^x$ whose graph is given.



(b)



$$f(x) = a^x$$

$$\underbrace{f(x)}_{\downarrow} = \underset{\uparrow = ??}{a}^x$$

$$y = a^x$$

$$25 = a^2$$

$$5^2 = a^2$$

$$\boxed{a = 5}$$

$$f(x) = a^x = 5^x$$

$$f(x) = a^x$$

$$y = a^x$$

$$\frac{1}{8} = a^3$$

$$\frac{1^3}{2^3} = a^3$$

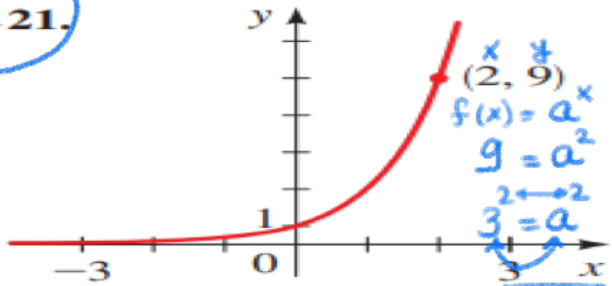
$$\left(\frac{1}{2}\right)^3 = a^3$$

$$a = \frac{1}{2}$$

$$f(x) = a^x = \frac{1}{2}^x$$

21–24 ■ Exponential Functions from a Graph Find the exponential function $f(x) = a^x$ whose graph is given.

21.



$x \quad y$

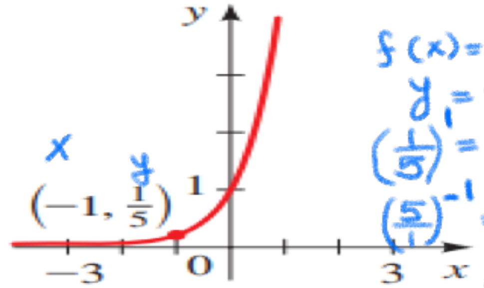
$f(x) = a^x$

$9 = a^2$

$3^2 = a^2$

$\boxed{a = 3}$

22.



$f(x) = a^x$

$y = a^x$

$\left(\frac{1}{5}\right) = a^{-1}$

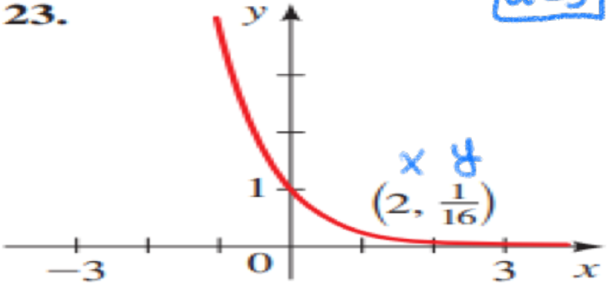
$\left(\frac{5}{1}\right)^{-1} = a^{-1}$

$5^{-1} = a^{-1}$

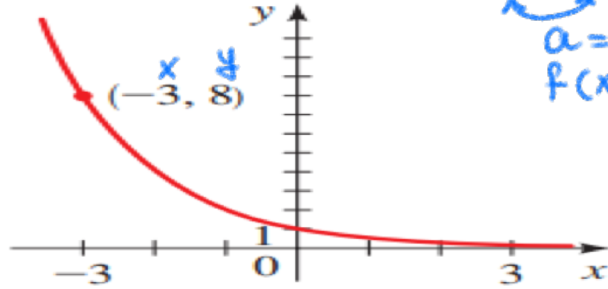
$a = 5$

$f(x) = a^x = 5^x$

23.



24.



The graphs of some functions can be constructed by translating, stretching and reflecting another graph or by combining these techniques.

$f(x) = a^x$

$y = a^x$

$\frac{1}{16} = a^2$

$\frac{1^2}{4^2} = a^2$

$\left(\frac{1}{4}\right)^2 = a^2$

$a = \frac{1}{4}$

$f(x) = a^x = \frac{1}{4}^x$

$f(x) = a^x$

$y = a^x$

$8 = a^{-3}$

$\frac{2^3}{1} = a^{-3}$

$\left(\frac{1}{2}\right)^{-3} = a^{-3}$

$\boxed{a = \frac{1}{2}}$

$f(x) = a^x = \frac{1}{2}^x$