$$\lim_{x \to n} \frac{\checkmark}{\checkmark} = \frac{0}{0}$$

Form 0/0 → Intermediate form

عملية تحليل لكل من البسط والمقام ويتم حذف الاصفار (العنا) عند التحليل

Find $\lim_{x \to -1} \frac{x^2 - 1^2}{x + 1}$. $= \lim \frac{(-1)^2 - 1}{(-1) + 1} = \frac{0}{0}$

$a^2 - b^2 = (a+b)(a-b)$

$$= \lim_{x \to -1} \frac{(x+1)(x-1)}{(x+1)}$$

$$= \lim_{x \to -1} (x-1) = -1-1 = -2$$

27. $\lim_{x \to -2} \frac{x+2}{x^2 - 4} = \frac{-2+2}{(-2)^2 - 4} = \frac{0}{0}$

$x^2 - 2^2$

$$\lim_{x \to -2} \frac{x+2}{(x+2)(x-2)} = \lim_{x \to -2} \frac{1}{x-2}$$

$$= \frac{1}{-2-2} = \frac{1}{-4} = -\frac{1}{4}$$

23. $\lim_{x\to 2}\frac{x^2-x-2}{x-2} = \frac{2^2-2-2}{2-2} = \frac{0}{0}$

$\lim_{x\to 2}\frac{(x-2)(x+1)}{x-2}$

$\lim_{x\to 2}(x+1) = 2+1 = 3$

24. $\lim_{t\to 0}\frac{t^3+3t^2}{t^3-4t^2} = \frac{0^3+3(0)^2}{0^3-4(0)^2} = \frac{0}{0}$

$\lim_{t\to 0}\frac{t^2(t+3)}{t^2(t-4)} = \lim_{t\to 0}\frac{t+3}{t-4} = \frac{0+3}{0-4} = \frac{3}{-4}$

$= -\frac{3}{4}$

25. $\lim_{x\to 3}\frac{x^2-1x-6}{x-3} = \frac{0}{0}$

$\lim_{x\to 3}\frac{(x-3)(x+2)}{x-3} = \lim_{x\to 3} x+2 = 3+2$

$= 5$

26. $\lim_{t\to 2}\frac{t^2-4}{t-2} \to \frac{2^2}{} = \frac{0}{0}$

$\lim_{t\to 2}\frac{(t+2)(t-2)}{t-2} = \lim_{t\to 2} t+2$

$= 2+2 = 4$

28. $\lim_{x\to 0} \dfrac{x^2-2x}{x} = \dfrac{0}{0}$

$$\lim_{x\to 0} \frac{x(x-2)}{x} = \lim_{x\to 0} x-2 = 0-2 = -2$$

29. $\lim_{x\to 4} \dfrac{x^2-9x+20}{x^2-3x-4} = \dfrac{0}{0}$

$$\lim_{x\to 4} \frac{(x-4)(x-5)}{(x-4)(x+1)}$$

$$\lim_{x\to 4} \frac{x-5}{x+1} = \frac{4-5}{4+1} = \frac{-1}{5}$$

30. $\lim_{x\to -3} \dfrac{x^4-81}{x^2+8x+15} = \dfrac{0}{0}$

$$\lim_{x\to -3} \frac{x^4-9^2}{x^2+8x+15} \qquad (3^2)^2$$

3	81
3	27
3	9
3	3
	1

$$\lim_{x\to -3} \frac{(x^2+9)(x^2-9)}{(x+3)(x+5)}$$

$$\lim_{x\to -3} \frac{(x^2+9)(x-3)(x+3)}{(x+3)(x+5)} = \frac{((-3)^2+9)(-3-3)}{(-3+5)}$$

$$= \frac{18\times -6}{2} = -54$$

34. $\lim_{x \to 0} \dfrac{(x+2)^2 - 2^2}{x}$

$$\lim_{x \to 0} \frac{(x+2+2)(x+2-2)}{x}$$

$$= \lim_{x \to 0} \frac{(x+4)x}{x} = \lim_{x \to 0} x+4$$

$$= 0+4 = 4$$

PROBLEMS 10.1

In Problems 9–34, find the limits.

9. $\lim_{x\to 2} 16$

10. $\lim_{x\to 3} 2x$

11. $\lim_{t\to -5} (t^2 - 5)$

12. $\lim_{t\to 1/3} (2t + 7)$

13. $\lim_{x\to -2} (3x^3 - 4x^2 + 2x - 3)$

14. $\lim_{r\to 9} \dfrac{4r - 3}{11}$

15. $\lim_{t\to -3} \dfrac{t - 2}{t + 5}$

16. $\lim_{x\to -6} \dfrac{x^2 + 6}{x - 6}$

17. $\lim_{t\to 0} \dfrac{t}{t^3 - t + 7}$

18. $\lim_{z\to 0} \dfrac{z^2 - 5z - 4}{z^2 + 1}$

19. $\lim_{p\to 4} \sqrt{p^2 + p + 5}$

20. $\lim_{y\to 15} \sqrt{y + 3}$

21. $\lim_{x\to -2} \dfrac{x^2 + 2x}{x + 2}$

22. $\lim_{x\to 1} \dfrac{x - 1}{x - 1}$

23. $\lim_{x\to 2} \dfrac{x^2 - x - 2}{x - 2}$

24. $\lim_{t\to 0} \dfrac{t^3 + 3t^2}{t^3 - 4t^2}$

25. $\lim_{x\to 3} \dfrac{x^2 - x - 6}{x - 3}$

26. $\lim_{t\to 2} \dfrac{t^2 - 4}{t - 2}$

27. $\lim_{x\to -2} \dfrac{x + 2}{x^2 - 4}$

28. $\lim_{x\to 0} \dfrac{x^2 - 2x}{x}$

29. $\lim_{x\to 4} \dfrac{x^2 - 9x + 20}{x^2 - 3x - 4}$

30. $\lim_{x\to -3} \dfrac{x^4 - 81}{x^2 + 8x + 15}$

31. $\lim_{x\to 2} \dfrac{3x^2 - x - 10}{x^2 + 5x - 14}$

32. $\lim_{x\to 2} \dfrac{x^2 - x - 2}{x^2 - 4}$

33. $\lim_{h\to 0} \dfrac{(2 + h)^2 - 2^2}{h}$

34. $\lim_{x\to 0} \dfrac{(x + 2)^2 - 4}{x}$

Solutions:

9. $\lim_{x\to 2} 16 = 16$

10. $\lim_{x\to 3} 2x = 2(3) = 6$

11. $\lim_{t\to -5} \left(t^2 - 5\right) = (-5)^2 - 5 = 25 - 5 = 20$

12. $\lim_{t\to 1/2}(3t-5)=3\left(\frac{1}{2}\right)-5=\frac{3}{2}-\frac{10}{2}=-\frac{7}{2}$

13. $\lim_{x\to -2}(3x^3-4x^2+2x-3)$
$=3(-2)^3-4(-2)^2+2(-2)-3$
$=-24-16-4-3$
$=-47$

14. $\lim_{r\to 9}\frac{4r-3}{11}=\frac{4(9)-3}{11}=\frac{36-3}{11}=\frac{33}{11}=3$

15. $\lim_{t\to -3}\frac{t-2}{t+5}=\frac{\lim_{t\to -3}(t-2)}{\lim_{t\to -3}(t+5)}=\frac{-3-2}{-3+5}=\frac{-5}{2}=-\frac{5}{2}$

16. $\lim_{t\to -6}\frac{x^2+6}{x-6}=\frac{\lim_{x\to -6}\left(x^2+6\right)}{\lim_{x\to -6}(x-6)}=\frac{(-6)^2+6}{(-6)-6}$
$=\frac{42}{-12}=-\frac{7}{2}$

17. $\lim_{t\to 0}\frac{t}{t^3-4t+3}=\frac{\lim_{t\to 0}t}{\lim_{t\to 0}(t^3-4t+3)}$
$=\frac{0}{0^2-4(0)+3}$
$=\frac{0}{3}$
$=0$

18. $\lim_{z\to 0}\frac{z^2-5z-4}{z^2+1}=\frac{\lim_{z\to 0}(z^2-5z-4)}{\lim_{z\to 0}(z^2+1)}$
$=\frac{0^2-5(0)-4}{0^2+1}$
$=-4$

19. $\lim_{p\to 4}\sqrt{p^2+p+5}=\sqrt{\lim_{p\to 4}\left(p^2+p+5\right)}$
$=\sqrt{4^2+4+5}=\sqrt{25}=5$

20. $\lim_{y\to 15}\sqrt{y+3}=\sqrt{\lim_{y\to 15}(y+3)}=\sqrt{15+3}=\sqrt{18}$
$=3\sqrt{2}$

21. $\lim_{x\to -2}\frac{x^2+2x}{x+2}=\lim_{x\to -2}\frac{x(x+2)}{x+2}=\lim_{x\to -2}x=-2$

22. $\lim_{x\to -1}\frac{x^2-1}{x^2-1}=\lim_{x\to -1}\frac{(x+1)(x-1)}{(x+1)(x-1)}=\lim_{x\to -1}1=1$

23. $\lim_{x\to 2}\frac{x^2-x-2}{x-2}=\lim_{x\to 2}\frac{(x-2)(x+1)}{x-2}$
$=\lim_{x\to 2}(x+1)=3$

24. $\lim_{t\to 0}\frac{t^3+3t^2}{t^3-4t^2}=\lim_{t\to 0}\frac{t^2(t+3)}{t^2(t-4)}=\lim_{t\to 0}\frac{t-3}{t-4}=-\frac{3}{4}$

25. $\lim_{x\to 3}\frac{x^2-x-6}{x-3}=\lim_{x\to 3}\frac{(x-3)(x+2)}{x-3}$
$=\lim_{x\to 3}(x+2)$
$=5$

26. $\lim_{t\to 2}\frac{t^2-4}{t-2}=\lim_{t\to 2}\frac{(t+2)(t-2)}{t-2}=\lim_{t\to 2}(t+2)=4$

27. $\lim_{x\to -4}\frac{x+4}{x^2-16}=\lim_{x\to -4}\frac{x+4}{(x+4)(x-4)}$
$=\lim_{x\to -4}\frac{1}{x-4}$
$=-\frac{1}{8}$

28. $\lim_{x\to 0}\frac{x^2-2x}{x}=\lim_{x\to 0}\frac{x(x-2)}{x}=\lim_{x\to 0}(x-2)=-2$

29. $\lim_{x\to 4}\frac{x^2-9x+20}{x^2-3x-4}=\lim_{x\to 4}\frac{(x-4)(x-5)}{(x-4)(x+1)}$
$=\lim_{x\to 4}\frac{x-5}{x+1}=-\frac{1}{5}$

30. $\lim_{x\to -3}\frac{x^4-81}{x^2+8x+15}=\lim_{x\to -3}\frac{(x^2+9)(x^2-9)}{(x+3)(x+5)}$
$=\lim_{x\to -3}\frac{(x^2+9)(x+3)(x-3)}{(x+3)(x+5)}$
$=\lim_{x\to -3}\frac{(x^2+9)(x-3)}{x+5}$
$=-54$

31. $\lim_{x\to 2}\frac{3x^2-x-10}{x^2+5x-14}=\lim_{x\to 2}\frac{(3x+5)(x-2)}{(x+7)(x-2)}$
$=\lim_{x\to 2}\frac{3x+5}{x+7}=\frac{11}{9}$